Explaining the Structure of CEO Incentive Pay with Decreasing Relative Risk Aversion

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Abstract:
It is established that the standard principal-agent model cannot explain the structure of commonly used CEO compensation contracts if CRRA preferences are postulated. However, we demonstrate that this model has potentially a high explanatory power with preferences with decreasing relative risk aversion, in the sense that a typical CEO contract is approximately optimal for plausible preference parameters.

Keywords: CEO pay, principal-agent model, corporate governance, stock-options

JEL Classification: G30, M52
1 Introduction

This paper argues that the explanatory power of the principal-agent model for the structure of CEO incentive pay is potentially high if CEOs are assumed to have preferences with hyperbolic absolute risk aversion (HARA), so that their risk tolerance is linear in wealth. We show that, in a model with a lognormally distributed stock price and a CEO with this type of von Neumann-Morgenstern preferences, a typical CEO compensation contract can be approximately optimal. We begin by corroborating the finding of Hall and Murphy (2002) and Dittmann and Maug (2007) that CEO incentive pay, which currently mostly consists of a mix of restricted stock grants and stock-options, is inefficient if CEOs are assumed to have preferences with constant relative risk aversion (CRRA), which is a special case of HARA. However, we show that there exists a subset of HARA utility functions such that this form of compensation is approximately optimal. These HARA utility functions are characterized by prudence, decreasing absolute risk aversion, and decreasing relative risk aversion, which is consistent with the empirical evidence on individual preferences. The associated levels of risk aversion and prudence are also consistent with empirical estimates. Hence, we regard this type of HARA utility function as a plausible representation of CEO preferences. With these preferences, the standard principal-agent model can explain reasonably well the form of a typical CEO compensation contract.

The assumptions of prudence and decreasing absolute risk aversion are now well-accepted and are discussed further down. The hypothesis of decreasing relative risk aversion is more controversial, but the empirical evidence points in this direction. Arrow (1965) shows that the wealth elasticity of investments at the risk-free rate is smaller than unity with decreasing relative risk aversion, thus generating a testable prediction. Cohn, Lewellen, Lease, and Schlarbaum (1975) and Kessler and Wolff (1991) show that the proportion of risky assets in household portfolios is strongly increasing in wealth, a finding corroborated by Levy (1994) in an experimental study, and by Siegel and Hoban (1982) and Morin and Suarez (1983) for wealthy households – although in a similar study, Blume and Friend (1975) cannot reject the hypothesis of constant relative risk aversion. In a survey of income and consumption at the household level, Ogaki and Zhang (2001) also conclude in favor of decreasing relative risk aversion.
The mix of stocks and stock-options in CEO compensation is a central yet unresolved issue in corporate governance.\textsuperscript{2} Hall and Murphy (2002), Jenter (2002), and Dittmann and Maug (2007) demonstrate that the principal-agent model cannot explain the use of stock-options in compensation contracts. They show that there exists a contract which would provide the CEO with the same expected utility as a typical CEO contract and would give the same effort incentives, but which would cost significantly less to the firm than a typical CEO contract. Crucially, these papers assume that the stock price is lognormally distributed, that the CEO has CRRA preferences, and that he can improve the distribution of stock returns in the sense of first-order stochastic dominance.

These findings motivated the search for an alternative model of CEO pay which could explain current CEO compensation contracts. Two routes are currently explored within the principal-agent model of efficient contracting. The first consists in modifying technological assumptions. For example, a risk averse CEO can be assumed to control the volatility of stock returns (Feltham and Wu (2001), Dittmann and Yu (2009)). Alternatively, stock returns can be assumed to follow a gamma distribution, as opposed to a lognormal distribution (Hemmer, Kim and Verrecchia (2000)). The second consists in relaxing the assumption of CRRA preferences. In Dittmann, Maug and Spalt (2010), CEOs are assumed to be loss averse and risk-loving in losses. With these preferences, observed CEO contracts can be shown to be approximately optimal.\textsuperscript{3} This paper adopts a similar approach, although we only consider concave and continuously differentiable utility functions (with no kinks). We show that a typical CEO contract is not significantly suboptimal with properly calibrated HARA preferences.

An alternative hypothesis is that CEO pay is inefficient, and cannot be explained by a model of efficient contracting. For example, Yermack (1995) presents evidence in support of

\textsuperscript{2}More generally, the importance of CEO incentive pay for corporate governance and corporate policies is underlined in Jensen and Murphy (1990), Smith and Watts (1992), and Mehran (1995).

\textsuperscript{3}See also de Meza and Webb (2007) for a model of optimal contracting with loss aversion.
this hypothesis with regards to grants of stock-options, while Bebchuk and Fried (2004) argue forcefully that CEO pay is inefficient (see Weisbach (2007) for a review). However, as the survey of Edmans and Gabaix (2009) demonstrates, recent research has proposed and tested hypotheses which potentially reconcile the efficient contracting paradigm with the observed characteristics of CEO pay. Most notably, Gabaix and Landier (2008) explain the rise of levels of CEO pay over time with a matching model, while Edmans, Gabaix and Landier (2009) explain the levels of pay-performance-sensitivities (i.e., the level of CEO incentives, not their structure) by treating leisure as a normal good and by assuming that CEO effort has a multiplicative effect on firm value. These results suggest that optimal contracting models can explain important features of CEO pay. This paper contributes to this literature by focusing on the structure of CEO incentives.

