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When (not) to Segment Markets

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Abstract:
A monopoly decides whether to segment two separate markets. Demand depends on stochastic shocks ad some buyers are uninformed about the quality of the good. Contrary to the case of complete information, we show that it is not always more profitable for the firm to segment the markets in an environment in which some buyers have incomplete information. The reason is that the presence of uninformed buyers provides the firm with the incentive to engage in noisy price-signaling. Indeed, if the benefit from price flexibility (through market segmentation) is offset by the cost of signaling quality through two distinct prices, then it is optimal not to segment the markets and to use uniform pricing.

Keywords: Market integration, Market segmentation, Learning, Monopoly, Profits, Noisy signaling, third-degree price discrimination

JEL Classification: D82, D83, L12, L15

1 Introduction

In an environment of complete information, it is always profitable for a monopoly to segment markets and engage in (third-degree) price discrimination.[1] The reason is that setting different prices – a lower price in the market segments with greater price elasticity and a higher price in those with lower price elasticity – allows the firm to capture more of the consumer surplus in each market. However, little is known on whether market segmentation is always profitable under incomplete information on the part of buyers.

In this paper, we show that in an environment of incomplete information, market segmentation is not necessarily the more profitable pricing strategy. Specifically, we show that segmenting the markets is not always profitable when some buyers do not know the quality of the good and the firm reacts by engaging in price signaling. Indeed, when confronted with incomplete information on the demand side, the firm faces a trade-off in choosing to segment or integrate the markets. On the one hand, market segmentation yields more flexibility and the ability to capture a greater share of the consumer surplus. On the other hand, market segmentation implies that the firm signals quality with two prices instead of one. Hence, two prices are distorted from their complete information counterpart, whereas only one price is when markets are integrated. If the signaling cost (due to the distortion in the prices) is higher under market segmentation than under market integration, then it is possible that the loss due to signaling outweighs the benefit from price flexibility. We find that the higher the number of informed buyers, the more similar the market segments have to be for market integration to be the more profitable option. We also find that it is more likely that market integration be optimal when uninformed buyers are numerous and originate from the market segment with the higher willingness to pay.

The question of whether market integration is optimal is closely related

[1] Third-degree price discrimination is feasible as long as there is some easily observable characteristic by which a firm can group buyers and arbitrage can be prevented. See Schmalensee (1981) and Tirole (1988) for a detailed discussion on third-degree price discrimination.

to the question of whether uniform pricing for differentiated goods is optimal. In both problems, the benefits of the increased price flexibility need to be compared to the costs of charging different prices. Some recent papers (McMillan, 2007; Orbach and Einav, 2007; Chen, 2009; Chen and Cui, 2013; Richardson and Stähler, 2013) study such question in the context of differentiated goods. The present paper contributes partially to this strand of the literature by identifying a cost to charging different prices in a signaling context. Hence, we provide a glimpse to what incomplete information can yield when goods are differentiated.

At last, our work is related to several papers in international economics investigating the non-optimality of charging different prices for different markets (Friberg, 2001; Asplund and Friberg, 2000). However, these papers do not study the optimality of market segmentation (or integration) in a noisy signaling environment. Friberg (2001) studies whether a firm selling in regions with different currencies should segment the markets with an emphasis on the impact of the exchange rate, whereas Asplund and Friberg (2000) focus on the transportation cost from one region to another.[2] We do not explore these issues here, but rather provide an information-based reason for the profitability of market integration.

The remainder of the paper is organized as follows. Section 2 presents the model and Section 3 provides conditions under which it is more profitable to integrate the markets. Finally, Section 4 concludes.

2 The Model

In this section, we present a model in which a monopoly decides whether or not to segment the market for a good whose quality is unknown to some buyers. Our model has two stages. At the first stage, the firm decides whether or not to split the market into two separate markets. At the second stage, the firm sets one price if there is no market segmentation and two prices otherwise. In either case, the firm takes into account the fact that prices can

[2] Other papers such as Friberg (2003), Friberg and Martensen (2001) and Gallo (2010) study the profitability of market segmentation in the context of a duopoly.

provide partial information about quality to the uninformed buyers. We first describe the markets and then present the decisions of the firm at each stage.

Consider a firm selling a good of quality $\mu > 0$ in markets A and B. In market A, the buyers are informed, i.e., they know μ. Aggregate demand in market A is given by

$$Q_A(P_A, \mu, \eta_A) = \mu - P_A + \eta_A \tag{1}$$

where η_A is a demand shock that is unobserved by the buyers. The difference in demand between markets A and B is two-fold. The first difference concerns information. Unlike market A, market B is composed of both informed and uninformed buyers. Specifically, a fraction $\lambda \in [0, 1]$ of the buyers knows μ and thus a fraction $1 - \lambda$ does not know μ. Although the uninformed buyers have prior beliefs about μ, they also extract partial information about quality from observing prices, i.e., noisy price signaling. That is, upon observing prices, the uninformed buyers' posterior mean for quality is $\int x\hat{\xi}(x|P_A, P_B)\mathrm{d}x$ where $\hat{\xi}(\cdot|P_A, P_B)$ is the posterior p.d.f. of $\tilde{\mu}$ given P_A and P_B.[3] The second difference is that conditional on μ, the buyers in market B have a reservation price $\gamma\mu$ where $\gamma > 0$ reflects the disparity in demand between the two markets (unless $\gamma = 1$). Aggregate demand in market B is thus given by

$$Q_B(P_B, \mu, \hat{\xi}(\cdot|P_A, P_B), \eta_B) = \lambda(\gamma\mu - P_B) + (1-\lambda)\left(\gamma \int x\hat{\xi}(x|P_A, P_B)\mathrm{d}x - P_B\right) + \eta_B \tag{2}$$

where η_B is a demand shock that is unobserved by the buyers and $\int x\hat{\xi}(x|P_A, P_B)\mathrm{d}x$ is the posterior mean of μ and reflects the learning activity of the uninformed buyers.

