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A PRUDENT CENTRAL BANKER

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RÉSUMÉ

Ce mémoire étudie la politique monétaire dans une économie où les préférences de la Banque centrale sont asymétriques autour de l'inflation optimale. Plus précisément, les déviations positives de l'optimum peuvent avoir une plus grande ou une plus faible pondération dans la fonction de perte que les déviations négatives en ce qui concerne les preneurs de décisions des politiques. Sous des préférences asymétriques, il a été démontré que l'incertitude pouvait entraîner un comportement prudent de la part de la Banque centrale. Ainsi, les motifs de prudence peuvent être assez grands pour annuler le biais d'inflation, une politique monétaire optimale peut être réalisée même en l'absence de règles, de réputation ou de mécanismes contractuels. Pour certaines valeurs de paramètres, un biais de déflation peut survenir en équilibre.

Mots clés : délégation, préférences asymétriques, prudence, déflation

ABSTRACT

This paper studies monetary policy in an economy where the central banker's preferences are asymmetric around optimal inflation. In particular, positive deviations from the optimum can be weighted more, or less, severely than negative deviations in the policy maker's loss function. It is shown that under asymmetric preferences, uncertainty can induce a prudent behavior on the part of the central banker. Since the prudence motive can be large enough to override the inflation bias, optimal monetary policy could be implemented even in the absence of rules, reputation, or contractual mechanisms. For certain parameter values, a deflationary bias can arise in equilibrium.

Key words : delegation, asymmetric preferences, prudence, deflation

1 Introduction

This paper studies monetary policy in an economy where the central banker's preferences are asymmetric around the optimal rate of price inflation. The preference specification permits different weights for positive and negative inflation deviations from the optimum, and includes as a special case the quadratic loss function employed by previous literature.[1] In contrast to the quadratic model, where the loss associated with a deviation depends solely on its magnitude, under asymmetric preferences both magnitude and sign matter to the policy maker. This more general preference specification has nontrivial implications for the design of monetary policy and modifies some of the earlier conclusions derived under the assumption of symmetry. This point is illustrated here in a simple game-theoretical model of monetary policy in the spirit of Barro and Gordon (1983). It is shown that relaxing certainty equivalence induces a prudent behavior on the part of the central banker that can reduce or eliminate the inflation bias associated with discretionary monetary policy. Moreover, there exists values of the preference parameters for which a deflationary bias can arise in equilibrium.

As the "conservative" central banker in Rogoff (1985), the "prudent" central banker here is characterized by preferences that can differ from the ones of the average member of society. The former attaches a larger relative weight to inflation stabilization in her loss function than society does. The latter weights more, or less, severely positive than negative inflation deviations from the optimal rate. However, while the "conservative" central banker reduces, but does not remove, the inflation bias, there exist values of the preference parameters for which a "prudent" central banker can eliminate the bias and maximize social welfare. Hence, optimal monetary policy could be implemented even without rules, reputation, or contractual mechanisms.

In order to see the intuition of this result, notice that relaxing the assumption of quadratic preferences means that certainty equivalence no longer holds. Then, the expected marginal cost of departing from optimal inflation is nonlinear in inflation. In particular, when the central banker associates a larger loss to positive than negation inflation deviations from the optimum, uncertainty raises the expected marginal cost and induces a prudent behavior

[1] See, among others, Kydland and Prescott (1977), Barro and Gordon (1983), Rogoff (1985), Walsh (1995), Svensson (1997, 1999), Beetsma and Jensen (1998), Clarida, Galí, and Gertler (1999), and Eijffinger, Hoeberichts, and Schaling (2000).

on the part of the monetary authority. Prudence then moderates the policy maker's incentive to create surprise inflation.

For certain preference parameter values, this prudence motive is large enough that inflation is on average below its optimal rate. Hence, asymmetric preferences can provide a theoretical foundation for Stanley Fischer's observation [Fischer (1994)] that a deflationary bias can be a possible outcome in the actual practice of monetary policy.

Asymmetric disappointment-aversion preferences whereby individuals treat differently expected gains and losses have been proposed in the literature by Epstein and Zin (1989) and Gul (1991). Regarding a government or central bank, their different attitude *vis a vis* booms and recessions and the more downward- than upward-rigidity in prices might plausibly induce asymmetries in their loss function. Consistent with this idea are findings by Clarida and Gertler (1997), Goldfajn and Valdés (1999) and Ruge-Murcia (2000). Clarida and Gertler estimate a reaction function for the Bundesbank and find that the central bank raises the day-to-day interest rate when inflation is above its steady-state trend value but barely responds when it is below. Goldfajn and Valdés examine more than 200 episodes of currency overvaluations and undervaluations, and document that while the former usually finish with sudden changes in the nominal exchange rate, the latter end by the smooth adjustment of prices and wages. Ruge-Murcia derives implicit bounds for the Canadian inflation target zone using data on market-determined nominal interest rates and reports that while the public might perceive the band to be of approximately the same width as announced, it appears asymmetrically distributed around the official mid-value of 2% per year.

