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Trade Structure, Transboundary Pollution and Multilateral Trade Liberalization : the Effects on Environmental Taxes and Welfare

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Abstract:

This paper considers a trade situation where the production activities of potentially heterogeneous countries generate pollution which can cross borders and harm the well-being of all the countries involved. In each of those countries the policy market levies pollution taxes on the polluting firms and a tariff on imports in order to correct that distortion. The purpose of the paper is to investigate the effect of a reduction in the tariff on equilibrium pollution taxes and welfare. The existing literature has investigated this problem for trade between two identical countries. This paper analyzes the problem in the more realistic context where countries are not necessarily identical and trade can be multilateral. It becomes possible to show what bias is introduced when those two realities are neglected. I find that a tariff reduction can actually lower output; it can also lower welfare even if pollution is purely local.

Keywords: Trade liberalization, Pollution taxes, Transboundary pollution, Heterogeneous countries, Imperfect markets

Résumé:

Ce papier s'inscrit dans un contexte où les activités de production des pays potentiellement hétérogènes génèrent de la pollution qui peut traverser les frontières et nuire au bien-être des pays impliqués. Dans chacun de ces pays, l'état s'impose des taxes sur la pollution aux firmes polluantes et des tarifs à l'importation afin de corriger cette distorsion. Ce papier a pour but d'évaluer les effets que pourrait avoir une diminution des tarifs douaniers sur la production, les taxes sur la pollution et le bien-être de ces pays. La littérature existante a étudié ce problème, mais seulement dans le cadre d'un commerce bilatéral entre pays identiques. Cet article fournit un cadre d'analyse plus réaliste dans lequel les pays ne seront pas nécessairement identiques et où le commerce pourra être multilatéral. Il devient alors possible de mettre en évidence le biais introduit en négligeant ces deux facteurs. Dans ce nouveau contexte, je montre qu'une réduction des tarifs d'importation n'augmente pas nécessairement l'output; elle peut aussi nuire au bien-être, même si la pollution est purement locale.

Mots clés: Libéralisation du commerce, taxes sur la pollution, pollution transfrontalière, pays hétérogènes, marchés imparfaits

Classification JEL: D43, F18, H23, Q58

1 Introduction

There is a growing concern among environmentalists about the negative effects of freer international trade on environment. The central point is that competitive pressures incurred by freer trade may oblige governments to dilute their environmental instrument. What is unfortunate is that despite the large difference among countries, papers that investigate the impact of trade liberalization on pollution taxes and welfare work only under the restrictive assumption of identical countries. However, we frequently observe that ”small” countries trade with ”big” partners. In such situations, taking into account the trade structure is important to best characterize the equilibrium.

The goal of this paper is to examine how a reduction in trade barriers between potentially heterogenous open economies affects their environmental policies. Specifically, we consider a finite number of trading countries divided into two groups. Countries are identical within each group but differ between groups by the number of firms in their industry. We assume that in each country production entails pollution and that a fraction of pollution emitted in the country flows into the other countries. The governments use tariffs on imports and pollution taxes in order to correct the distortion created by this global pollution. We are interested in how, in this context, a tariff reduction can affect the equilibrium output, the equilibrium pollution taxes and the equilibrium social welfare.

The problem described above will be modeled as an oligopolistic trade game where the tariff will be assumed to be the same for all the countries. In a first stage, in each country, the relevant authority chooses unilaterally the pollution tax that maximizes the social welfare of the country. In a second stage, given the tariff on export and the pollution tax rates, each firm decides how much to produce for the home market and how much for the foreign market.

A number of studies have examined the issue of global pollution in an international oligopolistic setting. Among them, Barrett (1994), Kennedy (1994) and Markusen (1975) ask how strategic environmental policies compared to the first best outcome. Their common result is that the pollution taxes set unilaterally are in general not socially optimal. Those studies assume free trade and identical countries in their analysis. In this paper, we relax these two assumptions and focus on the analysis of the effects of multilateral-trade liberalization on equilibrium pollution taxes, equilibrium output and equilibrium welfare of countries.

The model used is closely related to that of Burguet and Sempere (2003) and to that of Baksi and Chaudhuri (2009). Burguet and Sempere (2003) explore the impacts of a uniform tariff reduction on welfare and environmental policy. They show that a bilateral tariff reduction can affect environmental policy through two channels. First, they find that a bilateral tariff reduction always increases output which in turn lowers price and increases marginal damages of output. This incites governments to raise their environmental protection level by increasing the pollution taxes. Second, a bilateral tariff reduction diminishes revenues from imports and reduces the cost of exports, hence encouraging governments to dilute their environmental protection. The net effect of a tariff reduction on environmental policy depends on which channel outweighs the other. They also show that when the environmental policy is a pollution tax, a bilateral tariff reduction always improves welfare. The limitations of that paper are that it considers only bilateral trade between identical countries, with a monopoly in each country, and considers local pollution only.

Baksi and Chaudhuri (2009) extend that paper to an arbitrary number of firms in each of the two trading countries and also allow for many types of pollution. On the one hand, they show that trade liberalization always increases the output level in each country. It also increases the pollution tax when pollution is sufficiently harmful. On the other hand, they find that trade liberalization always improves social welfare when pollution is purely local.

In this paper, as in Baksi and Chaudhuri (2009), we consider varying degree of spillover of pollution to other countries, going from purely local pollution to totally global pollution. However, our approach is more general in some key respects: there is an arbitrary number of countries involved in trade; there is an arbitrary number of countries divided into two groups according to the number of firms in their industry; the number of countries in each group can differ. We focus on the impacts of this type of asymmetry on multilateral environmental policies.

To do this, we derive the Nash equilibrium pollution taxes, the equilibrium output and the equilibrium social welfare. We examine the effects of a tariff reduction on these equilibrium outcomes and compare them with those obtained when all the countries are identical. In particular, we compare the results related to the situation in which two types of trading partners coexist on the world market with the one where all trading partners are identical.

Unlike the case of identical countries in which trade liberalization always increases output, two situations may arise when the two types of countries coexist. Trade liberalization may increase output of the countries in one group while lowering output of the countries in the other group. It may also increase the output of all the countries. As in Baksi and Chaudhuri (2009), we find that in the identical countries setting, trade liberalization increases the pollution taxes when the pollution is sufficiently harmful. Moreover, social welfare is concave in the trade tariff and trade liberalization always increases social welfare when the pollution is purely local. However, in the presence of asymmetry, these results may not hold, depending on the range of asymmetry and the number of actors involved in trade.

The remainder of this paper is organized as follows. Section 2 sets out and solves the model. Section 3 presents the outcomes of the model obtained with identical countries. Section 4 compares the results of the asymmetric model to those derived with identical countries. Section 5 concludes.

2 The model

Consider a world of $N \geq 1$ countries, divided into two groups. Countries are identical within each group but differ between groups by the number of firms in their industry. The first group is made of N_1 countries and the industry of each country in that group has n_1 firms. The second group is constituted of N_2 countries and each country in that group has n_2 firms.

Industries are assumed to produce a homogenous good. They use the same technology of production and c is their constant unit cost of production.

A single firm that resides in country j produces and ships quantity y_j^i of the good to the market of country i. For simplicity, there is not storage. Firms compete in quantities in the market of their own country and in each foreign market, like in the reciprocal dumping game by Brander and Krugman (1983). The inverse demand is the same for countries in both groups and is given by

$$P(y^i) = a - y^i; \;\; a > c, \tag{1}$$

where y^i denotes the total quantity demanded in country i.

Each country levies a tariff z on each unit of import from foreign countries. The tariff is exogenous and is the same in both groups of countries. Multilateral trade liberalization is defined as a uniform reduction of the tariff in the N countries.[1]

During their activity of production, firms emit pollution that damages a shared environmental resource. It is assumed that one unit of production generates one unit of pollution and that pollution is transboundary. We denote by a parameter $\lambda \in [0, 1]$ the fraction of pollution emitted in one country that damages the other countries with $\lambda = 0$ being strictly local pollution and $\lambda = 1$ being perfectly global pollution.

The damage cost function from the pollution of country j is assumed to be quadratic, convex, and increasing in the pollution level:

$$D_j = \frac{\gamma}{2}\left(y_j + \lambda \sum_{k \in N_1 \setminus \{j\}} y_k + \lambda \sum_{k \in N_2 \setminus \{j\}} y_k \right)^2,$$

where $\gamma \geq 0$ is the damage cost parameter and, for all $k = 1, ..., N$,

$$y_k = n_i \sum_{i_1 \in N_1} y_k^{i_1} + n_i \sum_{i_2 \in N_2} y_k^{i_2}, \tag{2}$$

is the total output produced in country k; where, $i = 1$ if $k \in N_1$ and $i = 2$ if $k \in N_2$.