Since the form of individual preferences has not been precisely determined by the literature so far, it is usual to consider a range of utility functions with desirable properties (a set of “plausible” utility functions). In line with most of the literature, Hall and Murphy (2002) and Dittmann and Maug (2007) only consider CRRA utility functions. However, it is legitimate to relax this strong assumption, and to ask whether less stringent, potentially more accurate preferences, enable the model to better match the data.\textsuperscript{4} This is why we consider risk-averse and prudent HARA utility functions characterized by decreasing absolute risk aversion (this includes CRRA utility as a special case).\textsuperscript{5}

We find that, with plausible CEO preferences, the difference between the cost of a typical CEO contract and the cost of the optimal contract can be less than 1\% of the cost of the former. Furthermore, with these preferences, we find that the upside participation and the downside participation

\textsuperscript{4}It is worth pointing out that this approach has already been applied to the equity premium puzzle. It was addressed with preferences developed by Epstein and Zin (1989), and with the habit-formation preferences proposed by Constantinides (1990).

\textsuperscript{5}The hypothesis of decreasing absolute risk aversion is well-established, and goes back to Pratt (1964) and Arrow (1965). The hypothesis of prudence (Leland (1968) and Kimball (1990)) has been empirically validated by Kraus and Litzenberger (1976), Browning and Lusardi (1996), and Gourinchas and Parker (2001).
participation associated with the optimal contract are roughly in line with those associated
with a typical CEO compensation contract. This is in contrast with the optimal contracts
associated with CRRA preferences, which tend to feature too much downside participation
and too little upside participation relative to typically observed contracts. Finally, allowing
for HARA preferences with decreasing relative risk aversion rather than CRRA preferences
enables to capture two important features of CEO preferences which received some empirical
support, namely low and decreasing relative risk aversion (low risk aversion follows from high
wealth and decreasing relative risk aversion).

We also determine with numerical simulations how the form of the optimal contract relates
to the crucial parameters of the model. We find that the degree of convexity of the compensa-
tion profile tends to be increasing in CEO incentives, but decreasing in the level of reservation
utility and the wealth of the CEO. This generates cross-sectional predictions which provide a
good joint test of the principal-agent model for CEO pay and of the hypothesis of decreasing
relative risk aversion. We also argue that these predictions may shed some light on the shift
away from stock-options from the year 2000 documented by Frydman and Jenter (2010).

We conclude that the explanatory power of a properly calibrated principal-agent model
of efficient contracting is potentially strong. Thus, even though they may shed some light
on some aspects of CEO compensation, alternative technological assumptions or non-von
Neumann-Morgenstern preferences are not necessary for the structure of a typical CEO con-
tract to be successfully explained by the principal-agent model. More generally, our results
also provide indirect evidence in support of the hypothesis of decreasing relative risk aversion.

The paper proceeds as follows. Section 2 presents the model. Section 3 calibrates a
CRRA-lognormal model to the representative CEO of Dittmann and Maug (2007). Section
4 calibrates a HARA-lognormal model to the same representative CEO, and identifies the
preference parameters for which a typical CEO contract is approximately optimal. Section 5
derives the optimal contract for a range of other parameters, and establishes some predictions.
Section 6 concludes.
2 The Model and the Empirical Methodology

We use the same principal-agent model as Dittmann and Maug (2007), which is standard in the CEO compensation literature, except that we do not restrict attention to CRRA preferences, and we let the CEO be protected by limited liability. Risk-neutral shareholders offer a compensation contract to the CEO at time $-1$, which specifies his pay $W$ as a function of the stock price at time $T$, $P_T$. The CEO exerts effort $e$ at time 0, which affects firm value. The objective of the shareholders is to implement a given level of effort $e^\ast$ at the minimum cost. Thus, we take the level of incentives as given, and we focus on the form of the optimal compensation contract which delivers this level of incentives.

The time $T$ stock price is assumed to be lognormally distributed. It is a function of CEO effort, $e$, and of some noise:

$$P_T(u, e) = f(e) \exp \left\{ \left( r_f - \frac{\sigma^2}{2} \right) T + u \sqrt{T} \sigma \right\}$$

where $\bar{u}$ is the standard normal variable, $r_f$ is the risk-free interest rate, $\sigma$ is the stock price volatility, and $f(e)$ is an increasing and concave function of $e$. Thus, CEO effort affects the probability distribution of the time $T$ stock price in the sense of first-order stochastic

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6Dittmann and Maug assume that the contract could force the CEO to give up his preexisting wealth, so that his end-of-period wealth is only equal to his contractual payment. First, such a mechanism is rarely observed in practice. Second, it is common in principal-agent models to assume limited liability. Third, we need the CEO to have a minimum level of wealth in all states of the world to calibrate the HARA utility function in section IV of the paper. Fourth, imposing this constraint allows us to verify that Dittmann and Maug’s results are not driven by the absence of a limited liability constraint for the CEO. Since limited liability implies a higher lower bound on payments, it is possible that it would make the optimal contract more convex: roughly speaking, since it is may not be possible to provide adequate incentives with sticks, carrots may be used instead. This being said, as the analysis that follows will show, the main results of Dittmann and Maug are robust to the inclusion of a limited liability constraint.

7More precisely, we only consider the first step of optimal contracting in Grossman and Hart (1983), which consists in minimizing the agency cost of implementing a given effort $e^\ast$. Any optimal contract is a solution to the first step problem, for a given level of effort.
dominance.

The CEO’s wealth at time $T$ consists of his preexisting wealth $\omega$ capitalized for $T$ years at the risk-free rate of interest, and his contractual payment $W(P_T)$. To alleviate notations, we define $\hat{\omega} \equiv \omega \exp\{r_f T\}$. The CEO’s objective function is additively separable in wealth and effort cost and is given by

$$U(\hat{\omega} + W(P_T)) - C(e)$$

(2)

The utility function $U$ is characterized by $U'(W) > 0$, $U''(W) < 0$, $U'''(W) > 0$, and by decreasing absolute risk aversion. The function $C(e)$, which measures the future value of the cost of effort, is increasing and convex in $e$.