Before proceeding with the behavior of the firm at each stage, it is useful to present the timing of all decisions and the information available to all agents. Except for the quality parameter μ and the demand shocks η_A and η_B, all the other parameters of the model (including the uninformed

[3]Note that $\hat{\xi}(\cdot|P_A, P_B)$ is the general expression for posterior beliefs upon observing two signals. If there is no market segmentation, then the uninformed buyers receive two *identical* signals, i.e., $P \equiv P_A = P_B$. In that case, posterior beliefs can be simplified to $\hat{\xi}(\cdot|P)$.

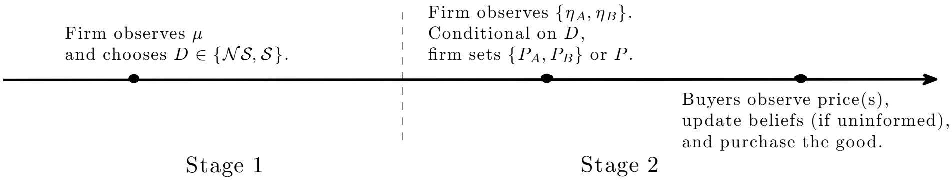


Figure 1: Timeline

buyers' prior beliefs and the distribution of the demand shocks) are public knowledge. More specifically, at the first stage, the firm observes the quality, but not the demand shocks.[4] The firm decides whether or not to segment the markets by comparing the expected profit under segmentation with the expected profit under no segmentation, rationally anticipating the demand shocks as well as the learning activity of the uninformed buyers. Formally, let $D \in \{\mathcal{NS}, \mathcal{S}\}$ be the firm's decision in the first stage. If $D = \mathcal{NS}$, then there is *No market Segmentation*, whereas $D = \mathcal{S}$ stands for *market Segmentation*. At the second stage, the firm observes the demand shocks and sets the price(s). The uninformed buyers do not know the quality and do not observe the demand shocks.[5] Upon observing the price(s), the buyers update beliefs (if uninformed), and purchase the good. Figure 1 summarizes the timeline.

We now describe formally the behavior of the firm at each stage.[6] We begin with the second stage. If the markets are not segmented, then the firm sets one price. Using (1) and (2) evaluated at $P \equiv P_A = P_B$, stage-2 maximization problem (given $D = \mathcal{NS}$) is

$$\Pi_{\mathcal{NS}}(\mu, \eta_A, \eta_B) = \max_P \left\{ P \cdot \Big(Q_A(P, \mu, \eta_A) + Q_B(P, \mu, \hat{\xi}_{\mathcal{NS}}(\cdot|P), \eta_B) \Big) \right\}. \quad (3)$$

If the markets are segmented, then the firm sets a price in each market.

[4]This reflects the idea that the firm faces some uncertainty in demand before making a decision about market segmentation.

[5]The fact that the buyers do not observe the demand shocks conveys the idea that the firm knows more about demand than the buyers do. Moreover, this informational asymmetry enables prices to provide partial (noisy) information about the quality of the good.

[6]A definition of the perfect Bayesian equilibrium is provided in Appendix A.

Using (1) and (2), given that the firm has decided to segment the market at stage 1, stage-2 maximization problem (given $D = \mathcal{S}$) is

$$\Pi_{\mathcal{S}}(\mu, \eta_A, \eta_B) = \max_{P_A, P_B} \left\{ P_A \cdot Q_A(P_A, \mu, \eta_A) + P_B \cdot Q_B(P_B, \mu, \hat{\xi}_{\mathcal{S}}(\cdot|P_A, P_B), \eta_B) \right\}. \tag{4}$$

Note that from (3) and (4), the firm's expected profits are influenced by the uninformed buyers' posterior mean. In particular, the p.d.f.'s $\hat{\xi}_{\mathcal{NS}}(\cdot|P)$ and $\hat{\xi}_{\mathcal{S}}(\cdot|P_A, P_B)$ are different functions because the uninformed buyers rationally anticipate the firm's decision to segment or integrate the market at the first stage. In the next section, it is shown that in equilibrium, the buyers correctly conjecture whether the market is segmented because the firm's decision at the first stage is independent of the quality μ and the realized demand shocks η_A and η_B.[7]

Next, at the first stage, the firm decides whether to segment the market by comparing expected profits under no market segmentation and market segmentation. To see this, let the tuple $\{\{P^*_{\mathcal{NS}}(\mu, \eta_A, \eta_B), \{P^*_{A,\mathcal{S}}(\mu, \eta_A, \eta_B), P^*_{B,\mathcal{S}}(\mu, \eta_B, \eta_A)\}\}, \{\hat{\xi}^*_{\mathcal{NS}}(\cdot|P_A, P_B), \hat{\xi}^*_{\mathcal{S}}(\cdot|P)\}\}$ define the equilibrium at the second stage. Specifically, $P^*_{\mathcal{NS}}(\mu, \eta_A, \eta_B)$ and $\{P^*_{A,\mathcal{S}}(\mu, \eta_A, \eta_B), P^*_{B,\mathcal{S}}(\mu, \eta_B, \eta_A)\}$ are the firm's price strategies under no market segmentation and market segmentation, respectively. The terms $\hat{\xi}^*_{\mathcal{NS}}(\cdot|P)$ and $\hat{\xi}^*_{\mathcal{S}}(\cdot|P_A, P_B)$ are the uninformed buyers' posterior beliefs under no market segmentation and market segmentation, respectively. Given these strategies and posterior beliefs at stage 2, the expected profits of the firm under no market segmentation and market segmentation are

$$\begin{aligned}\mathbb{E}[\Pi^*_{\mathcal{NS}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)] = \mathbb{E}\Big[P^*_{\mathcal{NS}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B) \cdot \Big(& Q_A(P^*_{\mathcal{NS}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B), \mu, \tilde{\eta}_A) \\ & + Q_B(P^*_{\mathcal{NS}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B), \mu, \hat{\xi}^*_{\mathcal{NS}}(\cdot|P^*_{\mathcal{NS}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)), \tilde{\eta}_B)\Big)\Big]\end{aligned} \tag{5}$$