[1] This situation prevails for instance in NAFTA where member countries are asked to diminish uniformly their trade tariff over a defined calendar of time.

The environmental instrument in each country is a pollution tax imposed by its government to its domestic firms. Denote by t_i the pollution tax per unit of pollution in country i.

In a first stage, the relevant authority in each country decides the tax level that maximizes the country's social welfare considering as given the tax level of the remaining countries. It also considers the common tariff z (per unit of export) in both groups of country as given. In a second stage, each firm decides the output level that maximizes its profits. In that decision, it considers the output level of the remaining $n_1N_1 + n_2N_2 - 1$ firms and the set of taxes in both groups of countries as given. The subgame perfect Nash equilibrium is derived using backward induction.

2.1 The second stage of the game: output decision of firms

The typical firm that operates in country j chooses the output strategy $\{y_j^i\}_{i=1}^{i=N}$ that maximizes its profit, namely:

$$\max_{\{y_j^i\}_{i=1}^{i=N}} \sum_{i=1}^{N} y_j^i(a - y^i) - \sum_{i=1, i\neq j}^{N} z y_j^i - (c + t_j) \sum_{i=1}^{N} y_j^i, \tag{3}$$

where

$$y^i = n_1 \sum_{j_1 \in N_1} y_{j_1}^i + n_2 \sum_{j_2 \in N_2} y_{j_2}^i \tag{4}$$

denotes the total quantity sold in country i, for $i = 1, ..., N$.

Assuming an interior solution, the first-order conditions for this problem are:

$$a - y^i - y_j^i = z + c + t_j, \quad \forall i \neq j \tag{5a}$$

$$a - y^j - y_j^j = c + t_j, \quad \forall j = 1, ..., N. \tag{5b}$$

Define the home (export) augmented marginal cost as the marginal cost c plus tax t_j (marginal cost c plus tax t_j and tariff z). The left-hand sides of (5a) and (5b) are respectively marginal revenue from export and from domestic sales. These fist-order conditions say that a given firm allocates for export (home) the output level for which the marginal revenue of production for export (home) equals to its export (home) augmented marginal cost of production.

For later reference, we calculate in what follows the equilibrium output and consumption for each country. Solving the system of equations (4), (5a) and (5b), we obtain the total sales of the good for country i_1 in the first group:

$$y^{i_1} = [(n_1N_1 + n_2N_2)(a - c) - z(n_1(N_1 - 1) + n_2N_2) - n_1 \sum_{j_1 \in N_1} t_{j_1} - n_2 \sum_{j_2 \in N_2} t_{j_2}]/d, \quad (6a)$$

where $d = 1 + n_1N_1 + n_2N_2$. Using a similar reasoning, we verify that total sales of the good for country i_2 in the second group is:

$$y^{i_2} = [(n_1N_1 + n_2N_2)(a - c) - z(n_1N_1 + n_2(N_2 - 1)) - n_1 \sum_{j_1 \in N_1} t_{j_1} - n_2 \sum_{j_2 \in N_2} t_{j_2}]/d. \quad (6b)$$

Using (6a) and (6b) we get: $y^{i_1} - y^{i_2} = z(n_1 - n_2)$, which is positive if and only if $n_1 > n_2$. Thus, in the case of pure asymmetry ($n_1 \neq n_2$), each country in the group which has the largest number of firms consumes more than a country in the other group.

Substituting (6a) into (5b), we get the quantity produced and consumed in country i of the first group:

$$y_i^i = -t_i + \{a - c + z[(N_1 - 1)n_1 + n_2N_2] + n_1 \sum_{j_1 \in N_1} t_{j_1} + n_2 \sum_{j_2 \in N_2} t_{j_2}\}/(1 + n_1N_1 + n_2N_2). \quad (6c)$$

Similarly, substituting (6b) into (5b), we obtain the output level produced and consumed in country i of the second group:

$$y_i^i = -t_i + \{a - c + z[n_1N_1 + (N_2 - 1)n_2] + n_1 \sum_{j_1 \in N_1} t_{j_1} + n_2 \sum_{j_2 \in N_2} t_{j_2}\}/(1 + n_1N_1 + n_2N_2). \quad (6d)$$

Now, substituting (6a) into (5a), we derive the quantity of the good produced by a firm in country j and shipped to country i of the first group:

$$y_j^i = -t_j + \{a - c - (1 + n_1)z + n_1 \sum_{j_1 \in N_1} t_{j_1} + n_2 \sum_{j_2 \in N_2} t_{j_2}\}/(1 + n_1N_1 + n_2N_2), \text{ for all } j \neq i. \quad (6e)$$

Substituting (6b) into (5a) yields the quantity of the good produced by a firm in a given country j and shipped to a given country i belonging to the second group:

$$y_j^i = -t_j + \{a - c - (1 + n_2)z + n_1 \sum_{j_1 \in N_1} t_{j_1} + n_2 \sum_{j_2 \in N_2} t_{j_2}\}/(1 + n_1N_1 + n_2N_2), \text{ for all } j \neq i. \quad (6f)$$

Substituting (6c)-(6f) in (2), we get the total output produced by a country j_1 in the first group:

$$y_{j_1} = -n_1 N t_{j_1} + n_1 N(a-c)/d - (N-1)n_1 z/d + n_1 N[n_1 \sum_{j\in N_1} t_j + n_2 \sum_{j \in N_2} t_j]/d. \quad (6g)$$

Likewise, plugging (6c)-(6f) in (2) we obtain the total output produced by a country j_2 in the second group which is given by:

$$y_{j_2} = -n_2 N t_{j_2} + n_2 N(a-c)/d - (N-1)n_2 z/d + n_2 N[n_1 \sum_{j\in N_1} t_j + n_2 \sum_{j \in N_2} t_j]/d. \quad (6h)$$

From (6g) and (6h) we observe that while an exogenous increase of the national tax always lowers national production, an exogenous increase of the foreign taxes raises the national production.

Each country's net import is the difference between its total consumption and its total production. Using (6a), (6b), (6g) and (6h), we derive the expressions for the net import of each country in the first and in the second group, which are respectively given by:

$$y^{i_1} - y_{i_1} = n_1 N t_{i_1} - (1+n_1 N)[n_1 \sum_{j_1 \in N_1} t_{j_1} + n_2 \sum_{j_2 \in N_2} t_{j_2}]/d + (a-c-z)N_2(n_2-n_1)/d,$$
$$y^{i_2} - y_{i_2} = n_2 N t_{i_2} - (1+n_2 N)[n_1 \sum_{j_1 \in N_1} t_{j_1} + n_2 \sum_{j_2 \in N_2} t_{j_2}]/d + (a-c-z)N_1(n_1-n_2)/d.$$

In each country, the net import is increasing in its own pollution tax and is decreasing in the foreign pollution tax. Note that in the case of symmetric industry size ($n_1 = n_2$), the net import does not depend on the tariff. However, in the case of asymmetric industry sizes ($n_1 \neq n_2$), the net imports of countries with higher industry size are affected negatively by a tariff reduction, while the reverse is true for countries with the lower industry size. This is the extension to asymmetry of a result by Baksi and Chaudhuri (2009) and Burguet and Sempere (2003). Recall that both papers investigate the effects of a tariff reduction on the optimal pollution tax for two identical trading countries. They find, among other things that the net import of each country does not depend on the trade tariff.

2.2 First stage: environmental policy

In the first stage of the game, the government of each country chooses the pollution tax that maximizes the country's welfare, considering as given the tariff level and the pollution tax of the other countries.[2] Welfare for each country is the sum of the consumer surplus, the producer surplus, the tariff revenue and the pollution tax revenue, minus the pollution damage. Its expression for country $j \in N_1 \cup N_2$ is given by:[3]

$$SW_j(t_j, t_{-j}, z) = CS_j + PS_j + TR_j + ER_j - D_j, \tag{7a}$$

where

$CS_j = \int_0^{y^j} p(x)dx - p(y^j)y^j$ is the consumer surplus,

$PS_j = n_k \sum_{i=1}^{N} y_j^i p(y^i) - (c + t_j)y_j - n_k z \sum_{i=1, i\neq j}^{N} y_j^i$ is the producer surplus,

$TR_j = zn_1 \sum_{k_1 \in N_1 \setminus \{j\}} y_{k_1}^j + zn_2 \sum_{k_2 \in N_2 \setminus \{j\}} y_{k_2}^j$ is the tariff revenue,

$ER_j = t_j n_k \sum_{i_1 \in N_1} y_j^{i_1} + n_k t_j \sum_{i_2 \in N_2} y_j^{i_2} = t_j y_j$ is the pollution tax revenue, where the y_j^i, y^j are given by (6a)-(6f), and where $k = 1$ if $j \in N_1$ and $k = 2$ if $j \in N_2$. Using these results, Condition (7a) can be rewritten as

$$SW_j(t_j, t_{-j}, z) = \int_0^{y^j} p(x)dx + [n_k \sum_{i=1}^{N} y_j^i p(y^i) - p(y^j)y^j] + z[y^j - y_j] - cy_j - D(w_j), \tag{7b}$$

where $w_j = y_j + \lambda \sum_{k \in N_1 \setminus \{j\}} y_k + \lambda \sum_{k \in N_2 \setminus \{j\}} y_k$ represents the total emissions discharged in country j; $D(x) = \gamma x^2/2$ for all $x \geq 0$ and where $k = 1$ if $j \in N_1$ and $k = 2$ if $j \in N_2$. In Expression (7b), the first right-hand side term represents the gross (domestic) consumer surplus. The second term represents the net balance of trade. That is, the total revenue of exports for country j net of the value of imports.[4] The third term is the net tariff revenue. The last two terms represent the social cost of production.