The incentive constraint guarantees that $e^*$ maximizes the CEO’s objective function given his contract. Using the first-order approach, it reduces to

$$E[W'(\hat{P}_T)U'(\hat{\omega} + W(\hat{P}_T))] = \frac{C'(e^*)}{f'(e^*)}$$

(3)

A contract that satisfies this constraint is said to be incentive-compatible. Notice that the right-hand-side of (3) is increasing in $e^*$.

The CEO’s outside option gives him an expected utility net of effort cost of $\bar{U}$. The contract offer therefore satisfies the CEO’s participation constraint at the equilibrium level of effort, and the contract is said to be individually rational, if and only if, in equilibrium,

$$E[U(\hat{\omega} + W(\hat{P}_T))] \geq \bar{U} + C(e^*)$$

(4)

Finally, we let the CEO be protected by limited liability, so that negative payments are

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*For any given contract $W$ such that the second derivative of $E[U(W(\hat{P}_T))]$ with respect to $e$ is finite, there exists a sufficiently convex cost function such that the first-order approach is valid. Put differently, the set of contracts for which the first-order approach is valid can be enlarged as needed by increasing $C''(e)$, i.e., by increasing the convexity of the cost function. Since in this paper we consider given contracts, it is always possible to ensure that the first-order approach holds in each case. Dittmann, Maug and Spalt (2010) use the same argument. Admittedly, this approach would not be appropriate for selecting the most efficient contract in an unbounded set of contracts.*
not feasible.

\[ W(P_T) \geq 0 \quad \text{for any } P_T \] (5)

The cost of a contract is defined as its expected payoff in equilibrium. The problem of the shareholders is to minimize the cost of implementing effort \( e^* \) subject to the incentive constraint, the participation constraint, and the limited liability constraint:

\[
\min_{W(P_T)} E[W(\tilde{P}_T)] \quad \text{s.t. (3), (4), and (5)}. \] (6)

We follow the same empirical methodology as Dittmann and Maug (2007), and we use their “representative CEO” to facilitate comparability. More details on the methodology can be found in Dittmann, Zhang, Maug, and Spalt (2011).

First, for each utility function considered, we derive the form of the optimal compensation contract, as explained in the Appendix. It gives the optimal mapping from the end-of-period stock price \( P_T \) to the payment \( W \) to the CEO as a function of two parameters, \( \alpha_0 \) and \( \alpha_1 \).

Second, for each utility function considered, we set the contract parameters \( \alpha_0 \) and \( \alpha_1 \) such that the optimal contract (denoted by \( W_O \)) is individually rational and incentive-compatible, in the sense that it gives the CEO the same expected utility and induces the same level of effort as the observed contract (denoted by \( W_D \)):

\[
E[U(\tilde{\omega} + W_O(\tilde{P}_T))] = E[U(\tilde{\omega} + W_D(\tilde{P}_T))] \] (7)

\[
E[W'_O(\tilde{P}_T)U'(\tilde{\omega} + W_O(\tilde{P}_T))] = E[W'_D(\tilde{P}_T)U'(\tilde{\omega} + W_D(\tilde{P}_T))] \] (8)

Third, for each utility function considered, we compute the difference between the cost of the observed contract and the cost of the optimal contract whose parameters were determined in the second step. Since by definition the optimal contract minimizes the cost of CEO pay, the model’s explanatory power is high if and only if this (positive) difference is relatively small.

The rest of the paper successively applies this methodology to CRRA and HARA utility functions.
3 CRRA Preferences

In this section, we assume CRRA preferences with positive coefficient of relative risk aversion $\gamma$:

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma}$$  \hspace{1cm} (9)

for $\gamma \neq 1$ and

$$U(W) = \ln(W)$$  \hspace{1cm} (10)

for $\gamma = 1$.

The CRRA-lognormal setting is commonly used in models of executive compensation.\textsuperscript{9} Admittedly, the CRRA utility function exhibits desirable properties, such as risk aversion, prudence, and decreasing absolute risk aversion. The purpose of this section is to show that the explanatory power of the principal-agent model is unsatisfactorily low when CRRA preferences with plausible values of relative risk aversion are postulated. However, we also emphasize that the explanatory power of the principal-agent model largely depends on the postulated value of relative risk aversion, which suggests that CEO preferences are crucial, and should be chosen carefully.

Dittmann and Maug (2007) acknowledge that there is no consensus regarding the value of the coefficient of relative risk aversion $\gamma$. For values of $\gamma$ between 0.5 and 10, they show that switching from the observed CEO contracts to the optimal contracts would generate large cost savings. This implies that, for any of these preference parameters, the model fails to explain the data.

We apply the aforementioned three steps approach to the representative CEO of Dittmann and Maug with CRRA utility. Results are reported in Figure 1 for selected values of $\gamma$, including $\gamma = 0.1$.

As Dittmann and Maug (2007), we find that the potential savings associated with a

Figure 1: The observed CEO compensation contract, and optimal compensation contracts for different coefficients of RRA.
switch to the optimal contract are monotonically increasing in $\gamma$, and become so low as to be economically insignificant for very low values of $\gamma$. For example, potential savings are $1.7\%$ with $\gamma = 0.1$, $10.0\%$ with $\gamma = 0.5$, $18.8\%$ with $\gamma = 1$, and $32.9\%$ with $\gamma = 2$. With CRRA utility, an implausibly low coefficient of relative risk aversion is needed for the observed contract to be approximately efficient. The findings of the next section will shed light on this puzzle.

Arguably, the optimal contract with low values of $\gamma$ could be approximately optimal because all contracts are approximately optimal for low values of $\gamma$. This is not the case: the cost of certain contracts remains relatively high even for low values of $\gamma$. For example, for $\gamma = 0.1$, an individually rational and incentive-compatible stock-option-like contract with at-the-money call options and a fixed wage may be derived. It costs $5.0\%$ more to the firm than the optimal contract. This example shows that not all contracts which satisfy the participation constraint and the incentive constraint are associated with low potential savings for low coefficients of relative risk aversion.

