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Resource Extraction under Heterogeneous Growth in Demand

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Abstract:
We study the effect of heterogeneous growth in demand on resource extraction. Using the Great Fish War framework of Levhari and Mirman (1980), we show that heterogeneity in demand growth has a profound effect on both cooperative and non-cooperative solutions.

Keywords: Common-pool resource, Dynamic games, Heterogeneous growth, Strategic extraction, Tragedy of the commons

JEL Classification: Q20, C72, C71, C73

1 Introduction

Since the beginning of the 20th century, the use of global materials has increased 8-fold (Krausmann et al., 2009). This increase in world demand ranges from natural resources such as fish to energy-related resources. See Figures 1 and 2 in Appendix A. Moreover, there is a lot of heterogeneity about the growth rates for demand of natural resources. For instance, the annual fish consumption growth rate for the years 1999-2013 is only 1.06% for the US, but 3.43% for China. Similarly, for total primary energy consumption, the annual growth rate for the years 2006-2013 is negative for the US (-0.44%), but positive for both India (5.14%) and China (7.16%). Figures 3 and 4 in Appendix A further illustrate this heterogeneity of demand growth among countries for both fish and primary energy consumption. In view of such heterogeneity with the particular case of China's exploding demand for resources, it is important to understand how the anticipation of growing demand affects extraction.

In this paper, we study the effect of heterogeneous growth in demand on extraction. To that end, we extend the Great Fish War framework (Levhari and Mirman, 1980) to a situation in which demand for the resource grows exogenously and heterogeneously.[1] Specifically, we consider the case of two countries with heterogeneous growth in demand. The growth in demand is assumed exogenous in order to identify clearly the effect of growing demand on behavior, thereby abstracting from the effect of natural resource utilization on demand growth.

We consider both non-cooperative and cooperative solutions. Under non-cooperation, a higher growth rate in demand for one country leads to lower extraction whereas a higher growth rate in demand from rivalrous countries leads to higher extraction. The presence of cooperation alters this result. The results change from the non-cooperative case. Specifically, under cooperation, a higher growth from any of the countries leads systematically to lower extraction of the resources for *both* countries. The presence of hetero-

[1]See Long (2011) for an exhaustive survey of models of dynamic games in the exploitation of renewable and exhaustible resources. None considers exogenous growth in demand with the possibility of heterogeneity in the growth rates, as in our paper.

geneous growth in demand increases the tragedy of commons because the anticipation of higher future demand from rivalrous countries induces each country to increase present extraction.

The rest of the paper is organized as follows. In Section 2, we present the model. Section 3 provides both non-cooperative and cooperative solutions, which are then analyzed in Section 4.

2 The Model

Consider the Great Fish War (Levhari and Mirman, 1980) dynamic game in which two countries derive utility from the utilization of a common-pool resource. Let y_t be the stock of the resource at time t. In the absence of extraction, the stock evolves according to the following rule,

$$y_{t+1} = y_t^{\alpha} \tag{1}$$

where $\alpha \in (0,1]$. From (1), the evolution of the stock applies to both renewable resources (i.e., $\alpha \in (0,1)$) and depletable resources (i.e., $\alpha = 1$).

At time t, for $i = 1, 2$, country i utilizes $q_{i,t}$ units of the stock. Using (1), the evolution of the stock under exploitation is given by

$$y_{t+1} = (y_t - q_{1,t} - q_{2,t})^{\alpha} \tag{2}$$

where a total of $q_{1,t} + q_{2,t}$ is utilized at time t. For country i at time t, utilizing $q_{i,t}$ yields utility $u_i(q_{i,t}) = g_{i,t} \ln q_{i,t}$ where $g_{i,t} > 0$ reflects country i's present level of demand.[2]

In order to study the effect of exogenous and heterogeneous growth in demand on behavior, we assume that the demand parameter evolves over time. For $i = 1, 2$ and $t = 1, 2, \ldots$,

$$g_{i,t+1} = \lambda_i g_{i,t} + \theta_i \tag{3}$$

[2]In Levhari and Mirman (1980), $g_{i,t} = 1$ for all i and t.

where $\lambda_i \in [0,1)$ and $\theta_i > 0$ are country-specific parameters. Given the initial value $g_{i,0} > 0$,[3] the complete solution to (3) is

$$g_{i,t} = \lambda_i^t \left(g_{i,0} - \frac{\theta_i}{1-\lambda_i} \right) + \frac{\theta_i}{1-\lambda_i} \tag{4}$$

and the system converges asymptotically to the steady state

$$\overline{g}_i = \frac{\theta_i}{1-\lambda_i} > 0. \tag{5}$$

Hence, from (5), the difference in demand between the two countries converges asymptotically to $\left| \frac{\theta_1}{1-\lambda_1} - \frac{\theta_2}{1-\lambda_2} \right|$.

