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Probability and Severity of Recessions [*]

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Résumé/abstract

This paper advances beyond the prediction of the probability of a recession by also considering its severity in terms of output loss and duration. First, Probit models are used to estimate the probability of a recession at period $t + h$ from the information available at period t. Next, a Vector Autoregression (VAR) augmented with diffusion indices and an inverse Mills ratio (IMR) is fitted to selected measures of real economic activity. The latter model is used to generate two forecasts: an average forecast, and a forecast under the pessimistic assumption that a recession occurs at the forecast horizon. The severity of recessions is then predicted as the gap between these two forecasts. Finally, a zero-inated Poisson model is fitted to historical durations of recessions. Our empirical results suggest that U.S. recessions are fairly predictable, both in terms of occurrence and severity. Out-of-sample experiments suggest that the inclusion of the IMR in the VAR model significantly improves its forecasting performance.

Mots clés : Duration of recessions, Forecasting Real Activity, Probability of Recessions, Probit, Vector Autoregression, Zero Inated Poisson.

Codes JEL *:* C3, C5, C35, E27, E37

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HIGHLIGHTS

1. Predicting the probability and severity of US recessions using a large data set.
2. Modeling economic activity using a vector autoregression (VAR) model augmented with diffusion indices (i.e., PCA factors) and an inverse Mills ratio (IMR).
3. Generating forecasts that are conditional on the future state of the economy (pessimistic and optimistic scenarios).
4. Predicting the depth of a recession as the gap between the forecasts associated with the pessimistic and the average scenario.
5. Predicting the duration of a recession using a zero-inflated Poisson model.
6. The inclusion of the IMR and PCA factors in the VAR model improves its out-of-sample forecast performance by up to 40%.

1 Introduction

The world economic history consists of an endless succession of business cycles characterized by swings across peaks and troughs of real economic activity. A period running between any given peak and the next trough is called a recession while a period between a trough and the next peak is an expansion. Although quite simple, this definition raises two practical issues. The first issue concerns the precise meaning of the expression "*real economic activity*". The Business Cycle Dating Committee of the National Bureau of Economic Research (NBER) does not provide a precise definition of this expression. Rather, the NBER defines a recession as "*a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales.*" The second issue concerns the identification of the peaks and troughs of real economic activity from observed data. The Business Cycle Dating Committee does provide a precise response to the latter issue by regularly publishing recession dates with six months to one year lag[1].

The general objective of this paper is to examine the predictability of economic recessions in the United States from three different perspectives. The first perspective concerns the probability of recessions, i.e., how likely is a recession to occur at a given forecast horizon. The two other perspectives pertains to the expected severity of a forthcoming recession. The severity of a recession can be measured in terms of its duration, but also in terms of its impact on output growth and unemployment rate. Each of the dimensions of a recession identified above is analyzed by combining well-established econometric methods. Thus, the contribution of our paper resides in the approach to combine simple econometric techniques to address important forecasting issues rather than in the novelty of the techniques themselves.

One strand of the literature related to this topic focuses on predicting the probability of a recession using financial indicators. For example, (Estrella & Mishkin 1998) examined

[1]The announcement dates can be found at http://www.nber.org/cycles.html.

the individual performance of financial variables such as interest rates, spreads, stock prices, and monetary aggregates in predicting the probability of a recession. They found that stock prices are good predictors of recessions at one to three quarters horizon while the slope of the yield curve emerges as a better predictor beyond one quarter horizon. (Anderson & Vahid 2001) applied nonlinear models to predict the probability of U.S. recession using the interest-rate spread and growth in M2 as leading indicators. Using (Fair 1993) definitions of a recession, they found that "*conditional on the spread, the marginal contribution of M2 growth in predicting the probability of recessions is negligible*"[2].

Another strand of the literature focuses on the computation of coincident and leading indicators of economic activity. Coincident indicators are aimed at nowcasting the current state of the economy while leading indicator are devised for forecasting future economic activity[3]. For instance, (Issler & Vahid 2006) used the information content in the NBER dates to construct a coincident and leading index of economic activity. Their coincident (leading) index is devised as a fixed-weight linear combination of coincident (leading) series. (Stock & Watson 1989) combined these two approaches to construct a recession index. An excellent overview of the literature on real time prediction of the state of the economy is provided by (Hamilton 2011).

Our paper advances one step beyond the prediction of the probability of a recession by also considering its severity. As mentioned earlier, the severity of a recession can be measured in terms of output loss or increased unemployment. Of two recessions with six months duration, the more severe is arguably the one that resulted in a higher output loss and unemployment rate. Alternatively, the severity of a recession can also be measured in terms of its duration. A recession that lasts 12 months is more harmful than a recession that lasts six months, assuming that output and jobs are destroyed at the same pace for both

[2](Fair 1993) defines a recession as either "at least two consecutive quarters of negative growth in real GDP over the next five quarters" or "at least two quarters of negative growth in real GDP over the next five quarters." This definition is not retained by the NBER.

[3]For the definition of coincident and lead economic indicators, see (Burns & Mitchell 1946) and (Stock & Watson 1989).

recessions.

In this paper, we shall not attempt to identify the business cycle turning points as done in (Chauvet 1998), (Chauvet & Hamilton 2006), (Chauvet & Piger 2008), (Stock & Watson 2010) or (Stock & Watson 2012), nor shall we try to identify the variables that lead future economic activity as in (Ng & Wright 2013). Rather, we devise a black-box that predicts the probability and severity of a recession as indicated by the NBER dates. More precisely, we employ the Principal Component Analysis to summarize a large number of candidate predictors into a few number of factors, which is in line with (Ng 2013). Identifying the candidates predictors individually is appealing in theory but difficult in practice because the transmission mechanisms might be different across recessions. Our approach circumvents this difficulty by letting the statistical method choose a linear combination of candidates predictors. The list of predictors considered includes the most widely used variables in this literature, as well as the realized volatility and skewness of the SP500 and DJIA. The principal components selected at this step are then used in subsequent models as leading indicators of real economic activity.

We design an empirical framework in three steps for analyzing the NBER business cycles. The first step relies on a Probit model that predict the probability of a recession. Regressors observed on a quarterly basis are used to predict the probability of a recession up to two years ahead. The second step uses a Vector Autoregression (VAR) model to predict future economic activity. Rather than constructing a single index of real economic activity, we consider predicting different measures of economic activity jointly. The estimating equation from our second step is a VAR augmented with PCA factors (i.e., diffusion indices) that summarize a large number of candidate predictors plus an inverse Mills ratio (IMR) deduced from the Probit. The inclusion of the IMR allows us to generate forecasts that are conditional on the future state of the economy. As a result, the model is capable of generating an optimistic forecast (assuming that an expansion occurs at the forecast horizon) as well as a pessimistic forecast (assuming that a recession occurs at the forecast horizon). Our IMR augmented VAR framework provides a simple approach to incorporate qualitative information in a standard VAR model and as such, it is reminiscent of the (Dueker & Wesche 2010)'s Qual VAR approach to forecasting macro variables. The third step of the analysis is concerned with the prediction of the severity of a recession in terms of its duration. In real life, the NBER recession durations are known only ex-post. However, historical data can be used to investigate the ex-ante predictability of the duration of recessions. We construct the duration variable N_t as the number of recessionary periods lying ahead at time t. We fit a zero-inflated Poisson model to this variable and use the output to construct an estimate of the conditional expected duration of a recession, which combines the probability and the (unconditional) expected duration of a recession into a single index.

We apply our methodology to U.S. macroeconomic and financial series. Our results support that U.S. recessions are predictable to a great extent, both in terms of occurrence and severity. Recession dates are reasonably well predicted up to 5 quarters ahead. The model is capable of correctly predicting 40% of recession dates at up to 2 years horizon while delivering a very low percentage of bad shots. The unemployment rate, employment growth, GDP deflator inflation, and industrial production growth are predictable at quite long horizons. GDP growth and SP500 returns appear to be less predictable. Interestingly, the Great Recession of 2007-2009 has been more severe than our pessimistic forecast. Also, the SP500 has outperformed our optimistic forecast during the period that preceded the Great Recession (approx. 2003-2007).

We perform an out-of-sample analysis to assess the extrapolation capabilities of the proposed empirical framework. The inclusion of the IMR and diffusion indices in the VAR model reduces the out-of-sample mean square forecast error by up to 38% for measures of real activity (GDP, Industrial production and Employment growth, and Unemployment rate) and by 41% for GDP Deflator inflation.

The remainder of the paper is organized as follows. Section 2 presents our framework. Section 3 presents our empirical application and Section 4 summarizes our findings. Estimation outputs and graphs are shown in the appendix.

2 The Framework

This section presents the three-step empirical framework that we employ to analyze the U.S. recession episodes.

2.1 Probability of Recessions

Let R_t be a variable such that $R_t = 1$ if the NBER Dating Committee identifies period t as a recession time and $R_t = 0$ otherwise. We would like to use a large number of economic indicators gathered in a N-dimensional vector X_t to predict recessions. Ideally, X_t should contain real economic activity as well as financial activity indicators, and coincident as well as leading indicators. The candidate predictors may be partially redundant or highly correlated (e.g., GDP deflator versus CPI inflation, or SP500 versus DJIA), but they should all be observable at time t or a few periods before time $t+h$, where h is the forecast horizon. In order to reduce the dimensionality of X_t and by the same token avoid multicolinearity issues, we consider summarizing X_t into a smaller number (q) of principal components F_t. By abuse of language, we refer to F_t as factors although we do not pretend that X_t obeys a formal factor model. We interpret each factor F_t by examining the three variables to which it correlates with the most.

To fix ideas, we assume that the data are observed on a quarterly basis. We augment F_t with a constant variable (so that subsequently, $F_t \in \mathbb{R}^{q+1}$) and assume the existence of a latent leading index $Z_{h,t}$ such that:

$$Z_{h,t} = \gamma_h F_t + u_{h,t}, \text{ for all } t \tag{1}$$

where $u_{h,t} \sim N(0, 1)$ for all $h = 1, 2, ...$ and h is a forecast horizon. The latent leading index

$Z_{h,t}$ predicts the state of the economy h periods ahead such that:

$$R_{t+h} = \begin{cases} 1 & \text{if } Z_{h,t} > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

This probabilistic approach has been used by (Estrella & Mishkin 1998) to identify lead indicators of U.S. recession at horizons ranging from 1 to 8 quarters. Hence, it allows us to predict the probability of a recession in h periods as:

$$\Pr\left(R_{t+h} = 1|X_t\right) = \Phi\left(\gamma_h F_t\right), \text{ for all } \ h, \tag{3}$$

where Φ is the CDF of the standard normal distribution. The predicted probability of recession may be interpreted as a recession index, as suggested by (Stock & Watson 1989). Model (1)-(2) can be estimated by Probit based on historical data. If release lags exist, the Probit model above can still be used for forecasting purposes as long as the release lags are shorter enough than the horizon h.

2.2 Severity of a Recession: an IMR-Augmented DI-VAR Approach

In order to assess the severity of a recession, we consider the Diffusion Index VAR model (DI-VAR) of (Stock & Watson 2002) as starting point:

$$y_{i,t+h} = \alpha_{i,h} Y_t + \beta_{i,h} F_t + \delta_{i,h,1} R_{t+h} + v_{i,t+h}, t = 1, ..., T-h \tag{4}$$

where $y_{i,t}$, $i = 1, \ldots, M$, the i^{th} coordinate of Y_t, is a measure of economic activity and $v_{i,t+h} \sim N(0, \sigma^2_{i,h})$ is uncorrelated with F_t and Y_t. As a reminder, we recall that F_t includes a constant variable and R_{t+h} is the indicator of recession at $t+h$.

The error term $v_{i,t+h}$ is allowed to be correlated with the future state of the economy. In order to avoid the resulting endogeneity problem, we consider taking the expectation y_{t+h} conditional on the information set at time t, that is:

$$E\left(y_{i,t+h}|Y_t, X_t\right) = \alpha_{i,h}Y_t + \beta_{i,h}F_t + \delta_{i,h,1}\Phi\left(\gamma_h F_t\right) \equiv \widehat{y}_{t+h}. \tag{5}$$

This allows us to represent y_{t+h} as follows:

$$y_{i,t+h} = \alpha_{i,h}Y_t + \beta_{i,h}F_t + \delta_{i,h,1}\Phi\left(\gamma_h F_t\right) + \widetilde{v}_{i,t+h}, \tag{6}$$

where $\widetilde{v}_{i,t+h} \equiv v_{i,t+h} + R_{t+h} - \Phi\left(F_t\gamma_h\right)$.