4 HARA Preferences

While CRRA preferences have desirable properties, the hypothesis of constant relative risk aversion is strong, and rules out a large set of plausible utility functions. In addition, we have shown in the preceding section that the performance of the principal-agent model is very sensitive to the postulated value of relative risk aversion, which suggests that it may also be sensitive to the postulated functional form for the utility function. Consequently, we now relax the assumption of constant relative risk aversion, and use HARA preferences instead. We are going to show that this markedly improves the explanatory power of the
principal-agent model.\textsuperscript{10}

A HARA utility function is defined by its parameters $a$ and $b$, and takes the form:\textsuperscript{11}

$$U(W) = \left(a + \frac{W}{b}\right)^{1-b}$$

where $b \neq 0$ and $W \geq 0$. Its coefficient of relative risk aversion is

$$R(W) = W \left(a + \frac{W}{b}\right)^{-1}$$

Relative risk aversion is decreasing in wealth if and only if $a$ is negative, whereas it is constant if $a = 0$. To ensure that our assumptions that $U' > 0$, $U'' < 0$, $U''' > 0$ and decreasing absolute risk aversion are satisfied, we must impose the conditions that $0 < b < 1$ and $W > -ab$. We refer to the preference parameters $a$ and $b$ which satisfy these restrictions as “eligible”.

We apply the aforementioned three steps approach to the representative CEO of Dittmann and Maug with HARA utility and eligible preference parameters. Potential savings associated with a switch to the optimal contract are reported in Table 1, for a range of values of $a$ and $b$.\textsuperscript{12} In addition, for $b = 0.25$ and $a = -3.9$, potential savings amount to 4.9% of the cost of compensation of the representative CEO; for $b = 0.1$ and $a = -1.85$, potential savings are only 0.96%.\textsuperscript{13} Thus, there exists some eligible preference parameters $a$ and $b$ such that

\textsuperscript{10}Rabin (2000) emphasizes that the attitude toward risk represented by a utility function may only apply to risks of a certain (adequate) “size”, and that it may not accurately represent the attitude toward risks of an altogether different size. Here we are only concerned about the risk preferences of CEOs as they relate to their level of pay.

\textsuperscript{11}More complex specifications could alternatively be used. This one is the simplest for our purposes.

\textsuperscript{12}For a given value of $b$, the algorithm does not converge when $a$ is too low. The lower $b$ is, the higher the lower bound for $a$ is. This explains the NA values in the table. For some values of $b$, we hereafter report the potential savings when $a$ takes the lowest possible value such that the algorithm converges. Notice that potential savings, as reported above in the CRRA case and in Table 1 for the HARA case, differ for $a = 0$, when the values of $\gamma$ and $b$ coincide. This is because, even though such utility functions share the same relative risk aversion, their forms nevertheless differ, as can easily be seen by comparing (9) to (11) with $a = 0$.

\textsuperscript{13}Preference parameters in a neighborhood of these points tend to be associated with similar potential savings.
the difference between the cost of the observed contract and the cost of the optimal contract is less than 1% of the cost of the observed contract. This indicates that the principal-agent model with HARA preferences can be calibrated in such a way that a typical CEO contract is not significantly inefficient.

Holding \( a \) constant, Table 1 suggests that potential savings are an increasing function of \( b \). We already obtained this result in the CRRA case (with \( a = 0 \)), but it seems to hold more generally for any value of \( a \). The suboptimality of the observed contract is therefore minimized for very low values of \( b \). On the contrary, the model performs poorly for higher values of \( b \): potential savings then remain large for any value of \( a \).

For low values of \( b \), savings are an increasing function of \( a \). For \( b = 0.1 \) (respectively \( b = 0.25 \)), assuming that \( a = -1.85 \) (respectively \( a = -3.9 \)) instead of relying on the hypothesis of constant relative risk aversion (\( a = 0 \)) reduces potential savings by 59% (respectively 15%). Postulating a decreasing relative risk aversion (a negative \( a \)) significantly reduces the inefficiency of the observed contract. In addition, as shown in Figure 2 and Table 2, it makes the optimal contract less concave than with constant relative risk aversion, thus reducing the difference in curvature between the optimal contract and the observed contract. For example, for \( a = -1.85 \) and \( b = 0.1 \), the optimal payment increases by 102% if the realized stock price is one standard deviation above the mean (against 130% for the observed contract), whereas it increases by only 52% with the hypothesis of constant relative risk aversion (\( a = 0, b = 0.1 \)). In sum, having preferences with decreasing relative risk aversion (\( a < 0 \)) in addition to decreasing absolute risk aversion (\( b > 0 \)) significantly improves the explanatory power of the model.
Figure 2: The observed CEO compensation contract, and optimal compensation contracts for different preference parameters.
Admittedly, the inefficiency of the observed contract relative to the optimal contract could be minimized simply because any contract will be approximately optimal when the risk aversion of the agent is sufficiently low. There are two answers to this argument. The first is that inefficient contracts remain significantly more costly than the optimal contract even at low levels of risk aversion, as already noted in the last paragraph of the previous section. The second answer is that the preference parameters which minimize the suboptimality of the observed contract also generate an optimal contract which closely resembles a typical CEO compensation contract, as can be seen in Figure 2. This is in contrast to the optimal contracts generated by the model with other preference parameters (see Figures 1 and 3 for examples).

