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Government Spending Multipliers and the Zero Lower Bound in an Open Economy^{*}

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Résumé/abstract

What is the size of the government-spending multiplier in an open economy when the Zero Lower Bound (ZLB) on the nominal interest rate is binding? Using a theoretical framework, in a closed economy, Christiano, Eichenbaum, and Rebelo (2011), show that, when the nominal interest rate is binding, the government-spending multiplier can be close to four. Their theory helps us to understand the government spending multiplier in ZLB, but it is difficult to match that theory with the data. We propose a dynamic stochastic general equilibrium in open macroeconomics, with market imperfections, wage and price rigidities and endogenous smoothing monetary policy. We argue that, in a closed economy and in the presence of ZLB, there is no crowding out effect through interest rates. We also argue that in an open economy, there is another channel for the crowding out effect via the real exchange rate. For an open economy, the multiplier falls to the levels usually observed in small, closed economies for which the ZLB is not binding.

Mots clés/Key words: Government-spending multiplier, zero lower bound, sticky price, sticky wages, Taylor rule.

Codes JEL : E52, E62, F41, F44.

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1 INTRODUCTION

What is the path followed by the fiscal multiplier in an open economy when the nominal interest rate reaches the Zero Lower Bound (ZLB)? Using a theoretical framework, in a closed economy, Christiano, Eichenbaum, and Rebelo (2011), show that, when the nominal interest rate is binding, the government-spending multiplier can be close to four. This theory helps us to understand the dynamics of an economy in ZLB after the increase in government spending, but it is difficult to match this theory with the data. For example, during the financial crisis and the recession that followed in 2007, the interest rate in the United States and in European countries, reached their lowest levels. Many significant budget plans emerged; the American Recovery and Reinvestment Act (ARRA) in the United States (831 billion from 2009 to 2019) and the European Economic Recovery Plan (EERP) in the European Union (€ 200 billion from 2008 to 2010). However, the ratio of debt to GDP increased on average by 40.5% before 2008 and by 80% after 2008 in the United States (see Boskin, 2012).

In this paper, we suggest that the real exchange rate is a channel that can explain why increasing government spending in ZLB, may not in some cases lead to a large government spending multiplier. We propose a theoretical open macroeconomics framework with market imperfections, wage and price rigidities and endogenous smoothing monetary policy. In our framework, we introduce a shock on the discount factor that pushes the nominal interest rate to its minimum level. We then compute the path followed by the fiscal multiplier due to increases of government spending in ZLB.

We argue that, in a closed economy and in the presence of ZLB, there is no crowding out effect through interest rates. We show, that in an open economy, there is another channel for the crowding out effect via the real exchange rate. For an open economy, the multiplier falls to the levels usually observed in small, closed economies for which the ZLB is not binding. We show that increasing government spending increases aggregate demand, which leads to appreciation of the real exchange rate that is greater than the appreciation that we would have had in the situation where the lower nominal interest rate was not binding. The appreciation of the real exchange rate then reduces the fiscal multiplier.

Our results are consistent with those of Perotti (2004) which shows empirically that the government spending effect on GDP tends to be lower for open economies. Our results also agree with those of Karras (2012) which shows that the fiscal multiplier decreases with the degree of openness of the economy (increased openness of the economy by 10% reduces the value of the multiplier of about 5% (data - 62

countries from 1951 to 2007)). Even if the analysis of Perotti (2004) and those of Karras (2012) do not take into account the special case of ZLB, they still give a good picture on what could happen in ZLB.

1.1 Literature review

As Amano and Shukayev (2010) argue, the ZLB constrains monetary policy. The real interest rate affects behavior of consumers and firms more than the nominal interest rate. A low real interest rate encourages more consumption and investment. In the Zero Lower Bound (ZLB) case, monetary authorities and governments must find another way to increase aggregate demand. The monetary authorities can, for example, convince agents that prices will increase in the future, and the government can increase spending.

Christiano, Eichenbaum, and Rebelo (2011), using a theoretical model, find that the government-spending multiplier can be much larger than one (close to four) while the nominal interest rate reaches the ZLB. However, the framework built in a closed economy cannot take into account the effect of government spending on the real exchange rate, or its effect on the level of the trade balance deficit. These effects can have a real impact on the cost of imported goods and consumption and therefore on the multiplier of public spending. The mechanism explaining the size of the government spending multiplier in a closed economy in ZLB is described by Christiano, Eichenbaum, and Rebelo (2011) as follows: “Following an increase in government spending, there is an increase in production marginal cost and expected inflation. This causes a decrease in the real interest rate, and households consume more. The increase in household spending increases output, marginal cost and expected inflation. This further decreases the real interest rate and so on, which in turn leads to a significant increase in production”. The theory may differ in an open economy.

Concerning the theory, the basic framework is developed by Mundell (1963). The model predicts that in a small open economy with flexible exchange rates, a fiscal policy is ineffective¹ if capital mobility is perfect. Indeed, an increase in government spending financed by borrowing, creates an excess demand for goods, which tends to increase income. This increases the demand for money and the interest rate, attracting foreign capital. The exchange rate then appreciates, which in turn leads to an equivalent decrease² in income through a trade imbalance. Even if the Mundell

¹Following an increase in public spending, the IS curve shifts to the right. As the central bank does not intervene, the LM curve does not shift. The interest rate increases and the real exchange rate appreciates. The appreciation of the real exchange rate penalizes exports and stimulates imports, which theoretically re-shifts the IS curve to its initial position.

²According to the Mundell (1963) model, income cannot change when the money supply and

(1963) framework is very restrictive³, the results described in the model are simple and understandable.

Concerning empirical analysis, using different empirical methodologies, many authors find government spending multipliers to be more or less close to one. The size of the multiplier depends on the method, period, and on the government spending indicator taken into account. However it is clear from empirical literature that the multiplier turns out to approximately one. Fisher and Ryan (2010) uses as an indicator, of government spending, the impact on income of the largest companies with government contracts in the military sector. They find a multiplier of government spending equal to 1.5 over a period of 5 years. Fisher and Ryan (2010) shows that a positive shock to government spending is associated with an increase in output, hours worked and consumption. They find that wages decrease initially and then increase one year after the shock.

The narrative approach⁴ developed by Ramey and Shapiro (1998) identifies the response of the economy due to a sustained and unpredictable increase of exogenous government spending. The narrative approach better approximates the period corresponding to the spending shock. Military spending is not theoretically explained by economic history. The narrative approach appears better than VAR for predicting periods of exogenous shocks and sustained military spending. Ramey and Shapiro (1998) shows that the government spending multiplier is sector-specific. Ramey and Shapiro (1998) finds that production and consumption decline following a government spending shock⁵.

interest rate are constant (LM does not change because the central bank does not intervene and $i = i^*$ does not change in the model since our country is small and there is perfect capital mobility). Therefore national income remains unchanged. As savings and taxes do not change because of the balance of the property market, the increase in public spending is exactly equal to the import surplus.

³As noted by Andrew (2000), Mundell's (1963) model has a lack of realism: domestic and foreign capital are perfectly substitutable. Sticky prices and aggregate supply are not modeled. There are no microeconomics foundations in the model. The model is static and there is neither wealth nor capital accumulation. Domestic and foreign interest rates are assumed to be identical.

⁴The Narrative approach uses newspaper information to identify periods of historical shocks. Increased military expenditure is then included as a dummy variable in an AR model to estimate the response of the economy.

⁵However, some reservations should be made when considering these results: the shocks, as specified, are only positive. The results imply good precision when estimating the period of sustained increase in public expenditure.

2 Methodology

To take into account the specifics of the open economy, the framework is as follows: the final goods are produced by competitive firms using a quantity of national aggregate goods and a quantity of imported aggregate goods. The national aggregate good is produced by competitive firms using a continuum of differentiated national intermediate goods produced by domestic firms in monopolistic competition. The aggregate imported good is produced by national competitive firms using a continuum of differentiated imported goods, produced by foreign firms in monopolistic competition. The aggregate national intermediate goods and aggregated imported intermediate goods are imperfectly substitutable. One part of the national aggregated good is exported and the rest is combined with the aggregated imported good for the production of the final good. The aggregate imported good cannot be consumed directly; this imported aggregate good is used only in the production process of the national final good. Households are characterized by different types of work, and act in monopolistic competition in the labor market. The final good is used for consumption, government spending and is also used as input in national production of intermediate goods. There is nominal rigidity of wages and prices: prices and wages are sticky in the sense of Calvo (1983). Prices of intermediate goods (domestic and foreign) and wages are set in advance. There is a continuum of types of job offers with constant elasticity of substitution. The labour is offered by a continuum of households in monopolistic competition on wages. This framework is common in the literature of open economy (see Ambler, Dib and Rebei 2004, Galí and Monacelli 2005).