Beside the average scenario forecast provided by (5), two other forecasts can be constructed. The first forecast is based on the pessimistic assumption that we will effectively witness a recession at period $t+h$, i.e. :

$$E\left(y_{i,t+h}|Y_t, X_t, R_{t+h}=1\right) = \alpha_{i,h}Y_t + \beta_{i,h}F_t + \delta_{i,h,1} + \delta_{i,h,2}\frac{\phi\left(\gamma_h F_t\right)}{\Phi\left(\gamma_h F_t\right)} = \underline{y}_{i,t+h}. \tag{7}$$

This expression is obtained by assuming that $(u_{h,t}, \widetilde{v}_{i,t+h})$ are jointly Gaussian, where $u_{h,t}$ is the error term of the latent variable governing the corresponding Probit. The second forecasting formula is based on the optimistic assumption that there will be an expansion at period $t+h$:

$$E\left(y_{i,t+h}|Y_t, X_t, R_{t+h}=0\right) = \alpha_{i,h}Y_t + \beta_{i,h}F_t - \delta_{i,h,2}\frac{\phi\left(F_t\gamma_h\right)}{1-\Phi\left(\gamma_h F_t\right)} = \overline{y}_{i,t+h}, \tag{8}$$

The estimating equation that allows us to identify δ is:

$$y_{i,t+h} = \alpha_{i,h}Y_t + \beta_{i,h}F_t + \delta_{i,h,1}\Phi\left(\gamma_h F_t\right) + \delta_{i,h,2}IMR_{t,h} + \widetilde{\widetilde{v}}_{i,t+h}, \tag{9}$$

where $IMR_{t,h} = \frac{\phi(\gamma_h F_t)}{\Phi(\gamma_h F_t)}$ if $R_{t+h}=0$, and $IMR_{t,h} = \frac{-\phi(\gamma_h F_t)}{1-\Phi(\gamma_h F_t)}$ if $R_{t+h}=1$ is the inverse Mills ratio, and $\widetilde{\widetilde{v}}_{i,t+h} \equiv \widetilde{v}_{i,t+h} - E(\widetilde{v}_{i,t+h}|Y_t, X_t, R_{t+h})$. We shall estimate Equation (9) to identify the parameters and use Equations (5), (7) and (8) for forecasting purposes.

The parameters $\delta_{i,h,1}$ and $\delta_{i,h,2}$ are both expected to be negative if $y_{i,t}$ is cyclical and they should be positive otherwise. For instance, if $y_{i,t}$ denotes the GDP growth, the term $\delta_{i,h,1}\Phi(\gamma_h F_t) + \delta_{i,h,2} IMR_{t,h}$ is expected to be negative as it represents the expected output loss during a recession. However, if $y_{i,t}$ is the unemployment rate, the term $\delta_{i,h,1}\Phi(\gamma_h F_t) + \delta_{i,h,2} IMR_{t,h}$ is expected to be positive as it measures the rise in unemployment due to recession.

2.3 Severity of a Recession: a Duration Approach

An alternative approach to gauge the severity of a recession consists of converting the indicator variable R_t into count data[4]. Based on historical data, it is possible to tell for any month t if there had been a recession at month $t+1$. Thus, let N_t be an indicator variable such that $N_t = 0$ if there is an expansion at period $t+1$ and $N_t > 0$ otherwise. If a recession starts at period $t+1$, then we let $N_t = r$, where r is the duration of that particular recession as indicated by historical data. Afterward, we let N_{t+h} denote the number of recession periods to go starting from $t+h+1$. Hence, for a recession that starts at $t+1$ and ends at $t+r$, we have:

$$N_{t+h} = r - h, \text{ for } h = 0, ..., r, \tag{10}$$

Accordingly, an expansion episode starts at period $t+r+1$. Hence,

$$N_{t+h} = 0 \text{ for } r \leq h \leq r + e - 1,$$

where e is the duration of the expansion episode.

Our idea is to fit a zero-inflated Poisson model to the count process N_t. By noting that

[4](Watson 1994) has investigated several explanations for the postwar duration stability. Here, our focus is to forecast the duration.

$R_{t+1} = 0$ if and only if $N_t = 0$ and $R_{t+1} = 1$ otherwise, we have:

$$\begin{aligned}
\Pr(N_t = 0) &= 1 - \Phi(\gamma_1 F_t) \text{ and} && (11)\\
\Pr(N_t = n | X_t, N_t \geq 1) &= \frac{\exp(-\lambda_t)}{1 - \exp(-\lambda_t)} \frac{\lambda_t^n}{n!}, n \geq 1, && (12)
\end{aligned}$$

where $\Phi(\gamma_1 F_t) \equiv \Pr(R_{t+1} = 1 | X_t)$, $\lambda_t = \exp\left(\theta \tilde{F}_t\right)$ is the time-varying parameter of the Poisson model and $\tilde{F}_t$ is a subset of F_t. $\Phi(\gamma_1 F_t)$ is already obtained from the Probit model of Section 2.1. Hence, it only remains to estimate (12) using strictly positive realizations of N_t. The sample on which the estimation of θ relies is much shorter than the original sample. Thus, we shall not use too many predictors in model (12) for the sake of degrees-of-freedom. Interestingly, our multi-step estimation procedure allows us to use different number of factors at each step: more factors in the Probit model and less factors in the Poisson model.

Equation (12) gives the probability of the next n consecutive periods belonging to a recessionary episode under the pessimistic assumption that recession is unavoidable at the next period. If no a priori assumption is made, the probability of a recession is computed as:

$$\Pr(N_t = n | X_t) = \frac{\Phi(\gamma_1 F_t) \exp(-\lambda_t)}{1 - \exp(-\lambda_t)} \frac{\lambda_t^n}{n!}, n \geq 1, \quad (13)$$

At any period t, the expected duration of a recession can be calculated under the pessimistic assumption that recession is unavoidable. This leads to the conditional expected duration given by

$$E(N_t | X_t, N_t \geq 1) = \frac{\lambda_t}{1 - \exp(-\lambda_t)}. \quad (14)$$

The *unconditional* expected duration of a recession is given by:

$$E(N_t | X_t) = \frac{\Phi(\gamma_1 F_t) \lambda_t}{1 - \exp(-\lambda_t)} \equiv \overline{N}_t. \quad (15)$$

The probability $\Phi(\gamma_1 F_t)$ is the recession index proposed by (Stock & Watson 1989). Our indicator $\overline{N}_t$ goes one step further in that it combines the probability of a recession with its expected severity as measured by the conditional expected duration.

One may consider undertaking the counterfactual exercise which consists of calculating $E(N_t|X_t, N_t \geq 1)$ for periods where $N_t = 0$ has actually been observed. However, this exercise tends to predict exaggerately high conditional durations at periods where $N_t = 0$. This happens because:

$$E(N_t|X_t, N_t \geq 1) = \frac{E(N_t|X_t)}{\Phi(\gamma_1 F_t)}.$$

As an estimator of the probability of a recession one period ahead, $\Phi(\gamma_1 F_t)$ exploits very imminent information when the data are quaterly. Hence, its tends to be very small compared to $E(N_t|X_t)$ when no recession is actually going to occur at quater $t+1$. In our empirical application, (14) is computed only when N_t is strictly positive while (15) is constructed for the all sample.

3 Application to NBER Recession

For this application, we use the quarterly NBER recession indicator from the FRED2 database. The time series included in X_t as candidate predictors are presented in Table 9. The time span starts in 1967Q2 and ends in 2012Q3[5]. We perform a full sample analysis as well as an out-of-sample exercise to assess the forecasting abilities of our methodology.

3.1 Full Sample Analysis

The first subsection below presents the results of the principal component analysis of X_t. The second subsection is devoted to the prediction of NBER recessions using a Probit model. The third subsection presents the estimation results of our IMR-augmented diffusion index VAR (IMR-DI-VAR) model while the fourth subsection presents the results for the zero-inflated

[5]We start only in 1967Q2 since one of important leading indicators, that is often used to assess the probability of a recession, Initial Claims (IC4WSA) is available from that date. We stop at 2012Q3 because of the availability of the Consumer Sentiment (ConsMICH) computed by the University of Michigan.

Poisson model.

3.1.1 Principal Component Analysis

We employ the Principal Component Analysis to reduce the dimensionality of X_t. Subsequently, F_t consists of 16 principal components of X_t. [6] Table 1 contains the three series with which each principal component is correlated the most.

The first factor (F_1) is mainly related to employment growth and external finance premium. The second factor (F_2) is in line with inflation and FED funds rate. The third factor (F_3) is correlated with consumption and stock prices. Echoing (Hamilton 2011), (Ng & Wright 2013) noted that "*the track record of forecasting models using asset prices is not good, or at least not consistent.*" However, the fact that stock prices greatly contribute to the formation of F_3 suggests that robust predictors of the state of the economy can be obtained by taking their linear combinations with other variables. The fourth factor (F_4) captures money growth as measured by M1 and unemployment rate. The fifth factor (F_5) is mostly correlated with money growth as measured by M2 and stock market volatility. The sixth factor (F_6) combines GS5 and stock market returns' skewness. The oil price growth is highly correlated with F_7, along with house starts and construction permits. The factor F_8 recombines again the M2 growth and the stock market skewness. The unemployment rate in the mining sector and the GS1-FFR spread appear to be strongly correlated with F_9. F_{10} is mainly correlated with M1 growth, unemployment rate in the mining sector and also the investment growth rate.

The remaining factors are mainly recombinations of the variables mentioned above. Hence, selecting the 16 most important principal components ensures us that we bring the most relevant part of the information content of X_t to our forecasting models. It is interesting to note that the stock market returns, volatility and skewness are related to different factors, namely, F_3 , F_5 and F_6 respectively. Likewise, the monetary aggregates M1 and M2

[6]At this step, the number 16 is selected in order to obtain a good in-sample fit. Later on, the choice of the number of principal components to include in F_t is based on out-of-sample performance criteria.

are related to different factors, the most important ones being F_4 and F_5 respectively.

Table 1: Most correlated variables with PCs

F_1	MANEMPgr	BAA-GS10	PAYEMSgr
F_2	CPIAUCSLinfl	FEDFUNDS	CPILFELinfl
F_3	RPCEgr	DJIAret	SP500ret
F_4	NAPMPRI	M1SLgr	UNRATE
F_5	M2SLgr	SP500-RV	DJIA-RV
F_6	GS5	DJIA-SK	SP500-SK
F_7	OILPRICEgr	PERMIT	HOUST
F_8	M2SLgr	SP500-SK	DJIA-SK
F_9	GS1-FFR	NAPMSDI	USMINEgr
F_{10}	M1SLgr	INVESTgr	USMINEgr
F_{11}	SP500ret	OILPRICEgr	INVESTgr
F_{12}	RPCEgr	INVESTgr	RPCEDGgr
F_{13}	M2SLgr	DJIAret	DJIA-RV
F_{14}	INVESTgr	M2SLgr	OILPRICEgr
F_{15}	AWOTMAN	USMINEgr	M1SLgr
F_{16}	PCEPIgr	CPIAUCSLinfl	USMINEgr

3.1.2 Predicting the Probability of a Recession

Table 2 presents the in-sample goodness-of-fit of the Probit model that predicts the probability of a recession. Detailed estimation results are presented in Table 10 of the Appendix. The goodness-of-fit measures considered are McFadden's pseudo R^2, Estrella's R^2, the percentage of good shots (calling for a recession when $R_t = 1$) and the percentage of bad shots (calling for a recession when $R_t = 0$).

Table 2: Predicting NBER recessions: in full-sample goodness-of-fit

Quarter	$h = 1$	$h = 2$	$h = 3$	$h = 4$	$h = 5$	$h = 6$	$h = 7$	$h = 8$
McFadden pseudo-R2	0,753	0,619	0,614	0,657	0,474	0,390	0,346	0,395
Estrella R2	0,692	0,558	0,554	0,598	0,422	0,345	0,306	0,352
% of good shots	0,778	0,667	0,815	0,778	0,630	0,407	0,296	0,407
% of bad shots	0,026	0,033	0,020	0,020	0,027	0,027	0,047	0,034

We call for a recession when the estimated probability is higher than 0.5. The forecasting horizons considered are $h = 1$ through $h = 8$ quarters. Recession dates are reasonably well predicted up to 5 quarters ahead. The model is capable of correctly predicting 40% of recessions dates at up to 2 years horizon while delivering a very low percentage of bad shots (see Figure 1).