More precisely, we compute in Table 2 some measures of upside participation and downside participation associated with the observed contract and the optimal contract, for a range of preference parameters. We can see that these two measures tend to differ widely, with an optimal contract generally characterized by too much downside participation and too little upside participation relative to the observed contract. However, in the case of a low $b$ and a negative $a$, the downside participation and upside participation of the optimal contract is roughly comparable to the observed contract’s. Moreover, for a given low $b$, having a negative $a$ (decreasing relative risk aversion) instead of a zero $a$ (constant relative risk aversion) markedly improves the fit of the model on this dimension, by almost doubling the upside participation of the optimal contract. Intuitively, this is because the hypothesis of decreasing relative risk aversion tends to reduce the concavity of the utility function for high payments, which reduces the discounting associated with high payments. This in turn makes it more desirable to use rewards (high payments for superior performance) for incentive purposes.

Because of the restrictions imposed on the utility function, we know that the HARA utility functions considered have plausible qualitative properties, namely risk aversion, prudence, and decreasing absolute risk aversion. The property of decreasing relative risk aversion (DRRA) associated with $a < 0$ is more controversial, but there is some evidence that relative risk aversion is indeed decreasing with wealth. Arrow (1965) shows that the wealth elasticity of
investments at the risk-free rate is smaller than unity with DRRA. The empirical evidence points in this direction: Cohn, Lewellen, Lease, and Schlarbaum (1975) and Kessler and Wolff (1991) show that the proportion of risky assets in household portfolios is strongly increasing in wealth, a finding corroborated by Levy (1994) in an experimental study, and Siegel and Hoban (1982) for wealthy households – although in a similar study, Blume and Friend (1975) cannot reject the hypothesis of constant relative risk aversion. In a survey of income and consumption at the household level, Ogaki and Zhang (2001) also conclude in favor of DRRA.

We now argue that the HARA utility functions that we have identified as having a high explanatory power also have plausible quantitative properties, in the sense that they tend to generate plausible values of relative risk aversion – we address prudence in the next paragraph. With \( a = -1.85 \) and \( b = 0.1 \), a CEO with a wealth of $0.2m would have a relative risk aversion of 1.33,\(^{14}\) while a CEO with a wealth of $10m or more would have a relative risk aversion of 0.1.\(^ {15}\) Thus, the hypothesis of decreasing relative risk aversion may not only yield plausible values of relative risk aversion for individuals with more common levels of wealth,\(^ {16}\) but also low values of relative risk aversion for wealthy CEOs. This is in line with the finding of Graham, Harvey, and Puri (2009) that CEOs tend to be significantly less averse to a

\(^{14}\)The coefficient of relative risk aversion would be even higher for poorer households. However, the utility function associated with these values of \( a \) and \( b \) is only defined for \( W \geq 0.185m \). According to the 2009 Survey of Consumer Finances, the mean (respectively median) U.S. household had a net worth of $0.56m (resp. $0.12m) in 2007.

\(^{15}\)The coefficient of relative risk aversion is equal to 0.102 at the representative CEO preexisting wealth level of $9.1m, and it converges toward 0.1 as wealth tends to infinity.

\(^{16}\)Kydland and Prescott (1982) need a relative risk aversion between 1 and 2 to replicate the observed fluctuations in consumption and investment. In a model of consumption behavior over the life cycle, Gourinchas and Parker (2002) estimate that the coefficient of relative risk aversion of households is between 0.5 and 1.4. Epstein and Zin (1991) estimate that relative risk aversion is around one, and Campbell, Lo and MacKinlay (1997), as well as Ait-Sahalia and Lo (2000) summarize estimates obtained in the macroeconomic literature. In experiments, Harrison, Lau and Rutstrom (2007) obtain an average value of relative risk aversion of 0.67, while Bombardini and Trebbi (2007) obtain an average value of 1. The latter also review the experimental literature.
multiplicative risk than the average individual, which implies that they have lower relative risk aversion.\textsuperscript{17}

Lastly, even though the HARA utility that we consider is markedly less risk averse than a CRRA utility for a range of commonly used values of $\gamma$, it is not much less prudent. Indeed, the coefficient of absolute prudence, which measures the extent of prudence at a given level of wealth $w$ (Kimball (1990)) is equal to $\frac{\gamma + 1}{w}$ in the case of CRRA utility, and to $\frac{b + 1}{aw + w}$ in the case of HARA utility. In a study of precautionary savings, Gourinchas and Parker (2001) use CRRA utility and find that a coefficient of relative risk aversion $\gamma$ of 0.51 best matches the data. When we compare the associated coefficient of absolute prudence with the one associated with the HARA utility function that we use, we find that they differ by less than 27\% for any level of wealth larger than 0.5m (the difference is only 16\% for a wealth of 0.5m). Thus, the degree of prudence implied by our utility function does not seem to differ much from empirical estimates.\textsuperscript{18}

Results in Table 1 highlight that the explanatory power of the model is sensitive to the parameterization of the CEO utility function. This emphasizes the importance of the calibration of CEO preferences. This also explains why simplifying assumptions like constant relative risk aversion may seriously undermine the performance of the model. Finally, if relative risk aversion is indeed decreasing with wealth, calibrating a misspecified model with CRRA preferences to the data will yield a very low implied coefficient of relative risk aversion

\textsuperscript{17}This result alone suggests either that CRRA preferences do not accurately represent individual preferences, or that CEOs are intrinsically (for any level of wealth) less risk averse than the general population, possibly because of a selection effect. This latter hypothesis is plausible, since there is evidence that risk preferences are heterogeneous across the population (Barsky, Juster, Kimball, and Shapiro (1997)). However, we have argued above that postulating that relative risk aversion is not only low (for high levels of wealth) but also decreasing significantly improves the explanatory power of the model.