Before proceeding, it is important to mention closely related research by Nobay and Peel (1998) who study optimal commitment and discretion in monetary policy using the same loss function employed here. This project extends and complements their analysis by fully characterizing the theoretical and empirical implications of the model. At the theoretical level the properties of the central banker's reaction are derived, conditions for the existence and uniqueness of the Nash equilibrium are established, and the government's problem of optimal delegation is formulated and solved At the empirical level, it is shown that asymmetric preferences (*i*) generate a positive *nonlinear* relation between inflation and natural rate of unemployment, and (*ii*) predict that the conditional variance of inflation helps forecast its mean.

The paper is organized as follows: section 2 presents and solves a game-theoretical model of monetary policy with asymmetric preferences; section 3 establishes its theoretical and empirical implications; and section 4 concludes and discusses future research.

2 The Model

2.1 The Economic Environment

Following the literature, the relation between inflation and unemployment is described by an expectations-augmented Phillips curve:

$$u = u^n - \lambda(\pi - \pi^e) + \eta, \qquad \lambda > 0, \tag{1}$$

where u is the rate of unemployment, u^n is the natural rate of unemployment, π is the rate of inflation, π^e is the public's previously-constructed inflation forecast, and η is a supply shock. The public is assumed to form its expectations rationally:

$$\pi^e = E(\pi|I), \tag{2}$$

where E is the expectations operator and I is the public's information set. Because there is no private information in the model, the government's and central banker's information sets coincide with the public's and are also given by I.

Rather than choosing the rate of inflation directly, the policy maker is assumed to affect π through a policy instrument (as in Walsh (1995) and Ireland (1999)). The instrument, whether a monetary aggregate or a short-term nominal interest rate, is imperfect in the sense that in a stochastic world it cannot determine inflation completely and its effect takes place with a one period lag:

$$\pi = f(i) + \epsilon, \tag{3}$$

where $f(\cdot)$ is a monotonic, continuous, and differentiable function, i is the policy instrument, and ϵ is a control error that represents imperfections in the conduct of monetary policy. Since i is assumed to be chosen in the previous period, it follows that $i \in I$.

Finally, collect the model's structural disturbances in the 2×1 vector $\boldsymbol{\xi}$. It is assumed that $\boldsymbol{\xi}$ is serially uncorrelated and normally distributed with zero mean:

$$\boldsymbol{\xi}|I = \left[\begin{array}{c} \eta \\ \epsilon \end{array}\right] \Bigg| \quad I \quad \sim N(\mathbf{0}, \boldsymbol{\Omega}),$$

where $\boldsymbol{\Omega}$ represents a 2×2 positive-definite variance-covariance matrix.

Kydland and Prescott (1977) and Barro and Gordon (1983) show that in the above setup – where surprise inflation can produce levels of unemployment above the natural rate – a government with preferences over inflation and unemployment might be unable to credibly commit to the optimal monetary policy. Instead, the strategic interaction between the public and the government yields a suboptimal Nash equilibrium where unemployment corresponds to the natural rate but inflation is strictly higher than the socially-optimal rate. A welfare-improving solution involves the delegation of monetary policy to a separate agent or central banker.[2] It is easy to show that if the central banker has exactly the same preferences as the government, the inefficient outcome remains unchanged. Hence, the idea that delegation solves or reduces the temporal inconsistency of monetary policy is only meaningful if the agent's preferences are, either by nature or by design, different from the government's. A classic example of the first case is the "conservative" central banker in Rogoff (1985) where the agent has quadratic preferences (as does the government) but attaches a proportionally smaller weight to unemployment in her loss function than the government. Among the second class of models, Persson and Tabellini (1993) and Walsh (1995) propose the use of a linear contract that modifies the agent's loss function by appending a penalty that increases as inflation deviates from the optimal rate. Green (1996) and Svensson (1997) consider an explicit inflation target that shifts the central banker's loss function such that the marginal benefit and cost of inflation to the agent are equalized at the socially-optimal inflation rate.