3 Non-cooperative vs. cooperation solutions

In this section, we first characterize the non-cooperative solution, i.e., the feedback-Nash equilibrium. We then provide the solution when the two countries cooperate.

Definition 3.1 states the feedback-Nash equilibrium in the infinite-horizon case.[4] To simplify notation, we drop the subscript t, and use instead a hat sign to mark the evolution over time. Specifically, g_i and $\hat{g}_i$ represent the level of demand today and tomorrow, respectively. Analogously, y and $\hat{y} = (y - q_1 - q_2)^\alpha$ are stock today and tomorrow, respectively. Let $\delta \in (0,1)$ be the discount factor. The superscript N stands for *Non-cooperation.*

[3]The restrictions on $\lambda_i \in [0,1)$ and $\theta_i > 0$ ensure that, for any $g_{i,0} > 0$, the system converges asymptotically to a positive steady state. If $\lambda_i = 1$, then the system explodes since $g_{i,t} = g_{i,0} + t\theta_i$ and $\theta_i > 0$.

[4]The conjecture can be inferred by solving the problem recursively as done in Levhari and Mirman (1980). By solving recursively, one realizes that the value function is always linear in $\ln y$. Moreover, the limit of the solution for the t-period game as t goes to infinity is the solution to the infinite-horizon game that we consider.

Definition 3.1. *The tuple* $\{Q_1^N(y, g_1, g_2), Q_2^N(y, g_2, g_1)\}$ *is a feedback-Nash equilibrium if*

1. *For* $i, j = 1, 2,\ i \neq j,$ *given* $Q_j^N(y, g_i, g_j)$,

$$Q_i^N(y, g_i, g_j) = \arg\max_{q_i} \{g_i \ln q_i + \delta V_i^N \left((y - q_i - Q_j^N(y, g_i, g_j))^\alpha, \lambda_i g_i + \theta_i, \lambda_j g_j + \theta_j\right)\} \tag{6}$$

such that $q_i \in (0, y - Q_j^N(y, g_j, g_i))$ *and where, for any* $\{y', g_i', g_j'\}$,

$$V_i^N(y', g_i', g_j') = g_i \ln Q_i^N(y', g_i', g_j') + \delta V_i^N((y' - Q_1(y', g_1', g_2') - Q_2(y', g_2', g_1'))^\alpha, \lambda_i g_i' + \theta_i, \lambda_j g_j' + \theta_j). \tag{7}$$

Proposition 3.2 presents the non-cooperative solution.

Proposition 3.2. *There exists a unique feedback-Nash equilibrium. In equilibrium, for* $i, j = 1, 2, i \neq j$,

$$Q_i^N(y, g_i, g_j) = \frac{g_i/A_i^N}{g_i/A_i^N + g_j/A_j^N + \alpha\delta} y \tag{8}$$

where

$$A_i^N \equiv \frac{\lambda_i g_i + \frac{\theta_i}{1-\alpha\delta}}{1 - \alpha\delta\lambda_i}. \tag{9}$$

Proof. See Appendix B. □

Remark 3.3. *In the non-cooperative steady state,*

$$\overline{Q}_i^N(\overline{y}^N, \overline{g}_i, \overline{g}_j) = \frac{1 - \alpha\delta}{2 - \alpha\delta} \overline{y}^N. \tag{10}$$

where $\overline{y}^N$ *is the stock value in the non-cooperative steady state, i.e.,*

$$\overline{y}^N = \frac{\alpha\delta \overline{A}_1^N \overline{A}_2^N}{\overline{g}_2 \overline{A}_1^N + \alpha\delta \overline{A}_1^N \overline{A}_2^N + \overline{g}_1 \overline{A}_2^N} \tag{11}$$

and $\overline{A}_2^N$ *is* A_2^N *evaluated at steady state.*

Having characterized the non-cooperative solution, we now turn to the case of cooperation. When countries cooperate, individual extractions is chosen so as to maximize the sum of present and future discounted utilities, i.e., $\{Q_1^C(y, g_1, g_2), Q_2^C(y, g_2, g_1)\}$ are the optimal solutions consistent with the Bellman equation

$$\begin{aligned} V^C(y, g_1, g_2) = & \max_{q_1, q_2 \in (0,y)} \{g_1 \ln q_1 + g_2 \ln q_2 \\ & + \delta V^C((y - q_1 - q_2)^\alpha, \lambda_1 g_1 + \theta_1, \lambda_2 g_2 + \theta_2)\}. \end{aligned} \tag{12}$$

Here, the superscript C stands for *Cooperation.*

Proposition 3.4 characterizes the cooperative solution.