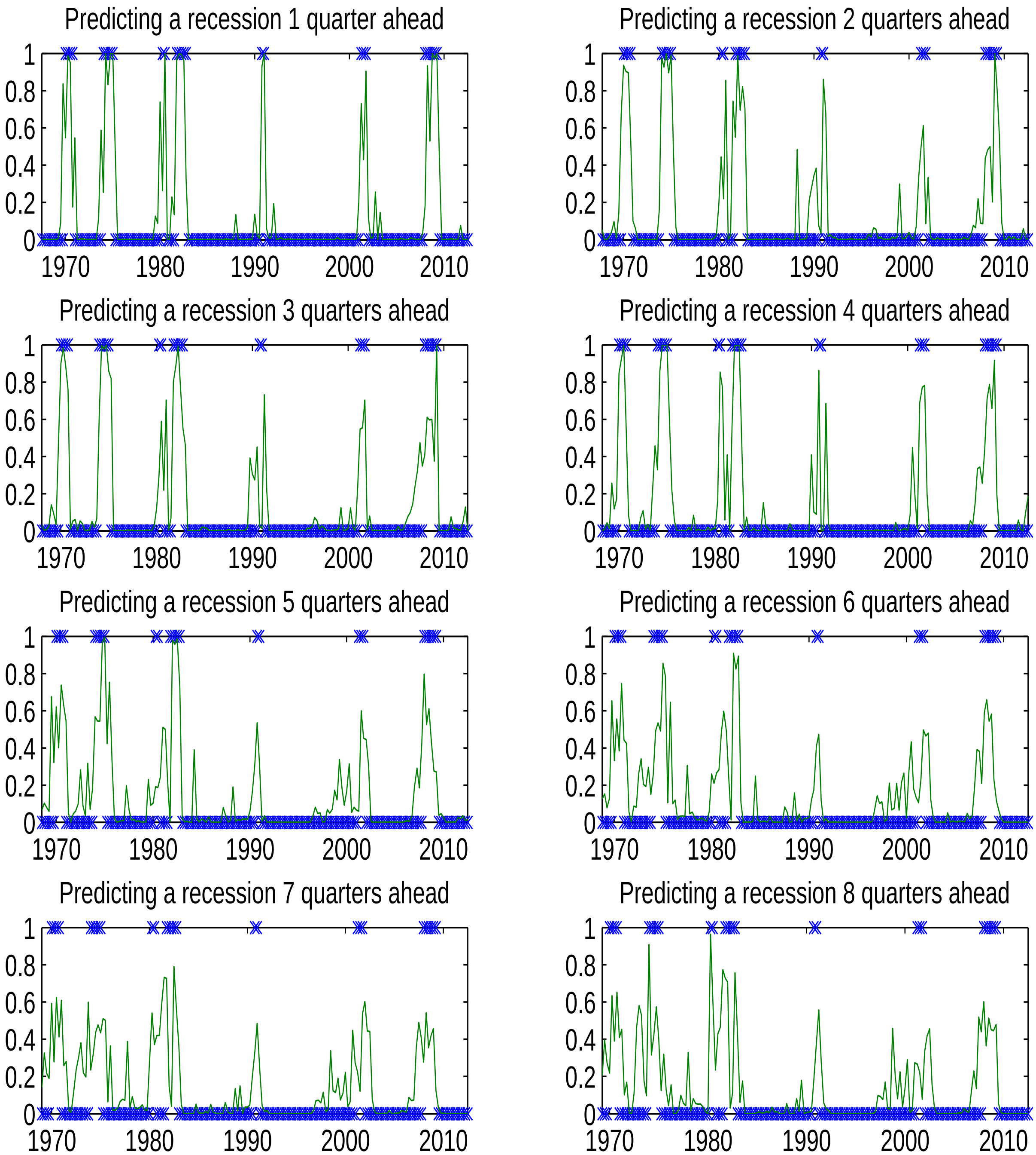
3.1.3 Predicting the Severity of a Recession using the IMR-DI-VAR

We fit the predictive equation (9) to the data by SURE. The variables included in Y_t are: real GDP growth rate (GDP growth), industrial production growth rate (INDPRO growth), unemployment rate (UNRATE), employment growth rate (EMPL growth), GDP Deflator growth rate (GDPDEF inflation) and SP500 returns. The first four variables measure the real activity while the last two variables are proxies for the price level and the financial activity. Table 3 presents the R^2s of the regressions for each of the measures of economic activity. At one year horizon, the most predictable variable is the unemployment rate (92.6%), followed by employment growth (79.4%), GDP deflator inflation (77.9%) and industrial production growth (66.8%). The R^2 of the prediction of GDP growth and SP500 returns one year ahead are 45.9% and 21.1% respectively. Figures 11 and 12 in appendix plot the actual and predicted values of all six series at horizons $h = 4$ and $h = 8$.

Table 3: Predicting economic activity: R^2 from DI-VAR predictive regressions

	1 quarter	2 quarters	3 quarters	4 quarters
GDP growth	0,574	0,523	0,494	0,459
INDPRO growth	0,704	0,638	0,621	0,668
UNRATE	0,989	0,972	0,950	0,926
EMPL growth	0,877	0,825	0,805	0,794
GDPDEF inflation	0,847	0,821	0,817	0,779
SP500 returns	0,395	0,309	0,279	0,211
	5 quarters	6 quarters	7 quarters	8 quarters
GDP growth	0,451	0,440	0,433	0,448
INDPRO growth	0,631	0,592	0,579	0,595
UNRATE	0,906	0,869	0,823	0,765
EMPL growth	0,766	0,718	0,678	0,657
GDPDEF inflation	0,754	0,729	0,711	0,700
SP500 returns	0,202	0,188	0,207	0,160

Figure 1: Predicting US recessions



Notes: Predicted in-sample probabilities of NBER recessions from Probit models.

We also compute the *"optimistic"* and *"pessimistic"* scenarios according to equations (7) and (8), respectively. Figure 2 presents the graphs for $h = 2$. The graphs for horizons $h = 1$,

$h = 4$, and $h = 8$ are shown in the Appendix. For illustrative purposes, we focus on the period 2000-2012 which covers the last two recessions. The Great Recession of 2007-2009 has been more severe than our pessimistic forecast. It is also interesting to note that the financial market has outperformed our optimistic forecast during the period that preceded that recession (approx. 2003-2007). This result suggests perhaps a simple methodology to identify financial bubbles, which consists of examining the gap between the actual value of the market index and its most optimistic forecast based on real activity measures.

Figure 3 presents the estimated cost of recessions at horizon $h = 2$, as defined in the paragraph following Equation (9).[7] The drop in output (GDP and industrial production) growth predicted by the model for the Great Recession appears to be smaller than the one predicted for the 2001 downturn. Likewise, the rise in unemployment rate predicted for the Great Recession is smaller than in 2001. However, Figure 2 shows that the Great Recession has been more severe. This suggest that the severity of the latter recession is less predictable than that of the preceding recession.

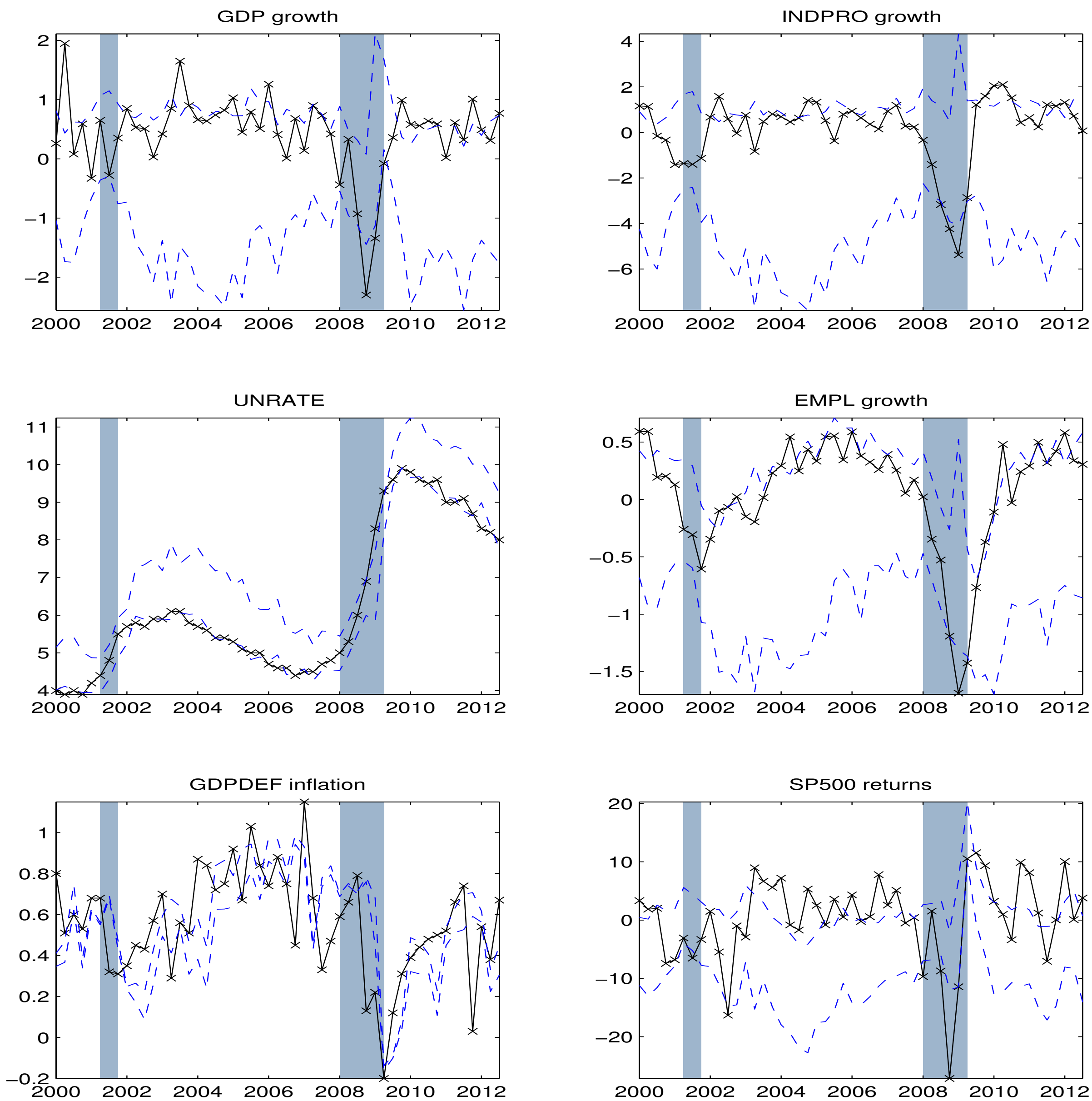
3.1.4 Predicting the Severity of a Recession using the Duration Approach

We now present the estimation results for the duration approach. As discussed in Section 2.3, we fit an inflated Poisson to the NBER durations. The estimation is done by setting $\lambda_t = \exp\left(\theta \tilde{F}_t\right)$ where $\tilde{F}_t$ consists of the first three principal components and a constant. We use much less factors than in the Probit because the sample contains only 27 observations with strictly positive duration (i.e. we only have 27 recessionary dates in the sample). The expected duration of a recession conditional on the event $R_{t+1} = 1$ is $E\left(N_t | X_t, N_t \geq 1\right)$. The expected duration can also be calculated unconditionally, that is, as $E\left(N_t | X_t\right)$. Although both expectations are conditional on X_t, we refer subsequently to $E\left(N_t | X_t, N_t \geq 1\right)$ as the "conditional expected duration" and to $E\left(N_t | X_t\right)$ as simply the "expected duration."

Note that $N_t \geq 1$ when $R_{t+1} = 1$. Consequently, $E\left(N_t | X_t, N_t \geq 1\right)$ is always larger than

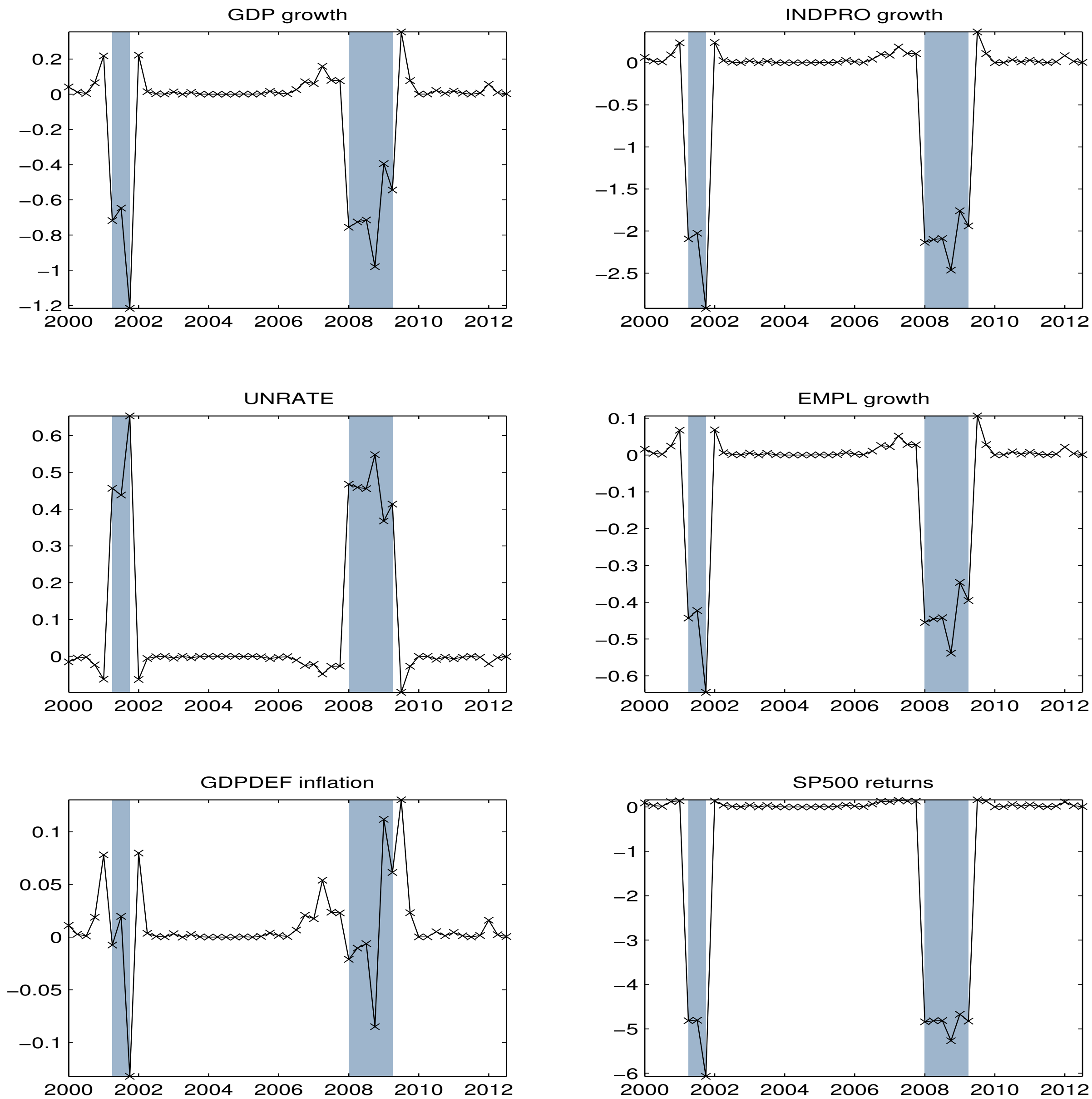
[7] See the Appendix for horizons $h = 1$, $h = 4$ and $h = 8$.