We allow our model to generate a time-variant discount factor that will help us to push interest rate to its lowest level. To allow the fiscal multiplier to be greater than one, we consider a non separable utility function so that the marginal utility of consumption will depend positively on hours worked. We also consider an endogenous monetary policy. The policy states that, “due to the shock, the monetary authorities set the nominal interest rate such that it converges smoothly to the lowest level, but remains positive and differentiable at all points”. This is a modified version of monetary policy used by Christiano, Eichenbaum and Rebelo (2011) and it will help us to use an existing program to solve the model. When the nominal interest rate reaches the lowest level, the government increases spending in order to stimulate the economic activity. We then compute the path followed by the government spending multiplier.

2.1 The household

The population is represented by a continuum of agents on a unit interval indexed by j . The utility function of household j is defined as follows:

$$U(j) = E_0 \sum_{t=0}^{\infty} (\beta_t)^t u \left(C_t(j), \frac{M_t(j)}{P_t}, h_t(j), G_t \right) \quad (1)$$

The discount factor (β_t) is time-variant. This is the only type of shock in the absence of capital and a risk premium on capital that could push the interest rate to its lower level (see Amano and Shukayev 2009). Since we do not have capital in our analytical framework, we have to consider the time variable discount factor. In this work, it is the shock on discount factor that will push the economy to ZLB.

For each period, household j chooses the amount of money to hold, consumption, the amount of domestic and foreign assets, and the salary level if required to maximize its inter-temporal utility function, taking into account their budget constraint, the demand of labor type j and the transversal condition⁶ on assets.

The instantaneous utility function is:

$$u(\cdot) = \frac{\left(\left[\left(C_t(j)^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} \left(\frac{M_t(j)}{P_t} \right)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} \right]^{\alpha} [1 - h_t(j)]^{1-\alpha} \right)^{1-\sigma}}{1 - \sigma} \quad (2)$$

In the closed economy analysis, Christiano, Eichenbaum, and Rebelo (2011) shows (specifying different types of utility functions) that to have a fiscal multiplier greater than one, it is necessary to consider a utility function for which the marginal utility of consumption depends positively on hours worked. It is therefore necessary to have a non separable utility function.

E_0 is the expectation operator conditional on the time 0, $C_t(j)$ the household consumption at the end of period t , and $M_t(j)$ the net amount of currency held by the agent at the end of period t . P_t is the price index at time t , h_t the number of hours worked by the household at time t and G the government spending. $\alpha \in (0, 1)$, $\gamma > 0$, $\sigma > 0$ and u is a concave function.

b_t is the shock on money demand. This shock evolves according to the following AR(1) process:

⁶Among the possible solutions, we choose the one for which the amount (in value) of assets held by the agent, at the end of the period, is zero. It would be suboptimal to finish with a positive stock in asset value since more consumption improves well-being.

$$\log(b_t) = (1 - \rho_b) \log(b) + \rho_b \log(b_{t-1}) + \epsilon_{bt} \quad (3)$$

with ϵ_{bt} i.i.d.

The household's budget constraint is given by:

$$P_t C_t(j) + M_t(j) + \frac{D_t^g(j)}{R_t} + e_t \frac{B_t^*(j)}{\kappa_t R_t^*} = (1 - \tau_t) W_t(j) h_t(j) + M_{t-1}(j) + D_{t-1}^g + e_t B_{t-1}^*(j) + T_t + D_t^d + D_t^m \quad (4)$$

where $W_t(j)$ is the nominal wage set by the household. τ_t is the labor tax. D_t^g is domestic obligation purchased by household at time t , which is used by the government to finance its deficit. B_t^* represents foreign bonds, purchased by a household at time t , and e_t is the nominal exchange rate. R_t and R_t^* are respectively the domestic and foreign nominal interest rates between time t and $t + 1$. D_t^d is the nominal profit received by domestic firms and D_t^m is the nominal profit received by firms that import intermediate goods. T_t is the lump-sum transfer from the government.

κ_t is the risk premium that adjusts the uncovered interest rate parity. κ_t corrects the problem of the random walk followed by consumption around the equilibrium when domestic and foreign interest rates are assumed equal. Ambler, Dib and Rebei (2004) define the risk premium as depending on the ratio of net foreign assets and domestic production.

$$\log(\kappa_t) = \varphi \left[\exp \left(\frac{e_t B_t^*}{P_t^d Y_t} \right) - 1 \right] \quad (5)$$

where P_t^d is the domestic price index.

The foreign interest rate R_t^* follows the following AR(1) process:

$$\log(R_t^*) = (1 - \rho_{R^*}) \log(R^*) + \rho_{R^*} \log(R_{t-1}^*) + \epsilon_{R_t^*}, \quad (6)$$

$\epsilon_{R_t^*}$ is i.i.d with zero mean and variance σ_{R^*}

Note σ_h , the elasticity of substitution between different types of work, aggregate labor is defined by:

$$h_t = \left(\int_0^1 h_t(j)^{\frac{\sigma_h - 1}{\sigma_h}} dj \right)^{\frac{\sigma_h}{\sigma_h - 1}} \quad (7)$$

The demand for labor of type j is therefore⁷

⁷See Dixit, Stiglitz (1977) for more details

$$h_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\sigma_h} h_t \quad (8)$$

where the aggregate wages W_t is given by:

$$W_t = \left(\int_0^1 W_t(j)^{1-\sigma_h} dj \right)^{\frac{1}{1-\sigma_h}} \quad (9)$$

The first order conditions⁸ of household j 's problem, concerning consumption, money, purchases of national obligation and purchases of foreign bonds are written as:

$$\alpha(1 - h_t(j))^{(1-\alpha)(1-\sigma)} C_t(j)^{-\frac{1}{\gamma}} \left[\left(C_t(j)^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} \left(\frac{M_t(j)}{P_t} \right)^{\frac{\gamma-1}{\gamma}} \right) \right]^{\frac{\alpha\gamma(1-\sigma)}{\gamma-1}-1} = \Lambda_t(j) \frac{P_t}{P_t^d} \quad (10)$$

$$\begin{aligned} \alpha(1 - h_t(j))^{(1-\alpha)(1-\sigma)} b_t^{\frac{1}{\gamma}} \left(\frac{M_t(j)}{P_t} \right)^{\frac{-1}{\gamma}} \frac{P_t^d}{P_t} \left[\left(C_t(j)^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} \left(\frac{M_t(j)}{P_t} \right)^{\frac{\gamma-1}{\gamma}} \right) \right]^{\frac{\alpha\gamma(1-\sigma)}{\gamma-1}-1} \\ = \Lambda_t(j) - \beta_t E_t \left[\frac{P_t^d}{P_{t+1}^d} \Lambda_{t+1}(j) \right] \end{aligned} \quad (11)$$

$$\frac{\Lambda_t(j)}{R_t} = \beta_t E_t \left[\frac{P_t^d}{P_{t+1}^d} \Lambda_{t+1}(j) \right] \quad (12)$$

$$\frac{\Lambda_t(j)}{\kappa_t R_t^*} = \beta_t E_t \left[\frac{P_t^d}{P_{t+1}^d} \frac{e_{t+1}}{e_t} \Lambda_{t+1}(j) \right] \quad (13)$$

Consider the following notation:

$$p_t = P_t/P_t^d, m_t = M_t/P_t, p_t^m = P_t^m/P_t^d, \tilde{p}_t^d = \tilde{P}_t^d/P_t^d, \pi_t = P_t/P_{t-1}, \pi_t^d = P_t^d/P_{t-1}^d, w_t = W_t/P_t^d, \pi_t^* = P_t^*/P_{t-1}^*, s_t = e_t P_t^*/P_t^d, tr_t = T_t/P_t^d$$

⁸It is consistent to divide the two sides of budget constraint by domestic price index p_t^d when writing the Lagrangian

2.2 Discount factor shock and ZLB

As we said, the discount factor is the only type of shock in the absence of capital and risk premium on capital that could push the interest rate to its lower bound (see Amano and Shukayev 2009). Since we do not have capital in our analytical framework, we have to consider the time-variant discount factor. In this work, it is the shock on the discount factor that will push the economy to ZLB. The discount factor shock increases the propensity to save. The practical mechanism is relatively the same as the one presented by Christiano, Eichenbaum, and Rebelo (2011). It is as follows:

Initially (time -1) the economy is in the steady state, driven by the Taylor rule and the macroeconomic framework presented above ($\beta_{(-1)} = \frac{1}{R}$). Then there is a positive shock on the discount factor (at time 0 ($\beta_0 = 1$)). Subsequently, the discount factor gradually returns to its equilibrium value. Let R_t be the interest rate at time t induced by the shock to the discount factor at time 0. For simplicity, after the shock on the discount factor, the interest rate may remain at its threshold level (R^l) with probability (p_r), or it may return to its steady state (R) with probability ($1 - p_r$); in the latter case it remains at the stationary level forever.