[2] Jensen (1997) argues that since delegation itself is discretionary, it might not solve the temporal inconsistency of monetary policy unless there are reappointment costs. McCallum (1995) and Blinder (1998, p. 46) question the government's incentive to enforce contractual arrangements with the central banker.

2.2 The Central Banker

In the spirit of Rogoff (1985), this paper studies the implementation of monetary policy by an agent endowed by nature with different preferences than the average member of society. Without imposing restrictions on the relative weight attached to unemployment stabilization, the central banker's objective function is generalized to allow different losses for positive and negative inflation deviations from its optimal rate:

$$L(\pi, u) = (\exp(\alpha(\pi - \pi^*)) - \alpha(\pi - \pi^*) - 1)/\alpha^2 + (\phi/2)(u - ku^n)^2, \quad (4)$$

where is α is a nonzero real number, π^* is the socially optimal inflation rate, ku^n is the desired rate of unemployment, $0 < k < 1$, and ϕ is a positive constant that measures the relative importance of unemployment stabilization. The possibly nonzero value π^* can be interpreted, for example, as the one associated with the optimal inflation tax, that might reduce the need for distorting income taxes or other type of non-lump-sum taxes.

The inflation component in (4) is described by the linex function originally introduced by Varian (1974) and examined by Zellner (1986), Granger and Pesaran (1996), and Christoffersen and Diebold (1997) in the context of optimal forecasting. This function is plotted in figure 1(a) for the special case where $\alpha > 0$. Notice that for inflation rates above π^*, the exponential term eventually dominates and the loss associated with a positive deviation rises exponentially. For $\pi < \pi^*$, it is the linear term that becomes progressively more important as inflation decreases and the loss rises linearly. This asymmetry can be easily seen by considering, for example, the loss associated with a ± 1 inflation deviation from π^*. It is apparent that even though their magnitudes are the same, the -1 deviation delivers a smaller loss than the $+1$ deviation. Consequently, positive deviations are weighted more heavily than negative ones in the central banker's objective function. (Converse results are obtained when $\alpha < 0$.) In contrast to the usual quadratic loss function, where only the magnitude of the deviation matters to the policy maker [see figure 1(b)], under asymmetric preferences both the magnitude and sign affect the central banker's loss.

In the special case where α tends to zero, the linex component in (4) reduces to the quadratic form $(\pi - \pi^*)^2/2$. To see this, take the limit of $(\exp(\alpha(\pi - \pi^*)) - \alpha(\pi - \pi^*) - 1)/\alpha^2$ as α goes to 0 and use L'Hôpital's rule twice. This result is important because it formally proves that the quadratic loss function employed in previous literature is a special case of the one proposed here. It also suggests that the hypothesis that the central banker's preferences are symmetric over inflation could be evaluated by testing whether α is significantly different from zero.[3]

2.3 Nash Equilibrium

The problem of the central banker is to choose the value of the instrument that minimizes her expected loss. Formally,

$$\underset{\{i\}}{Min} \quad E[L(\pi, u)\,|I],$$

subject to the expectations-augmented Phillips curve and taking π^e as given. The first-order condition is

$$E[(\partial L/\partial \pi)(\partial \pi/\partial i) + (\partial L/\partial u)(\partial u/\partial \pi)(\partial \pi/\partial i)|I] = 0,$$

and is satisfied by the value of i that equates the marginal cost of higher unemployment with the marginal benefit of lower inflation. Since the loss function is globally convex, this value corresponds to a unique minimum. Computing the partial derivatives:

$$E[(\exp(\alpha(\pi - \pi^*)) - 1)/\alpha - \lambda\phi(u - ku^n)|I] = 0. \tag{5}$$

In order to find the conditional expectation of $\exp(\alpha(\pi - \pi^*))$, the following result is useful:

Proposition 1. *The assumption of normally distributed disturbances implies that, conditional on the information set, inflation is normally distributed.*
Proof. Taking conditional expectations of both sides of (3), noting that i forms part of the public's information set, and substituting back into (3) yields

$$\pi = E(\pi|I) + \mathbf{A}\boldsymbol{\xi}, \tag{6}$$

[3] In principle, one could also consider a more general model with asymmetric preferences over both inflation and unemployment. However, the solution is only slightly different from the one derived here, and the basic predictions of the model remain unchanged.

where $\mathbf{A} = (0, 1)$. Then, since $E(\pi|I)$ forms part of I and ξ is jointly normally distributed by assumption, it follows that

$$\pi|I \sim N(E(\pi|I), \mathbf{A}\Omega\mathbf{A}').\P$$

The above result implies that, conditional on I, the variable $\exp(\alpha(\pi - \pi^*))$ is distributed log normal. Then, $E[\exp(\alpha(\pi - \pi^*))|I] = \exp(\alpha(E(\pi|I) - \pi^*) + \alpha^2\sigma_\pi^2/2)$ where $\sigma_\pi^2 = \mathbf{A}\Omega\mathbf{A}'$ is the conditional variance of inflation.