[7] The fact that the firm's decision in the first stage is independent of μ is a consequence of the distributional assumptions and of the demand specification. The independence from the realized demand shocks follows from the fact that the firm does not observe η_A and η_B at the first stage.

and

$$\mathbb{E}[\Pi^*_{\mathcal{S}}(\mu,\tilde{\eta}_A,\tilde{\eta}_B)] = \mathbb{E}[P^*_{A,\mathcal{S}}(\mu,\tilde{\eta}_A,\tilde{\eta}_B)\cdot Q_A(P^*_{A,\mathcal{S}}(\mu,\tilde{\eta}_A,\tilde{\eta}_B),\mu,\tilde{\eta}_A)] + \mathbb{E}[P^*_{B,\mathcal{S}}(\mu,\tilde{\eta}_B,\tilde{\eta}_A) \cdot Q_B(P^*_{B,\mathcal{S}}(\mu,\tilde{\eta}_B,\tilde{\eta}_A),\mu,\hat{\xi}^*_{\mathcal{S}}(\cdot|P^*_{A,\mathcal{S}}(\mu,\tilde{\eta}_A,\tilde{\eta}_B),P^*_{B,\mathcal{S}}(\mu,\tilde{\eta}_B,\tilde{\eta}_A)),\tilde{\eta}_B)], \tag{6}$$

respectively. Here, $\mathbb{E}[\cdot]$ is the expectation operator over $\{\tilde{\eta}_A,\tilde{\eta}_B\}$ where a tilde sign is used to distinguish a random variable from its realization. Hence, at the first stage, using (5) and (6), the firm chooses not to split the two markets (i.e., $D^* = \mathcal{NS}$) when

$$\mathbb{E}[\Pi^*_{\mathcal{NS}}(\mu,\tilde{\eta}_A,\tilde{\eta}_B)] > \mathbb{E}[\Pi^*_{\mathcal{S}}(\mu,\tilde{\eta}_A,\tilde{\eta}_B)]. \tag{7}$$

3 On the Profitability of Market Integration

Having presented the model, we now provide conditions under which (7) holds. Specifically, we show that the presence of uninformed buyers (inducing the firm to engage in noisy signaling) makes it possible for the firm to obtain higher expected profits by not segmenting the market. To characterize the equilibrium, we make the following assumption.

Assumption 3.1. *Prior beliefs are $\tilde{\mu} \sim N(\rho,\sigma_\mu^2)$, with $\rho > 0$. Distributions of demand shocks are $\tilde{\eta}_A \sim N(0,\sigma_\eta^2), \tilde{\eta}_B \sim N(0,\sigma_\eta^2)$ such that $\mathbb{E}[\tilde{\eta}_A\tilde{\eta}_B] = 0$.*

Before proceeding with the analysis, we discuss the distributional assumption for the uninformed buyers' prior beliefs and the random demand shocks. From Assumption 3.1, we rely on the fact that the family of normal distributions with an unknown mean is a conjugate family for samples from a normal distribution.[8] With the normality assumption, we obtain a unique

[8]Normal assumption combined with linear demand yields closed-form equilibrium values and makes the analysis tractable by focusing on the mean and variance of price and posterior beliefs. See Grossman and Stiglitz (1980), Kyle (1985), Judd and Riordan (1994), and Mirman et al. (2013) for the use of normal distributions to study the informational role of prices in single-agent problems (without market segmentation). See also Vives (2011) for the use of normal distributions in a rational expectations environment.

linear equilibrium, i.e., an equilibrium in which the uninformed buyers' posterior mean is linear in the price-signals. Although negative demand shocks can yield a negative price or a negative posterior mean, the values of the parameters of the model can be restricted to ensure that the probability of such events be arbitrarily close to zero. Moreover, it turns out that, for any parameters, equilibrium values for mean prices are always positive.

We now provide the equilibrium expected profits for each possible state in stage 2 (i.e., $D = \mathcal{NS}$ and $D = \mathcal{S}$) when the uninformed buyers have unbiased beliefs about the unknown quality (i.e., $\rho = \mu$).[9] Proposition 3.2 shows that regardless of the firm's decision to segment or integrate the markets, second-stage expected profits are the sum of two components. The first component is the *full-information* expected profits, i.e., when all buyers are informed (i.e., $\lambda = 1$). The second component is a cost that emanates from the firm's need to signal quality via prices. Indeed, in order to signal the quality of the good to the uninformed buyers, the firm alters prices. This distortion in prices translates into a loss in expected profits. Formally, from (9) and (11) in Proposition 3.2, $-C^*_{\mathcal{NS}}\mu^2 \leq 0$ and $-C^*_{\mathcal{S}}\mu^2 \leq 0$ denote the loss in expected profits (due to signaling) under no market segmentation and market segmentation, respectively.