Proposition 3.4. *From* (12)*, for* $i, j = 1, 2, i \neq j$,

$$Q_i^C(y, g_i, g_j) = \frac{g_i / A^C}{\frac{g_1 + g_2}{A^C} + \alpha\delta} y \tag{13}$$

where

$$A^C \equiv \frac{\lambda_1 g_1 + \frac{\theta_1}{1 - \alpha\delta}}{1 - \alpha\delta\lambda_1} + \frac{\lambda_2 g_2 + \frac{\theta_2}{1 - \alpha\delta}}{1 - \alpha\delta\lambda_2}. \tag{14}$$

Proof. See Appendix B. □

Remark 3.5. *In the cooperative steady state,*

$$\overline{Q}_i^C(\overline{y}^C, \overline{g}_i, \overline{g}_j) = \frac{\overline{g}_i (1 - \alpha\delta)}{\overline{g}_i + \overline{g}_j} \overline{y}^C. \tag{15}$$

where $\overline{g}_1, \overline{g}_2$ *are given by* (5) *and* $\overline{y}^C$ *is the cooperative steady state stock, i.e.,*

$$\overline{y}^C = \frac{\alpha\delta\overline{A}^C}{\overline{g}_1 + \overline{g}_2 + \alpha\delta\overline{A}^C} \tag{16}$$

and $\overline{A}^C$ *is given by* (14) *and evaluated at steady state.*

From Propositions 3.2 and 3.4, it follows that non-cooperation induces both countries to extract more than cooperation, which yields a lower level of the resource stock in the steady state. Remark 3.6 restates the tragedy of the commons in the context of heterogeneous growth in demand.

Remark 3.6. *From* (8) *and* (13),

$$Q_1^C(y, g_1, g_2) + Q_2^C(y, g_2, g_1) \leq Q_1^N(y, g_1, g_2) + Q_2^N(y, g_2, g_1). \tag{17}$$

4 Discussion

In this section, we study how noon-cooperation affects behavior in the presence of heterogeneous growth in demand. We begin by considering the intermediate case in which the level of demand is different across the two countries, i.e., $g_1 \neq g_2$, but without growth in demand. Proposition 4.1 states that in the presence of differences in demand, non-cooperation distorts the allocation of the resource between the two countries in favor of the smaller country. Specifically, the cooperative solution allocates more resource toward the largest country whereas in the non-cooperative solution, each country extracts the same amount each period, regardless of size of demand.

Proposition 4.1. *Suppose that* $\lambda_i = 1$ *and* $\theta_i = 0$*. Then,*

1. *Under cooperation, for* $i = 1, 2$,

$$Q_i^C(y, g_1, g_2) = \frac{g_i\,(1 - \alpha\delta)}{g_i + g_j} y. \tag{18}$$

2. *Under non-cooperation, for* $i = 1, 2$,

$$Q_i^N(y, g_i, g_j) = \frac{1 - \alpha\delta}{2 - \alpha\delta} y. \tag{19}$$

Proof. Evaluating (13) and (8) at $\lambda_i = 1$ and $\theta_i = 0$ yields (18) and (19), respectively. □

Having considered the intermediate case of heterogeneity in demand with no growth, we now study how the non-cooperative solution compares to the cooperative solution when there is heterogeneous growth in demand. Proposition 4.2 states that in the presence of heterogeneous growth in demand, similar to the case of no growth, the cooperative solution considers higher extraction level for the country with larger present-demand size, i.e., larger g_i. However, unlike the no-growth case, under non-cooperation, countries with heterogeneous growth in demand extract unequally. Nevertheless, the allocation of the resource between the two countries is distorted in a different way. Indeed, under non-cooperation, countries take account of growing demand (i.e., the terms A_i^N and A_j^N) for their current consumption decisions.

Proposition 4.2. *Suppose that* $\lambda_i \in [0,1)$ *and* $\theta_i > 0$*. Then,*

1. *Under cooperation, from* (13) *and* (14)*,* $Q_i^C(y, g_1, g_2) > Q_j^C(y, g_1, g_2)$ *if and only if* $g_i > g_j$ *.*

2. *Under non-cooperation, from* (8) *and* (9)*,* $Q_i^N(y, g_1, g_2) > Q_j^N(y, g_1, g_2)$ *if and only if*
$$\frac{g_i}{A_i^N} > \frac{g_j}{A_j^N}. \tag{20}$$

Proposition 4.3 provides the effect of an increase in the size of demand on the cooperative and the non-cooperative solutions. Consider first an increase in the current demand of one country (part (a)). Under cooperation, such change increases extraction for the growing country, but reduces extraction of the other country. However, under non-cooperation, both countries increase their resource extraction. Consider next an increase in the future demand one country (part (b)). Under cooperation, each country reduces present extraction to preserve the stock of resources for the future enlarged demand. Under non-cooperation, the country whose future demand size has increased, cuts current resource extraction. However, the other country increases extraction in anticipation to lower availability in future. Hence, heterogeneity in demand growth has an effect in over-exploitation of the resources and the tragedy of the commons. The reason is that, under non-cooperation, countries' competition to extract resources is exacerbated when they anticipate higher future demand from their competitor.