The conditional expectation of unemployment can be found by taking $E[\cdot|I]$ in both sides of (1) to obtain:

$$E(u|I) = u^n - \lambda(E(\pi|I) - \pi^e).$$

With these intermediate results, the first-order condition can be written as

$$(\exp(\alpha(E(\pi|I) - \pi^*) + \alpha^2\sigma_\pi^2/2) - 1)/\alpha + \lambda^2\phi(E(\pi|I) - \pi^e) - \lambda\phi(1-k)u^n = 0.$$

Recall that in the quadratic model, the first-order condition of the central banker's minimization problem is linear and can be solved explicitly to obtain her reaction function in terms of the public's inflation forecast, π^e.[4] In contrast, under asymmetric preferences, the first-order condition only defines the reaction function implicitly:

$$\begin{aligned} &g(E(\pi|I), \pi^e) \\ &\quad = (\exp(\alpha(E(\pi|I) - \pi^*) + \alpha^2\sigma_\pi^2/2) - 1)/\alpha + \lambda^2\phi(E(\pi|I) - \pi^e) - \lambda\phi(1-k)u^n), \\ &\quad = 0. \end{aligned} \tag{7}$$

However, using the implicit function theorem, it is possible to show that

$$\partial E(\pi|I)/\partial\pi^e = \lambda^2\phi/(\lambda^2\phi + \exp(\alpha(E(\pi|I) - \pi^*) + \alpha^2\sigma_\pi^2/2)) \in (0, 1),$$

for all values of α. Hence, as in the quadratic model, the central banker's reaction is a monotonically increasing function of the public's inflation forecast. Also

$$\begin{aligned} &\partial^2 E(\pi|I)/\partial(\pi^e)^2 = \\ &\qquad -\alpha\lambda^4\phi^2\exp(\alpha(E(\pi|I) - \pi^*) + \alpha^2\sigma_\pi^2/2)/(\lambda^2\phi + \exp(\alpha(E(\pi|I) - \pi^*) + \alpha^2\sigma_\pi^2/2))^3, \end{aligned}$$

[4] Strictly speaking, the reaction function relates the policy instrument, i, and π^e, both of which are determined in the previous period. However, in what follows it will be convenient to work with $E(\pi|I)$ rather than i. Since these two variables are monotonically related by the function $f(\cdot)$, this approach entails no loss of generality.

that is less than zero for $\alpha > 0$, equal to zero for $\alpha \to 0$, and larger than zero for $\alpha < 0$. Hence, for $\alpha > 0$ ($\alpha < 0$) the central banker's reaction is a concave (convex) function of π^e.

In order to develop further the reader's intuition and to illustrate future theoretical results, it is useful to plot the central banker's reaction function for different values of the preference parameter α. This is done in figure 2 under the assumption that the optimal rate of inflation is $\pi^* = 0$, and remaining parameters are $\lambda = 2$, $\phi = 0.5$, $k = 0.8$, $u^n = 5$, and $\sigma_\pi^2 = 2.5^2$. The figure also includes the reaction function of the quadratic central banker that is obtained when $\alpha \to 0$, and the public's reaction function that is summarized by the rational expectations relation (2). Graphically, the Nash equilibrium is the point where (7) and (2) intersect. Treating all parameters as fixed, the "prudent" central banker's reaction was computed by solving numerically the implicit function (7) for 80 equally-spaced values of π^e between -8 and 8.

Notice that although in all cases the central banker's reaction is an increasing function of the public's inflation forecast, her willingness to accommodate the public's expectations depends on the preference parameter α. Consider first the case where $\alpha < 0$. Note that in this situation the central banker responds to π^e at an increasing rate and the inflation bias is larger than under quadratic preferences. For values of $\alpha \leq -1/(\lambda\phi(1-k)u^n)$, there is no finite rate of inflation at which (7) and (2) intersect, and the Nash equilibrium will not exist [see proposition 2 below].