Proposition 3.2. *Suppose that Assumption 3.1 holds. Then, there exists an equilibrium in the second stage. Suppose further that prior beliefs are unbiased, i.e., $\rho = \mu$. Then, stage-2 expected profits are*

1. *For $D^* = \mathcal{NS}$,*

$$\mathbb{E}[\Pi^*_{\mathcal{NS}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)] = \frac{(1+\gamma)^2\mu^2}{8} - C^*_{\mathcal{NS}}\mu^2, \tag{8}$$

where

$$C^*_{\mathcal{NS}} = \frac{(1-\lambda)^2(1+\gamma)^2(1+\gamma\lambda)^2\gamma^2\sigma_\mu^4}{8(2\sigma_\eta^2 + (1+\gamma)(1+\gamma\lambda)\sigma_\mu^2)^2}. \tag{9}$$

[9] The firm's expected profits may also be higher under market integration when buyers' beliefs are biased. However, the firm's decision to split the market depends on prior beliefs as well as quality.

2. *For* $D^* = \mathcal{S}$,

$$\mathbb{E}[\Pi^*_{\mathcal{S}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)] = \frac{(1+\gamma^2)\mu^2}{4} - C^*_{\mathcal{S}}\mu^2, \tag{10}$$

where

$$C^*_{\mathcal{S}} = \frac{(1-\lambda)^2(1+\gamma^2\lambda^2)\gamma^4\sigma_\mu^4}{4(\sigma_\eta^2 + 2(1+\gamma^2\lambda)\sigma_\eta^2\sigma_\mu^2 + (1+\gamma^2)(1+\gamma^2\lambda^2)\sigma_\mu^4)}. \tag{11}$$

Proof. See Appendix B. □

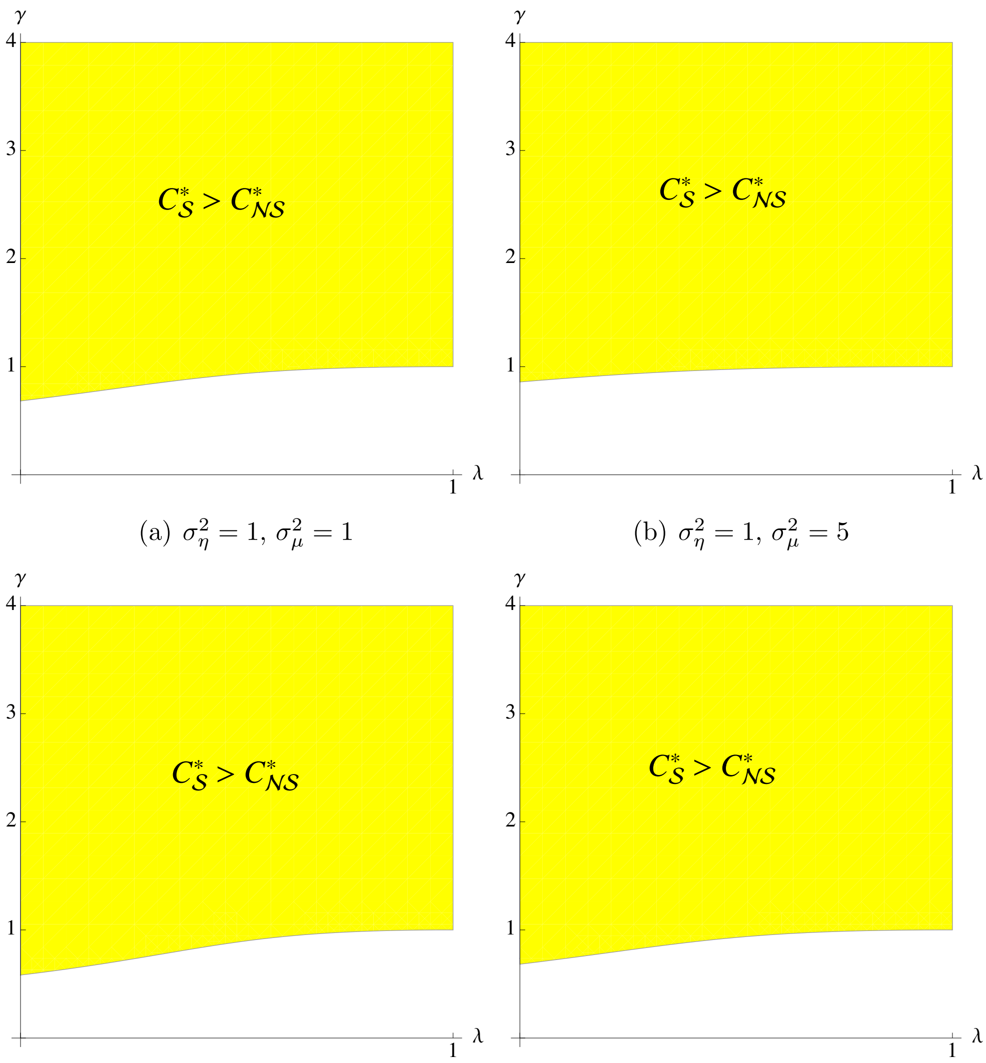
One comment about Proposition 3.2 is warranted. From (8) and (10), expected profits are linear in μ^2. As a result, the firm's decision to segment or integrate the market is independent of μ. As noted earlier in Footnote 7, this implies that the uninformed buyers correctly conjecture whether the market is segmented. It also implies that observing two different prices and thus inferring that the markets are segmented brings no additional information about μ since the firm's decision at stage 1 is uninformative about μ.[10]

Using Proposition 3.2, we now turn to our main result, i.e., under noisy signaling, it is possible for the firm to choose *not* to segment the market. We begin by stating the well-known benchmark case of full information when all buyers are informed, i.e., $\lambda = 1$. If every buyer is informed, then it is always profitable for a firm to segment the market. Indeed, in that case, there is no loss in expected profits due to signaling. That is, from (9) and (11), $C^*_{\mathcal{NS}}|_{\lambda=1} = C^*_{\mathcal{NS}}|_{\lambda=1} = 0$. Hence, the flexibility of using two prices always yields higher expected profits.