The stochastic process describing the behavior of interest rates after the shock can be as follows (see Christiano, Eichenbaum, and Rebelo 2011):

$$\begin{aligned} Pr [R_{t+1} = R^l | R_t = R^l] &= p_r, \\ Pr [R_{t+1} = R | R_t = R^l] &= 1 - p_r, \\ Pr [R_{t+1} = R^l | R_t = R] &= 0, \\ Pr [R_{t+1} = R | R_t = R] &= 1, \end{aligned} \tag{14}$$

We can easily write the discount factor process as an AR (1):

$$\beta_t = p_r \beta_{t-1} + (1 - p_r) \beta + \epsilon_{\beta t} \tag{15}$$

The parameter β is the steady state value of the discount factor. It is calibrated to the standard value of 0.99.

2.3 Salaries

Salary is defined in Calvo framework (see Calvo 1983). With probability ($1 - d_w$), household j is allowed to adjust the salary. Otherwise the previous period salary remains in place. When considering all households, a proportion ($1 - d_w$) of households re-optimize their wages, and the other proportion d_w maintains the previous salary.

The aggregate wage index can be written as

$$W_t = \left(\int_0^1 W_t(j)^{1-\sigma_w} dj \right)^{\frac{1}{1-\sigma_w}} = \left((1-d_w)\tilde{W}_t^{1-\sigma_w} + d_w W_{t-1}^{1-\sigma_w} \right)^{\frac{1}{1-\sigma_w}} \quad (16)$$

and rearranged as:

$$W_t^{1-\sigma_w} = (1-d_w)\tilde{W}_t^{1-\sigma_w} + d_w W_{t-1}^{1-\sigma_w} \quad (17)$$

The Salary is the value that maximizes the expected utility of the household under the budget constraint for the expected time period where wages remain fixed. This salary will remain valid until the next authorization of wage readjustment.

The Lagrangian associated with the wage problem is as follows:

$$\begin{aligned} \mathcal{L} = \max_{\tilde{W}_t} \sum_{l=0}^{\infty} (\beta_t d_w)^l & \left[\frac{\left(\left[\left(C_{t+l}(j)^{\frac{\gamma-1}{\gamma}} + b_{t+l}^{\frac{1}{\gamma}} \left(\frac{M_{t+l}(j)}{P_{t+l}} \right)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} \right]^{\alpha} [1-h_{t+l}(j)]^{1-\alpha} \right)^{1-\sigma}}{1-\sigma} - 1 \right] \\ & + \sum_{l=0}^{\infty} (\beta_t d_w)^l \left(\frac{\wedge_{t+l}}{P_{t+l}^d} \right) \left[P_{t+l} C_t(j) + M_{t+l}(j) + \frac{D_{t+l}^g(j)}{R_{t+l}} + e_t \frac{B_{t+l}^*(j)}{\kappa_{t+l} R_{t+l}^*} \right] \\ & - \sum_{l=0}^{\infty} (\beta_t d_w)^l \left(\frac{\wedge_{t+l}}{P_{t+l}^d} \right) \left[(1-\tau_{t+l})\tilde{W}_t(j)h_{t+l}(j) + M_{t+l-1}(j) + D_{t+l-1}^g + e_{t+l} B_{t+l-1}^*(j) + T_{t+l} + D_{t+l}^d + D_{t+l}^m \right] \end{aligned} \quad (18)$$

Throughout the period of fixed wage, the household is subject to the following constraint:

$$h_{t+l}(j) = \left(\frac{\tilde{W}_t(j)}{W_{t+l}} \right)^{-\sigma_h} h_{t+l} \quad (19)$$

When the household is allowed to adjust the salary, the optimal level is as follows.

$$\tilde{W}_t(j) = (1-\alpha) \frac{\sigma_h}{\sigma_h - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta_t d_w)^l \left[\left(C_{t+l}(j)^{\frac{\gamma-1}{\gamma}} + b_{t+l}^{\frac{1}{\gamma}} \left(\frac{M_{t+l}(j)}{P_{t+l}} \right)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\alpha\gamma(1-\sigma)}{\gamma-1}} h_{t+l}(j) [1-h_{t+l}(j)]^{(1-\alpha)(1-\sigma)-1} \right]}{E_t \sum_{l=0}^{\infty} (\beta_t d_w)^l \left(\frac{\wedge_{t+l}}{P_{t+l}^d} \right) (1-\tau_{t+l})h_{t+l}(j)} \quad (20)$$

2.4 Production of national intermediate goods

Firms producing intermediate goods use final goods as inputs. The production function of firm i producing intermediate goods is:

$$Y_t(i) = X_t(i)^\phi (A_t h_t(i))^{1-\phi} \quad (21)$$

where $h_t(i)$ is labor, $X_t(i)$ the quantity of final good used by firm i , and A_t the technology shock that follows the auto-regressive process below:

$$\log(A_t) = (1 - \rho_A) \log(A) + \rho_A \log(A_{t-1}) + \epsilon_{At} \quad (22)$$

where ϵ_{At} is i.i.d with zero mean and variance σ_A . Prices are set by Calvo, firm i re-optimizes its price $\tilde{P}_t^d(i)$ with probability $(1 - d_p)$, and chooses the quantities of final goods and labor that maximize its expected profit. This is represented by the value of stock shares it issues. The price set in period t remains for l period with probability $(d_p)^l$. Let Λ_t represent the marginal utility of wealth, that is the Lagrange multiplier of the household problem. Let P_t represent the price of the final good and P_t^d the price index of national intermediate goods.

A firm producing the intermediate good solves the following problem:

$$\max_{\{X_t(i), h_t(i), \tilde{P}_t^d(i)\}} E_t \left[\sum_{l=0}^{\infty} (\beta_t d_p)^l \left(\frac{\Lambda_{t+l}}{\Lambda_t} \right) \frac{\left(\tilde{P}_t^d(i) Y_{t+l}(i) - W_{t+l} h_{t+l}(i) - P_{t+l} X_{t+l}(i) \right)}{P_{t+l}^d} \right], \quad (23)$$

subject to the following production function:

$$Y_t(i) = X_t(i)^\phi (A_t h_t(i))^{1-\phi} \quad (24)$$

and subject to the demand for the intermediate good i in the production of the final good.

Let $(-\theta)$ represent the demand elasticity for the intermediate good. Demand for the intermediate good i is given by:

$$Y_{t+l}(i) = \left(\frac{\tilde{P}_{t+l}^d(i)}{P_{t+l}^d} \right)^{-\theta} Y_{t+l}, \quad (25)$$

Lets $\xi_t(i)$ denote the Lagrange multiplier associated with production function constraint.

The first order conditions are given by:

$$\frac{W_t}{P_t^d} = \xi_t(i)(1 - \phi)A_t (X_t(i))^\phi (A_t h_t(i))^{-\phi} = \xi_t(i)(1 - \phi) \frac{Y_t(i)}{h_t(i)} \quad (26)$$

$$\frac{P_t}{P_t^d} = \xi_t(i)\phi (X_t(i))^{\phi-1} (A_t h_t(i))^{1-\phi} = \xi_t(i)\phi \frac{Y_t(i)}{X_t(i)} \quad (27)$$

$$\tilde{P}_t^d(i) = \left(\frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{l=0}^{\infty} (\beta_t d_p)^l \binom{\wedge_{t+l}}{\wedge_t} \xi_{t+l}(i) Y_{t+l}(i)}{E_t \sum_{l=0}^{\infty} (\beta_t d_p)^l \binom{\wedge_{t+l}}{\wedge_t} \left(\frac{Y_{t+l}(i)}{P_{t+l}^d} \right)} \quad (28)$$

2.5 Imported intermediate good

There is monopolistic competition on imported goods. There is a continuum of firms and each firm imports a differentiated good in unit intervals. These imported goods are imperfectly substitutable, and are used in the production of the composite good imported, noted Y_t^m , and produced by a representative firm.

With probability $(1 - d_m)$ the firm that imported the intermediate good re-optimizes its price \tilde{P}_t^m so as to maximize its expected weighted profit under its demand constraint. The problem can be written as:

$$\max_{\{\tilde{P}_t^m(i)\}} E_t \left[\sum_{l=0}^{\infty} (\beta_t d_m)^l \binom{\wedge_{t+l}}{\wedge_t} \left(\frac{\tilde{P}_t^m(i) - e_{t+l} P_{t+l}^*}{P_{t+l}^d} \right) \left(\frac{\tilde{P}_t^m(i)}{P_{t+l}^m} \right)^{-\vartheta} Y_{t+l}^m \right],$$

P_t^* is the price index of imported goods in foreign currency, and $-\vartheta$ the elasticity of demand of imported goods i .