On the other hand, when $\alpha > 0$ the central banker accommodates the public's inflation forecast at a decreasing rate and the inflation bias will always be smaller than under quadratic preferences. Proposition 4 below shows that there exists a unique, positive value of α, say α^*, such that the Nash equilibrium is obtained at the optimal rate of inflation. (The reaction function for this case is labeled as "optimal" in figure 2.) Hence, an optimally "prudent" central banker can implement optimal monetary policy, even in the absence of rules, reputation or contractual agreements with the government.

Finally, for values of $\alpha > \alpha^*$, a deflationary, rather than an inflationary, bias can arise in equilibrium.[5] Thus, asymmetric preferences can provide a theoretical foundation for Fischer's (1994) observation that a deflationary

[5]In what follows, deflation refers not only to a negative rate of price inflation, but, more generally, to a rate of inflation below the optimal one.

bias is a possible outcome in the practice of monetary policy.[6]

Conditions for the existence and uniqueness of the Nash equilibrium are presented in the following proposition:

Proposition 2. *Provided that* $1 + \alpha\lambda\phi(1-k)u^n > 0$, *there exists a unique* $\pi^e = E(\pi|I)$, *such that* $g(E(\pi|I), \pi^e) = 0$.

Proof. To prove existence, construct a

$$\pi^e = E(\pi|I) = \pi^* - \alpha\sigma_\pi^2/2 + (1/\alpha)\ln(1 + \alpha\lambda\phi(1-k)u^n). \tag{8}$$

Plugging (8) into (7) and using $\pi^e = E(\pi|I)$ delivers $g(E(\pi|I), \pi^e) = 0$. To show uniqueness, assume there exists a second inflation forecast, say $\hat{\pi}^e = \pi^* - \alpha\sigma_\pi^2/2 + (1/\alpha)\ln(1+\alpha\lambda\phi(1-k)u^n) + x$, that also lies on the 45° line on the plane $(\pi^e, E(\pi|I))$ and satisfies $g(E(\pi|I), \pi^e) = 0$. It will be shown that the only way this can happen is if $x = 0$. Replace $\hat{\pi}^e$ in (7). Then,

$$((1 + \alpha\lambda\phi(1-k)u^n)\exp(\alpha x) - 1 - \alpha\lambda\phi(1-k)u^n)/\alpha = 0.$$

Simplifying,

$$(1 + \alpha\lambda\phi(1-k)u^n)(\exp(\alpha x) - 1)/\alpha = 0.$$

Since $1 + \alpha\lambda\phi(1-k)u^n > 0$ and $\alpha \neq 0$, then it must be the case that $x = 0$.¶

Notice in (8) that depending on the sign of the preference parameter α, inflation is an increasing or decreasing function of its conditional variance. In order to understand this result, it is useful to recall that when the loss function is quadratic, certainty equivalence holds. Hence, the model solution is the same regardless of whether there is uncertainty or not. On the other hand, when the loss function is asymmetric on inflation, the marginal cost of departing from π^* is not linear in inflation, but convex (when $\alpha > 0$) or concave (when $\alpha < 0$). Thus, when $\alpha > 0$, an increase in uncertainty increases the expected marginal cost of deviating from π^*. (The converse result is obtained when $\alpha < 0$.) A comparable result can be found in the literature on precautionary savings. When the assumption of quadratic utility is relaxed and labor-income risk in nondiversifiable, then uncertainty increases the expected marginal utility of future consumption. To satisfy the

[6] To my knowledge this point was first made by Nobay and Peel (1998).

Euler condition, prudent households decrease current consumption compared to future consumption and increase their savings [see Caballero (1990)].

Finally, note that as usual the unemployment rate in equilibrium does not differ systematically from the natural rate, nor does it depend on the central banker's preferences. To see this, plug (6) into (1) and use the assumption of rational expectations to obtain:

$$u = u^n + \mathbf{B}\boldsymbol{\xi}, \tag{9}$$

where $\mathbf{B} = (1, -\lambda)$.

3 Implications

This section examines a number of important features of the model solution. First, notice that the solution under asymmetric preferences can be specialized to the one obtained using a quadratic loss function. To see this, take the limit of (8) when $\alpha \to 0$ to obtain

$$E(\pi|I) = \pi^* + \lambda\phi(1-k)u^n.$$

This equation corresponds to the one originally derived by Barron and Gordon (1983, p. 597). In this case the inflation bias, $\lambda\phi(1-k)u^n$, is strictly positive and, consequently, the realized rate of inflation is systematically above π^*. Because the inflation bias is increasing on ϕ, a "conservative" central banker that attaches a smaller weight to unemployment stabilization in her loss function unambiguously reduces its magnitude. However, since the optimal value of $\phi \in (0, \infty)$ [see Rogoff (1985, p. 1178)], the "conservative" central banker reduces but cannot eliminate the inflation bias.