Remark 3.3. *Suppose that Assumption 3.1 holds. Then, from* (8) *and* (10), $\mathbb{E}[\Pi^*_{\mathcal{NS}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)]|_{\lambda=1} < \mathbb{E}[\Pi^*_{\mathcal{S}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)]|_{\lambda=1}$.

Remark 3.3 implies that a necessary condition for the firm to prefer not to segment the market is the presence of uninformed buyers, which is related to the loss (due to signaling) in expected profits. Indeed, in order to offset the benefit from price flexibility (by segmenting the market), it is necessary (but

[10]In other words, the ordering of (8) and (10) is independent of μ.



(a) $\sigma^2_\eta = 1$, $\sigma^2_\mu = 1$

(b) $\sigma^2_\eta = 1$, $\sigma^2_\mu = 5$

(c) $\sigma^2_\eta = 5$, $\sigma^2_\mu = 1$

(d) $\sigma^2_\eta = 5$, $\sigma^2_\mu = 5$

Figure 2: Comparison of $C^*_{\mathcal{NS}}$ and $C^*_{\mathcal{S}}$. The shaded area $C^*_{\mathcal{S}} > C^*_{\mathcal{NS}}$ regroups the set of pairs $\{\gamma, \lambda\}$ for which the loss in expected profits (due to signaling) is greatest under market segmentation.

not sufficient) for the loss in expected profits under market segmentation to be greater than the loss in expected profits under no market segmentation. For $\lambda \in (0,1)$, it is possible that $C^*_{\mathcal{S}} > C^*_{\mathcal{NS}}$. Figure 2 depicts the region of the parameters space $\{\lambda, \gamma\}$ corresponding to $C^*_{\mathcal{S}} > C^*_{\mathcal{NS}}$.

Proposition 3.4 establishes the condition under which the firm chooses not to segment the market. Condition (12) compares the gains and losses in expected profits from integrating the markets. Intuitively, the firm faces a trade-off. On the one hand, market segmentation yields more flexibility and the ability to capture more of the consumer surplus. On the other hand, the firm also has to incur a signaling cost, i.e., the distortion needed to signal quality via prices depends on whether the market is integrated or separated. Specifically, the firm does not segment the market if there is a reduction in cost due to signaling (i.e., $C^*_{\mathcal{S}} - C^*_{\mathcal{NS}} > 0$) which is greater than the loss from price flexibility (i.e. $(1-\gamma)^2/8$). While there is always a loss from price flexibility (unless the markets are identical), the reduction in cost due to signaling depends on the parameter values.

Proposition 3.4. *Suppose Assumption 3.1 holds and that* $\lambda \in (0,1)$. *Then, at the first stage, the firm does not segment the market (i.e.,* $D^* = \mathcal{NS}$*) if and only if*

$$C^*_{\mathcal{S}} - C^*_{\mathcal{NS}} \geq (1-\gamma)^2/8 \tag{12}$$

where $C^*_{\mathcal{NS}}$ *and* $C^*_{\mathcal{S}}$ *are given by* (9) *and* (11), *respectively.*

Proof. From (8) and (10),

$$\mathbb{E}[\Pi^*_{\mathcal{NS}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)] - \mathbb{E}[\Pi^*_{\mathcal{S}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)] = \frac{\mu^2}{8}\left[-(1-\gamma)^2 - 8C^*_{\mathcal{NS}} + 8C^*_{\mathcal{S}}\right], \tag{13}$$

which implies (12). □

Note that market integration is optimal even in the case of identical reservation prices across the two markets. i.e., $\gamma = 1$.[11] In that case, the loss in expected profits due to signaling is always larger under market segmentation.

[11] There is still a difference between the two markets because there are some uninformed buyers in market B.

That is, from (9) and (11), $C^*_{\mathcal{S}}|_{\gamma=1} > C^*_{\mathcal{NS}}|_{\gamma=1} > 0$. Hence, since there is no benefit in price flexibility when $\gamma = 1$, the firm always prefers to integrate the markets.[12]

It is convenient to depict the condition stated in Proposition 3.4. Figure 3 illustrates Proposition 3.4 by showing the region of the parameters space $\{\lambda, \gamma\}$ corresponding to $\mathbb{E}[\Pi^*_{\mathcal{NS}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)] > \mathbb{E}[\Pi^*_{\mathcal{S}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)]$.[13] The firm chooses not to segment the markets when the fraction of informed buyers is low enough and the reservation price on market B is either almost similar to the one of market A, or higher. In terms of the parameters, this implies that λ is low and γ is either just a little below 1, or above. This is consistent with the decomposition of expected profits provided in Proposition 3.2. Indeed, as noted, the firm faces a trade-off between a benefit from price flexibility and a cost from having to signal quality from prices.