Proposition 4.3. *For $i, j = 1, 2$, $i \neq j$,*

1. *Under cooperation, from* (13) *and* (14),
   (a) $\frac{\partial Q_i^C(y,g_i,g_j)}{\partial g_i} > 0$, $\frac{\partial Q_j^C(y,g_i,g_j)}{\partial g_i} < 0$,
   (b) $\frac{\partial Q_i^C(y,g_i,g_j)}{\partial \hat{g}_i} < 0$, $\frac{\partial Q_j^C(y,g_i,g_j)}{\partial \hat{g}_i} < 0$.
2. *Under non-cooperation, from* (8) *and* (9),
   (a) $\frac{\partial Q_i^N(y,g_i,g_j)}{\partial g_i} > 0$, $\frac{\partial Q_j^N(y,g_i,g_j)}{\partial g_i} > 0$,
   (b) $\frac{\partial Q_i^N(y,g_i,g_j)}{\partial \hat{g}_i} < 0$, $\frac{\partial Q_j^N(y,g_i,g_j)}{\partial \hat{g}_i} > 0$.

In order to understand better the distortion resulting from non-cooperation pointed out in Proposition 4.3, we can consider each country's share of extraction. Let r_i^C and r_i^N be country i's share of extraction under cooperation and non-cooperation, respectively. Hence, from (13) and (8), for $i, j = 1, 2, i \neq j$,

$$r_i^C = \frac{Q_i^C(y, g_i, g_j)}{Q_1^C(y, g_i, g_j) + Q_2^C(y, g_i, g_j)} = \frac{g_i}{g_i + g_j}, \tag{21}$$

$$r_i^N = \frac{Q_i^N(y, g_i, g_j)}{Q_1^N(y, g_i, g_j) + Q_2^N(y, g_i, g_j)} = \frac{g_i A_j^N}{g_i A_j^N + g_j A_i^N}. \tag{22}$$

As it is presented in part 1.a of Remark 4.4, under cooperation, an increase in the demand size of a country increases his own share of extraction, and thus reduces the other country's share of extraction. These effects hold under non-cooperation as well (part 2.a). In other words, even though part 2.a of Proposition 4.3 states that, under non-cooperation, in response to an increase in the demand size of country i, country j increases its extraction, its share decreases due to the greater increase in the demand size of its rival, i.e., country i. Under the cooperative solution, an increase in the future demand size of a country does not affect the countries extractions shares today. However, under non-cooperation, the share of the country whose future demand size has increased will decrease due to the strategic reaction of its rival (parts (1-b) and (2- b) of Remark 4.4).

Remark 4.4. *For* $i, j = 1, 2$, $i \neq j$,

1. *Under cooperation, from* (21)
   (a) $\frac{\partial r_i^C}{\partial g_i} > 0$, $\frac{\partial r_j^C}{\partial g_i} < 0$,
   (b) $\frac{\partial r_i^C}{\partial \hat{g}_i} = 0$, $\frac{\partial r_j^C}{\partial \hat{g}_i} = 0$.
2. *Under non-cooperation, from 22*
   (a) $\frac{\partial r_i^N}{\partial g_i} > 0$, $\frac{\partial r_j^N}{\partial g_i} < 0$,
   (b) $\frac{\partial r_i^N}{\partial \hat{g}_i} < 0$, $\frac{\partial r_j^N}{\partial \hat{g}_i} > 0$.