Note that quadratic preferences predict a linear and positive relationship between inflation and the natural rate of unemployment. Instead, the model with asymmetric preferences (*i*) generates a positive *nonlinear* relation between π and u^n, (*ii*) predict that the conditional variance of inflation helps forecast its mean, and (*iii*) allows either an inflation or a deflationary bias depending on the central banker's preferences parameters. To see this later point, note that the nonlinear component $(1/\alpha)\ln(1 + \alpha\lambda\phi(1-k)u^n))$ is always positive. However, since the term $-\alpha\sigma_\pi^2/2$ in (8) can be positive or negative depending on the sign of α, inflation could be on average above or below π^*.

Consider first the case where the preference parameter, α, is negative. In this case, $-\alpha\sigma_\pi^2/2 > 0$ and the model predicts an even larger positive departure of inflation from π^* than the specification with quadratic preferences. In addition, the model implies a positive relationship between the level and the conditional variance of inflation. This prediction is in line with empirical literature that reports a positive relation between the first and second inflation moments in moderate- to high-inflation episodes [see, for example, Baillie, Chung, and Tieslau (1996)].

Consider now the more plausible situation where $\alpha > 0$. That is, positive deviations from π^* are weighted more heavily than negative ones in the central banker's loss function. Then, $-\alpha\sigma_\pi^2/2 < 0$ and it might be possible for this component to be sufficiently large (in absolute value) to overcome the inflation bias. This observation suggests that there exists a value of α, for which a "prudent" central banker can eliminate the bias associated with discretionary policy.

This issue can be formally examined by considering the problem of a government concerned with delegating monetary policy to a "prudent" central banker. In order to allow a direct comparison between this model and previous literature, assume that the government's (or interpreted more broadly, society's) preferences are described by the usual quadratic loss function:

$$G(\pi, u) = (1/2)(\pi - \pi^*)^2 + (\phi/2)(u - ku^n)^2. \tag{10}$$

Then, government's problem is

$$\underset{\{\alpha\}}{Min} \quad E[G(\pi(\alpha), u)|I],$$

for $\alpha \neq 0$, where the minimization takes (8) as given.[7] Recalling that unemployment does not depend on α, and noting that $\partial\pi/\partial\alpha \neq 0$, the first-

[7] As in preceding literature [see, for example, Rogoff (1985) and Svensson (1997)], this paper preserves the simplifying assumption that the central banker's preferences are public information. To motivate this assumption, it could be argued that the track record and previously-expressed opinions of individuals might provide information about their preferences when in office. Beestma and Jensen (1998) and Muscatelli (1999) examine the relative merits of inflation targets and linear contracts in a model where the central banker's preferences are unobserved by the government at the moment of delegation. The uncertainty pertains only to the inflation/unemployment weights that are stochastic, in an otherwise standard quadratic loss function.

order condition is

$$E[(\partial G/\partial \pi)(\partial \pi/\partial \alpha)|I] = E(\pi|I) - \pi^* = 0.$$

Using (8), it can be seen that the value α^* that maximizes social welfare by delivering $E(\pi|I) = \pi^*$ is the one that solves

$$h(\alpha) = -\alpha\sigma_\pi^2/2 + (1/\alpha)\ln(1 + \alpha\lambda\phi(1-k)u^n) = 0. \tag{11}$$

While this equation has no closed-form solution, proposition 4 below shows that an α^* does exist and is unique.[8] In proving proposition 4, the following result is useful

Proposition 3. *$h(\alpha)$ is monotonically decreasing on $\alpha \in (-1/(\lambda\phi(1-k)u^n)), 0) \cup (0, \infty)$.*
Proof. See Appendix A.

Consider now:

Proposition 4. *There exists a unique $\alpha^* \in (0, \infty)$ such that $E(\pi|I) = \pi^*$.*
Proof. Recall that the set of admissible values for α lay in the interval $(-1/(\lambda\phi(1-k)u^n)), 0) \cup (0, \infty)$. However, since $(1/\alpha)\ln(1 + \alpha\lambda\phi(1-k)u^n)$ and σ_π^2 are strictly positive, the root of $h(\alpha)$ can only lay in the range $(0, \infty)$. Consider the limits

$$\lim_{\alpha \to 0^+} h(\alpha) = \pi^* + \lambda\phi(1-k)u^n > 0$$

and

$$\lim_{\alpha \to +\infty} h(\alpha) = \; = -\infty,$$

and notice $h(\alpha)$ is continuous on α in $(0, \infty)$. Hence, there exists at least one value of α that solves (11). Since, by proposition 3, $h(\alpha)$ is monotonically decreasing on α, this value must be unique.¶

[8]The reaction function labeled as "optimal" in figure 2, was indeed constructed using the value of α that solves (11) given the remaining structural parameters. The fixed point was found using the Newton method.