When γ is low, markets A and B are very different and there is thus a great gain from splitting the market and capturing the consumer surplus. That is, the first component in (8) is higher than the first component in (10). When the markets are similar, i.e., $\gamma = 1$, then in addition to generating no benefit, signaling through the use of two prices is more costly than using a single price. Consequently, the firm is better off by integrating the markets. The same is true when γ is just a little below 1 as the benefit from segmentation is small in comparison of the additional signaling cost $C^*_{\mathcal{S}} - C^*_{\mathcal{NS}}$. When γ is above 1, for low level of λ, then the cost difference $C^*_{\mathcal{S}} - C^*_{\mathcal{NS}}$ increases more rapidly than the benefits. Hence, the firm is still better off by integrating the markets. Finally, as λ decreases, the effect of signaling on profit increases. When distorting two prices is more costly than distorting one price, the second component in (8) is higher than the component in (10).

[12] That is, from (8) and (10), $\mathbb{E}[\Pi^*_{\mathcal{NS}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)]|_{\gamma=1} > \mathbb{E}[\Pi^*_{\mathcal{S}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)]|_{\gamma=1}$.

[13] From Figures 2 and 3, if the firm chooses not to segment the market, then the loss (due to signaling) in expected profits is always greater under market segmentation.

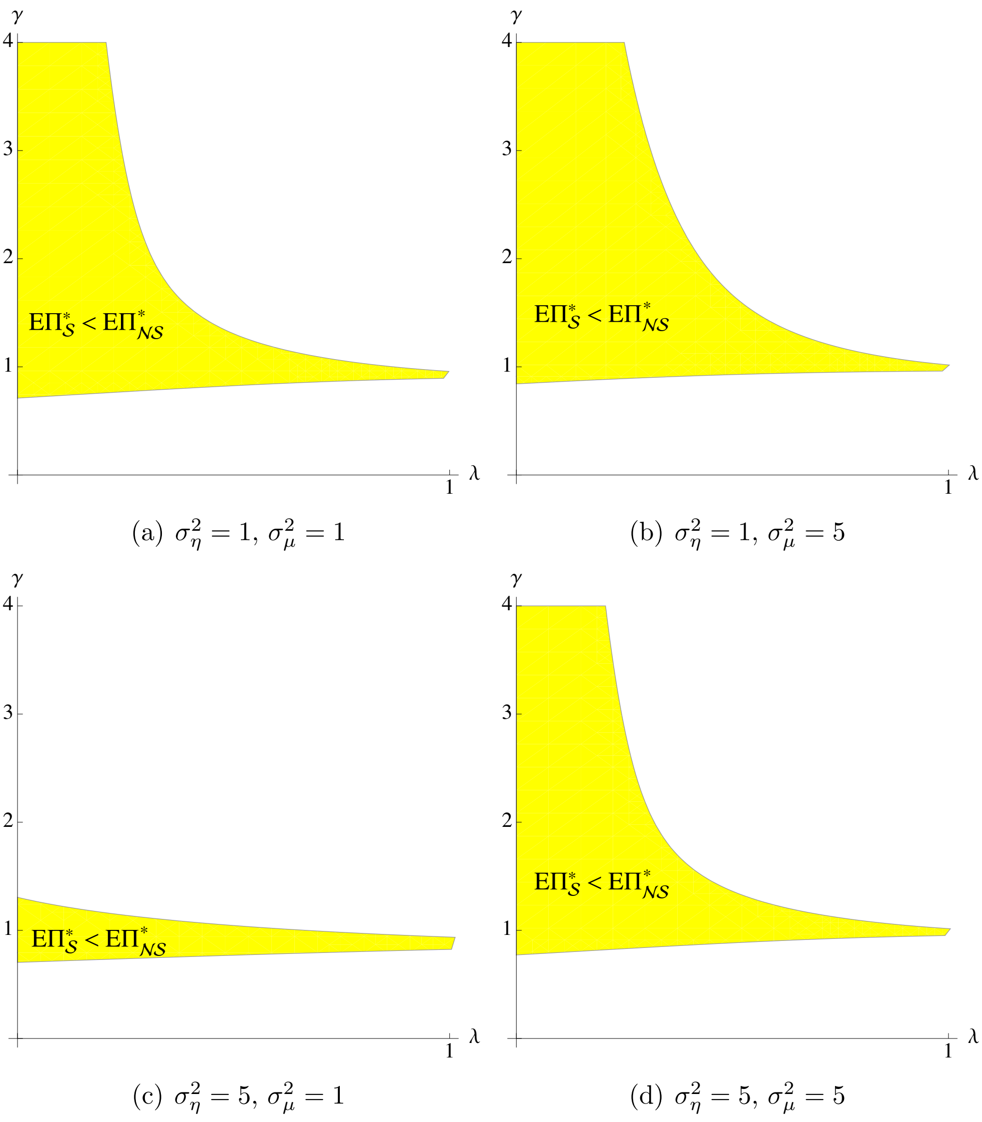


Figure 3: Comparison of $\mathbb{E}[\Pi^*_{\mathcal{NS}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)]$ and $\mathbb{E}[\Pi^*_{\mathcal{S}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)]$. The shaded area $\mathbb{E}[\Pi^*_{\mathcal{S}}] < \mathbb{E}[\Pi^*_{\mathcal{NS}}]$ regroups the set of pairs $\{\gamma, \lambda\}$ for which the expected profits is greatest under no market segmentation.

4 Conclusion

Under complete information on the part of buyers, a monopoly obtains a higher expected profit by charging difference price for market segments having different price elasticities. We show that this conclusion does not hold when some buyers have incomplete information about the quality of the good they consider purchasing and the firm engages in price signaling. The analysis presented in this paper complements the analysis in Gendron-Saulnier and Santugini (2013). They both outline an important difference regarding the effect of market segmentation between complete and incomplete information. Indeed, Gendron-Saulnier and Santugini (2013) shows that the uninformed buyers obtain an informational benefit when the firm segments the markets. Hence, the conclusion that under complete information the firm gains while all the buyers lose from market segmentation (if both markets are served under market integration) are shown to be reversed under incomplete information.