A Figures

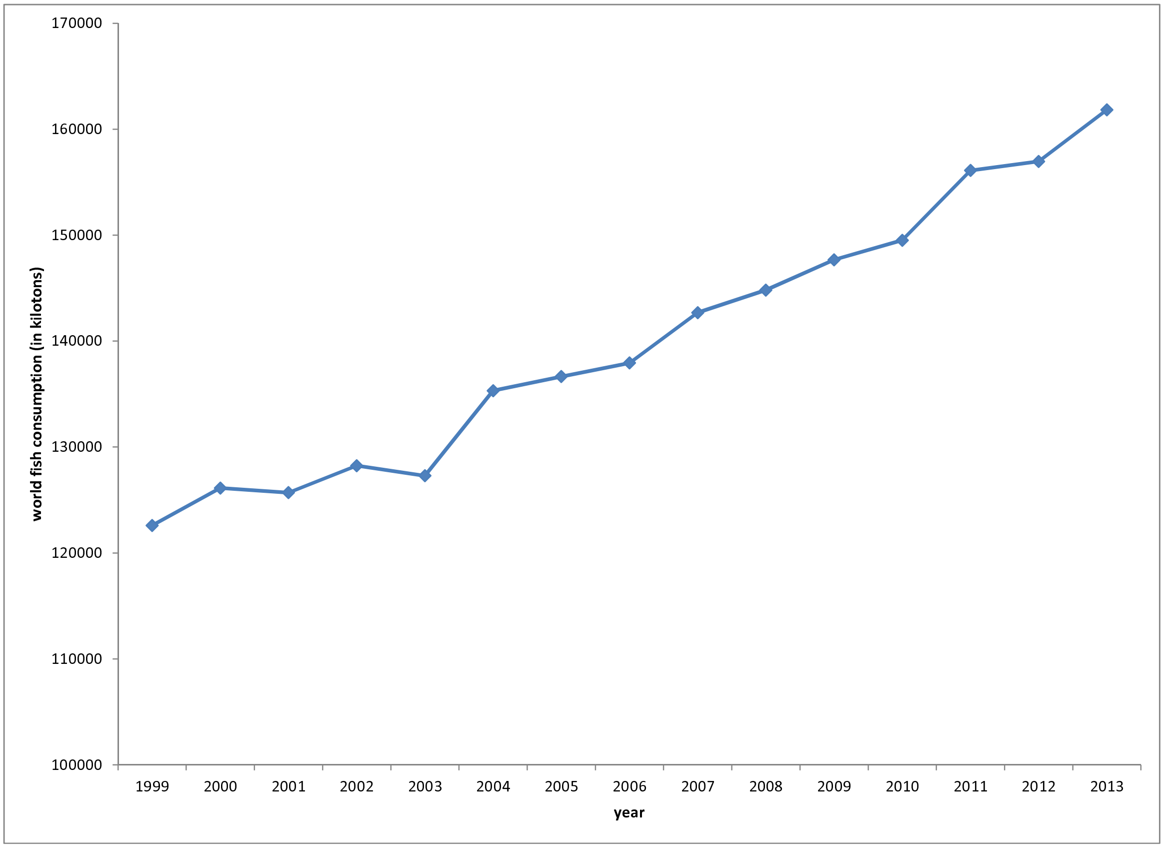

Figure 1: World Fish Consumption. Source: OECD/FAO (2013),"OECD-FAO Agricultural Outlook: Highlights 2013," OECD Agriculture Statistics (database).