The first-order conditions for the maximization of (7b) yield the best-response pollution tax for country j. The expression for the equilibrium tax of country j, $t_j(t_{-j})$ that depends

[2]This results in a Nash equilibrium pollution tax which is not in general socially efficient as pointed out by Kennedy (1994).

[3]In the expression $SW_j(t_j, t_{-j})$, t_{-j} represents the vector of taxes of the countries other than j.

[4]To see this, remark that $n_k \sum_{i \in N} y_j^i p(y^i) - p(y^j)y^j = n_k \sum_{i \in N \setminus \{j\}} y_j^i p(y^i) - p(y^j)[n_1 \sum_{k_1 \in N_1 \setminus \{j\}} y_{k_1}^j + n_2 \sum_{k_2 \in N_2 \setminus \{j\}} y_{k_2}^j]$.

on taxes of other countries and on parameters of the model. The second-order condition for welfare maximization is verified since we have the following inequality:

$$\frac{\partial^2 SW_j}{\partial(t_j)^2}(t_j, t_{-j}, z) = A_k - \gamma n_k^2 N^2[-1 + \frac{n_k(1-\lambda) + \lambda(n_1 N_1 + n_2 N_2)}{1 + n_1 N_1 + n_2 N_2}]^2 < 0, \quad (7c)$$

where $A_k = n_k^2[1 + 2(N_1 + N_2)(n_k - 1 - n_1 N_1 - n_2 N_2)]/(1 + n_1 N_1 + n_2 N_2)^2 < 0$, and where $k = 1$ if $j \in N_1$ and $k = 2$ if $j \in N_2$.[5]

The tax policies at the equilibrium for the first group and the second group of countries are given respectively by:[6]

$$t_1 = [z(v_1\hat{e}_2 - e_2\hat{v}_1) + (a - c)(v_2\hat{e}_2 - e_2\hat{v}_2)]/(e_1\hat{e}_2 - e_2\hat{e}_1), \quad (8a)$$

$$t_2 = [z(\hat{v}_1 e_1 - \hat{e}_1 v_1) + (a - c)(\hat{v}_2 e_1 - \hat{e}_1 v_2)]/(e_1\hat{e}_2 - e_2\hat{e}_1), \quad (8b)$$

where, $e_i, \hat{e}_i, v_i, \hat{v}_i$ for all $i = 1, 2$ are given in the appendix. Hereafter we restrict our attention to the set of parameters for which the equilibrium output and taxes are positive.

In each country, the equilibrium pollution tax results in the strategic interaction of five sources of market failure. First, the rent capture effect that tends to lower the equilibrium pollution tax. Since the market is imperfect, each government has the incentive to provide an edge to its domestic firms so that they can gain more rent through their exports. Second, the pollution-shifting effect increases the equilibrium pollution taxes as each country tends to shift output and its associated pollution to the foreign countries.[7] Third, the transboundary externality effect that tends to lower the equilibrium pollution tax, as each country does not care about the damages associated to its pollution on the well being of the other countries. Notice that the last two effects vanish when the good is clean ($\gamma = 0$). The tariff may also affect the equilibrium tax. Indeed, for any positive tariff z, countries collect revenues from their imports and pay fees on their exports. Thus, they may have an incentive to substitute

[5] $A_1 < 0$ indeed: in its expression, denote by $g(n_1)$ the quantity in square brackets. Since $g'(n_1) = 2(N_1 + N_2)(1 - N_1) \leq 0$ and $g(1) = 1 - 2(N_1 + N_2)(N_1 + n_2 N_2) < 0$, it follows that $g(n_1) < 0$ for all $n_1 \geq 1$. Using a similar reasoning, we can show that $A_2 < 0$.

[6] We show in the appendix how to derive the expressions of equilibrium taxes (8a) and (8b).

[7] The expression of the rent capture effect and the one of the pollution shifting effect are given in the appendix.

foreign production for local production and then increase the tax on local production. Finally, the "price effect" that makes prices to differ across groups also impacts the pollution taxes. When asymmetry prevails, the consumption level varies across groups, and as a result, according to (1), so does the price. Each low market price country will then have a greater incentive to soften its environmental tax in order to gain foreign rents as compared to each high market price country. This induces different rent capture effects across groups. Notice that the price effect exists only in the presence of asymmetry.

3 Symmetric equilibrium

Setting in (8a) $N_1 = N$, $N_2 = 0$ and $n_1 = n$, we get the tax level for the symmetric equilibrium, which is given by:

$$t_s = \frac{(N-1)\{1+nN(1+n)-\gamma nN[1+\lambda(N-1)][1+n(N-1)(1-\lambda)]\}z+\tau_s}{nN^2[1+n(N-1)+\gamma(1+\lambda(N-1))+n(N-1)\gamma(1-\lambda^2)]}, \tag{9}$$

where the subscript s stands for the symmetric equilibrium and where
$\tau_s = (a-c)N\{n-1-nN+\gamma nN[1+\lambda(N-1)][1+n(N-1)(1-\lambda)]\}$.

Substituting (9) into (6g), we get the total production of each country when all the countries are identical, given by:

$$y_s = \frac{(a-c)(1+n(N-1))-(N-1)(1+nN)z}{N[1+\gamma(1-\lambda+N\lambda)+n(N-1)(1+\gamma(1-\lambda)(1+\lambda(N-1)))]}. \tag{10}$$

Since y_s is linear in z and has a negative slope, a reduction of the tariff results in an increase of the national production of each country.

3.1 Effect of tariff reduction on the equilibrium tax: the symmetric case

The effect of a tariff reduction on the pollution tax depends on how this reduction affects each of the effects detailed above and the resulting interaction.

Using (9), we derive

$$\frac{\partial t_s}{\partial z} = \frac{(N-1)\{1+nN(1+n)-\gamma nN[1+\lambda(N-1)][1+n(N-1)(1-\lambda)]\}}{nN^2[1+n(N-1)+\gamma(1+\lambda(N-1))+n(N-1)\gamma(1-\lambda^2)]}. \tag{11}$$

Since the denominator of (11) is positive, the sign of that expression is the same as that of its numerator. Solving the equation $\frac{\partial t_s}{\partial z} = 0$ for the transboundary pollution parameter, λ, we obtain two roots

$$\underline{\lambda} = [\gamma n N(N-1)(1+n(N-2)) - \sqrt{\Delta}]/(2n^2\gamma N(N-1)^2),$$
$$\overline{\lambda} = [\gamma n N(N-1)(1+n(N-2)) + \sqrt{\Delta}]/(2n^2\gamma N(N-1)^2),$$

where

$\Delta = N\gamma(n(N-1))^2(N\gamma(1+nN)^2 - 4(1+nN(1+n)))$.

The above roots are real if and only if $\Delta \geq 0$. This last condition is equivalent to

$$\gamma \geq \frac{4(1+nN(1+n))}{N(1+nN)^2} \equiv \gamma_1$$

Furthermore $\underline{\lambda} \geq 0$ if and only if

$$\gamma \leq \frac{1+nN(1+n)}{nN(1+n(N-1))} \equiv \gamma_2$$

and $\overline{\lambda} \leq 1$ if and only if

$$\gamma \leq \frac{1+nN(1+n)}{nN^2} \equiv \gamma_3$$

These computations lead to the following proposition.

Proposition 1 *Under symmetry, we have:* (*i*) *if* $\gamma < \gamma_1$ *then* $\frac{\partial t_s}{\partial z} > 0$. *When the damage cost parameter is sufficiently small, multilateral trade liberalization lowers the equilibrium pollution tax, regardless to the remaining feasible parameters of the model.* (*ii*) *If* $\gamma \in [\gamma_1, \gamma_2]$, *then* $\frac{\partial t_s}{\partial z} > 0$ *if and only if* $\lambda \leq \underline{\lambda}$ *or* $\lambda \geq \overline{\lambda}$. (*iii*) *if* $\gamma \in [\gamma_2, \gamma_3]$, *then* $\frac{\partial t_s}{\partial z} > 0$ *if and only if* $\lambda \geq \overline{\lambda}$. (*iv*) *if* $\gamma > \gamma_3$ *then* $\frac{\partial t_s}{\partial z} < 0$.