As before, the first-order condition gives:

$$\tilde{P}_t^m(i) = \left(\frac{\vartheta}{\vartheta - 1} \right) \frac{E_t \sum_{l=0}^{\infty} (\beta_t d_m)^l \binom{\wedge_{t+l}}{\wedge_t} Y_{t+l}^m(i) e_{t+l} \left(\frac{P_{t+l}^*}{P_{t+l}^d} \right)}{E_t \sum_{l=0}^{\infty} (\beta_t d_m)^l \binom{\wedge_{t+l}}{\wedge_t} \left(\frac{Y_{t+l}^m(i)}{P_{t+l}^d} \right)} \quad (29)$$

2.6 Aggregated national good

The national good is produced by a representative firm from domestic intermediate goods.

The national good is an aggregate of a continuum of intermediate goods $Y_t(i)$ produced locally. The production function is a constant elasticity of substitution technology function:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \quad (30)$$

The firm producing the national good solves the following problem:

$$\max_{\{Y_t(i)\}} P_t^d Y_t - \int_0^1 P_t^d(i) Y_t(i) di \quad (31)$$

The first order condition gives:

$$P_t^d(i) = (Y_t(i))^{-\frac{1}{\theta}} \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{1}{\theta-1}} P_t^d = \left(\frac{Y_t(i)}{Y_t} \right)^{-\frac{1}{\theta}} P_t^d \quad (32)$$

The demand function for intermediate good is:

$$Y_t(i) = \left(\frac{P_t^d(i)}{P_t^d} \right)^{-\theta} Y_t \quad (33)$$

By aggregating both sides of demand equation, we obtain the price index for domestic goods.

$$P_t^d = \left(\int_0^1 P_t^d(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \quad (34)$$

$(1 - d_p)$ is defined as the proportion of firms that readjust their prices. We can therefore split domestic firms into two groups: those which are allowed to adjust their prices in period t and those which continue to apply the price of the previous period $(t - 1)$.

The price index in period t can be written as:

$$P_t^d = \left[d_p (P_{t-1}^d)^{1-\theta} + (1 - d_p) (\tilde{P}_t^d)^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (35)$$

where \tilde{P}_t^d is the price index set by firms that have the right to readjust their prices in period t .

2.7 Aggregate exported good

One part of the national aggregated good is exported⁹, while the other¹⁰ is combined with imported goods to produce national final goods. We have:

$$Y_t = (1 - \alpha_x)Y_t + \alpha_x Y_t \quad (36)$$

$$Y_t = Y_t^d + Y_t^x \quad (37)$$

$$Y_t^d = (1 - \alpha_x)Y_t$$

and

$$Y_t^x = \alpha_x Y_t$$

$\alpha_x > 0$ is the proportion of the national good that is exported.

The exported good is part of a continuum of imperfectly substitutable goods that are used by a foreign representative company. The elasticity of substitution between goods is noted $-\iota$. The foreign demand of the national good is represented by the following equation:

$$Y_t = \left(\frac{P_t^d}{e_t P_t^*} \right)^{-\iota} Y_t^*$$

we deduce that

$$Y_t^x = \alpha_x \left(\frac{P_t^d}{e_t P_t^*} \right)^{-\iota} Y_t^* \quad (38)$$

As we assume, our economy is small, it therefore does not influence the foreign price index or aggregate foreign production.

The foreign price and foreign production follow the following processes:

$$\log \left(\frac{P_t^*}{P_{t-1}^*} \right) = (1 - \rho_{\pi^*}) \log(\pi^*) + \rho_{\pi^*} \log \left(\frac{P_{t-1}^*}{P_{t-2}^*} \right) + \epsilon_{\pi^* t} \quad (39)$$

$$\log(Y_t^*) = (1 - \rho_{y^*}) \log(Y^*) + \rho_{y^*} \log(Y_{t-1}^*) + \epsilon_{y^* t} \quad (40)$$

where π^* and Y^* are respectively inflation and foreign production in the steady state.

⁹ $Y_t^x = \alpha_x Y_t$

¹⁰ $Y_t^d = (1 - \alpha_x)Y_t$

2.8 Aggregate imported good

With a constant elasticity of substitution production function, a representative firm produces a good using a continuum of imported goods $Y_t^m(i)$. The production function is:

$$Y_t^m = \left(\int_0^1 Y_t^m(i)^{\frac{\vartheta-1}{\vartheta}} di \right)^{\frac{\vartheta}{\vartheta-1}} \quad (41)$$

The representative firm solves the following problem:

$$\max_{\{Y_t^m(i)\}} P_t^m Y_t^m - \int_0^1 P_t^m(i) Y_t^m(i) di \quad (42)$$

The first order condition gives the demand equation for the imported intermediate good i :

$$Y_t^m(i) = \left(\frac{P_t^m(i)}{P_t^m} \right)^{-\vartheta} Y_t^m \quad (43)$$

We can deduce the price index for imported intermediate goods:

$$P_t^m = \left(\int_0^1 P_t^m(i)^{1-\vartheta} di \right)^{\frac{1}{1-\vartheta}} \quad (44)$$

A proportion $(1 - d_m)$ firms importing the intermediate good re-optimize their price at time t , while the other portion d_m keep the price of previous period $(t - 1)$.

We can rewrite the price index of imported goods as follows:

$$P_t^m = \left[d_m (P_{t-1}^m)^{(1-\vartheta)} + (1 - d_m) \left(\tilde{P}_t^m \right)^{(1-\vartheta)} \right]^{\frac{1}{1-\vartheta}} \quad (45)$$

where \tilde{P}_t^m is the price index of firms that re-optimize their prices in period t .

2.9 The final good

The final good Z_t is produced by a representative firm that uses the aggregated national good as well as the aggregated imported good. The technology used to produce the final good is the constant elasticity of substitution production function:

$$Z_t = \left[\alpha_d^{\frac{1}{v}} (Y_t^d)^{\frac{v-1}{v}} + \alpha_m^{\frac{1}{v}} (Y_t^m)^{\frac{v-1}{v}} \right]^{\frac{v}{v-1}} \quad (46)$$

The firm producing the final good solves the following problem:

$$\max_{\{Y_t^d, Y_t^m\}} P_t Z_t - P_t^d Y_t^d - P_t^m Y_t^m \quad (47)$$

The first order conditions gives:

$$Y_t^d = \alpha_d \left(\frac{P_t^d}{P_t} \right)^{-v} Z_t \quad (48)$$

and

$$Y_t^m = \alpha_m \left(\frac{P_t^m}{P_t} \right)^{-v} Z_t \quad (49)$$

then the price of the final good can be written as:

$$P_t = [\alpha_d (P_t^d)^{1-v} + \alpha_m (P_t^m)^{1-v}]^{\frac{1}{1-v}} \quad (50)$$

As a reminder, the final good is used for consumption, in the process to produce intermediate goods, and is also used for government spending.

$$Z_t = C_t + X_t + G_t \quad (51)$$

2.10 Trade balance equilibrium

Net profits of assets purchased abroad in local currency are equal to the net value of goods purchased abroad in local currency.

Income from foreign assets purchased in a preceding period minus assets purchased in the current period are equal to imported goods minus exported goods.

The trade balance can be summarized by the following equation

$$e_t B_{t-1}^* - e_t \frac{B_t^*}{\kappa_t R_t^*} = e_t P_t^* Y_t^m - P_t^d Y_t^x \quad (52)$$

where

$$s_t = e_t \frac{P_t^*}{P_t^d} ; b_t^* = \frac{B_t^*}{P_t^*}$$

The balance of payments can be represented by the following equation:

$$\frac{b_t^*}{\kappa R_t^*} - \frac{b_{t-1}^*}{\pi_t^*} = \frac{Y_t^x}{s_t} - Y_t^m \quad (53)$$

2.11 Transformation of variables

To solve the model, it is necessary to use stationary variables.

We have the following notations:

$$p_t = P_t/P_t^d, m_t = M_t/P_t, p_t^m = P_t^m/P_t^d, \tilde{p}_t^d = \tilde{P}_t^d/P_t^d, \pi_t = P_t/P_{t-1}, \pi_t^d = P_t^d/P_{t-1}^d, w_t = W_t/P_t^d, \pi_t^* = P_t^*/P_{t-1}^*, s_t = e_t P_t^*/P_t^d, d_t^g = D_t^g/P_t^d$$

2.12 New Phillips curves

The Phillips curve defines inflation dynamics as a function of future inflation and real marginal costs. The Phillips curve can also describe current inflation as a function of the expected inflation and the output gap.

We will have one Phillips curve for wages on the labor market, one for the price of the domestic intermediate goods market, one for price in imported intermediate goods market, and from these we can then easily derive a Phillips curve for price in the final goods market.