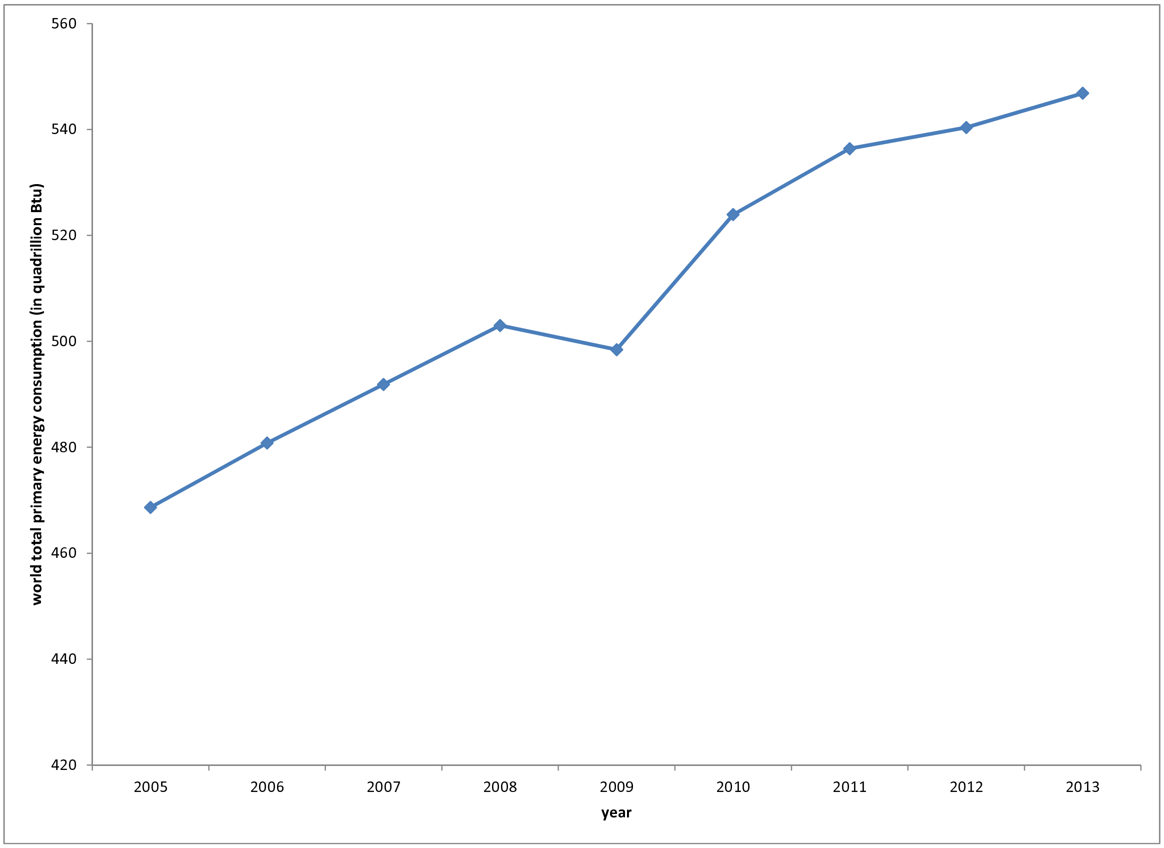

Figure 2: World Total Primary Energy Consumption. Source: U.S. Energy Information Administration (EIA); International Energy Statistics database (as of November 2012); and International Energy Agency, "Balances of OECD and Non-OECD Statistics" (2012).

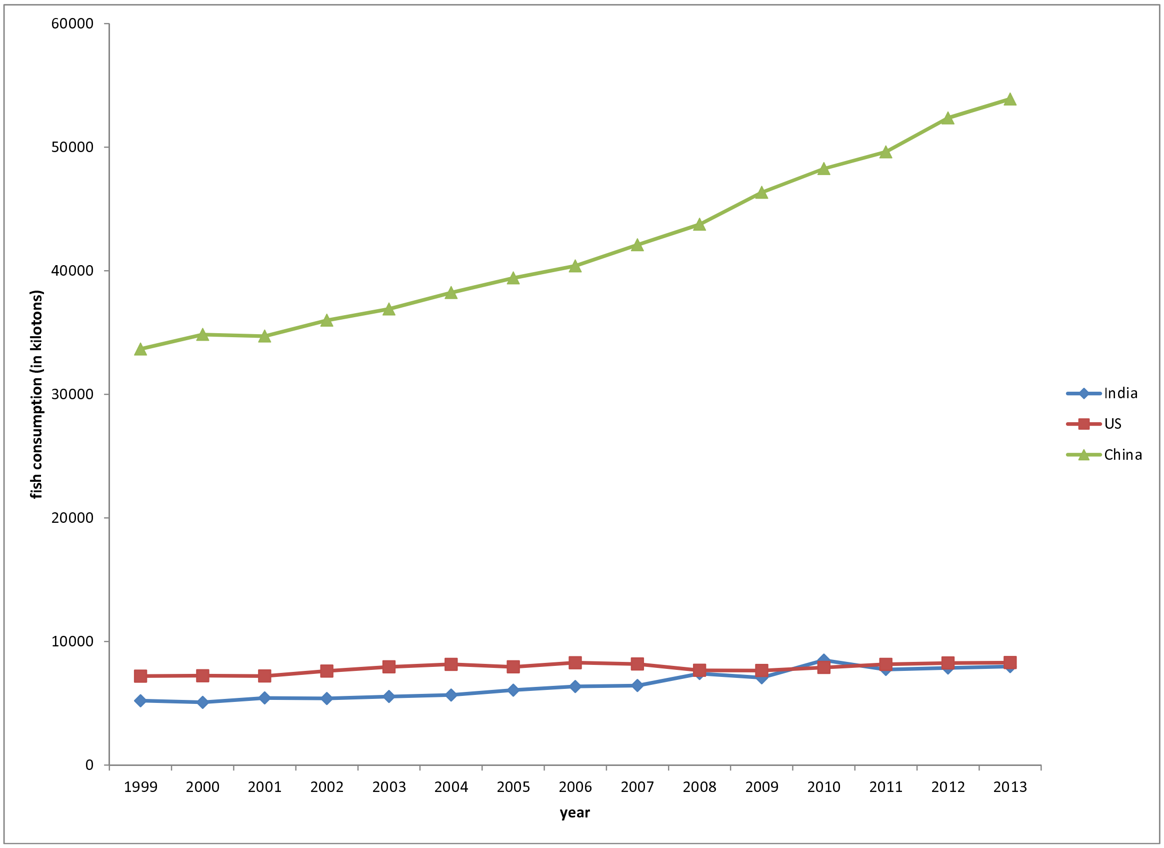

Figure 3: Fish Consumption by Country. Source: OECD/FAO (2013), "OECD-FAO Agricultural Outlook: Highlights 2013," OECD Agriculture Statistics (database).