Proof. See the appendix.■

Notice that the above thresholds of the damage cost parameter have the following features. First they satisfy the inequalities $\gamma_3 \geq \gamma_2 \geq \gamma_1$. They are also decreasing functions in the N number of countries participating in trade. In addition, each of them goes to zero as N goes to infinity so that only case (iv) of Proposition 1 is likely to hold when each γ_i goes to zero. As a result, for the symmetric equilibrium, when the number of countries involved in trade becomes sufficiently large, multilateral trade liberalization is more likely to increase the environmental pollution tax. The result (iv) in Proposition 1 can be seen as the "mitigation effect". Indeed, it states that if the damages are too harmful, countries must raise their environmental tax in response to a tariff reduction. This in turn will lower the national production of the dirty good in each country (see, Equation 6g or Equation 6h). The overall effect will be the mitigation of the damages incurred from the global pollution.

The intuition underlying the results of Proposition 1 is as follows. The tariff reduction can either lower or increase the pollution tax depending on which of the two opposite forces resulting from such a reduction outweighs the other. (i) By increasing output, the tariff reduction lowers prices, raises environmental damages and reduces economic rents, inducing tougher environmental policies. (ii) By lowering the revenue from imports and the tariff cost on exports, the tariff reduction diminishes the incentive to substitute foreign output for domestic production by raising the tax. This incentive tends to diminish the equilibrium pollution tax.

3.2 Effect of tariff reduction on welfare: the symmetric case

Substituting (9) into (7b) yields SW_s, the expression for the welfare of the typical country in the symmetric setting that depends on the tariff z and the remaining parameters of the model. Its derivative with respect to z is given by

$$\frac{\partial SW_s}{\partial z} = \frac{(N-1)^2(1+nN)^2[(a-c)N\gamma\lambda(1-\lambda+N\lambda) - z(1+\gamma(1+\lambda(N-1))^2)]}{N^2[-1+\gamma(-1+\lambda-N\lambda) - n(N-1)(1+\gamma(1-\lambda)(1+\lambda(N-1)))]^2}. \quad (12)$$

Proposition 2 *Under symmetry, there exists a tariff threshold* $\hat{z} \equiv \frac{(a-c)N\gamma\lambda[1+\lambda(N-1)]}{1+\gamma[1+\lambda(N-1)]^2}$*, under which a tariff reduction lowers the well-being of each country. Above that threshold, a tariff reduction improves the well-being (i.e.*$\frac{\partial SW_s}{\partial z} \leq 0$ *if and only if* $z \geq \hat{z}$*).*

To better understand the results of Proposition 2, remark that $y_j = y^j = y_s$ for $j = 1, ..., N$, which implies that the second and the third right-hand side terms of (7b) vanish at the equilibrium. Consequently, the equilibrium social welfare can be rewritten as

$$SW_s(z) \equiv SW_j(t_j(z), t_{-j}(z), z) = \int_0^{y_s} p(x)dx - cy_s - D(w_s), \tag{13}$$

where $w_s = y_s(1 + \lambda(N-1))$ and where y_s is defined by (10). Differentiating (13) with respect to z yields

$$\frac{\partial SW_s}{\partial z}(z) = \frac{\partial y_s}{\partial z}[p(y_s) - c - (1 + \lambda(N-1))D'(w_s)].$$

Since $\frac{\partial y_s}{\partial z} < 0$, this relation shows that a marginal tariff reduction increases welfare only when the price is initially greater than the marginal social cost of production. Proposition 2 suggests that such a condition holds only when the initial tariff is sufficiently large.

In the case of purely local pollution, we have $\hat{z} = 0$, which by Proposition 2 implies that a reduction of the tariff always increases the payoff of each country. Furthermore, differentiating $\hat{z}$ with respect to N, we get:

$$\frac{\partial \hat{z}}{\partial N} = (a-c)N\gamma\lambda \frac{1 + (2N-1)\lambda + \gamma(1-\lambda)(1+\lambda(N-1))^2}{[1+\gamma(1+\lambda(N-1))^2]^2} > 0 \text{ for } \lambda\gamma > 0.$$

This inequality, combined with the results of Proposition 2, suggest that for a spillover pollution problem, as the number of countries involved in trade increases, trade liberalization is less likely to improve the well being of each country.

Notice that this section represents an immediate extension of Baksi and Chaudhuri (2009), which analyze the particular case where $N = 2$. It serves as a benchmark for identifying channels by which trade liberalization affect output, environmental policies and welfare. We next investigate the role of asymmetry.

4 Effects of asymmetry

This section considers first the simple case of bilateral trade prevailing between country 1 and country 2 by allowing for the number of firms to differ across countries. Refer the former to "Home" and the latter to "Foreign". As we are interested in the effects of having the two types of countries involved in trade rather than having all the countries identical, we start by deriving the equilibrium under asymmetry and compare it with the equilibrium under the symmetric setting calculated in Section 3.

Studying variations of the pollution taxes in (8a) and (8b), for the case where $N_1 = N_2 = 1$, we get:

Proposition 3 *Assume that pollution is perfectly transboundary i.e.* $\lambda = 1$. *(i) If* $1 \leq n_1 \leq \bar{n}_1(n_2)$*, then a bilateral tariff reduction lowers the pollution taxes in Home even if pollution is harmful enough. (ii) If* $n_1 > \bar{n}_1(n_2)$*, then the bilateral tariff reduction diminishes the pollution taxes in Home if and only if*

$$\gamma < \bar{\gamma} \equiv \frac{2 + 2n_1^3 + n_2(8 + 5n_2) + 2n_1 n_2(3 + n_2) + n_1^2(1 + 4n_2)}{4[n_1(4n_1 + 5 + 2n_2) - n_2(3 + 2n_2)]}.$$

(iii) If $n_1 \geq \tilde{n}_1(n_2)$*, then the bilateral tariff reduction lowers the pollution taxes in Foreign even if pollution is harmful enough. (iv) If* $n_1 < \tilde{n}_1(n_2)$*, the tariff reduction lowers the pollution taxes in Foreign only when*

$$\gamma < \tilde{\gamma} \equiv \frac{2 + n_2^2(n_2 + 2) + n_1^2(5 + 2n_2) + n_1(8 + 6n_2 + 4n_2^2)}{4[n_1(-2n_1 + 2n_2 - 3) + n_2(5 + 4n_2)]},$$

where $\tilde{n}_1(n_2) > \bar{n}_1(n_2) \geq 0$ *are defined in the appendix.*

Proof. See the appendix.■

Proposition 3 highlights a new strategic interaction prevailing in the presence of asymmetry. Indeed, when all the countries are identical, the tariff reduction always lowers the pollution taxes for sufficiently harmful pollution. However, in the presence of asymmetry, the interaction of the price effect and the above mentioned forces lead to a reverse result.

As shown in the appendix, our analysis also suggests that when pollution is not perfectly transboundary *i.e.* $0 \leq \lambda < 1$, a bilateral tariff reduction always increases the pollution taxes when pollution is sufficiently harmful (*i.e.* for γ sufficiently large).

We will next investigate the effect of a bilateral tariff reduction on output.

Proposition 4 *Assume that pollution is perfectly transboundary. (i) The bilateral tariff reduction raises output in Home only when* $1 \leq n_1 < \hat{n}_1(n_2)$*. (ii) The bilateral tariff reduction* *raises output in Foreign only when* $n_1 > \breve{n}_1(n_2)$, *where* $\hat{n}_1(n_2)$ *and* $\breve{n}_1(n_2)$ *are defined in the appendix.*

Proof. See the appendix.■

Proposition 4 shows that contrary to the case where countries are identical, in the context of perfectly transboundary pollution, a bilateral tariff reduction actually increases output in a country and lowers the one of the other country for n_1 satisfying $1 \leq n_1 < \min(\hat{n}_1(n_2), \breve{n}_1(n_2))$ or $n_1 > \max(\hat{n}_1(n_2), \breve{n}_1(n_2))$. It raises output in all countries if $n_1 \in (\breve{n}_1(n_2), \hat{n}_1(n_2))$. It lowers output in all countries only when $n_1 \in (\hat{n}_1(n_2), \breve{n}_1(n_2))$.[8]

To gain the intuition underlying the results of Proposition 4, differentiating (6g) with respect to z, we get: $\frac{\partial y_1}{\partial z} < 0$ if and only if

$$\frac{\partial t_1}{\partial z} > \frac{n_2}{1+n_2}.\frac{\partial t_2}{\partial z} - \frac{1}{2(1+n_2)}. \tag{14a}$$