We find the optimal price levels when firms are allowed to change their prices. Firms allowed to re-optimize their prices take into account the probability of future price changes. The discount rate considered by firms takes into account the valuation of future consumption by households. Firms also take into account the real marginal costs and demand for goods they produce. We have found that the current price index level is a weighted average of the price index of the previous period and the price index set by firms allowed to optimize their price.

Previous versions of wages and prices contain infinite summations. In order to solve the model we have to make another transformation. We will combine the wages and prices that we obtained previously with the dynamics of wages and prices obtained in the Calvo framework. Furthermore, we will transform our variables, such that each new variable will be in percentage deviation from the steady state of the variable it represents. Thus, when the initial variables converge to the steady state, the new variables converge to zero. After that we can have a Taylor expansion series around zero (see Brook Taylor¹¹1715 also known as Mac-Lauren developments series) for all transformed variables. To illustrate the model, the Taylor expansion is in first order. But for better accuracy, in the computation algorithm we will extend the Taylor approximation to order 3.

- For the domestic intermediate good:

¹¹If a function f is differentiable at x_0 up to order n , we can write the polynomial function as follows:

$$f(x) = \sum_{k=0}^n \frac{f^{(k)}(x_0)}{k!} (x - x_0)^k + \epsilon_n(x) \text{ with } \lim_{x \rightarrow x_0} \epsilon_n(x) = 0$$

$$\hat{\pi}_t^d = \beta_t \hat{\pi}_{t+1}^d + \frac{(1 - \beta_t d_p)(1 - d_p)}{d_p} \hat{\xi}_t \quad (54)$$

We will see later that

$$\hat{\xi}_t = (1 - \phi) \hat{w}_t + \phi \hat{p}_t - (1 - \phi) \hat{a}_t \quad (55)$$

$\hat{\pi}_t^d = \hat{p}_t^d - \hat{p}_{t-1}^d$; $\hat{p}_t^d = \frac{p_t^d - \bar{p}^d}{\bar{p}^d}$ and $\hat{\xi}_t = \frac{\xi_t - \bar{\xi}}{\bar{\xi}}$. As a reminder, ξ_t is the Lagrange multiplier associated with the production function constraint. It therefore represents the real marginal cost of producing one unit of domestic intermediate good.

- For the imported intermediate good

$$\hat{\pi}_t^m = \beta_t \hat{\pi}_{t+1}^m + \frac{(1 - \beta_t d_m)(1 - d_m)}{d_m} \hat{s}_t \quad (56)$$

with $\hat{\pi}_t^m = \hat{p}_t^m - \hat{p}_{t-1}^m$; $\hat{p}_t^m = \frac{p_t^m - \bar{p}^m}{\bar{p}^m}$

e_t is the exchange rate. This is the value of a unit of foreign currency in terms of domestic currency.

P_t^* is the price index of imported goods in foreign currency

$s_t = e_t P_t^* / P_t^d$ is the real exchange rate and $\hat{s}_t = \frac{s_t - \bar{s}}{\bar{s}}$;

- For the wages

$$\begin{aligned} \hat{\pi}_t^w = & \beta_t \hat{\pi}_{t+1}^w \\ & + \frac{(1 - \beta_t d_w)(1 - d_w)}{d_w} \left[(1 - (1 - \alpha)(1 - \sigma)) \frac{h}{1 - h} \hat{h}_t + \frac{\tau}{1 - \tau} \hat{\tau}_t - \hat{\lambda}_t - \hat{w}_t \right] \\ & + \frac{(1 - \beta_t d_w)(1 - d_w)}{d_w} \left(\frac{\alpha \gamma (1 - \sigma) / (1 - \gamma)}{c^{\frac{\gamma-1}{\gamma}} + b^{\frac{1}{\gamma}} m^{\frac{\gamma-1}{\gamma}}} \right) \left[\frac{\gamma - 1}{\gamma} c^{\frac{\gamma-1}{\gamma}} \hat{c}_t + \frac{1}{\gamma} b^{\frac{1}{\gamma}} m^{\frac{\gamma-1}{\gamma}} \hat{b}_t + \frac{\gamma - 1}{\gamma} b^{\frac{1}{\gamma}} m^{\frac{\gamma-1}{\gamma}} \hat{m}_t \right] \end{aligned} \quad (57)$$

2.13 The IS-dynamic equation

We can write current aggregate production in terms of expected future aggregate production, current and expected future interest rate, as well as present and future government spending. To do this, we will use the aggregate resource constraint, the optimal conditions for the household problem, and the optimal conditions of firms producing the intermediate national goods.

We then have

$$\begin{aligned}
& [(\alpha - \alpha\sigma - 1)z + c(1 - \alpha)(1 - \sigma) + x(\alpha - \alpha\sigma - 1)] (\hat{z}_{t+1} - \hat{z}_t) \\
& = \left[\frac{(\alpha\gamma(1-\sigma) + (1-\gamma))}{c^{\frac{\gamma-1}{\gamma}} + b^{\frac{1}{\gamma}} m^{\frac{\gamma-1}{\gamma}}} b^{\frac{1}{\gamma}} m^{\frac{\gamma-1}{\gamma}} \right] \frac{(\hat{R}_{t+1} - \hat{R}_t)}{1-R} + G(\alpha - \alpha\sigma - 1) (\hat{G}_{t+1} - \hat{G}_t) \\
& \quad - (1 - \phi) \left[c(1 - \alpha)(1 - \sigma) \frac{h}{1-h} + x(\alpha - \alpha\sigma - 1) \right] (\hat{a}_{t+1} - \hat{a}_t) \\
& \quad + \left[c(1 - \alpha)(1 - \sigma) \frac{h}{1-h} \phi + x(\alpha - \alpha\sigma - 1)(1 - \phi) \right] \hat{\pi}_{t+1}^w \\
& + \left[c(1 - \alpha)(1 - \sigma) \frac{h}{1-h} (v - \phi) + x(\alpha - \alpha\sigma - 1)(v + \phi - 1) + 1 \right] \left(\frac{\alpha_m (p^m)^{1-v}}{p} \right) \hat{\pi}_t^m \\
& \quad - \hat{\kappa}_t - \hat{R}_t^* - \hat{s}_{t+1} + \hat{s}_t
\end{aligned} \tag{58}$$

with

$$\hat{\pi}_t = \frac{\alpha_m (p^m)^{1-v}}{p} \hat{\pi}_t^m \tag{59}$$

3 Monetary and government spending policies

Monetary policy follows the Taylor rule (see Taylor 1993) when the interest rate is strictly positive. The central bank sets the nominal interest rate in response to short-term fluctuations in inflation ($\pi_t = \frac{P_t}{P_{t-1}}$), fluctuations in the money ($\mu_t = \frac{M_t}{M_{t-1}}$), fluctuations in production (Y_t) and fluctuations in real exchange rate ($s_t = e_t \frac{P_t^*}{P_t}$) (see Ambler and al. 2003).

When the Taylor rule implies a negative interest rate (r_{taylor_t}), the monetary authorities systematically fix the nominal interest rate to the lowest level (see Christiano, Eichenbaum and Rebelo 2011).

Note $R_{taylor_t} = 1 + r_{taylor_t}$, and $R_t = r_t + 1$

Monetary policy can then be summarized in the following equation (see Amano and Ambler 2012)

$$\log(R_t) = \max \{ \log(R_{taylor_t}), 0 \} \tag{60}$$

with

$$\log(R_{taylor_t}) = -\log(\beta) + \phi_\pi \log(\pi_t/\pi) + \phi_y \log(Y_t/Y) + \rho_\mu \log(\mu_t/\mu) + \rho_s \log(s_t/s) + \epsilon_{Rt} \tag{61}$$

where Y , π , μ , and s denote the stationary state value of production, stationary state value of inflation, stationary growth rate value of money supply and stationary value of real exchange rate. ϵ_{Rt} is i.i.d shocks to monetary policy with zero mean

and variance σ_R . The central bank can only indirectly control short-term interest rates, by setting the Bank Rate. The error term reflects developments in money and financial markets that are not explicitly captured by our model.

Practical case

The function defining the level of nominal interest rates ($\log(R_t) = \max\{\log(R_{taylor_t}), 0\}$) in our analytical framework is not differentiable around the intersection between the Taylor rule function and the lowest nominal interest rate (ZLB). We will therefore smooth the previous interest rate function around the ZLB so that the new interest rate function can be differentiable at every point. As in Amano and Ambler (2012), we will write:

$$\log(R_t) = \max\{\log(R_{taylor_t}), 0\} \cong \frac{\sqrt{(\log(R_{taylor_t}))^2 + a^2} + \log(R_{taylor_t})}{2} \quad (62)$$

“a” is a smoothing parameter that defines the curvature of the new monetary interest rate policy function around the ZLB.