Figure 2: Predicting US economic activity: 2-quarters ahead scenarios



Notes: Predicted in-sample optimistic and pessimistic scenarios from DI-VAR forecasting models.

Figure 3: Predicting US economic activity: 2-quarters ahead loss due to recession

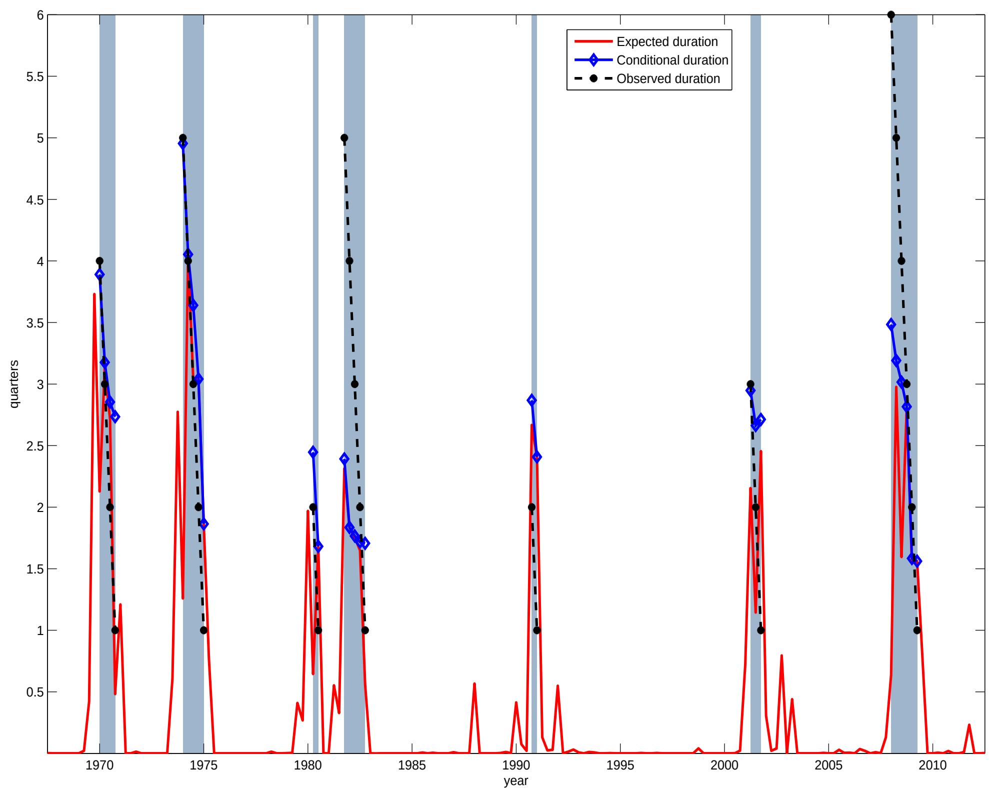


Notes: Predicted in-sample losses during recessions in US economic activity from DI-VAR forecasting models.

one. The expected duration is equal to the probability of a recession times the conditional expected duration. Figure 4 plots the expected duration and conditional expected duration as defined in equations (15) and (14) respectively, as well as the observed NBER duration.

Recall that the expected duration is calculated for the whole sample while the conditional expected duration is computed only for recession periods. The conditional expected duration better approximates the actual duration, particularly at the beginning of the recessionary episodes. The MSE of the conditional expected duration is 1.38 while the MSE of the expected duration is 3.01. This suggests that the conditional expected duration is more informative than its unconditional version when a recession is imminent.

Figure 4: Observed versus fitted NBER recession durations



Notes: We fit an inflated Poisson to NBER durations. The dashed asterisk black line is the observed NBER duration. The solid diamond blue line is the expected duration conditional on the next period belonging to a recessionary episode. Finally, the solid red line is the expected duration computed for the all sample. The MSE of conditional expected duration is 1.38 while the MSE of expected duration is 3.01.

3.2 Out-of-Sample Analysis

We perform an out-of-sample analysis aimed at assessing the extrapolation capabilities of the proposed empirical framework. For this purpose, we compare the IMR-DI-VAR model to alternative specifications. The competing models retained are:

- Our **IMR-DI-VAR** model, which is the multivariate version of (9):

$$Y_{t+h} = \alpha_h Y_t + \beta_h F_t + \delta_{h,1}\Phi\left(\gamma_h F_t\right) + \delta_{h,2} IMR_{t,h} + \varepsilon_{t+h}$$

- The **DI-VAR** model, an extension of the univariate diffusion indices model of (Stock & Watson 2002), that is a IMR-DI-VAR from which the fitted probability of recession and IMR are omitted:

$$Y_{t+h} = \alpha_h Y_t + \beta_h F_t + \varepsilon_{t+h}$$

- The **IMR-VAR**, which is another version of the IMR-DI-VAR without the factors:

$$Y_{t+h} = \alpha_h Y_t + \delta_{h,1}\Phi\left(\gamma_h F_t\right) + \delta_{h,2} IMR_{t,h} + \varepsilon_{t+h}$$

- And finally, the standard **VAR**:

$$Y_{t+h} = \alpha_h Y_t + \varepsilon_{t+h}.$$

The out-of-sample evaluation period is 2000Q1 - 2012Q3, which leaves us with 131 periods for model training, and 51 periods for forecasting evaluation. We use an expanding window scheme by re-estimating the model as a new quarter is added to the training period. The forecasting horizon is set at 4 quarters.

We consider selecting the number of factors (q_1) to include in the Probit independently from the number of factors (q_2) to include in the prediction of Y_{t+h}. Moreover, q_1 and q_2 are selected independently for each of the models above and for each target variable in Y_t. Note that only q_1 matters for IMR-VAR model while only q_2 matters for DI-VAR. We choose q_1 and q_2 that minimize the out-of-sample MSE.

The results are reported in Table (4). Interestingly, the IMR-DI-VAR model delivers the smallest out-of-sample MSE for all targets except the SP500 returns. The latter variable is better predicted by the IMR-VAR model. We perform Diebold-Mariano tests for the purpose of judging whether the observed differences in out-of-sample MSEs truly reflect different forecasting abilities across models. Table (5) presents the p-values computed for pairwise tests for the null case that two given models deliver identical forecasts. As the four models being compared are nested, we follow (Alquist & Kilian 2010) by adjusting the p-values as in (Clark & West 2006). We also compare our competing models to a random walk (RW). We find that the forecasts from all four models are mutually different and are also different from the predictions of a RW[8].

Table 4: Out-of-sample exercise: MSE results with expanding window

	GDP Growth			IP Growth		
	MSE	q_1	q_2	MSE	q_1	q_2
IMR-DI-VAR	**0.5077**	**15**	**4**	**1.9781**	**15**	**12**
DI-VAR	0.5555	.	18	2.3524	.	4
IMR-VAR	0.5305	15	.	2.1390	15	.
VAR	0.6708	.	.	2.6786	.	.
RW	0.8834	.	.	5.1295	.	.
	Unemployment Rate			Employment Growth		
	MSE	q_1	q_2	MSE	q_1	q_2
IMR-DI-VAR	**0.4826**	**13**	**17**	**0.1670**	**15**	**12**
DI-VAR	0.5272	.	17	0.2061	.	18
IMR-VAR	0.6174	9	.	0.2071	15	.
VAR	0.7729	.	.	0.2701	.	.
RW	1.4688	.	.	0.3772	.	.
	GDPD Inflation			SP500 Returns		
	MSE	q_1	q_2	MSE	q_1	q_2
IMR-DI-VAR	**0.0775**	**2**	**3**	52.3303	16	1
DI-VAR	0.1303	.	18	57.1233	.	3
IMR-VAR	0.0993	2	.	**51.2833**	**16**	.
VAR	0.1326	.	.	56.8302	.	.
RW	0.0999	.	.	107.5507	.	.

These empirical results suggest four important conclusions. First, the IMR is an important lead indicator of economic activity; its inclusion as the predictor in a VAR model permits the significant reduction of the out-of-sample error. In particular, adding only the IMR to the VAR improves the forecasting precision of the GDP growth rate one year ahead by 20% (ratio of IMR-VAR and VAR MSEs), while considering the full IMR-DI-VAR model reduces

[8] As discussed in (Diebold 2012), these tests do not compare models but the forecasts.

Table 5: Out-of-sample exercise: p-values for testing the equal predictive accuracy with expanding window

	GDP Growth				IP Growth			
	IMR-DI-VAR	DI-VAR	IMR-VAR	VAR	IMR-DI-VAR	DI-VAR	IMR-VAR	VAR
DI-VAR	0.0000	.	-	-	0.0000	.	-	-
IMR-VAR	0.0011	0.0000	.	-	0.0001	0.0010	.	-
VAR	0.0063	0.0000	0.0022	.	0.0003	0.0008	0.0033	.
RW	0.0208	0.0113	0.0275	0.0156	0.0171	0.0254	0.0166	0.0394
	Unemployment Rate				Employment Growth			
	IMR-DI-VAR	DI-VAR	IMR-VAR	VAR	IMR-DI-VAR	DI-VAR	IMR-VAR	VAR
DI-VAR	0.0000	.	-	-	0.0000	.	-	-
IMR-VAR	0.0011	0.0000	.	-	0.0001	0.0010	.	-
VAR	0.0063	0.0000	0.0022	.	0.0003	0.0008	0.0033	.
RW	0.0208	0.0113	0.0275	0.0156	0.0171	0.0254	0.0166	0.0394
	GDPD Inflation				SP500 Returns			
	IMR-DI-VAR	DI-VAR	IMR-VAR	VAR	IMR-DI-VAR	DI-VAR	IMR-VAR	VAR
DI-VAR	0.0000	.	-	-	0.0000	.	-	-
IMR-VAR	0.0011	0.0000	.	-	0.0001	0.0010	.	-
VAR	0.0063	0.0000	0.0022	.	0.0003	0.0008	0.0033	.
RW	0.0208	0.0113	0.0275	0.0156	0.0171	0.0254	0.0166	0.0394

Note: All p-values refer to a pairwise test of equal forecasts obtained from two models. Following (Alquist & Kilian 2010), we produce adjusted p-values based on (Clark & West 2006).

the MSE by 25%. In case of IP growth, Unemployment rate, and Employment growth, the improvements of IMR-DI-VAR specifications are 24%, 37%, and 38%, respectively. Second, including the IMR and principal components is especially important to forecast the inflation rate, which is a variable very difficult to predict, see discussion in (Faust & Wright 2012). Not only the IMR-DI-VAR improves over the VAR model by 41%, but it reduces the MSE with respect to the Random Walk by 22%. Third, the SP500 returns do not seem to be linearly predictable by the PCA factors. In fact, the inclusion of these factors as predictors in a VAR model increases the out-of-sample MSE. The best forecasts are delivered by the IMR-VAR model which reduces the MSE by 10% compared to the VAR. Finally, note that the out-of-sample evaluation period covers the biggest financial crisis and the global economic downturn since the Great Depression. Hence, the flexibility of IMR-DI-VAR approach in including information from different sources to a VAR is an important advantage.

To check the robustness of the results, we repeat the out-of-sample exercise but now using a rolling window scheme (as opposed to an expanding window). As shown by Table

(6), the IMR-DI-VAR model has the smallest out-of-sample tracking error for unemployment and inflation rates, as well as for the SP500 returns, while the IMR-VAR model is more successful at tracking the other targets. However, the performances of these two models are quite close.

We also consider using the mean absolute error (MAE) as loss function (as opposed to the MSE). This choice of loss function implicitly assumes that the quantity being tracked is the conditional median of the target while the models have been trained to correctly track its conditional mean. The results for the expanding window are shown in Table (7) while those for the rolling window are shown in Table (8). The results suggest that the IMR-DI-VAR model has superior performance at predicting all six targets under expanding windows. This model also dominates the other benchmarks under the rolling window scheme, except for the employment growth. The latter variable is better predicted by the IMR-VAR model. Overall, this robustness check exercise confirms the relevance of including the IMR (and possibly, diffusion indices) as predictor in VAR models. Our results can be related to those of (Dueker 2005) and (Dueker & Wesche 2010), who show that including information about a qualitative variable (i.e., the indicator of NBER recession) in a standard VAR improves the forecasts of the system.