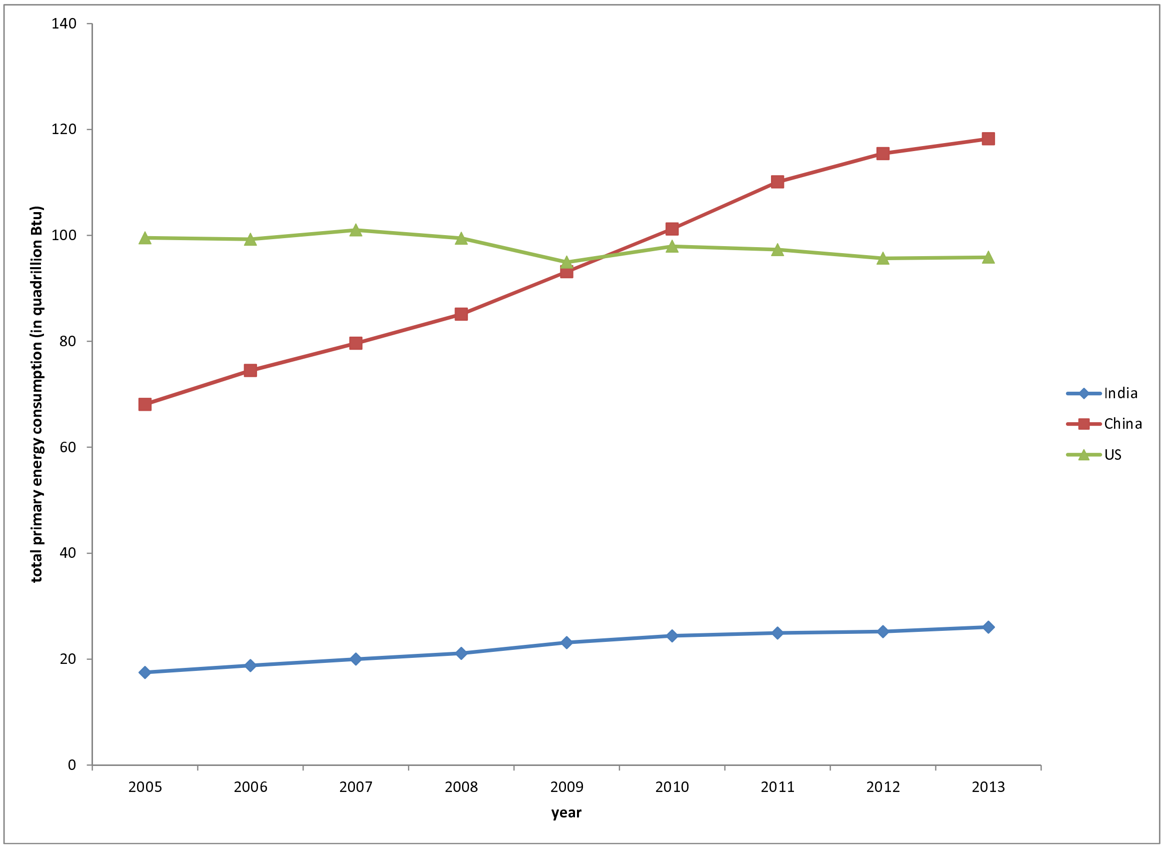

Figure 4: Total Primary Energy Consumption by Country. Source: U.S. Energy Information Administration (EIA); International Energy Statistics database (as of November 2012); and International Energy Agency, "Balances of OECD and Non-OECD Statistics" (2012).

B Proofs

Proof of Proposition 3.2. We conjecture that country i's value function has the form,

$$V_i^N(y, g_i, g_j) = (X_i^N g_i + Y_i^N) \ln y + \varphi_i^N(g_i, g_j). \tag{23}$$

Plugging (23) into the objective function of (6) yields the dynamic maximization problem

$$\max_{q_i}\{g_i \ln q_i + \delta(A_i \ln(y - q_1 - q_2)^\alpha + \varphi_i^N(\hat{g}_i, \hat{g}_j)\} \tag{24}$$

where $A_i^N = X_i^N \hat{g}_i + Y_i^N$. For $i, j = 1, 2, i \neq j$, given $q_j = Q_j^N(y, g_j, g_i)$, country i's first-order condition is

$$\frac{g_i}{q_i} = \frac{\alpha\delta A_i^N}{y - q_i - Q_j^N(y, g_j, g_i)}, \tag{25}$$

which yields $Q_i^N(y, g_i, g_j) = \omega_i^N y$ where

$$\omega_i^N = \frac{g_i/A_i^N}{g_i/A_i^N + g_j/A_j^N + \alpha\delta}. \tag{26}$$