The new monetary policy

Due to the shock, the monetary authorities set the nominal interest rate such that it converges smoothly to the lowest level, but remains positive and differentiable at all points

3.1 Government response to demand shock

The government budget constraint is as follows:

$$P_t G_t + T_t + D_{t-1}^g = \tau_t W_t h_t + M_t - M_{t-1} + \frac{D_t^g}{R_t} \quad (63)$$

To obtain this mathematically, we just rewrite the household’s budget constraint, taking into account the trade balance, replacing dividends by firms’ profits, and using the equation characterizing how the final resource is used in the economy ($Z = C + X + G$).

The equation above shows that government spending is well represented by the purchases, transfers and debt repayments. Government revenues are represented by taxes on wages, by money creation and by new borrowing.

There are no Ponzi games, i.e:

$$\lim_{t \rightarrow \infty} \frac{D_t^g}{\prod_{i=0}^{\infty} R_i} = 0$$

This imply that:

$$\sum_{t=1}^{\infty} \left(\frac{P_t G_t + T_t}{\prod_{i=0}^{t-1} R_i} \right) + D_0^g = \sum_{t=1}^{\infty} \left(\frac{\tau_t W_t h_t + M_t - M_{t-1}}{\prod_{i=0}^{t-1} R_i} \right)$$

This means that the present value of government spending over the original debt is equal to the present value of government revenues. Discounted tax revenues are equal to discounted loans.

A positive shock to the discount factor (β_t) leads the nominal interest rate to the ZLB, then, the government reacts immediately or after a delay to this demand shock in order to stimulate economic activity.

The government's response to the demand shock caused by a positive shock on the discount factor is dictated by the following rule:

$$G_t = (1 - \rho_g) \cdot G + \rho_g \cdot G_{t-1} + \sum_{i=0}^T \rho_{g\beta_i} (\beta_{t-i} - \beta) + \epsilon_{gt} \quad (64)$$

i is the number of periods after impact.

$\rho_{g\beta_i}$ is the government's response at time t to the demand shock that occurs at time $t - i$.

G is the level of public expenditure in the steady state.

ϵ_{gt} is a zero-mean, serially uncorrelated government policy shock with standard deviation σ_g , ϵ_{gt} is a shock that represented the unpredictable government spending.

This specification will identify the government spending multiplier as a function of government time reaction.

The level of taxes follows an AR (1):

$$\log(\tau_t) = (1 - \rho_\tau) \log(\tau) + \rho_\tau \log(\tau_{t-1}) + \epsilon_{\tau t} \quad (65)$$

4 Analysis and results

4.1 Resolution method

We follow the Dynare algorithm, based on the perturbation method (see Collard and Juillard 2001, Schmitt-Ghore and Martín Uribe 2004), which allows for Taylor

expansion in any higher order. In our analysis, we have a smoothing interest rate policy that should be approached with precision. A second order polynomial approximation is better to smooth our monetary policy. For this reason we chose the perturbation method with second order Taylor expansion. Results in order 3 are the same. We therefore limited our Taylor expansion to the order two.

With the notation of Collard and Juillard (2001) and those of Schmitt-Ghore and Martín Uribe (2004), the equilibrium conditions of our rational expectations model can be summarized as follows:

$$E_t[f(y_{t+1}, y_t, x_{t+1}, x_t, u_t, u_{t+1})]=0$$

f defines the set of equilibrium equations, y is the vector defining the set of variables to predict, x is the set of predetermined variables and u is the shock vector.

At time t , the agents know the value of predetermined variables of time $t - 1$ and observe the shock at time t . Their decisions are based on beliefs that relate to variables y_{t+1} , all the current variables y_t and x_t .

The solution to this problem is a set of relationships between current variables, the predetermined variables and shocks that satisfy the original equation system (defining the equilibrium conditions of our model).

Solving this problem is the same as finding two functions g and h such that :

$$y_t = g(x_t)$$

$$x_t = h(x_{t-1}, u_t)$$

We can therefore rewrite the equilibrium condition in the form:

$$F(x_t) = E_t[f(g(h(x_t, u_{t+1})), g(x_t), h(x_t, u_{t+1}), x_t, u_t, u_{t+1})]=0$$

The strategy is to write the Taylor expansion for g and for h in the chosen order n , around the steady state, and then to find the coefficients of the n th-order polynomials considered. We also have to understand that F and its derivatives in any order are zero at all points (see Collard and Juillard 2001, Schmitt-Ghore and Martín Uribe 2004, for details).

4.2 Indicators of the fiscal multiplier

We can have different indicators for the government spending multiplier. These indicators cannot necessarily have identical values. Like any indicator, the goal is to make a comparison between two different situations.

In our framework we would like to know the path followed by the government spending multiplier indicator, following a demand shock.

Several authors use the impact multiplier (see Christiano and al. 2011, Montford and Uhlig 2009). This allows one to evaluate the change in output in a given period due to a change in government spending in the current period. This indicator is defined as follows:

$$Impact - multiplier(k) = \frac{\Delta Y_{t+k}}{\Delta G_t} \quad (66)$$

Montford and Uhlig (2009) and other works use the present value multiplier at lag k defined as follows:

$$present - value - multiplier - at - lag(k) = \frac{\sum_{t=0}^k (1+i)^{-t} \Delta Y_t}{\sum_{t=0}^k (1+i)^{-t} \Delta G_t} \quad (67)$$

This indicator is used to update the impact of fiscal policy on k periods.

To identify the evolution of the public expenditure multiplier, we considered the two following indicators:

$$\frac{\partial Y_{t+k}}{\partial G_t} \cong dY divdG(k) = \frac{\left(\frac{Y_{t+k}}{Y_{t+k-1}}\right)}{\left(\frac{G_t}{G_{t-1}}\right)} \quad (68)$$

$$\frac{\partial Z_{t+k}}{\partial G_t} \cong dZ divdG(k) = \frac{\left(\frac{Z_{t+k}}{Z_{t+k-1}}\right)}{\left(\frac{G_t}{G_{t-1}}\right)} \quad (69)$$

We can also use the following indicator

$$\frac{\partial Z_{t+k}}{\partial G_t} \cong dZ(k) divdG = \frac{\left(\frac{Z_{t+k}}{Z_t}\right)}{\left(\frac{G_t}{G_{t-1}}\right)} \quad (70)$$

By logarithmic transformation, our indicators are exactly the impact multipliers used by Christiano and al (2011).

Our indicators can be directly implemented in our algorithm in order to see their changes over time.

4.3 Calibration

Tables 1, 2 and 3 present calibration values. The stationary discount value, β , is standard and leads to a stationary annual real interest rate value equal to 4 %. The elasticity of substitution between different kinds of labor is also standard and is set at $\sigma_h = 6$. The probabilities of not adjusting wages are set so that wages adjust after six quarters and prices after two quarters, approximately. The parameters θ and ϑ that define elasticities of substitution between goods are set so that the markup in the marginal cost is approximately 14 %.

Table 1: The household

Parameters	Description
$\beta = 0,99$	Stationary discount parameter (Ambler and al., 2004).
$1 - \alpha = 0,71$	Leisure share in the utility function (Christiano and al., 2011).
$\gamma = 0,3561$	Weighted real-consumption cash flow (Ambler and al., 2004).
$\sigma = 2$	Parameter related to risk aversion (Christiano and al., 2011).
$\rho_b = 0,6450$	AR(1) parameter for shock on money demand (Ambler and al., 2004).
$\sigma_h = 6$	Elasticity of substitution between different labor types (Ambler and al., 2004).
$d_w = 0,8257$	Probability of not adjusting wages (Ambler and al., 2004).