4 Conclusion

We propose an empirical framework for analyzing the probability and severity of U.S. recessions in a data rich environment. The severity of a recession is measured in two dimensions: its depth (impact on economic activity) and its length (duration). We consider a framework where a large number of candidate predictors of economic activity is available to the econometrician. The range of candidate predictors considered includes financial spreads, consumption expenditures, monetary aggregates, and measures of stock market performance. This large number of predictors is summarized into a fewer number of principal components

Table 6: Out-of-sample exercise: MSE results with rolling window

	GDP Growth			IP Growth		
	MSE	q_1	q_2	MSE	q_1	q_2
IMR-DI-VAR	0.5047	15	1	2.1556	16	1
DI-VAR	0.5887	.	3	2.4212	.	1
IMR-VAR	**0.4937**	**17**	.	**2.0529**	**16**	.
VAR	0.6651	.	.	2.4744	.	.
RW	0.8834	.	.	5.1295	.	.
	Unemployment Rate			Employment Growth		
	MSE	q_1	q_2	MSE	q_1	q_2
IMR-DI-VAR	**0.5192**	**16**	**6**	0.1757	16	1
DI-VAR	0.5768	.	18	0.2312	.	1
IMR-VAR	0.5522	4	.	**0.1601**	**16**	.
VAR	0.7288	.	.	0.2376	.	.
RW	1.4688	.	.	0.3772	.	.
	GDPD Inflation			SP500 Returns		
	MSE	q_1	q_2	MSE	q_1	q_2
IMR-DI-VAR	**0.0817**	**2**	**3**	**53.6018**	**17**	**3**
DI-VAR	0.1104	.	1	60.8804	.	6
IMR-VAR	0.1040	2	.	55.3633	17	.
VAR	0.1120	.	.	59.4742	.	.
RW	0.0999	.	.	107.5507	.	.

Table 7: Out-of-sample exercise: MAE results with expanding window

	GDP Growth			IP Growth		
	MAE	q_1	q_2	MAE	q_1	q_2
IMR-DI-VAR	**0.5306**	**1**	**4**	**0.9568**	**1**	**4**
DI-VAR	0.5425	.	18	1.0679	.	1
IMR-VAR	0.5340	1	.	1.0432	1	.
VAR	0.6140	.	.	1.1503	.	.
RW	0.6914	.	.	1.5361	.	.
	Unemployment Rate			Employment Growth		
	MAE	q_1	q_2	MAE	q_1	q_2
IMR-DI-VAR	**0.4286**	**9**	**1**	**0.3055**	**1**	**18**
DI-VAR	0.4905	.	4	0.3286	.	12
IMR-VAR	0.4793	9	.	0.3134	10	.
VAR	0.6080	.	.	0.3587	.	.
RW	0.8216	.	.	0.4333	.	.
	GDPD Inflation			SP500 Returns		
	MAE	q_1	q_2	MAE	q_1	q_2
IMR-DI-VAR	**0.2232**	**2**	**2**	**5.1764**	**17**	**3**
DI-VAR	0.2748	.	17	5.5699	.	3
IMR-VAR	0.2520	2	.	5.1915	16	.
VAR	0.2875	.	.	5.6465	.	.
RW	0.2639	.	.	7.5608	.	.

Table 8: Out-of-sample exercise: MAE results with rolling window

	GDP Growth			IP Growth		
	MAE	q_1	q_2	MAE	q_1	q_2
IMR-DI-VAR	**0.5057**	**1**	**4**	1.0110	1	1
DI-VAR	0.5627	.	1	**1.0891**	.	**1**
IMR-VAR	0.5125	1	.	1.0166	1	.
VAR	0.5968	.	.	1.1193	.	.
RW	0.6914	.	.	1.5361	.	.
	Unemployment Rate			Employment Growth		
	MAE	q_1	q_2	MAE	q_1	q_2
IMR-DI-VAR	**0.4477**	**3**	**1**	0.3064	1	1
DI-VAR	0.4968	.	3	0.3376	.	1
IMR-VAR	0.4971	7	.	**0.3023**	**1**	.
VAR	0.6215	.	.	0.3421	.	.
RW	0.8216	.	.	0.4333	.	.
	GDPD Inflation			SP500 Returns		
	MAE	q_1	q_2	MAE	q_1	q_2
IMR-DI-VAR	**0.2242**	**2**	**2**	**5.3618**	**17**	**3**
DI-VAR	0.2670	.	1	5.5288	.	6
IMR-VAR	0.2615	4	.	5.5019	16	.
VAR	0.2669	.	.	5.7361	.	.
RW	0.2639	.	.	7.5608	.	.

or factors. Each factor is interpreted by examining the variables with which it is correlated the most. Among others, we find that the first factor is related to employment growth and external finance premium, the second factor is correlated with inflation and FED fund rate, and the third factor captures the movements of consumption and stock prices. This step provides a summary description of the data while eluding the multicolinearity problems that would have resulted from including highly correlated variables in the information set (e.g., GDP deflator and CPI inflation).

To analyze the probability of recessions, we advocate Probit models in which the information available at time t (as summarized by the factors) conditions the indicator of NBER recession at horizon $t + h$, where h varies between 1 and 8 quarters. Our in-sample analysis suggests that recession dates are fairly predictable at up to 5 quarters horizon. Second, we fit an IMR-DI-VAR model to the GDP growth, industrial production growth, unemployment rate, employment growth, inflation rate, and SP500 returns. The IMR-DI-VAR model is obtained by augmenting the standard diffusion index vector autoregression model of (Stock & Watson 2002) by an Inverse Mills Ratio deduced from the Probit models. This model allows us to construct an average forecast scenario as well as an optimistic and a pessimistic forecast scenarios. The severity of a recession is predicted as the gap between the forecasts associated with the pessimistic and the average scenarios. Third, we fit a zero-inflated Poisson model to the duration of NBER recessions. The estimation output is used to construct an estimate of the expected duration of a recession.

We perform an out-of sample experiment aimed at optimally selecting the number of factors to include in the analysis. The optimal numbers of factors to use in the Probit model and in the IMR-DI-VAR model are assumed to be distinct. We compare the IMR-DI-VAR model to three alternative specifications: the standard DI-VAR, the IMR-VAR (a VAR augmented with an IMR), and a standard VAR model. The results suggest that the IMR-DI-VAR has the best out-of-sample forecast performance for all target variables under an expanding window scheme and a quadratic loss function. Particularly, important findings include a reduction of up to 38% in MSE when forecasting several measures of real activity, and 41% in the case of GDP Deflator inflation. Under alternative schemes, the IMR-VAR outperforms the IMR-DI-VAR for certain targets (e.g., SP500 returns). The main lesson learned from this study is that the IMR conveys valuable information about future states of the economy.

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Appendix A: Description of the data

The transformation codes are: 1 - no transformation; 2 - first difference; 4 - logarithm; 5 - first difference of logarithm; 0 - variable not used in the estimation (only used for transforming other variables). A * indicate a series that is deflated by the Personal Consumption Expenditures: Chain-type Price Index. GDP and GDPDEF are observed quarterly and are not in X_t.

Table 9: Data used to construct the diffusion indices

INDPROgr	5	Industrial Production Index
UNRATE	1	Civilian Unemployment Rate
PAYEMSgr	5	All Employees: Total nonfarm
MANEMPgr	5	All Employees: Manufacturing
USMINEgr	5	All Employees: Mining and logging
IC4WSA	4	4-Week Moving Average of Initial Claims
RPCEgr	5*	Real Personal Consumption Expenditures
RPCEDGgr	5*	Real Personal Consumption Expenditures: Durable Goods
AWHMAN	1	Average Weekly Hours: Manufacturing
AWOTMAN	1	Average Weekly Overtime Hours: Manufacturing
NAPM	1	ISM Manufacturing: PMI Composite Index
NAPMOI	1	ISM Manufacturing: New Orders Index
NAPMEI	1	ISM Manufacturing: Employment Index
NAPMII	1	ISM Manufacturing: Inventories Index
NAPMSDI	1	ISM Manufacturing: Supplier Deliveries Index
NAPMPRI	1	ISM Manufacturing: Prices Index
CUmftg	1	Capacity Utilization: Manufacturing
CPILFEL	5	Consumer Price Index for All Urban Consumers: All Items
CPIAUCSL	5	Consumer Price Index for All Urban Consumers: All Items Less Food & Energy
PCEPI	5	Personal Consumption Expenditures: Chain-type Price Index
M1SL	5	M1 Money Stock
M2SL	5	M2 Money Stock
HOUST	4	Housing Starts: Total: New Privately Owned Housing Units Started
PERMIT	4	New Private Housing Units Authorized by Building Permits
ConsMICH	1	University of Michigan: Consumer Sentiment
OILPRICEgr	5	Spot Oil Price: West Texas Intermediate
FFR	1	Effective Federal Funds Rate
INVEST	5	Securities in Bank Credit at All Commercial Banks
TB3MS	1	3-Month Treasury Bill: Secondary Market Rate
GS1	0	1-Year Treasury Constant Maturity Rate
GS5	1	5-Year Treasury Constant Maturity Rate
GS10	0	10-Year Treasury Constant Maturity Rate
SP500	5	S&P 500 Stock Price Index
DJIA	5	Dow Jones Industrial Average
BAA	0	Moodyś Seasoned Baa Corporate Bond Yield
AAA	0	Moodyś Seasoned Aaa Corporate Bond Yield
BAA-GS10	1	
BAA-AAA	1	
BAA-FFR	1	
GS10-TB3MS	1	
GS5-FFR	1	
GS1-FFR	1	
SP500-RV	1	S&P500: realized volatility
SP500-SK	1	S&P500: realized skewness
DJIA-RV	1	DJIA: realized volatility
DJIA-SK	1	DJIA: realized skewness
GDP	5	Real Gross Domestic Product
GDPDEF	5	GDP Deflator

Appendix B: Estimation results

Table 10: Predicting NBER recessions: full-sample results

	1 quarter ahead		2 quarters ahead		3 quarters ahead		4 quarters ahead	
	coefficients	p-value	coefficients	p-value	coefficients	p-value	coefficients	p-value
const	-2,934	0,000	-2,183	0,000	-2,208	0,000	-3,054	0,000
F_1	-0,328	0,001	-0,055	0,305	0,247	0,096	0,524	0,004
F_2	0,594	0,000	0,393	0,000	0,386	0,000	0,518	0,000
F_3	-0,714	0,001	-0,531	0,000	-0,330	0,029	-0,342	0,087
F_4	0,065	0,707	0,183	0,147	0,097	0,421	-0,010	0,951
F_5	-0,164	0,376	-0,097	0,509	-0,408	0,155	-0,281	0,395
F_6	-0,154	0,464	-0,128	0,456	-0,184	0,541	0,558	0,078
F_7	0,539	0,032	-0,131	0,494	0,098	0,687	-0,237	0,321
F_8	-0,082	0,753	-0,269	0,210	-0,580	0,062	-0,363	0,172
F_9	0,016	0,952	-0,238	0,290	-0,362	0,190	-0,227	0,422
F_{10}	0,430	0,142	-0,018	0,928	-0,047	0,858	-0,234	0,497
F_{11}	0,225	0,323	0,308	0,173	0,466	0,064	0,848	0,011
F_{12}	-0,231	0,495	-0,079	0,767	0,349	0,254	0,013	0,966
F_{13}	-0,371	0,249	0,225	0,319	-0,369	0,367	-1,278	0,033
F_{14}	0,281	0,395	0,107	0,705	-0,210	0,491	-1,402	0,001
F_{15}	0,342	0,447	0,206	0,566	-0,178	0,640	-0,440	0,277
F_{16}	0,731	0,078	0,985	0,008	1,126	0,006	1,701	0,001

	5 quarters ahead		6 quarters ahead		7 quarters ahead		8 quarters ahead	
	coefficients	p-value	coefficients	p-value	coefficients	p-value	coefficients	p-value
const	-1,953	0,000	-2,035	0,000	-2,277	0,000	-2,685	0,000
F_1	0,208	0,004	0,280	0,008	0,386	0,002	0,594	0,000
F_2	0,265	0,000	0,249	0,003	0,285	0,001	0,296	0,002
F_3	-0,143	0,121	0,096	0,573	0,155	0,396	0,467	0,051
F_4	0,035	0,706	0,259	0,086	0,340	0,044	0,521	0,012
F_5	0,124	0,366	0,089	0,748	0,311	0,258	0,107	0,727
F_6	0,370	0,024	0,588	0,010	0,415	0,091	0,340	0,312
F_7	-0,586	0,002	-0,283	0,186	0,128	0,563	0,334	0,186
F_8	-0,410	0,018	-0,169	0,436	-0,165	0,492	-0,353	0,297
F_9	-0,135	0,378	0,019	0,907	0,153	0,363	0,302	0,196
F_{10}	-0,392	0,045	-0,211	0,387	-0,502	0,036	-0,576	0,041
F_{11}	0,055	0,762	0,214	0,297	0,076	0,695	0,076	0,717
F_{12}	0,545	0,016	0,520	0,028	0,593	0,013	0,637	0,016
F_{13}	-0,158	0,517	-0,191	0,573	0,166	0,600	0,041	0,911
F_{14}	-0,369	0,152	-0,347	0,143	-0,361	0,116	-0,256	0,276
F_{15}	-0,626	0,033	-0,698	0,016	-0,753	0,009	-0,576	0,063
F_{16}	0,260	0,290	-0,191	0,457	-0,344	0,191	-0,093	0,735