It is important to note that the optimally "prudent" central banker with preference parameter implicitly defined by $h(\alpha^*) = 0$, implements the optimal monetary policy under complete discretion.

The relation between α^* and the conditional variance of inflation and the natural rate is outlined in the following proposition:

Proposition 5. *α^* is increasing on u^n and decreasing on σ_π^2.*
Proof. Use the implicit function theorem and Proposition 3 to obtain

$$\partial\alpha^*/\partial u^n = -(\partial h/\partial u^n)/(\partial h/\partial\alpha^*) > 0,$$

and

$$\partial\alpha^*/\partial\sigma_\pi^2 = -(\partial h/\partial\sigma_\pi^2)/(\partial h/\partial\alpha^*) < 0.¶$$

These results indicate that a more prudent central banker should be appointed when the natural rate of unemployment is higher or inflation is less volatile. In the first case, recall that the inflation bias is monotonically increasing on u^n. Then, to moderate the effect of u^n on the equilibrium through the component $-\alpha\sigma_\pi^2/2$, a larger degree of prudence is necessary. In the second case, note that when σ_π^2 is low, a comparatively larger value of α is required to override a given inflation bias.

Finally, for values of $\alpha > \alpha^*$, the term $-\alpha\sigma_\pi^2/2$ is larger (in absolute value) than $(1/\alpha)\ln(1+\alpha\lambda\phi(1-k)u^n)$. Thus, $-\alpha\sigma_\pi^2/2+(1/\alpha)\ln(1+\alpha\lambda\phi(1-k)u^n) < 0$ and the rate of inflation is systematically below its socially-optimal level. Hence a deflationary (rather than an inflationary) bias can arise in equilibrium.

4 Conclusions and Future Research

This paper presents initial results in a larger research project that examines monetary policy under asymmetric preferences. The relevance of this generalization of the central banker's loss function is illustrated here in the simplest possible setup. The central banker and the public play a one-shot game without private information. It is shown that although the central banker's reaction function is nonlinear on the public's inflation forecast, there are conditions under which the Nash equilibrium exists and is unique. More importantly, relaxing certainty equivalence means that uncertainty can induce prudence motive on the part of the monetary authority. Prudence then moderates the incentive to create surprise inflation. For certain parameter values optimal inflation can be obtained. Furthermore, in certain circumstances, a deflationary bias can arise in equilibrium. This can explain, for example, why the Bundesbank raises the day-to-day interest rate when inflation is above its steady-state trend value but barely responds when it is below [see Clarida and Gertler (1997)].

As part of the same research project, Ruge-Murcia (2001) develops a dynamic model of inflation targeting where the natural rate of unemployment is allowed to vary over time. The intention is to examine whether asymmetric preferences could account for the persistent undershooting of inflation targets observed in some countries. Preliminary estimates of the central banker's preference parameters for Canada, Sweden, and the United Kingdom are statistically different from the ones implied by the commonly-used quadratic loss function. Finally, since the notion of an asymmetric central bank's loss function has some intuitive appeal and empirical support, an important component of this research agenda is to provide adequate microfoundations for this preference specification.

A Appendix A: Proof of Proposition 3

Recall that $\alpha \neq 0$ and take

$$\partial h/\partial\alpha = -\sigma_\pi^2/2 + (1/\alpha^2)[\alpha\lambda\phi(1-k)u^n/(1+\alpha\lambda\phi(1-k)u^n) - \ln(1+\alpha\lambda\phi(1-k)u^n)].$$

Since $-\sigma_\pi^2/2 < 0$ and $(1/\alpha^2) > 0$, a sufficient condition for $\partial h/\partial\alpha < 0$ is that $\psi(\alpha) = \alpha\lambda\phi(1-k)u^n/(1+\alpha\lambda\phi(1-k)u^n) - \ln(1+\alpha\lambda\phi(1-k)u^n) < 0$. I will show that $\psi(\alpha) \to 0$ when $\alpha \to 0$ is a global maximum and, consequently, for all other values of α in $(-1/(\lambda\phi(1-k)u^n)), 0) \cup (0, \infty)$, $\psi(\alpha) < 0$. Taking

$$\partial\psi/\partial\alpha = -\alpha\lambda\phi(1-k)u^n/(1+\alpha\lambda\phi(1-k)u^n)^2.$$