Table 2: National intermediate goods

Parameters	Description
$\phi = 0,3788$	Share of final goods in the production of intermediate goods (Ambler & al 2004)
$\rho_A = 0,8795$	Coefficient of lagged variable in AR (1) of productivity
$d_p = 0,4398$	Probability of not adjusting prices (Ambler & al 2004)
$-\theta = -2,95$	Elasticity of demand for intermediate goods

Table 3: Others parameters

Parameters	Description
$d_m = 0.5508$	Probability of not adjusting prices of firms that import intermediate goods. (Ambler & al 2004)
$-\vartheta = -2.95$	Elasticity of substitution between imported intermediate goods(Ambler & al 2004)
$\rho_G = 0.8$	AR(1) parameter related to public spending (Christiano & al 2011)
$\rho_{G\beta} = 0.8$	AR (1) Parameter of public spending for discount factor
$\rho_{y^*} = 0.8835$	Coefficient of lagged variable in AR (1) of foreign production
$\rho_A = 0.8795$	Coefficient of lagged variable in AR (1) of productivity
$\rho_\tau = 0.4320$	Coefficient of lagged variable in AR (1) of tax rate (Ambler & al 2004)
$Y^* = 0.1$	Stationary value of foreign production
$R^* = 1.150$	Stationary value of foreign interest rate
$\pi^* = 1$	Stationary value of foreign inflation rate
$A = 1$	Stationary value of national productivity
$G = 0.0562$	Stationary level of Government spending (Ambler & al 2004)
$\tau = 0.29$	Stationary level of tax (Ambler & al 2004)
$p_r = 0.5$	Auto-regressive parameter of the discount factor after the shock
$\iota = 0.5962$	Elasticity of substitution between imported goods (Ambler & al 2004)
$\alpha_x = 0.074$	Share of exported domestic good product (Ambler & al 2004)
$\alpha_m = 0.3594$	Share of imported goods in the production of national final good (Ambler & al 2004)
$\nu = 0.5962$	Elasticity of substitution between aggregate imported goods and aggregated domestic goods(Ambler & al 2004)
$\phi_\pi = 0.73$	Inflation parameter in Taylor rule (Ambler & al 2003)
$\phi_\mu = 0.5059$	Parameter associated with growth rate of real cash in the Taylor rule (Ambler & al 2003)
$\phi_s = 0$	Parameter associated with real exchange rate in the Taylor rule (Ambler & al 2003)
$\phi_y = 0$	Parameter associated to production in the Taylor rule (Ambler & al 2003)
$v = 0.36$	Parameter related to production function for the final good
$\sigma_G = 0.0016$	Standard deviation of government spending shock(Ambler & al 2004)
$\sigma_\beta = 0.04$	Standard deviation of discount factor

4.4 Results

In the beginning, the nominal interest rate is binding due to the shock on the nominal discount factor. When the government reacts instantaneously, the aggregate demand increases, leading to appreciation of real exchange rate. The appreciation of the real exchange rate reduces the government spending multiplier. This result is not sensitive to the government spending multiplier indicator taken into account. The result is

also not sensitive to the time at which the government reacts to the discount factor shock.

Figure 1 displays the impulse response function of real exchange rate (s), the government spending multiplier ($DydivdG$) and other endogenous variables. The figure shows that after the increase in government spending, the real exchange rate appreciates, reaching approximately the level 0.027, relative to its stationary value.

Figure 2 displays the impulse response function that proves that the increase of government spending leads to less appreciation of the real exchange rate compared to the situation where the nominal interest rate is binding. When we compare figures 1 and 2, it is easy to observe that the appreciation of the real exchange rate is more important when the ZLB on the nominal interest rate is binding compared with the situation where it is not (0.027 compared to 0.02). However the difference between the two values is not too high, and econometric methods are needed to test whether this difference is significant or not. Nonetheless this small appreciation is sufficient to reduce the government spending multiplier. The maximum government spending multiplier is about 1.1. This value is clearly less than 4, which represents the maximum multiplier found by Christiano, Eichenbaum and Rebelo (2011).

Figure 3 displays the government spending multiplier path when considering the indicator in order 2 ($\frac{\partial Y_{t+2}}{\partial G_t}$). Our result is not sensitive to the indicator that we use¹².

Figure 4 displays results obtain when the government reacts after 5 periods. It confirms the fact that the appreciation of real exchange rate arises in period 5, exactly when the government reacts. It also shows the considerable reduction of government spending multiplier in period 5.

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$$\frac{\partial Y_{t+2}}{\partial G_t} \cong dZdivdG(2) = \frac{\left(\frac{Z_{t+2}}{Z_{t+1}}\right)}{\left(\frac{G_t}{G_{t-1}}\right)} \quad (71)$$

Figure 1: Discount factor shock coupled with increases of government spending (indicator $\frac{\partial Y_t}{\partial G_t} \cong dYdivdG(0)$)

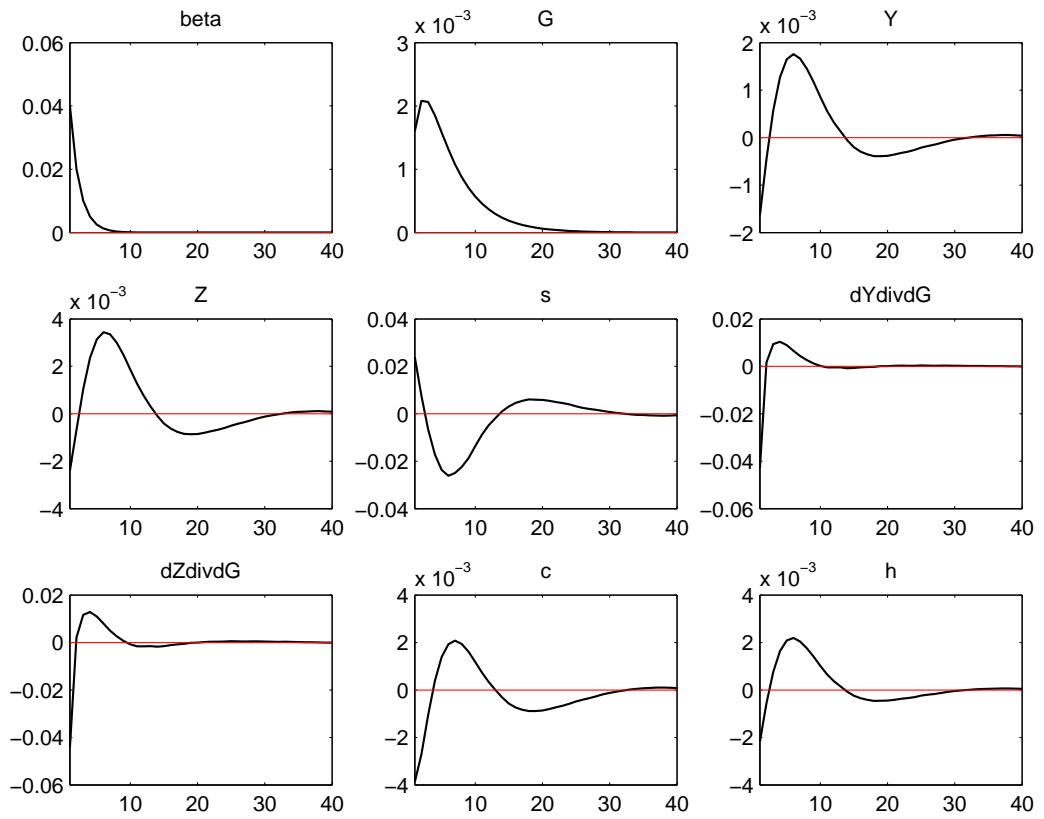


Figure 2: Increases of government spending without any discount factor shock (indicator $\frac{\partial Y_t}{\partial G_t} \cong dY \text{divd}G(0)$)

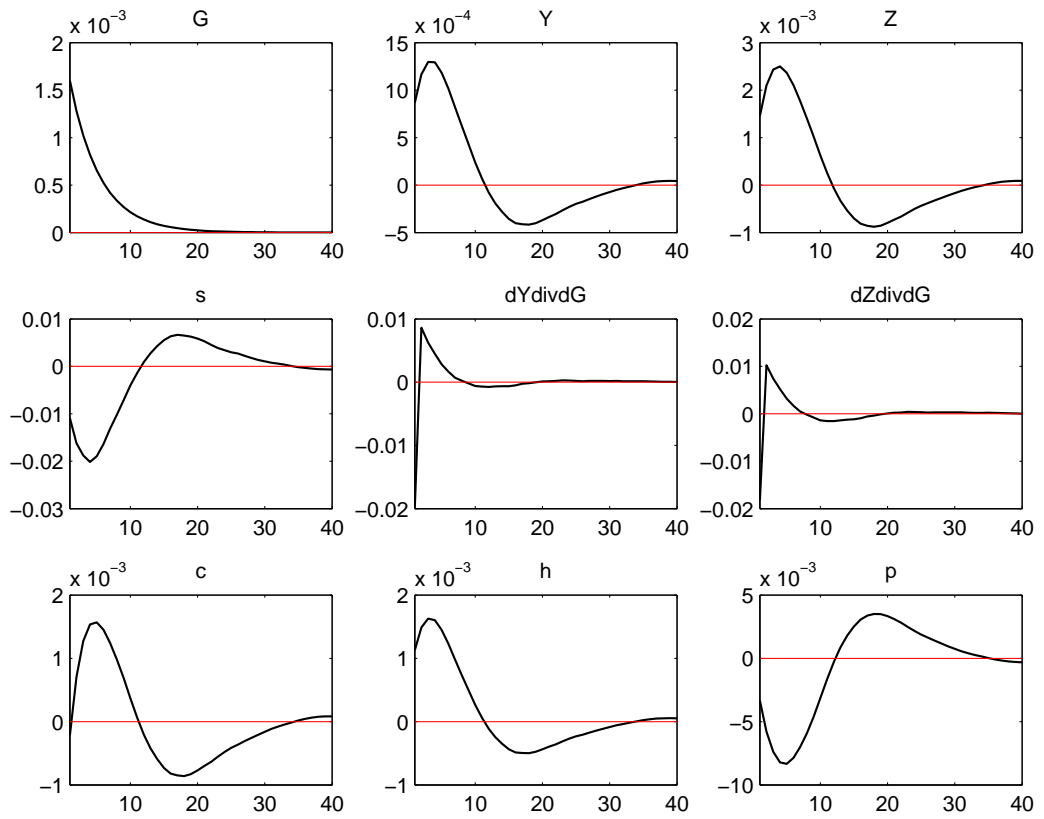


Figure 3: Discount factor shock coupled with increases of government spending (indicator $\frac{\partial Y_{t+2}}{\partial G_t} \cong dY \text{divd}G(2)$)