\textsuperscript{18}Dynan (1993) finds that the coefficient of relative prudence (which is equal to $\gamma + 1$ with CRRA utility) that best explains precautionary savings is less than one, which is admittedly puzzling. It may also suggest that consumers are not as prudent as a calibration of CRRA utility with a range of commonly accepted values for $\gamma$ would suggest, which is consistent with our findings.
for wealthy individuals (including CEOs), and a very high implied coefficient of relative risk aversion for poor individuals. This would in turn explain why the coefficient of relative risk aversion which best fits the data is implausibly low in the CRRA-lognormal model.

Even though HARA utility may allow for a significantly better representation of CEO preferences than CRRA utility, it should be stressed that it is not a perfect representation either. First of all because two parameters cannot generate any “plausible” utility function. But also because it is possible and even likely that different CEOs have at least slightly different preferences, so that they will not share exactly the same utility function. The preference parameters that we have derived for the representative CEO should be considered with this caveat in mind. The main point of this section of the paper is that relaxing the hypothesis of CRRA preferences, and in particular postulating a low and decreasing relative risk aversion, can significantly increase the explanatory power of the principal-agent model for CEO incentive pay.

5 Robustness and predictions

Our approach works well for the representative, “median” CEO, but one limitation is that it cannot be readily extended to a sample of CEOs. This is because the preference parameters that we infer in section 4 for the representative CEO of Dittmann and Maug (2007) are a corner solution: the algorithm does not converge, so that the optimal contract cannot be derived, for a lower $a$ and/or a lower $b$. The corner solution for $a$ and $b$ is obtained for the parameter values of the representative CEO. For other parameter values, the algorithm will converge in some cases, and not converge in other cases. Therefore, even if the model had a good explanatory power with the derived preference parameters for about one half of CEOs, say, it would be impossible to know whether or not the model fits the data – all the more that this set of CEOs would not be representative.

Another limitation of the calibration to a sample of CEOs is that neither the wealth nor
the portfolio of stock-options of the CEOs are observed per se – only estimates or proxies are available. This limitation does not matter much in Dittmann and Maug (2007), precisely because they find that the explanatory power of the principal-agent model with CRRA preferences is low for most parameter values. It does not matter much in Dittmann, Maug and Spalt (2010) either, because the convexity of CEO pay in this paper is primarily generated by the combination of loss aversion and a convex utility function for levels of wealth below the reference point. That is, their result that the optimal contract is convex on some interval is not very sensitive to parameter values. In our model with HARA preferences, however, the degree of convexity of the optimal contract is quite sensitive to parameter values. In this case, the unobservability of the wealth and stock-options portfolios of the CEOs become a significant issue.

For these two reasons, we adopt an alternative, indirect approach. With the Dittmann and Maug (2007) methodology that we use, the only effect of any given observed CEO contract is to determine the right-hand-sides of (3) and (4), respectively the incentive constraint and the participation constraint. In addition, given some CEO preferences, the right-hand-sides of (3) and (4) and the level of wealth of the CEO are the only three relevant parameters for the form of the optimal contract. Indeed, the optimal contract is derived in the Appendix, as a function of two parameters, \( \alpha_0 \) and \( \alpha_1 \), which are set to match the participation constraint and the incentive constraint of the CEO. Hence the following approach: given the CEO preference already derived in section 4, we compute the optimal contract for a range of values of these three crucial parameters. This enables us to at least partly address not only the effect of heterogeneity among CEOs on the form of the optimal contract, but also the changes in the structure of CEO incentive pay over time, and to establish predictions relating the form of the optimal contract to these three parameters.

First of all, leaving the wealth of the representative CEO unchanged, we compute the

---

\[19\] The sensitivity to preference parameters is illustrated in section 4. In this section, we will illustrate the sensitivity to other relevant parameters.
optimal contract for a level of incentives which is equal to its baseline level (i.e., for the representative CEO), to half this baseline level, and to twice this baseline level, and for a reservation utility which equal to its baseline level, to 75% of this baseline level (the algorithm does not converge at 50%), and to twice this baseline level. Next, we repeat this procedure for a level of wealth which is half the baseline level, and for a level of wealth which is equal to twice the baseline level. Results are reported in Figures 3, 4, and 5.

As expected given the corner solution limitation mentioned above, the algorithm does not converge in a number of cases, but these Figures nevertheless suggest certain relationships. First, for a given level of wealth and reservation utility, the degree of convexity of CEO compensation seems to be increasing in the level of CEO incentives. Second, for a given level of wealth and of CEO incentives, the degree of convexity of CEO compensation seems to be decreasing in the reservation utility of the CEO. Third, for a given reservation utility and a given level of CEO incentives, the degree of convexity of CEO compensation seems to be decreasing in the level of CEO wealth. These relationships in turn suggest some predictions, which we describe below. Whether or not these predictions are verified in the data provides a good test of the joint hypothesis of HARA utility with the preference parameters derived in section 4 and of the standard model of CEO compensation of Hall and Murphy (2002) and Dittmann and Maug (2007).