Likewise, using (6h), we derive: $\frac{\partial y_2}{\partial z} < 0$ if and only if

$$\frac{\partial t_2}{\partial z} > \frac{n_1}{1+n_1}.\frac{\partial t_1}{\partial z} - \frac{1}{2(1+n_1)}. \tag{14b}$$

These results show that the bilateral tariff reduction may increase or lower output in a country depending on how such a reduction affects the country's pollution tax as compared to that of the other country. When n_1 is small and $\gamma > \tilde{\gamma}$, by the results (i) and (iv) of Proposition 3, $\frac{\partial t_1}{\partial z} > 0$ and $\frac{\partial t_2}{\partial z} < 0$ so that by (14a), we have $\frac{\partial y_1}{\partial z} < 0$. Likewise, when n_1 is large and $\gamma > \bar{\gamma}$, results (ii) and (iii) of Proposition 3 suggest that $\frac{\partial t_1}{\partial z} < 0$ and $\frac{\partial t_2}{\partial z} > 0$, which by (14b) imply that $\frac{\partial y_2}{\partial z} < 0$. More generally, Conditions (14a) and (14b) read as follows: the tariff reduction raises output for a country only when such a reduction generates an increase in the pollution tax in that country greater than the one generated by such a reduction on the pollution tax of the other country. The latter increase should be adjusted by the foreign industry size. We provide a numerical example in the appendix in support to the above results.

[8] Notice that depending on values of γ and n_2, we can either have $\breve{n}_1(n_2) > \hat{n}_1(n_2)$ or $\hat{n}_1(n_2) > \breve{n}_1(n_2)$ or $\hat{n}_1(n_2) = \breve{n}_1(n_2)$.

The above findings suggest an interesting policy implication: the claim done by some environmentalists that trade liberalization generates more pollution is not necessarily true. However, it is important to mention that this result has been obtained in a particular context. Namely, we consider for simplicity a constant pollution intensity (defined as pollution per unit of output) and our analysis relies on specific preferences. We now investigate the effect of a tariff reduction on welfare.

Denote by $SW_1(z)$ and $SW_2(z)$ the equilibrium welfare in Home and in Foreign, respectively. What first clearly appears is that $SW_1(z)$ and $SW_2(z)$ are second degree polynomials in z and they can be either concave or convex.

Indeed, substituting (8a) and (8b) into (6a)-(6f), we get the equilibrium values for $\bar{y}^i, \bar{y}_i$, $\bar{y}_i^p$ produced by each country. Since the $\bar{y}^i, \bar{y}_i$, $\bar{y}_i^p$ are linear in z, the particular quadratic functional form of (7b) in $\bar{y}_i^p$ shows that $SW_1(z)$ and $SW_2(z)$ are second degree polynomials in z. Moreover, for $k = 1, 2$, SW_k is concave if and only if $\frac{\partial^2 SW_k}{\partial z^2}(z) < 0$. It is convex when $\frac{\partial^2 SW_k}{\partial z^2}(z) > 0$.

In order to give a support to the above results, consider the case where $n_1 = 1, n_2 = 2$ and for arbitrary values of a, λ, γ and c. Whether SW_k is concave or convex depends on values of the spillover parameter and the damage cost parameter. Figure 1 and 2 in the appendix illustrate these situations. For example, if $\lambda \in (0, 1)$ and $\gamma \in (0.3, 1)$ then $SW_1(z)$ and $SW_2(z)$ are both concave. If $\lambda \in (0, 1)$ and $\gamma \in (0, 0.1)$ then $SW_1(z)$ is convex and $SW_2(z)$ concave. In the symmetric model of Section 3, we have proved that the social welfare is necessarily concave in the tariff z as in Baksi and Chaudhuri (2009). This simple example shows that such is not the case in the presence of asymmetry.

Proposition 5 *Let $\bar{x}_k$ be the solution of the equation $\frac{\partial SW_k}{\partial z}(z) = 0$ and $\bar{z}_k = \max(0, \bar{x}_k)$.*
(i) When SW_k is concave, we have $\frac{\partial SW_k}{\partial z}(z) < 0$ if and only if $z > \bar{z}_k$. Trade liberalization improves welfare in country k only when the initial tariff is large.
(ii) When SW_k is convex, we have $\frac{\partial SW_k}{\partial z}(z) < 0$ if and only if $z < \bar{z}_k$. Trade liberalization increases welfare in country k only when the initial tariff is small.

For the particular case where pollution is purely local ($\lambda = 0$) and where $n_1 = 1, n_2 = 2$, and $\gamma = 1$, we get: $\frac{\partial SW_1}{\partial z}(z) > 0$ if and only if $z < 0.522(a-c)$ and $\frac{\partial SW_2}{\partial z}(z) > 0$ if and only if $z < 0.39(a-c)$. These results imply that bilateral trade liberalization lowers welfare of all the countries when the initial tariff is lower than $0.39(a-c)$. In addition, it reduces welfare, but only in Home when the initial tariff lies in the open interval $(0.39(a-c), 0.522(a-c))$. Recall that in the symmetric framework, as in Baksi and Chaudhuri (2009) and Burguet and Sempere (2003), we have shown that trade liberalization always improves welfare when pollution is purely local. This simple case highlights the limitation of such a finding in the presence of asymmetry.

In order to better understand these results, notice that Home's equilibrium social welfare (7b) for the case where $N_1 = N_2 = 1$, can be rewritten as

$$SW_1(z) = \int_0^{\bar{y}^1} p(x)dx + [n_1 \sum_{i=1}^{2} \bar{y}_1^i p(\bar{y}^i) - p(\bar{y}^1)\bar{y}^1] + z[\bar{y}^1 - \bar{y}_1] - c\bar{y}_1 - D(\bar{w}_1), \tag{15}$$

where $\bar{w}_1 = \bar{y}_1 + \lambda \bar{y}_2$ and where $\bar{y}_1, \bar{y}_2, \bar{y}^1, \bar{y}^2$ represent the equilibrium output and consumption.

Differentiating (15) with respect to z yields

$$\frac{\partial SW_1}{\partial z}(z) = n_1 \sum_{i=1}^{2} \left(\frac{\partial \bar{y}_1^i}{\partial z} p(\bar{y}^i) + \bar{y}_1^i \frac{\partial \bar{y}^i}{\partial z} p'(\bar{y}^i) \right) - \bar{y}^1 \frac{\partial \bar{y}^1}{\partial z} p'(\bar{y}^1) + z \frac{\partial \bar{y}^1 - \bar{y}_1}{\partial z} + (\bar{y}^1 - \bar{y}_1)$$
$$- c\frac{\partial \bar{y}_1}{\partial z} - \frac{\partial \bar{w}_1}{\partial z} D'(\bar{w}_1).$$

Notice that the first four right-hand side terms of this expression represent the marginal social benefit of production while the last two terms are the marginal social cost of production. Hence, the bilateral tariff reduction actually improves social welfare for country i (or equivalently $\frac{\partial SW_i}{\partial z}(z) < 0$) only when initially, the marginal social benefit is lower than the marginal social cost of production.[9]

Proposition 5 then shows that when social welfare for a given country is concave, the marginal social benefit is initially lower than the marginal social cost of production only

[9]The analysis done for Home can be repeated verbatim to obtain a similar result for Foreign.

when the initial tariff is sufficiently large. In such a case, a tariff reduction increases welfare. However, when the social welfare for a given country is convex, the marginal social benefit is initially lower than the marginal social cost of production only when the initial tariff is sufficiently small. In such a case, the tariff reduction raises welfare.

The outcomes of the tariff reduction on welfare differ from those of Section 3, which are obtained with identical countries for three main reasons. When all the countries are identical, for a given country, (*i*) in Equation 15, the second right-hand side term vanishes; (*ii*) the total consumption is equal to the total production so that in Equation 15, the third right-hand side term also vanishes; (*iii*) the tariff reduction always increases output. However, our analysis suggests the price effect acts in a way that these results do not hold in the presence of asymmetry.

We next extend the analysis done above to two layers of asymmetry. Countries are identical within groups, but differ between groups by the number of firms in their industry. In addition, the group sizes may also differ.

For the sake of tractability, assume that pollution is harmful enough and is perfectly transboundary. As proved in the appendix, there exist critical values n_1^{ss} and n_1^{**} for the industry size such that a tariff reduction raises the pollution taxes for countries in the first group only when their industry size is sufficiently large (*i.e.*, $n_1 > n_1^{**}$). Such a tariff reduction increases the pollution taxes for countries in the second group only when the industry size for countries in the other group is small (*i.e.*, $n_1 < n_1^{ss}$). These results are similar to those of Proposition 3. Their particularity comes from the fact that the critical values n_1^{ss} and n_1^{**} depend on the group sizes N_1 and N_2. On the other hand, output and welfare are determined by the pollution taxes (see for instance, Equations 6g, 6h and 7b). Consequently, the distribution of countries across groups impacts significantly the pollution taxes, output and welfare, outcome of a tariff reduction.