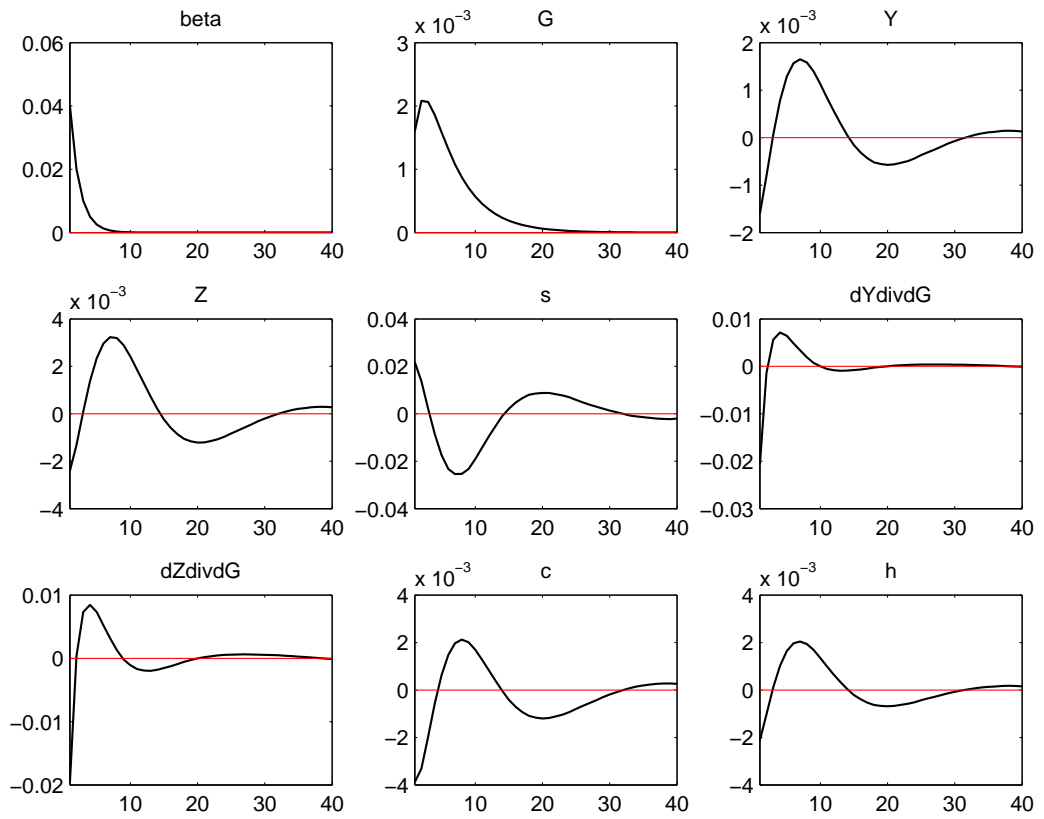
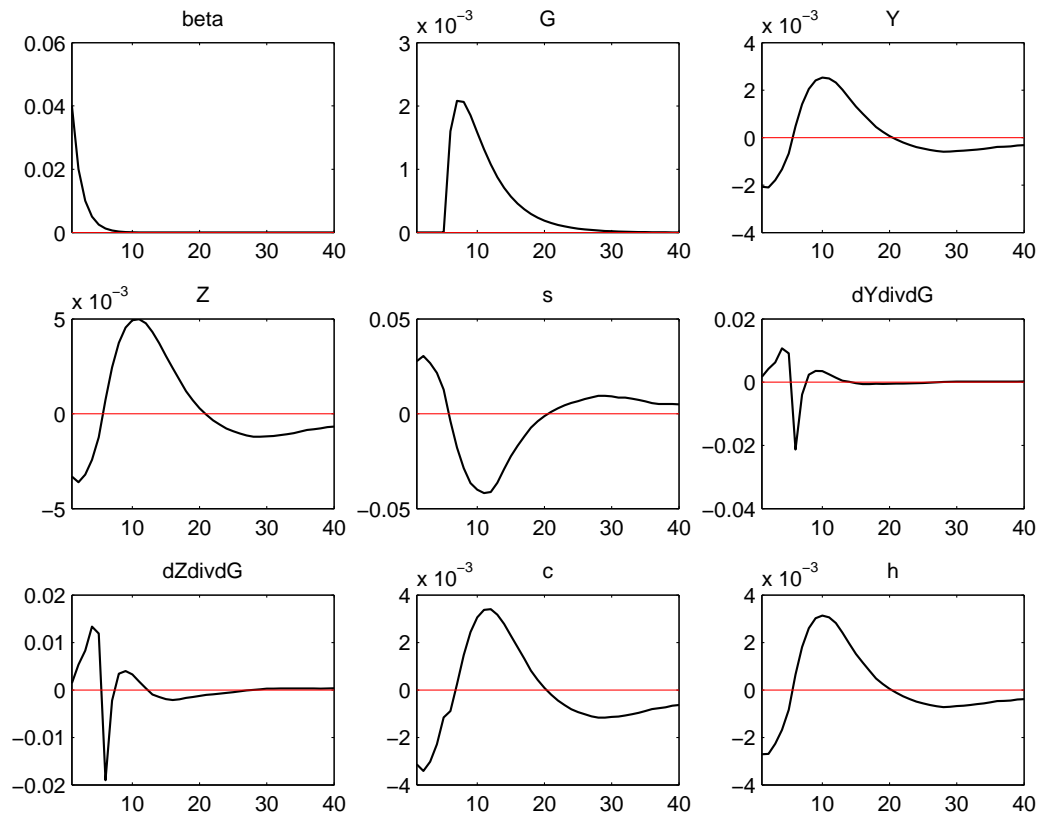


Figure 4: Discount factor shock coupled with increases of government spending after 5 periods



5 Conclusion

The aim of this paper was to identify the path followed by the government spending multiplier in an open economy when the Zero Lower Bound (ZLB) on nominal interest rate is binding. We have shown that increasing government spending in a ZLB results in appreciation of the real exchange rate, and a depreciation of the government spending multiplier. In fact, the increase in government spending increases aggregate demand, which leads to the appreciation of real exchange rate. The appreciation of real exchange rate reduces the government spending multiplier.

When we compare our economy in two different situations i.e. when the nominal interest rate is binding versus when it is not, we find that the appreciation of the real exchange rate is greater in the situation where the zero lower bound is binding.

Our result is not sensitive to the indicator used to evaluate the multiplier and it is also not sensitive to the date at which the government reacts to the discount factor shock.

Despite the use of a non-separable utility function (as required by Monacelli & Perotti 2007 and Christiano & al. 2011 and others), it was not possible to have a government spending multiplier larger than one due to the role played by the real exchange rate.

In conclusion, the ZLB removes the crowding out effect that passes through the interest rate¹³. In an open economy, there is another channel for the crowding out effect that passes through the real exchange rate. Thus, the multiplier drops to the normal level of the small closed economy when the nominal interest rate is not binding.

Our result is similar to the one obtain by Perotti (2004), that shows empirically that the effect of government spending on Gross Domestic Product (GDP) tends to be lower for open economies. Our result is also similar to the one found by Karras (2012) that proved that the multiplier tends to decrease with the degree of an economy's openness ($(X + M)/PIB$). Using data for 62 countries, for the period of 1951 to 2007, they found that an increase of openness by 10 % reduces the value of multiplier by 5 %.

Perotti (2004) and Karras (2012) studied periods when the nominal interest rate was not binding. An extension of our results can be to quantify empirically those results in ZLB (despite the short period of data).

¹³As the interest rate defined by the Taylor rule, is below the threshold interest rate, the higher interest rates caused by the crowding out effect may remain below the threshold and the interest rate applied by the central bank remains the threshold interest rate. This simply means that there is no crowding out effect.

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