We start with cross-sectional predictions. First, for a given level of wealth and reservation utility, CEOs who receive more incentives will tend to hold relatively more options than stocks. Thus, all else equal, the model predicts a positive relationship between the level of equity incentives and the convexity of the incentive package. This is consistent with the positive correlation between the mix of stock-options in CEO pay and the intensity of CEO equity incentives reported in Bryan, Hwang, and Lilien (2000). Second, CEOs with a higher reservation utility (which can be proxied by the total pay of the CEO in a given year) will tend to receive less convex compensation packages, i.e., they will hold relatively more stocks
Figure 3: The observed CEO compensation contract, and the optimal compensation contract with HARA utility for $a = -1.85$ and $b = 0.1$, for different levels of incentives and reservation utility. The central figure is the same as Figure 2. Upward, reservation utility is 100% higher. Downward, reservation utility is 25% lower. Leftward, incentives are 50% lower. Rightward, incentives are 100% higher.
Figure 4: The observed CEO compensation contract, and the optimal compensation contract with HARA utility for $a = -1.85$ and $b = 0.1$, for different levels of incentives and reservation utility, and a level of wealth 50% lower than in Figure 2. The (missing) central figure has the same levels of incentives and reservation utility as Figure 2. Upward, reservation utility is 100% higher. Downward, reservation utility is 25% lower. Leftward, incentives are 50% lower. Rightward, incentives are 100% higher.
Figure 5: The observed CEO compensation contract, and the optimal compensation contract with HARA utility for \( a = -1.85 \) and \( b = 0.1 \), for different levels of incentives and reservation utility, and a level of wealth 100% higher than in Figure 2. The central figure has the same levels of incentives and reservation utility as Figure 2. Upward, reservation utility is 100% higher. Downward, reservation utility is 25% lower. Leftward, incentives are 50% lower. Rightward, incentives are 100% higher. Notice the change of scale on the \( y \) axis for the first row.
than options, ceteris paribus. Third, proxies for wealth, such as age, should be negatively correlated with the degree of convexity of CEO compensation, ceteris paribus. Given current compensation practices, the degree of convexity can be measured by the ratio of options-to-stocks held by the CEO – as adjusted for the fact that stock-options granted in previous years are typically not at-the-money anymore. That is, the equity participation of young, less wealthy CEOs should be more in the form of options than the equity participation of old, more wealthy CEOs. This is consistent with the findings of Ryan and Wiggins (2001) that the percentage of stock-options in CEO compensation is decreasing in proxies for CEO wealth such as CEO tenure, CEO age, and CEO stock ownership. Still, more research would be needed to specifically test these predictions.

Using a taxonomy which is popular in the industrial organization literature (Fudenberg and Tirole (1984)), we may also say that the model suggests the following predictions. “Lean and hungry” CEOs, i.e. those with high incentives, low pay and low wealth, have the most convex compensation schedules, with lots of stock-options. “Puppy dogs”, i.e., CEOs with low incentives, low pay and low wealth, have a less convex compensation, which consists of both stocks and stock-options. “Tog dogs”, i.e., CEOs with high incentives, high pay and high wealth, have an even less convex compensation, which consists primarily of stocks. Finally, the model does not explain the equity-based compensation of “fat cats”, i.e., CEOs with low incentives, high pay and high wealth, since their compensation should typically be a concave function of their performance. An alternatively view is that the model predicts that the incentives for this latter type of CEOs will not be primarily provided in the form of stocks and stock-options, but instead with long-term incentive payments and bonuses.

Data on the wealth of American CEOs is not publicly available.

It should be noted that we do not ascribe the same meanings to those terms as Fudenberg and Tirole (1984).

This is arguably consistent with the finding of Ryan and Wiggins (2001) that the proportion of CEO pay in the form of bonuses is increasing and concave in CEO age. Chhaochharia and Grinstein (2009) also find that equity-based compensation decreases with tenure, but that the share of bonuses in CEO compensation
precisely, bonuses will be largely insensitive to firm performance as long as stock returns are not too low, but will be cut in the event of very low stock returns (cf. the graph on the top left corner of Figure 5 in this paper, which looks very much like a typical bonus plan with a low performance threshold, as displayed in Figure 5 in Murphy (1999)). This could explain the empirical fact that bonuses are typically only paid if performance exceeds a threshold, and that they are capped (Murphy (1999)).

We may also analyze the changes in the structure of CEO compensation over time in the light of these predictions. It is well-known (Frydman and Jenter (2010)) that the structure of CEO compensation varies a lot over time. In particular, from 1950 to 2000, the equity incentives and the pay of S&P 500 CEOs have risen tremendously, which suggests that CEO wealth has also increased over the period. Whereas the rise in incentives suggests an increase in the convexity of CEO compensation, the rise in pay and in wealth suggest a decrease in convexity. Given our predictions, the fact that options progressively became more important than stocks over the period\(^{23}\) can only be explained by the rise in CEO incentives, which must have outweighed the other two factors.

On the contrary, from 2000 to 2008, the data indicates that S&P 500 CEOs received slightly less incentives.\(^{24}\) In addition, their pay tended to remain approximately at the high level reached in 2000 (Frydman and Jenter (2010)). Since CEO pay increased markedly in the 1900s, and because stock returns have mostly been positive for big companies from 2000 to 2008, this in turn suggests that average CEO wealth increased at least slightly from increases with tenure.

\(^{23}\)Frydman and Jenter (2010) document that large US firms gave small but roughly similar amounts of stocks and options in the 1950s and the 1960s. Options then progressively became more popular relative to stocks in the 1970s, the 1980s, and in the late 1990s. In 2000, 49% of CEO pay consisted of options, while only 7% consisted of stocks in S&P 500 companies.

\(^{24}\)This is true whether the Jensen and Murphy (1990) or the Hall and Liebman (1998) statistic is used. The first is the dollar change in CEO wealth per dollar change in firm value. The second is the dollar change in CEO wealth per percent change in firm value. This is also true if we adjust for utility: to the extent that CEO wealth increases over the period, the decrease in CEO incentives is then even more pronounced.
2000 to 2008. Given these two changes – lower incentives and higher CEO wealth – the model unambiguously predicts a diminution in the convexity of CEO compensation. This is precisely what happened. Whether average CEO pay was 49% options and 7% stocks in 2000, it was 25% options and 32% stocks in 2008. While not conclusive, this change is at least consistent with the model’s predictions.