A Equilibrium Definition

Definition A.1 states the Perfect Bayesian Equilibrium. The equilibrium consists of the firm's strategy (a segmentation decision at stage 1 and prices at stage 2), the distribution of the price-signals conditional on any quality x, and the uninformed buyers' posterior beliefs about the quality upon observing any prices.[14] In equilibrium, the posterior beliefs are consistent with Bayes' rule and the equilibrium distribution of prices.

Definition A.1. *The tuple* $\{\{D^*, \{\{P^*_{\mathcal{NS}}(\mu, \eta_A, \eta_B), \{P^*_{A,\mathcal{S}}(\mu, \eta_A, \eta_B), P^*_{B,\mathcal{S}}(\mu, \eta_B, \eta_A)\}\}\}\}$, $\{\hat{\xi}^*_{\mathcal{NS}}(\cdot|P_A, P_B), \hat{\xi}^*_{\mathcal{S}}(\cdot|P)\}\}$ *is an equilibrium if, for all* $\mu > 0$,

1. *At stage 2,*

   (a) *For* $D^* = \mathcal{NS}$,

      i. *Given* $\hat{\xi}^*_{\mathcal{NS}}(\cdot|P)$*, and for any* η_A *and* η_B*, the firm's price strategy is*

      $$P^*_{\mathcal{NS}}(\mu, \eta_A, \eta_B) = \arg\max_P \left\{ P \cdot \Big(Q_A(P, \mu, \eta_A) + Q_B(P, \mu, \hat{\xi}^*_{\mathcal{NS}}(\cdot|P), \eta_B) \Big) \right\}. \tag{14}$$

      ii. *Given the distribution of* $\{\tilde{\eta}_A, \tilde{\eta}_B\}$, $\phi^*_{\mathcal{NS}}(P|x)$ *is the p.d.f. of the random price-signal* $P^*_{\mathcal{NS}}(x, \tilde{\eta}_A, \tilde{\eta}_B)$ *conditional on any quality* x.

      iii. *Given* $\phi^*_{\mathcal{NS}}(P|\cdot)$ *and prior beliefs* $\xi(\cdot)$*, the uninformed buyers' posterior beliefs upon observing any* P *is* $\tilde{\mu}^*|P$ *with p.d.f.*

      $$\hat{\xi}^*_{\mathcal{NS}}(x|P) = \frac{\xi(x)\phi^*_{\mathcal{NS}}(P|x)}{\int_{x' \in \mathbb{R}} \xi(x')\phi^*_{\mathcal{NS}}(P|x')\mathrm{d}x'}, \tag{15}$$

      $x \in \mathbb{R}$.

   (b) *For* $D^* = \mathcal{S}$,

[14] The variable μ refers to the true quality whereas x is used as a dummy variable for quality.

i. Given $\hat{\xi}^*_{\mathcal{S}}(\cdot|P_A, P_B)$, and for any η_A and η_B, the firm's price strategies are

$$\begin{aligned}\left\{P^*_{A,\mathcal{S}}(\mu, \eta_A, \eta_B), P^*_{B,\mathcal{S}}(\mu, \eta_B, \eta_A)\right\} = \arg\max_{P_A, P_B} \Big\{ & P_A \cdot Q_A(P_A, \mu, \eta_A) \\ & + P_B \cdot Q_B(P_B, \mu, \hat{\xi}^*_{\mathcal{S}}(\cdot|P_A, P_B), \eta_B)\Big\}.\end{aligned} \tag{16}$$

ii. Given the distribution of $\{\tilde{\eta}_A, \tilde{\eta}_B\}$, $\phi^*_{\mathcal{S}}(P_A, P_B|x)$ is the p.d.f. of the random price-signals $\left\{P^*_{A,\mathcal{S}}(x, \tilde{\eta}_A, \tilde{\eta}_B), P^*_{B,\mathcal{S}}(x, \tilde{\eta}_B, \tilde{\eta}_A)\right\}$ conditional on any quality x.

iii. Given $\phi^*_{\mathcal{S}}(P_A, P_B|\cdot)$ and prior beliefs $\xi(\cdot)$, the uninformed buyers' posterior beliefs about quality upon observing P_A and P_B is $\tilde{\mu}^*_{\mathcal{S}}|P_A, P_B$ with the p.d.f.

$$\hat{\xi}^*_{\mathcal{S}}(x|P_A, P_B) = \frac{\xi(x)\phi^*_{\mathcal{S}}(P_A, P_B|x)}{\int_{x' \in \mathbb{R}} \xi(x')\phi^*_{\mathcal{S}}(P_A, P_B|x')\mathrm{d}x'}, \tag{17}$$

$x \in \mathbb{R}$.