Table 11: Predicting economic activity 4 quarters ahead: estimation results

	GDP growth		INDPRO growth		UNRATE	
	coefficients	p-value	coefficients	p-value	coefficients	p-value
const	-1,878	0,061	-5,632	0,000	6,728	0,000
$F_{1,t}$	0,026	0,760	-0,097	0,436	-0,272	0,000
$F_{2,t}$	0,054	0,349	-0,001	0,989	0,121	0,003
$F_{3,t}$	-0,117	0,292	-0,317	0,047	0,163	0,039
$F_{4,t}$	0,283	0,007	0,659	0,000	-0,453	0,000
$F_{5,t}$	0,149	0,115	0,235	0,085	-0,292	0,000
$F_{6,t}$	0,057	0,498	0,149	0,217	-0,205	0,001
$F_{7,t}$	0,124	0,157	0,261	0,038	-0,275	0,000
$F_{8,t}$	-0,105	0,100	-0,346	0,000	0,176	0,000
$F_{9,t}$	0,027	0,720	0,068	0,539	-0,189	0,001
$F_{10,t}$	0,153	0,061	0,337	0,004	-0,070	0,228
$F_{11,t}$	-0,020	0,831	-0,092	0,500	0,181	0,007
$F_{12,t}$	0,112	0,305	0,176	0,265	0,020	0,799
$F_{13,t}$	0,206	0,093	0,338	0,056	-0,094	0,280
$F_{14,t}$	-0,082	0,305	0,093	0,418	0,058	0,308
$F_{15,t}$	-0,025	0,752	-0,235	0,039	0,123	0,029
$F_{16,t}$	0,000	0,998	-0,065	0,661	0,117	0,107
$y_{1,t}$	0,080	0,491	0,139	0,405	-0,094	0,255
$y_{2,t}$	0,249	0,018	0,448	0,003	-0,031	0,675
$y_{3,t}$	0,402	0,011	0,855	0,000	-0,053	0,639
$y_{4,t}$	-0,150	0,713	0,469	0,426	-0,226	0,437
$y_{5,t}$	0,150	0,556	0,733	0,046	0,063	0,728
$y_{6,t}$	-0,011	0,827	0,004	0,953	-0,028	0,431
$\Phi(F_t\gamma_h)$	-1,297	0,000	-2,244	0,000	0,346	0,108
IMR_t	-0,647	0,000	-1,463	0,000	0,386	0,000

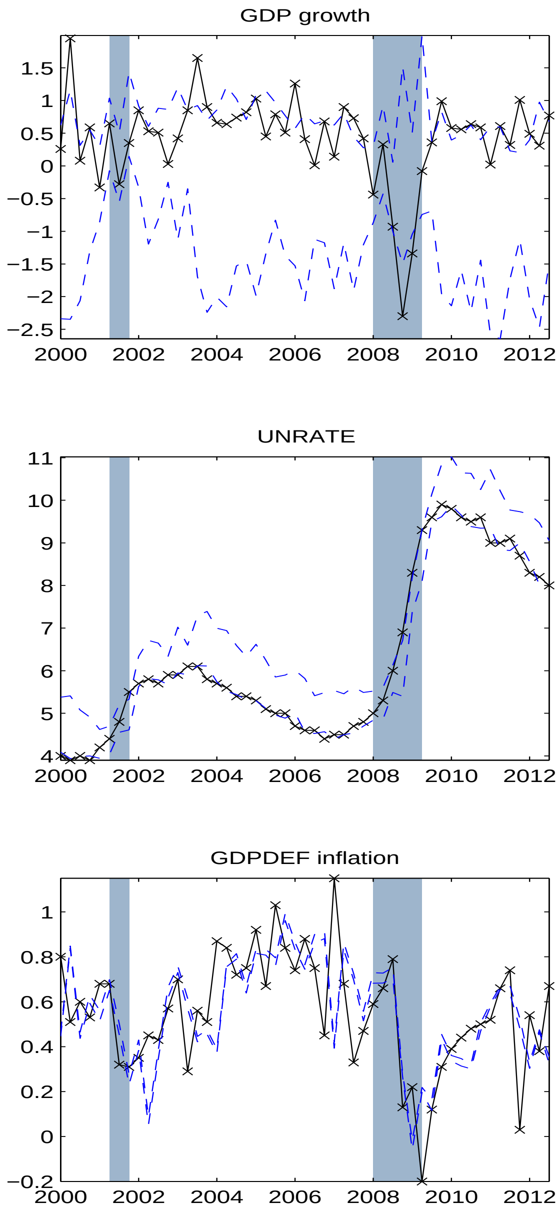
	EMPL growth		GDPDEF inflation		S&P500 returns	
	coefficients	p-value	coefficients	p-value	coefficients	p-value
const	-3,093	0,000	-0,217	0,639	-6,158	0,503
$F_{1,t}$	0,082	0,020	0,030	0,452	0,674	0,397
$F_{2,t}$	0,016	0,509	0,120	0,000	0,238	0,654
$F_{3,t}$	-0,158	0,000	-0,085	0,095	-0,771	0,448
$F_{4,t}$	0,257	0,000	-0,052	0,288	0,508	0,599
$F_{5,t}$	0,185	0,000	0,110	0,011	0,349	0,688
$F_{6,t}$	0,096	0,005	0,108	0,005	-0,850	0,268
$F_{7,t}$	0,175	0,000	0,129	0,001	0,520	0,517
$F_{8,t}$	-0,118	0,000	-0,061	0,040	0,474	0,421
$F_{9,t}$	0,107	0,001	0,075	0,034	0,331	0,637
$F_{10,t}$	0,176	0,000	0,107	0,005	-0,417	0,577
$F_{11,t}$	-0,071	0,065	0,018	0,684	-0,239	0,784
$F_{12,t}$	0,014	0,757	-0,048	0,340	-0,168	0,867
$F_{13,t}$	0,139	0,005	0,055	0,332	0,919	0,416
$F_{14,t}$	-0,009	0,778	0,059	0,107	-0,472	0,519
$F_{15,t}$	-0,098	0,002	-0,027	0,460	-0,293	0,686
$F_{16,t}$	0,008	0,847	-0,051	0,281	0,998	0,288
$y_{1,t}$	0,071	0,132	0,005	0,926	0,105	0,921
$y_{2,t}$	0,096	0,024	0,047	0,329	-0,809	0,401
$y_{3,t}$	0,515	0,000	0,116	0,114	1,448	0,321
$y_{4,t}$	0,274	0,098	0,261	0,166	-0,963	0,798
$y_{5,t}$	0,150	0,148	0,229	0,052	0,728	0,756
$y_{6,t}$	0,023	0,265	0,037	0,111	0,108	0,814
$\Phi(F_t\gamma_h)$	-0,703	0,000	0,135	0,334	-8,032	0,004
IMR_t	-0,365	0,000	-0,089	0,154	0,210	0,866

Table 12: Predicting economic activity 8 quarters ahead: estimation results

	GDP growth		INDPRO growth		UNRATE	
	coefficients	p-value	coefficients	p-value	coefficients	p-value
const	0,902	0,376	0,012	0,994	10,553	0,000
$F_{1,t}$	0,121	0,169	0,162	0,246	-0,107	0,323
$F_{2,t}$	-0,018	0,764	0,057	0,551	0,122	0,095
$F_{3,t}$	0,189	0,098	-0,017	0,926	0,255	0,068
$F_{4,t}$	0,080	0,458	0,080	0,641	-0,780	0,000
$F_{5,t}$	-0,064	0,510	0,000	0,998	-0,419	0,000
$F_{6,t}$	0,067	0,423	-0,016	0,905	-0,183	0,075
$F_{7,t}$	-0,093	0,311	0,055	0,704	-0,721	0,000
$F_{8,t}$	0,012	0,860	-0,063	0,557	0,208	0,012
$F_{9,t}$	-0,060	0,438	0,165	0,178	-0,324	0,001
$F_{10,t}$	-0,107	0,199	0,010	0,938	-0,228	0,025
$F_{11,t}$	-0,088	0,367	-0,136	0,379	0,093	0,437
$F_{12,t}$	0,000	1,000	-0,090	0,611	0,045	0,744
$F_{13,t}$	0,143	0,256	0,064	0,747	-0,367	0,017
$F_{14,t}$	-0,164	0,055	-0,110	0,415	-0,117	0,266
$F_{15,t}$	-0,007	0,932	-0,105	0,433	0,119	0,251
$F_{16,t}$	0,168	0,094	0,324	0,042	-0,039	0,751
$y_{1,t}$	-0,018	0,877	-0,140	0,452	0,155	0,283
$y_{2,t}$	-0,024	0,829	-0,117	0,504	-0,314	0,020
$y_{3,t}$	0,050	0,757	0,220	0,386	-0,583	0,003
$y_{4,t}$	-0,573	0,174	-0,457	0,494	-1,091	0,034
$y_{5,t}$	0,106	0,681	0,042	0,919	-0,015	0,963
$y_{6,t}$	-0,067	0,191	0,005	0,953	-0,042	0,507
$\Phi(F_t\gamma_h)$	-1,795	0,000	-3,482	0,000	0,638	0,176
IMR_t	-0,731	0,000	-1,691	0,000	0,376	0,003

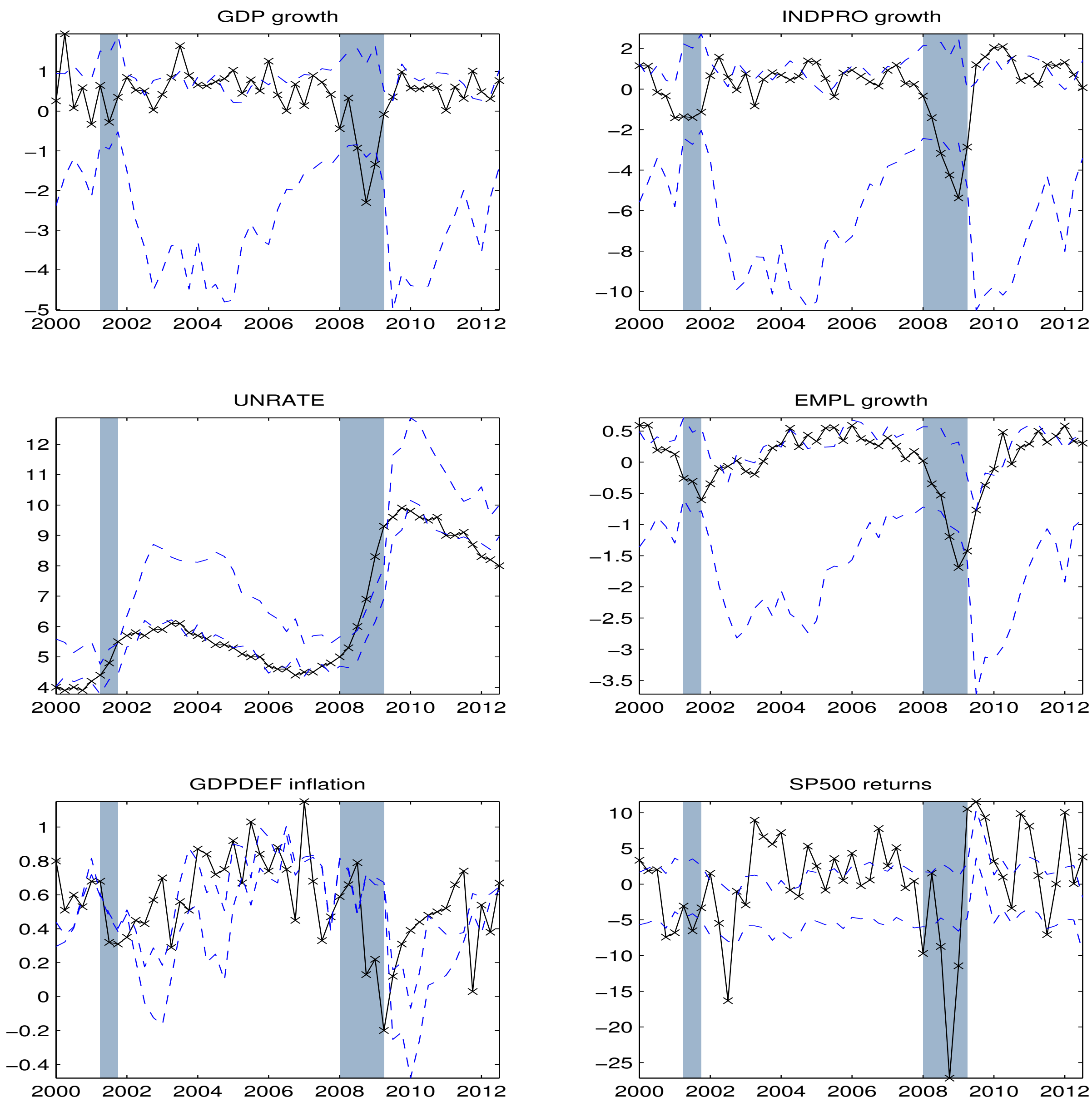
	EMPL growth		GDPDEF inflation		S&P500 returns	
	coefficients	p-value	coefficients	p-value	coefficients	p-value
const	-0,999	0,058	-0,407	0,456	3,939	0,681
$F_{1,t}$	0,003	0,942	0,085	0,073	0,007	0,994
$F_{2,t}$	0,024	0,443	0,051	0,113	0,356	0,527
$F_{3,t}$	-0,002	0,971	-0,036	0,558	-0,323	0,763
$F_{4,t}$	0,114	0,040	-0,052	0,370	-0,497	0,625
$F_{5,t}$	0,042	0,399	0,144	0,006	0,082	0,929
$F_{6,t}$	0,064	0,138	0,111	0,013	-0,014	0,986
$F_{7,t}$	0,106	0,026	0,133	0,007	-0,359	0,680
$F_{8,t}$	-0,063	0,073	-0,117	0,001	0,863	0,176
$F_{9,t}$	0,082	0,040	0,126	0,002	-0,278	0,703
$F_{10,t}$	0,071	0,099	0,185	0,000	0,120	0,878
$F_{11,t}$	-0,063	0,213	-0,069	0,186	-0,102	0,911
$F_{12,t}$	0,018	0,753	0,060	0,315	-0,346	0,743
$F_{13,t}$	0,159	0,014	0,009	0,895	0,324	0,784
$F_{14,t}$	-0,019	0,667	-0,044	0,338	0,071	0,929
$F_{15,t}$	-0,050	0,254	-0,037	0,414	1,010	0,204
$F_{16,t}$	0,166	0,001	-0,095	0,077	-0,422	0,655
$y_{1,t}$	-0,062	0,308	-0,018	0,772	0,891	0,422
$y_{2,t}$	0,099	0,084	0,013	0,828	0,125	0,904
$y_{3,t}$	0,222	0,007	0,185	0,032	-0,259	0,864
$y_{4,t}$	0,178	0,414	0,003	0,990	-0,448	0,910
$y_{5,t}$	0,108	0,419	0,148	0,285	0,005	0,999
$y_{6,t}$	-0,004	0,882	0,017	0,530	0,110	0,820
$\Phi(F_t\gamma_h)$	-1,232	0,000	0,227	0,271	-8,237	0,023
IMR_t	-0,518	0,000	0,056	0,311	-2,054	0,036