5 Conclusion

This paper has investigated the impacts of trade liberalization on equilibrium output, pollution taxes and welfare. Unlike the existing literature, we have considered the multilateral aspect of trade and have distinguished two types of countries according to the size of their industry, with the number of countries of each type potentially different. We have proved that asymmetry has a significant impact on the equilibrium outcome. This has been done by comparing the outcomes derived in the asymmetric setting to those in the symmetric setting where all the countries are identical.

In the symmetric setting, our results are similar to those obtained in Baksi and Chaudhuri (2009). Trade liberalization always increases output. If the pollution is harmful enough, countries will raise their environmental protection in response to trade liberalization as shown in Proposition 1. Furthermore, as in Burguet and Sempere (2003), trade liberalization always increases welfare when we have to do with a local pollution problem.

However, when asymmetry exists, the price effect comes out, altering those strategic interactions. In addition to the classical result of the symmetric model, trade liberalization may actually increase output only for the countries in one of the two groups, while decreasing output for the countries in the other group. Even if the pollution is harmful enough, it can be optimal to soften environmental policies in response to trade liberalization. Moreover, trade liberalization may not improve welfare even for strictly local pollution.

In this paper we have assumed an exogenous number of firms in each group of countries. In a symmetric setting with clean goods, Horstmann and Markusen (1992) have shown that allowing for endogenous firm location decisions might affect the analysis. Moreover, when pre-liberalization tariff rates differed across countries, a multilateral reduction in tariff could lead to different outcomes from those reported in this paper. Taking into account these effects could be an avenue for new research. However, our results still highlight clearly that asymmetry plays a crucial role in the outcome of trade liberalization.

Appendix

The first-order conditions for the pollution taxes are:

$$\frac{\partial SW_j}{\partial t_j}(t_j(z), t_{-j}(z), z) = 0 \quad \text{for all} \quad j \in N_1 \cup N_2. \tag{16}$$

For a symmetric equilibrium in each group, we have: $t_1 = t_j\ \forall j \in N_1$ and $t_2 = t_k\ \forall k \in N_2$. So, (16) can be rewritten as:

$$e_1 t_1 + e_2 t_2 \ = v_1 z + v_2(a - c), \tag{17a}$$

$$\hat{e}_1 t_1 + \hat{e}_2 t_2 \ = \hat{v}_1 z + \hat{v}_2(a - c), \tag{17b}$$

where

$e_1 = n_1^2 N_1(1 + 2n_1 N)/d^2 - N n_1^2(1 + N_1)/d + NB_1 n_1[1 - \lambda + \lambda N_1 + (1 - \lambda)n_2 N_2]/d;$

$e_2 = n_1 n_2 N_2(1 + 2n_1 N)/d^2 - n_1 n_2 N_2 N/d + NB_1 n_2 N_2[\lambda - n_1(1 - \lambda)]/d;$

$\hat{e}_1 = n_1 n_2 N_1(1 + 2n_2 N)/d^2 - n_1 n_2 N_1 N/d + NB_2 n_1 N_1[\lambda - n_2(1 - \lambda)]/d;$

$\hat{e}_2 = n_2^2 N_2(1 + 2n_2 N)/d^2 - N n_2^2(1 + N_2)/d + NB_2 n_2[1 - \lambda + \lambda N_2 + (1 - \lambda)n_1 N_1]/d;$

$v_1 = n_1[1 + n_1(2N - 1)]/d^2 + n_1(1 + n_1 - N - d)/d - (N - 1)B_1[(1 - \lambda)n_1 + \lambda(n_1 N_1 + n_2 N_2)]/d;$

$\hat{v}_1 = n_2[1 + n_2(2N - 1)]/d^2 + n_2(1 + n_2 - N - d)/d - (N - 1)B_2[(1 - \lambda)n_2 + \lambda(n_1 N_1 + n_2 N_2)]/d;$

$v_2 = -n_1(1 + 2n_1 N)/d^2 + n_1(1 + N)/d + NB_1[(1 - \lambda)n_1 + \lambda(n_1 N_1 + n_2 N_2)]/d;$

$\hat{v}_2 = -n_2(1 + 2n_2 N)/d^2 + n_2(1 + N)/d + NB_2[(1 - \lambda)n_2 + \lambda(n_1 N_1 + n_2 N_2)]/d;$

$B_1 = \gamma N n_1[-1 + (1 - \lambda)(n_1 - n_1 N_1 - n_2 N_2)]/d;$

$B_2 = \gamma N n_2[-1 + (1 - \lambda)(n_2 - n_1 N_1 - n_2 N_2)]/d;$

$d = 1 + n_1 N_1 + n_2 N_2.$

Solving (17a) and (17b), we get:

$$t_1 = [z(v_1\hat{e}_2 - e_2\hat{v}_1) + (a - c)(v_2\hat{e}_2 - e_2\hat{v}_2)]/(e_1\hat{e}_2 - e_2\hat{e}_1),$$

$$t_2 = [z(\hat{v}_1 e_1 - \hat{e}_1 v_1) + (a - c)(\hat{v}_2 e_1 - \hat{e}_1 v_2)]/(e_1\hat{e}_2 - e_2\hat{e}_1).$$

Impacts of the rent capture effect and the pollution shifting effect

In what follows, t_1 and t_2 are as defined in (8a) and (8b), respectively. The impact of the rent capture effect for a country in group i is: $REC_i \equiv t_{i|\gamma=z=0} = -(a - c)/n_i(N_1 + N_2)$.

The impact of the polluting shifting effect is defined as $\lim_{n_i \to \infty} t_{i|z=0}$. If the number of countries in group $i = 1, 2$ satisfies $N_i > 1$, then we will have

$$\lim_{n_i \to \infty} t_{i|z=0} = \frac{\gamma(a-c)(1-\lambda)(1-\lambda+\lambda(N_1+N_2))}{1+\gamma(1-\lambda)(1-\lambda+\lambda(N_1+N_2))}.$$

If $N_i = 1$, then we will have

$$\lim_{n_i \to \infty} t_{i|z=0} = \frac{\gamma(a-c)(1-\lambda)(1+\lambda N_k)[N_k+\gamma(1-\lambda)(1+N_k)^2(1+(1-\lambda)n_k N_k)]}{N_k+\gamma(1-\lambda)(b_0+b_1\gamma)},$$

where,

$b_0 = 1-\lambda+(3-\lambda+(1-\lambda)n_k)N_k+(1+\lambda+2(1-\lambda)n_k)N_k^2+(1-\lambda)n_k N_k^3$,

$b_1 = (1-\lambda)(1+N_k)^2(1+\lambda N_k)(1+(1-\lambda)n_k N_k)$,

and where $k = 1$ if $i \in N_2$; $k = 2$ if $i \in N_1$.

Proof of Proposition 1

In what follows, $\frac{\partial t_s}{\partial z}$ is defined in (11). Since the denominator of $\frac{\partial t_s}{\partial z}$ is positive, that fraction has the same sign as its numerator. Solving the equation $\frac{\partial t_s}{\partial z} = 0$ with respect to λ, we get two roots:

$$\underline{\lambda} = [\gamma n N(N-1)(1+n(N-2)) - \sqrt{\Delta}]/(2n^2\gamma N(N-1)^2),$$

$$\overline{\lambda} = [\gamma n N(N-1)(1+n(N-2)) + \sqrt{\Delta}]/(2n^2\gamma N(N-1)^2),$$

where

$\Delta = N\gamma(n(N-1))^2(N\gamma(1+nN)^2 - 4(1+nN(1+n)))$.

The above roots are real if and only if $\Delta \geq 0$. This last condition is equivalent to

$$\gamma \geq \frac{4(1+nN(1+n))}{N(1+nN)^2} \equiv \gamma_1.$$

In such a case, the roots satisfy: $\underline{\lambda} \leq \overline{\lambda}$. Furthermore $\underline{\lambda} \geq 0$ if and only if

$$\gamma \leq \frac{1+nN(1+n)}{nN(1+n(N-1))} \equiv \gamma_2$$

and $\overline{\lambda} \leq 1$ if and only if

$$\gamma \leq \frac{1+nN(1+n)}{nN^2} \equiv \gamma_3$$

Since $n \geq 1$, and $N \geq 2$ we have:

$$\frac{\gamma_2}{\gamma_3} = \frac{N}{1 + n(N-1)} \leq \frac{N}{1 + (N-1)} = 1.$$

We also have:

$$\frac{\gamma_1}{\gamma_2} = \frac{4n(1 + n(N-1))}{(1 + nN)^2} \equiv g(n).$$

So, in order to show that $\gamma_1 \leq \gamma_2$, it suffices to prove that g is bounded above by 1. Since we have: $g'(n) = 4/(1 + nN)^2 > 0$, $lim_{n \to +\infty} g(n)$ is an upper bound for g. But we have $lim_{n \to +\infty} g(n) = 4(N-1)/N^2 < 1$, the result then follows. The above inequalities suggest that $\gamma_1 \leq \gamma_2 \leq \gamma_3$.