Note that $\partial\psi/\partial\alpha \to 0$ when $\alpha \to 0$. For values of $\alpha \in (-1/(\lambda\phi(1-k)u^n)), 0)$, $\psi(\alpha)$ is increasing in α. For values of $\alpha \in (0, \infty)$, $\psi(\alpha)$ is decreasing in α. Since $\psi(\alpha) \to 0$ when $\alpha \to 0$, then for all values of α in $(-1/(\lambda\phi(1-k)u^n)), 0) \cup (0, \infty)$, $\psi(\alpha) < 0$ and $\partial h/\partial\alpha < 0$, as claimed.¶

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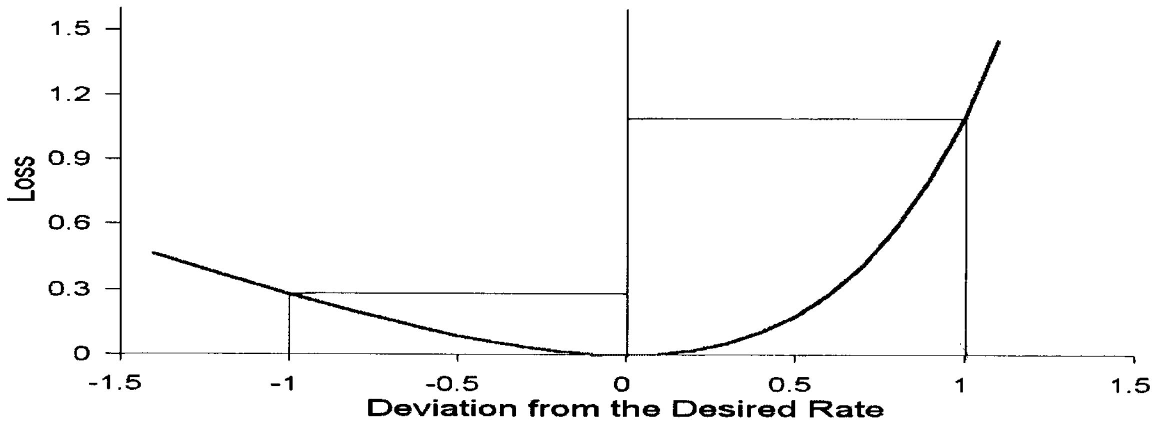
Figure 1: Preferences

(a) Asymmetric Preferences



(b) Quadratic Preferences

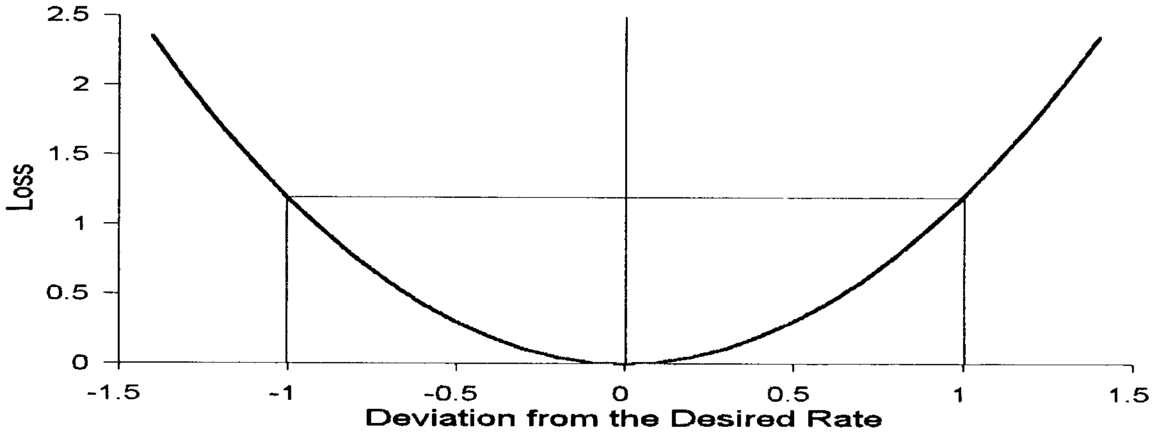


Figure 2: Reaction Functions