Figure 5: Predicting US economic activity: 1-quarter ahead scenarios



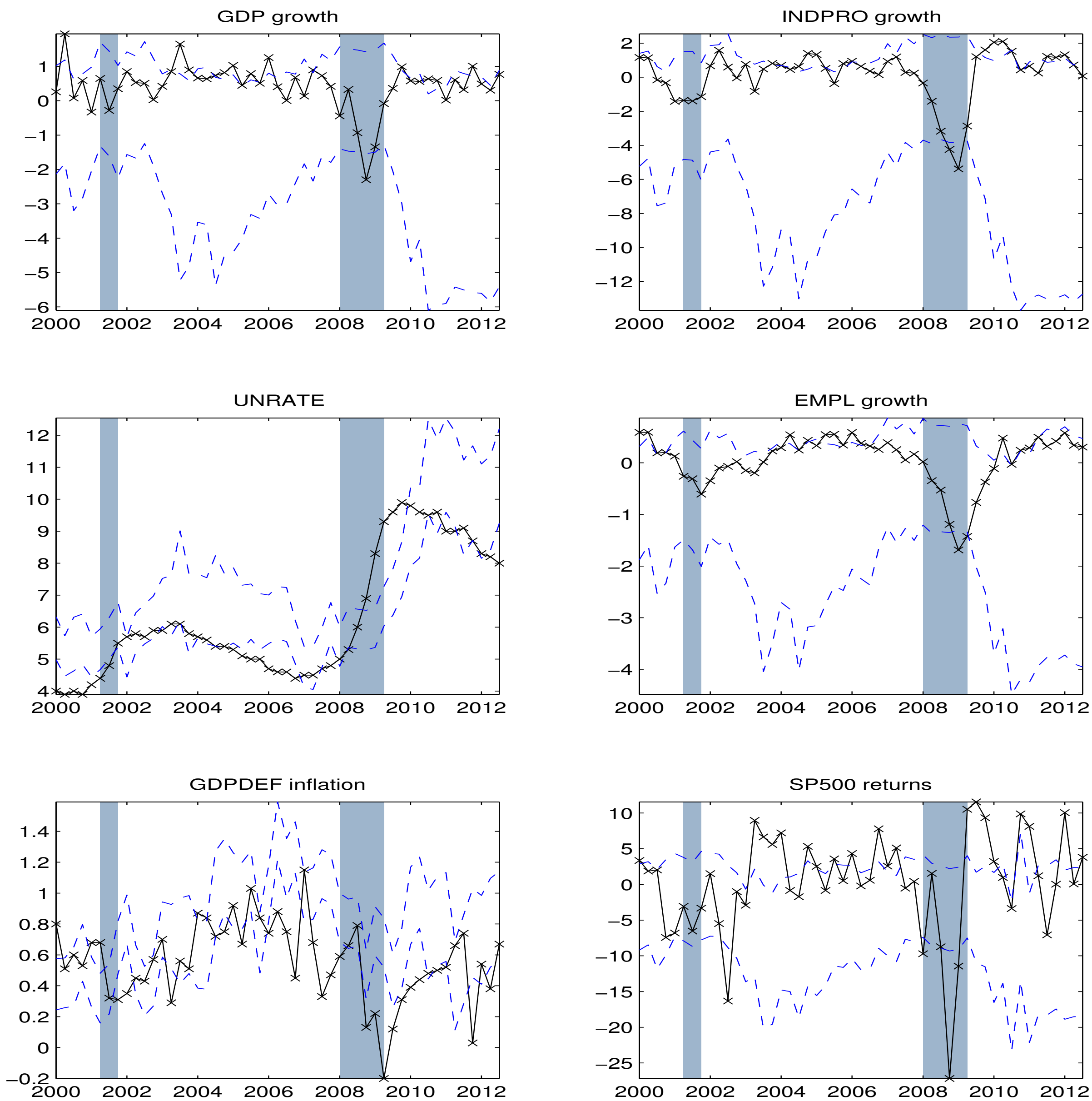
Notes: Predicted in-sample optimistic and pessimistic scenarios from DI-VAR forecasting models.

Figure 6: Predicting US economic activity: 4-quarters ahead scenarios



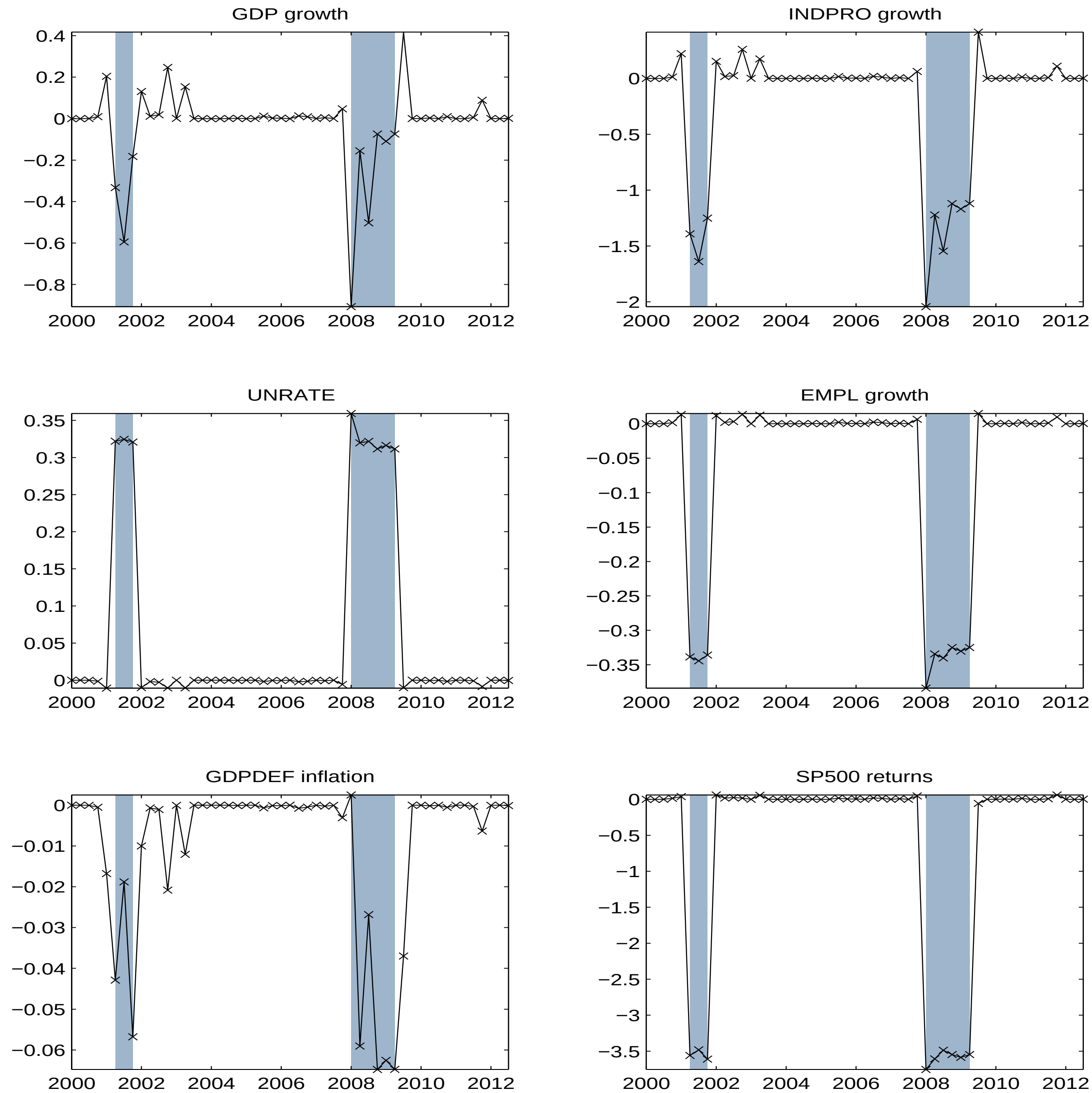
Notes: Predicted in-sample optimistic and pessimistic scenarios from DI-VAR forecasting models.

Figure 7: Predicting US economic activity: 8-quarters ahead scenarios



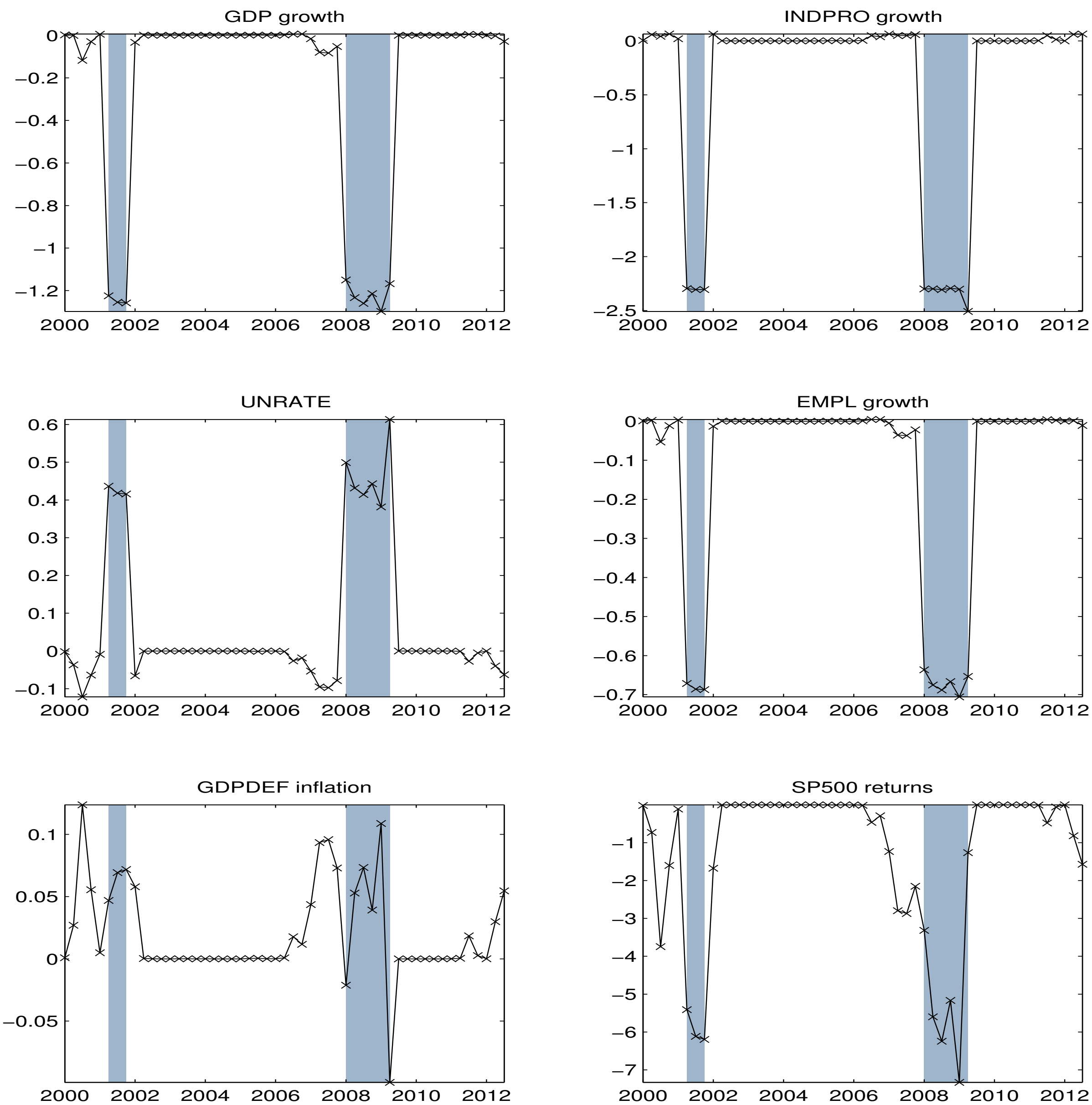
Notes: Predicted in-sample optimistic and pessimistic scenarios from DI-VAR forecasting models.