Notice that the numerator of $\frac{\partial t_s}{\partial z}$ is a second degree polynomial in λ. Hence, if we have $\gamma < \gamma_1$, then $\frac{\partial t_s}{\partial z} > 0$. When $\gamma \in [\gamma_1, \gamma_2]$, we get $\frac{\partial t_s}{\partial z} > 0$ if and only if $\lambda \leq \underline{\lambda}$ or $\lambda \geq \overline{\lambda}$. When $\gamma \in [\gamma_2, \gamma_3]$, the condition $\frac{\partial t_s}{\partial z} > 0$ holds if and only if $\lambda \geq \overline{\lambda}$. If we have $\gamma > \gamma_3$, then $\frac{\partial t_s}{\partial z} < 0$.

Proof of Proposition 3

Using (8a), (8b) for $\lambda = 1$, we get:

$$\frac{\partial t_1}{\partial z} = \frac{2 + 2n_1^3 + 8n_2 + 5n_2^2 + 2n_1 n_2(3 + n_2) + 4\gamma a_1(n_1)}{4n_1(1 + n_1 + n_2)(2 + 4\gamma + n_1 + n_2)}, \tag{18a}$$

$$\frac{\partial t_2}{\partial z} = \frac{2 + n_2^2(2 + n_2) + n_1^2(5 + 2n_2) + n_1(8 + 6n_2 + 4n_2^2) + 4\gamma a_2(n_1)}{4n_2(1 + n_1 + n_2)(2 + 4\gamma + n_1 + n_2)}, \tag{18b}$$

where $a_1(n_1) = -4n_1^2 - n_1(5+2n_2) + n_2(3+2n_2)$ and $a_2(n_1) = 2n_1^2 + n_1(3-2n_2) - n_2(5+4n_2)$. Set $\bar{n}_1(n_2) \equiv [-5 - 2n_2 + \sqrt{25 + 68n_2 + 36n_2^2}]/4$ the positive root of the equation $a_1(n_1) = 0$. We have: $a_1(n_1) \geq 0$ if and only if $n_1 \in [1, \bar{n}_1(n_2)]$. Likewise, set $\tilde{n}_1(n_2) \equiv [-3 + 2n_2 + \sqrt{9 + 28n_2 + 36n_2^2}]/4$ the positive root of: $a_2(n_1) = 0$. We have $a_2(n_1) \geq 0$ if and only if $n_1 \geq \tilde{n}_1(n_2)$. The above results suggest that for $n_1 \in [1, \bar{n}_1(n_2)]$, we have $a_1(n_1) \geq 0$, which by (18a) implies that $\frac{\partial t_1}{\partial z} > 0$. If the inequality $n_1 > \bar{n}_1(n_2)$ holds, then $a_1(n_1) < 0$. Hence, $\frac{\partial t_1}{\partial z} > 0$ if and only if $\gamma < \bar{\gamma} \equiv [2 + 2n_1^3 + 8n_2 + 5n_2^2 + 2n_1 n_2(3+n_2)]/4[4n_1^2 + n_1(5+2n_2) - n_2(3+2n_2)]$. The results (i) and (ii) then follow.

If we have $n_1 \geq \tilde{n}_1(n_2)$, then $a_2(n_1) \geq 0$. In such a case, by (18b) we get $\frac{\partial t_2}{\partial z} > 0$. However, if we have $n_1 < \tilde{n}_1(n_2)$, then $a_2(n_1) < 0$ so that using (18b) we obtain: $\frac{\partial t_2}{\partial z} > 0$ if and only if

$\gamma < \tilde{\gamma} \equiv [2+n_2^2(2+n_2)+n_1^2(5+2n_2)+n_1(8+6n_2+4n_2^2)]/4[n_1(-2n_1+2n_2-3)+n_2(5+4n_2)]$. The results (iii) and (iv) then follow.

Proof that a bilateral tariff reduction always increases the pollution taxes when pollution is harmful enough and $0 \leq \lambda < 1$

Making use of (8a), (8b), we show that:

$$\frac{\partial t_1}{\partial z} = [-8\gamma^2(1-\lambda^2)n_1(1+(1-\lambda)n_1)(1+(1-\lambda)n_2)(1+n_1+n_2)+s_1\gamma+s_2]/f,$$
$$\frac{\partial t_2}{\partial z} = [-8\gamma^2(1-\lambda^2)n_2(1+(1-\lambda)n_1)(1+(1-\lambda)n_2)(1+n_1+n_2)+s_3\gamma+s_4]/\tilde{f},$$

where $s_i, i = 1,2,3,4$ are terms depending on n_1, n_2, and λ, whereas f and $\tilde{f}$ are positive terms. When γ is sufficiently large, the signs of $\frac{\partial t_1}{\partial z}$ and $\frac{\partial t_2}{\partial z}$ are those of their respective coefficients of γ^2, which are negative for $0 \leq \lambda < 1$. The result then follows.

Proof of Proposition 4

Differentiating (6g) and (6h) with respect to z, we get: (a) $\frac{\partial y_1}{\partial z} < 0$ if and only if $h_1 \equiv \frac{\partial t_1}{\partial z} - \frac{n_2}{1+n_2}.\frac{\partial t_2}{\partial z} + \frac{1}{2(1+n_2)} > 0$; (b) $\frac{\partial y_2}{\partial z} < 0$ if and only if $h_2 \equiv \frac{\partial t_2}{\partial z} - \frac{n_1}{1+n_1}.\frac{\partial t_1}{\partial z} + \frac{1}{2(1+n_1)} > 0$. Making use of (8a) and (8b), we obtain:

$$h_1 = \frac{-(1+8\gamma)n_1^2+4n_1(-3\gamma+n_2)+4(2+3\gamma)n_2+(5+8\gamma)n_2^2+2}{4n_1(1+n_2)(2+4\gamma+n_1+n_2)},$$
$$h_2 = \frac{(5+8\gamma)n_1^2+4n_1(2+3\gamma+n_2)+2-(1+8\gamma)n_2^2-12\gamma n_2}{4n_2(1+n_1)(2+4\gamma+n_1+n_2)}.$$

These expressions have positive denominator. As a result, their sign is given by that of their respective numerators.

(i) Clearly, $h_1 > 0$ if and only if $1 \leq n_1 < \hat{n}_1(n_2)$, where $\hat{n}_1(n_2)$ is the positive root of the equation: $-(1+8\gamma)n_1^2+4n_1(-3\gamma+n_2)+(4(2+3\gamma)n_2+(5+8\gamma)n_2^2+2) = 0$.

(ii) Likewise, $h_2 > 0$ if and only if $n_1 > \breve{n}_1(n_2)$, where $\breve{n}_1(n_2)$ is the positive root of the equation:

$$(5+8\gamma)n_1^2+4n_1(2+3\gamma+n_2)+2-(1+8\gamma)n_2^2-12\gamma n_2 = 0. \tag{19}$$

Notice that when $0 \leq n_2 \leq \bar{n}_2 \equiv [-6\gamma+\sqrt{2(1+8\gamma+8\gamma^2)}]/(1+8\gamma)$, we have: $2-(1+8\gamma)n_2^2-12\gamma n_2 \geq 0$, which implies: $h_2 \geq 0$. As a result, we can set $\breve{n}_1(n_2) = 0$ in order to get the desired outcome.

A numerical example for $n_1 = 1, n_2 = 2, N_1 = N_2 = 1$ and arbitrary values of a, c, γ and λ.

For $n_1 = 1, n_2 = 2, N_1 = N_2 = 1$ and arbitrary values of a, c, γ and λ, using (8a) and (8b), we get:

$$\frac{\partial t_1}{\partial z} = [69 + 2\gamma(117 - 204\lambda + 89\lambda^2) - 32\gamma^2(1-\lambda^2)(3-2\lambda)(2-\lambda)]/16\rho(\lambda,\gamma), \tag{20a}$$

$$\frac{\partial t_2}{\partial z} = [67 + 2\gamma(278 - 632\lambda + 254\lambda^2) - 64\gamma^2(1-\lambda^2)(3-2\lambda)(2-\lambda)]/32\rho(\lambda,\gamma), \tag{20b}$$

where $\rho(\lambda, \gamma) = 5 + \gamma(29 - 36\lambda + 11\lambda^2) + 4\gamma^2(1-\lambda^2)(3-2\lambda)(2-\lambda) > 0$.