2. At stage 1,

$$D^* = \arg\max_{D \in \{\mathcal{NS}, \mathcal{S}\}} \mathbb{1}_{[D=\mathcal{NS}]} \cdot \mathbb{E}[\Pi^*_{\mathcal{NS}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)] + \mathbb{1}_{[D=\mathcal{S}]} \cdot \mathbb{E}[\Pi^*_{\mathcal{S}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)] \tag{18}$$

where

$$\begin{aligned}\mathbb{E}[\Pi^*_{\mathcal{NS}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)] = \mathbb{E}\Big[& P^*_{\mathcal{NS}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B) \cdot \Big(Q_A(P^*_{\mathcal{NS}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B), \mu, \tilde{\eta}_A) \\ & + Q_B(P^*_{\mathcal{NS}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B), \mu, \hat{\xi}^*_{\mathcal{NS}}(\cdot|P^*_{\mathcal{NS}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)), \tilde{\eta}_B)\Big)\Big]\end{aligned} \tag{19}$$

and

$$\mathbb{E}[\Pi^*_{\mathcal{S}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B)] = \mathbb{E}\Big[P^*_{A,\mathcal{S}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B) \cdot Q_A(P^*_{A,\mathcal{S}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B), \mu, \tilde{\eta}_A)\Big] + \mathbb{E}\Big[P^*_{B,\mathcal{S}}(\mu, \tilde{\eta}_B, \tilde{\eta}_A) \cdot Q_B(P^*_{B,\mathcal{S}}(\mu, \tilde{\eta}_B, \tilde{\eta}_A), \mu, \hat{\xi}^*_{\mathcal{S}}(\cdot | P^*_{A,\mathcal{S}}(\mu, \tilde{\eta}_A, \tilde{\eta}_B), P^*_{B,\mathcal{S}}(\mu, \tilde{\eta}_B, \tilde{\eta}_A)), \tilde{\eta}_B)\Big]. \tag{20}$$

B Proofs

Proof of Proposition 3.2. See Gendron-Saulnier and Santugini (2013) for the existence of the equilibrium at the second stage.

1. If the markets are not segmented, then from Proposition 2.5 (Gendron-Saulnier and Santugini, 2013), the firm's price strategy is

$$P^*_{\mathcal{NS}}(\mu, \eta_A, \eta_B) = \frac{\beta^*_{\mathcal{NS}}\gamma(1-\lambda) + (1+\gamma\lambda)\mu + \eta_A + \eta_B}{4 - 2\beta^*_{\mathcal{NS}}\gamma(1-\lambda)} \tag{21}$$

   and the uninformed buyers' posterior mean is

$$\int x\hat{\xi}^*_{\mathcal{NS}}(x|P)\mathrm{d}x = \beta^*_0 + \beta^*_1 P \tag{22}$$

   where

$$\beta^*_0 = \frac{2\rho\sigma^2_\eta}{2\sigma^2_\eta + \sigma^2_\mu(1+\gamma+\gamma\lambda+\gamma^2\lambda)}, \tag{23}$$

$$\beta^*_1 = \frac{4(1+\gamma\lambda)\sigma^2_\mu}{2\sigma^2_\eta + \sigma^2_\mu(1+2\gamma+2\gamma^2\lambda-\gamma^2\lambda^2)}. \tag{24}$$

   Plugging (21) and (22) (evaluated at $P = P^*_{\mathcal{NS}}(\mu, \eta_A, \eta_B)$) into (5), setting $\rho = \mu$ (for unbiased beliefs), and taking expectations over $\{\tilde{\eta}_A, \tilde{\eta}_B\}$ yields (8).

2. If the markets are segmented, then from Proposition 2.3 (Gendron-

Saulnier and Santugini, 2013), the firm's price strategies are

$$P^*_{A,\mathcal{S}}(\mu,\eta_A,\eta_B) = \frac{\delta^*_0\delta^*_1\gamma^2(1-\lambda)^2 + (2-2\delta^*_2\gamma(1-\lambda)+\delta^*_1\gamma^2\lambda(1-\lambda))\mu}{4-\delta^{*2}_1\gamma^2(1-\lambda)^2-4\delta^*_2\gamma(1-\lambda)} + \frac{(2-2\delta^*_2\gamma(1-\lambda))\eta_A+\delta^*_1\gamma(1-\lambda)\eta_B}{4-\delta^{*2}_1\gamma^2(1-\lambda)^2-4\delta^*_2\gamma(1-\lambda)} \tag{25}$$

and

$$P^*_{B,\mathcal{S}}(\mu,\eta_B,\eta_A) = \frac{2\delta^*_0\gamma(1-\lambda)+(\delta^*_1\gamma(1-\lambda)+2\gamma\lambda)\mu+\delta^*_1\gamma(1-\lambda)\eta_A+2\eta_B}{4-\delta^{*2}_1\gamma^2(1-\lambda)^2-4\delta^*_2\gamma(1-\lambda)}, \tag{26}$$

and the uninformed buyers' posterior mean is

$$\int x\hat{\xi}^*_{\mathcal{S}}(x|P_A,P_B)\mathrm{d}x = \delta^*_0+\delta^*_1P_A+\delta^*_2P_B, \tag{27}$$

where

$$\delta^*_0 = \frac{\rho\sigma^2_\eta}{\sigma^2_\eta+\sigma^2_\mu(1+\gamma^2\lambda)}, \tag{28}$$

$$\delta^*_1 = \frac{2\sigma^2_\mu}{\sigma^2_\eta+\sigma^2_\mu(1+\gamma^2\lambda)}, \tag{29}$$

$$\delta^*_2 = \frac{2\gamma(\lambda\sigma^2_\mu(\sigma^2_\eta+2\sigma^2_\mu)-\sigma^4_\mu(1-\gamma^2\lambda^2))}{(\sigma^2_\eta+\sigma^2_\mu(1+\gamma^2\lambda))(\sigma^2_\eta+\sigma^2_\mu(1+\gamma^2\lambda(2-\lambda)))}. \tag{30}$$

Plugging (25), (26), and (27) (evaluated at $P_A = P^*_{A,\mathcal{S}}(\mu,\eta_A,\eta_B)$ and $P_B = P^*_{B,\mathcal{S}}(\mu,\eta_B,\eta_A)$) into (6), setting $\rho=\mu$ (for unbiased beliefs), and taking expectations over $\{\tilde{\eta}_A,\tilde{\eta}_B\}$ yields (10).

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