Plugging $Q_i^N(y, g_i, g_j) = \omega_i^N y$, $A_i^N = X_i^N \hat{g}_i + Y_i^N$, and (23) into the objective function of (24) yields the value function

$$\begin{aligned} V_i^N(y, g_i, g_j) = g_i \ln \omega_i^N + g_i \ln y + \delta\alpha(X_i^N \hat{g}_i + Y_i^N) \ln(1 - \omega_i^N - \omega_j^N)) \\ + \delta\alpha(X_i^N \hat{g}_i + Y_i^N) \ln y + \varphi_i^N(\hat{g}_i, \hat{g}_j), \end{aligned} \tag{27}$$

which needs to agree with the conjecture as defined by (23), i.e.,

$$X_i^N g_i + Y_i^N = g_i + \delta\alpha\left(X_i^N(\lambda_i g_i + \theta_i) + Y_i^N\right) \tag{28}$$

and

$$\varphi_i^N(g_i, g_j) = g_i \ln \omega_i^N + \delta\alpha\left(X_i^N \hat{g}_i + Y_i^N\right) \ln(1 - \omega_i^N - \omega_j^N) + \delta\varphi_i^N(\hat{g}_i, \hat{g}_j). \tag{29}$$

Given that $\alpha\delta\lambda_i < 1$, equation (28) implies that

$$X_i^N = \frac{1}{1-\alpha\delta\lambda_i}, \tag{30}$$

$$Y_i^N = \frac{\alpha\delta\theta_i}{1-\alpha\delta}\frac{1}{1-\alpha\delta\lambda_i}, \tag{31}$$

which, using (26) and the fact that $A_i^N = X_i^N \hat{g}_i + Y_i^N$, yields (8) and (9).

Proof of Proposition 3.4. We conjecture that the cooperative value function has the form,

$$V^C(y, g_1, g_2) = \left(X_1^C g_1 + X_2^C g_2 + Y^C\right) \ln y + \varphi^C(g_1, g_2). \tag{32}$$

Plugging (23) into (12) yields

$$\begin{aligned} V^C(y, g_1, g_2) = & \max_{0<q_1,q_2<y} \{g_1 \ln q_1 + g_2 \ln q_2 \\ & + \delta(A^C \ln(y - q_1 - q_2)^\alpha + \varphi^C(\hat{g}_1, \hat{g}_2))\}. \end{aligned} \tag{33}$$

where $A^C = X_1^C \hat{g}_1 + X_2^C \hat{g}_2 + Y^C$. For $i, j = 1, 2, i \neq j$, the first-order condition for i yields

$$q_i = g_i \frac{y - q_j}{g_i + \alpha\delta A^C} \tag{34}$$

so that $Q_i^C(y, g_1, g_2) = \omega_i^C y$ where

$$\omega_i^C = \frac{g_i}{g_i + g_j + \alpha\delta A^C}. \tag{35}$$

Plugging $Q_i^C(y, g_i, g_j) = \omega_i^C y$, $A^C = X_1^C \hat{g}_1 + X_2^C \hat{g}_2 + Y^C$ and (32) into (33) yields the value function

$$\begin{aligned} V^C(y, g_1, g_2) = & (g_1 + g_2) \ln y + g_1 \ln \omega_1^C + g_2 \ln \omega_2^C \\ & + \delta\alpha \left(X_1^C \hat{g}_1 + X_2^C \hat{g}_2 + Y^C\right) \left(\ln y + \ln(1 - \omega_1^C - \omega_2^C)\right) \\ & + \delta\varphi^C(\hat{g}_1, \hat{g}_2), \end{aligned} \tag{36}$$

which needs to agree with the conjecture as defined by (32), i.e.,

$$X_1^C g_1 + X_2^C g_2 + Y^C = g_1 + g_2 + \alpha\delta \left(X_1^C \hat{g}_1 + X_2^C \hat{g}_2 + Y^C \right) \tag{37}$$

and

$$\begin{aligned} \varphi^C(g_1, g_2) = {} & g_1 \ln \omega_1^C + g_2 \ln \omega_2^C + \\ & \delta\alpha \left(X_1^C \hat{g}_1 + X_2^C \hat{g}_2 + Y^C \right) \left(\ln(1 - \omega_1^C - \omega_2^C) \right) + \delta\varphi^C(\hat{g}_1, \hat{g}_2). \end{aligned} \tag{38}$$

Solving equation (37) for X_1^C, X_2^C and Y^C yields

$$X_i^C = \frac{1}{1 - \alpha\delta\lambda_i}, \tag{39}$$

$$\begin{aligned} Y^C &= \frac{\alpha\delta}{1 - \alpha\delta} \left(X_1^{SP} \theta_1 + X_2^{SP} \theta_2 \right) \\ &= \frac{\alpha\delta}{1 - \alpha\delta} \left(\frac{\theta_1}{1 - \alpha\delta\lambda_1} + \frac{\theta_2}{1 - \alpha\delta\theta_1} \right), \end{aligned} \tag{40}$$

which, using (35) and the fact that $A^C = X_1^C \hat{g}_1 + X_2^C \hat{g}_2 + Y^C$, yields (13) and (14).

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