The analysis of the signs of (20a) and (20b) can be carried out by distinguishing two cases of transboundary pollution.

The first case is for totally global pollution (*i.e.* $\lambda = 1$). In this case, we get (a) $\frac{\partial t_1}{\partial z} > 0$ and (b) $\frac{\partial t_2}{\partial z} > 0$ if and only if $\gamma < 0.67$. Thus, while a tariff reduction lowers the pollution tax in Home even if pollution is harmful enough, it lowers the pollution tax in Foreign only when $\gamma < 0.67$.

The second case is for partially global pollution (*i.e.* $0 \leq \lambda < 1$). In this situation, for $i = 1, 2$, set $\hat{\gamma}_i(\lambda)$ the positive root of the equation $\frac{\partial t_i}{\partial z} = 0$. We have $\frac{\partial t_i}{\partial z} > 0$ if and only if $0 \leq \gamma < \hat{\gamma}_i(\lambda)$. Hence, a tariff reduction lowers the pollution taxes in all the countries if and only if $0 \leq \gamma < \min(\hat{\gamma}_1(\lambda), \hat{\gamma}_2(\lambda))$. It raises the pollution taxes in all the countries when $\gamma > \max(\hat{\gamma}_1(\lambda), \hat{\gamma}_2(\lambda))$. It raises the pollution tax but only in one country for $\gamma \in (\min(\hat{\gamma}_1(\lambda), \hat{\gamma}_2(\lambda)), \max(\hat{\gamma}_1(\lambda), \hat{\gamma}_2(\lambda)))$.

We next examine the effects of the tariff reduction on output.

$$\frac{\partial y_1}{\partial z} = (-45 - 4\gamma(41 - 55\lambda + 23\lambda^2))/8\rho(\lambda,\gamma),$$
$$\frac{\partial y_2}{\partial z} = (-19 + 2\gamma(-69 + 128\lambda - 4\lambda^2))/8\rho(\lambda,\gamma).$$

Notice that we have $\frac{\partial y_1}{\partial z} < 0$. Therefore, the tariff reduction always increases output in Home.

It can be shown that $\frac{\partial y_2}{\partial z} > 0$ if and only $\gamma > \gamma_{ex}(\lambda) \equiv \frac{19}{2(-69+128\lambda-4\lambda^2)}$ and $\lambda > 0.69$. Hence, the tariff reduction lowers output in Foreign for $\lambda > 0.69$ and $\gamma > \gamma_{ex}(\lambda)$.

Proof that $\frac{\partial t_1}{\partial z} < 0$ for $n_1 > n_1^{**}$ and $\frac{\partial t_2}{\partial z} < 0$ for $n_1 < n_1^{ss}$ when pollution is harmful enough and $\lambda = 1$.

Using (8a) and (8b) for arbitrary values of N_1, N_2 and $\lambda = 1$ yields:

$$\frac{\partial t_1}{\partial z} = \frac{b_0(n_1)\gamma + b_1}{n_1 d[(\gamma + n_1)N_1^2 + N_2(1 + n_2(N_2 - 1) + \gamma N_2) + N_1(1 + n_1(N_2 - 1) + (2\gamma + n_2)N_2)]},$$
$$\frac{\partial t_2}{\partial z} = \frac{c_0(n_1)\gamma + c_1}{n_2 d[(\gamma + n_1)N_1^2 + N_2(1 + n_2(N_2 - 1) + \gamma N_2) + N_1(1 + n_1(N_2 - 1) + (2\gamma + n_2)N_2)]},$$

where b_1 and c_1 do not depend on γ; $b_0(n_1)$ and $c_0(n_1)$ are second degree polynomials in n_1 defined as

$b_0(n_1) = -n_1^2 N_1[N_1^2 + N_2(2N_2 - 1) + N_1(3N_2 - 1)] + n_1[-N_1^2 + (-2 + n_2(1 - 2N_2))N_2^2 + N_1(1 + (-3 + n_2)N_2 - 2n_2 N_2^2] + n_2 N_2(1 + N(1 + n_2 N_2)),$

$c_0(n_1) = N_1^2 N n_1^2 + n_1 N_1[1 - 2n_2 N_1^2 + N_1(1 + n_2(1 - 2N_2)) - n_2[(1 + n_2 N_2)(N_2 + 2N_1^2 - 1) + N_1 N_2(3 + n_2(3N_2 - 1))].$

Denote by n_1^{**} the positive roots of the equation $b_0(n_1) = 0$ of and by n_1^{ss} the positive root of $c_0(n_1) = 0$. We have: (i) when pollution is harmful enough, $\frac{\partial t_1}{\partial z}$ and $\frac{\partial t_2}{\partial z}$ have the same sign as $b_0(n_1)$ and $c_0(n_1)$, respectively; (ii) $b_0(n_1) < 0$ if and only if $n_1 > n_1^{**}$; (iii) $c_0(n_1) < 0$ if and only if $n_1 < n_1^{ss}$. The result then follows.

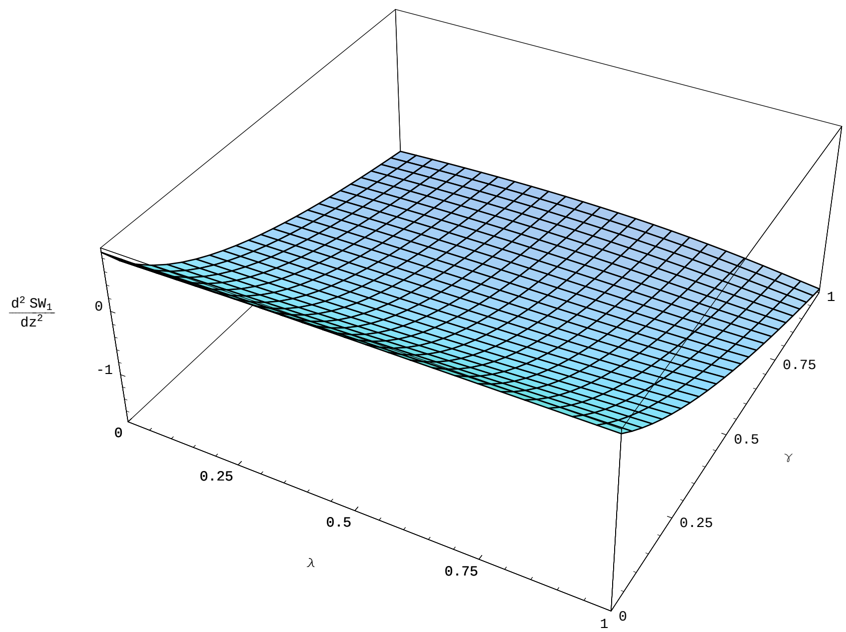

Figure 1: Sign of $\frac{\partial^2 SW_1}{\partial z^2}$ for $N_1 = N_2 = 1$, $n_1 = 1$ and $n_2 = 2$.

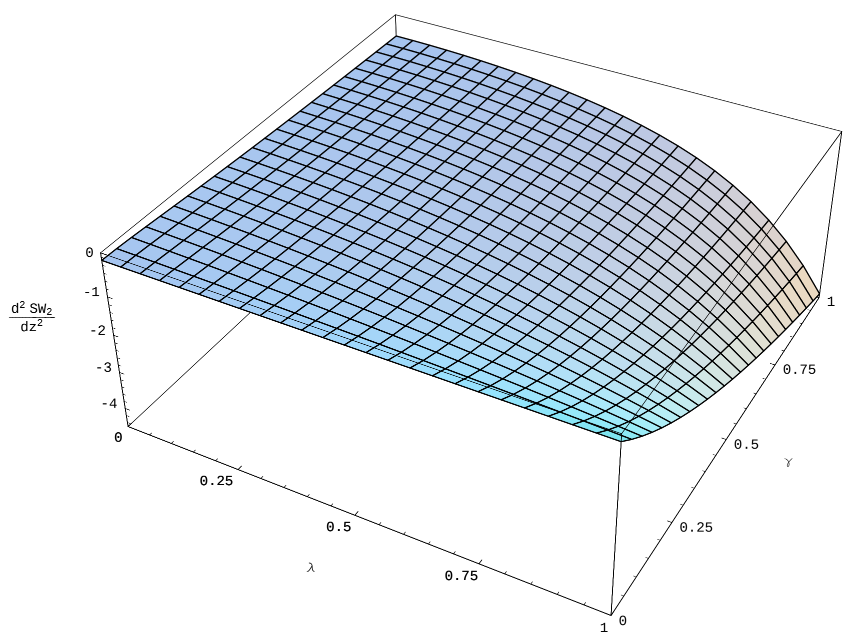

Figure 2: Sign of $\frac{\partial^2 SW_2}{\partial z^2}$ for $N_1 = N_2 = 1$, $n_1 = 1$ and $n_2 = 2$.

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