Figure 8: Predicting US economic activity: 1-quarter ahead loss due to recession



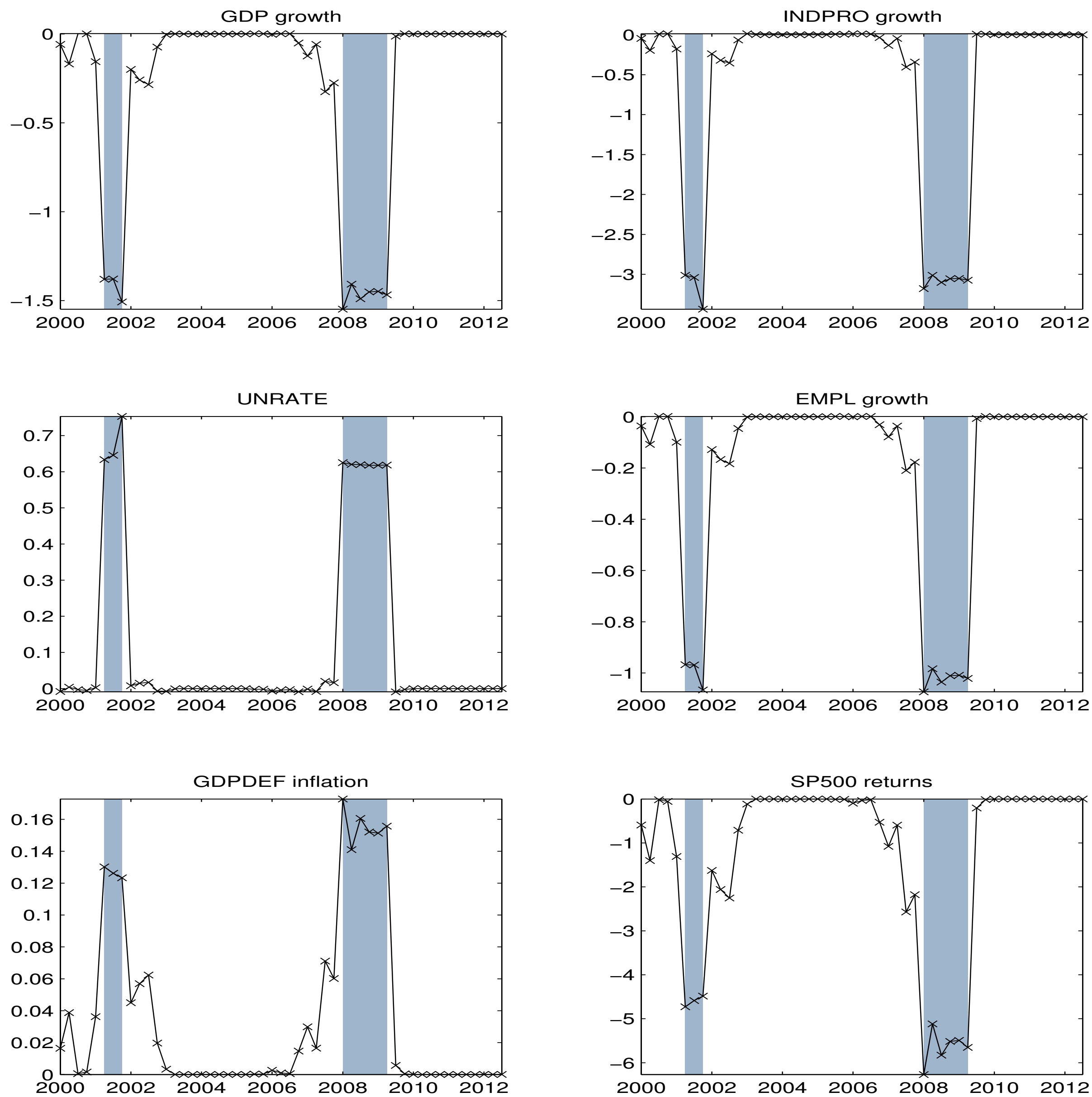
Notes: Predicted in-sample losses during recessions in US economic activity from DI-VAR forecasting models.

Figure 9: Predicting US economic activity: 4-quarters ahead loss due to recession



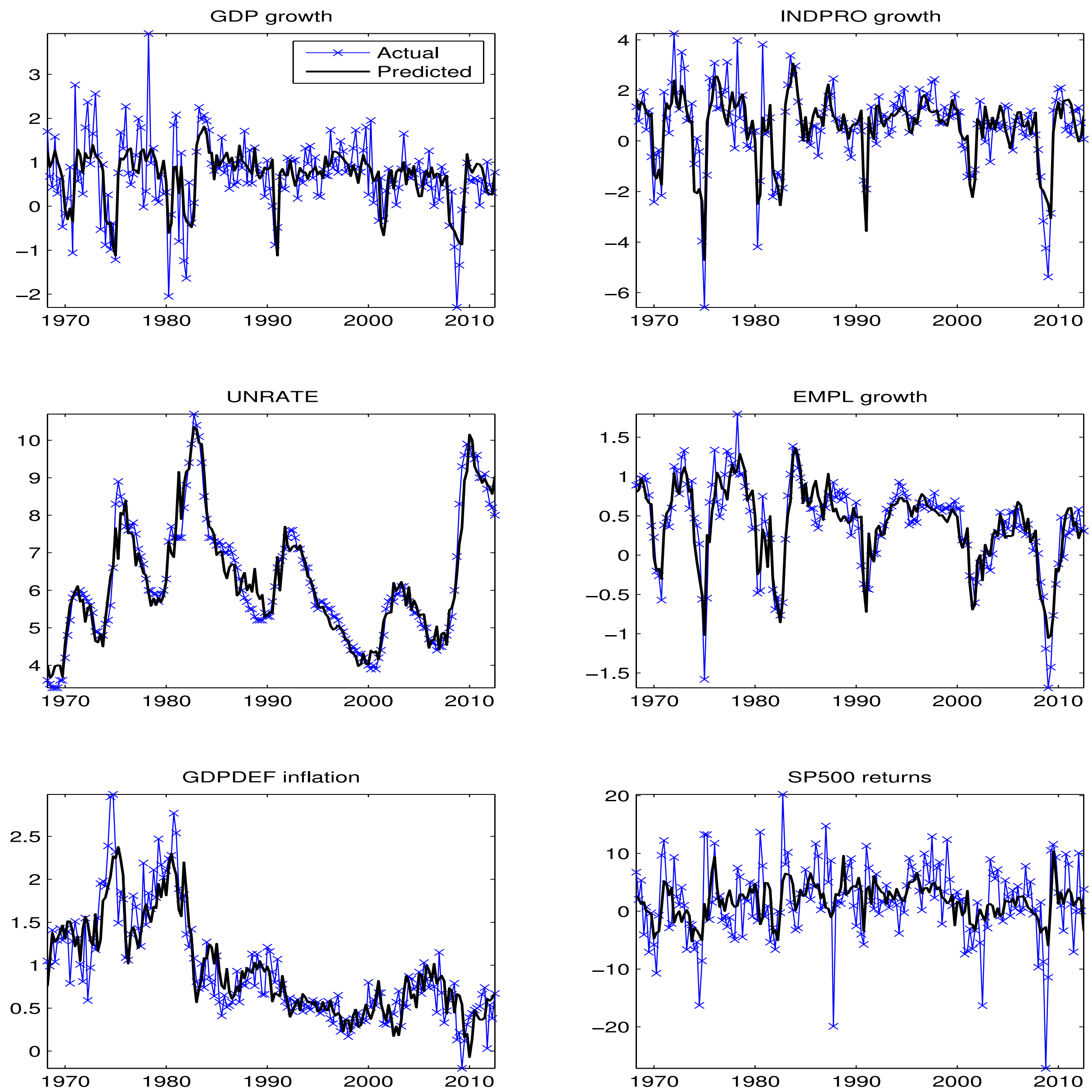
Notes: Predicted in-sample losses during recessions in US economic activity from DI-VAR forecasting models.

Figure 10: Predicting US economic activity: 8-quarters ahead loss due to recession



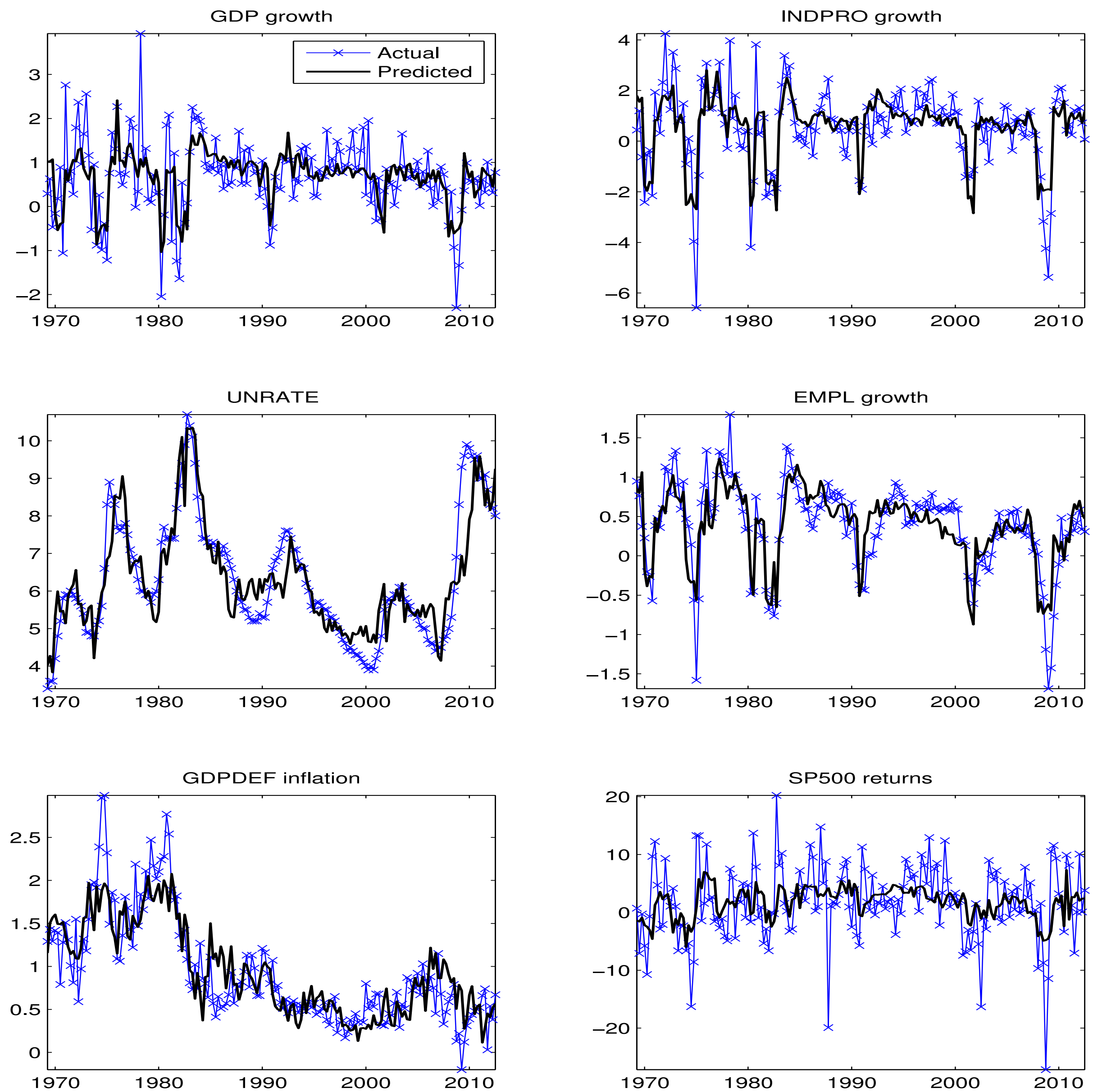
Notes: Predicted in-sample losses during recessions in US economic activity from DI-VAR forecasting models.

Figure 11: Predicting US economic activity: 4 quarters ahead



Notes: Predicted in-sample US economic activity series from DI-VAR forecasting models.

Figure 12: Predicting US economic activity: 8 quarters ahead



Notes: Predicted in-sample US economic activity series from DI-VAR forecasting models.