6 Conclusion

The explanatory power of the principal-agent model for the structure of CEO incentive pay largely depends on the postulated utility of wealth. While the hypothesis of CRRA preferences cannot explain the form of CEO incentive pay for plausible values of relative risk aversion (Dittmann and Maug (2007)), we have shown that there exists some HARA utility functions whose explanatory power can be high. More specifically, we have identified preference parameters such that the cost difference between a typical CEO contract and the optimal contract is less than 1% of CEO pay, and the optimal contract thus derived looks similar to a typical CEO contract. This can be achieved with a utility function which is risk averse, prudent, characterized by decreasing absolute risk aversion and decreasing relative risk aversion. The hypothesis of decreasing relative risk aversion can also explain why wealthy CEOs tend to have a low relative risk aversion (Graham, Harvey and Puri (2010)).

This paper does not argue that CEOs have a specific set of preferences that may be inferred from the observation of their compensation contract, if only because the assumption of HARA utility obviously remains quite restrictive. However, we have shown that relaxing the hypothesis of CRRA preferences enables to markedly improve the explanatory power of a standard version of the principal-agent model notably used by Hall and Murphy (2002) and Dittmann and Maug (2007). Given that the principal-agent model of efficient contracting has a high explanatory power for the structure of CEO incentive pay for some plausible CEO preferences, it is not clear that this model should be rejected.
7 Appendix

Let \( \varphi \) be the p.d.f. of \( P_T \). The Holmstrom (1979) condition below describes the optimal contract \( W(P_T) \) when the principal is risk-neutral and the first-order approach applies:

\[
\frac{1}{u'(\hat{\omega} + W(P_T))} = \lambda + \mu \frac{\varphi_e(P_T)}{\varphi(P_T)}
\]

where \( \lambda \) and \( \mu \) are the Lagrange multipliers on the participation constraint and the incentive constraint, respectively. With CRRA utility, using (13), the optimal contract takes the form:

\[
W(P_T) = \begin{cases} 
(a_0 + \alpha_1 \ln(P_T))^{\frac{1}{\gamma}} - \hat{\omega} & \text{if } P_T \geq \bar{P}_T \\
0 & \text{if } P_T < \bar{P}_T 
\end{cases}
\]

where \( \bar{P}_T \equiv \exp\{\frac{\hat{\omega} - a_0}{\alpha_1}\} \), \( a_0 \) and \( \alpha_1 \) are two constants which are determined to satisfy the participation constraint and the incentive constraint.

Likewise, with HARA utility, the optimal contract takes the form:

\[
W(P_T) = \begin{cases} 
\frac{1}{b} \left( \frac{1-b}{a} (a_0 + \alpha_1 \ln(P_T))^{\frac{1}{b}} - a \right) - \hat{\omega} & \text{if } P_T \geq \bar{P}_T \\
0 & \text{if } P_T < \bar{P}_T 
\end{cases}
\]

where \( \bar{P}_T \equiv \exp \left\{ \frac{1}{\alpha_1} \left( \frac{a}{1-b} (a + b\hat{\omega})^b - a_0 \right) \right\} \).

The “representative CEO” of Dittmann and Maug has a fixed wage of $1.2m, is endowed with 0.42% of his company’s equity and 0.50% in stock-options, and has an initial wealth unrelated to his company of 9.1m. The market value of equity of his company is $3.7bn, the options’ exercise price amounts to 63% of the time 0 stock price, \( T \) is 8.5 years, the volatility is 33.5%, and the risk-free rate is 6.6%. We use the same data to facilitate comparability.

8 References


Table 1
Potential cost savings for a range of preference parameters.
This table reports the cost savings associated with a switch to the optimal contract in the HARA-lognormal model, as a percentage of the cost of the actual contract of the representative CEO. Results are reported for a range of values of $a$ and $b$, the parameters of a HARA utility function. In addition, for $b = 0.25$ and $a = -3.9$, potential savings amount to 4.9% of the cost of compensation of the representative CEO. For $b = 0.1$ and $a = -1.85$, potential savings are only 0.96%.

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<tr>
<th>$b$</th>
<th>0.10</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>0.90</th>
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<tr>
<td>-10</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>21.69%</td>
<td>21.80%</td>
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<td>-5</td>
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<td>NA</td>
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<tr>
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<td>5.57%</td>
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<tr>
<td>0</td>
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<td>5.71%</td>
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<td>18.43%</td>
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</tr>
<tr>
<td>10</td>
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<td>5.83%</td>
<td>9.47%</td>
<td>13.63%</td>
<td>16.53%</td>
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Table 2
The shape of different contracts in the HARA-lognormal model.
This table reports the change in the wealth of the representative CEO for a given value of the stock price $P_T$ with respect to his wealth at the median of the distribution. For any given contract, the end-of-period wealth is calculated as the initial wealth capitalized at the risk-free rate of interest for 7 years plus any contractual payment. The median stock price is $P_T(0,e^*)$. The four columns correspond respectively to $u = -2, -1, 1, 2$. Results are reported for five different contracts, including the contract of the representative CEO (“observed contract”) and nine optimal contracts for different values of $a$ and $b$, the parameters of a HARA utility function.

<table>
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<tr>
<th>Change in the wealth of the representative CEO with respect to his wealth at the median stock price $P_T(0,e^<em>)$ if the stock price is $P_T(u,e^</em>)$</th>
<th>$P_T(-2,e^*)$</th>
<th>$P_T(-1,e^*)$</th>
<th>$P_T(1,e^*)$</th>
<th>$P_T(2,e^*)$</th>
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</thead>
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<td>-44%</td>
<td>130%</td>
<td>467%</td>
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<tr>
<td>Optimal contract, $b=0.1$, $a=-1.85$</td>
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<td>-41%</td>
<td>102%</td>
<td>335%</td>
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<td>Optimal contract, $b=0.1$, $a=0$</td>
<td>-60%</td>
<td>-36%</td>
<td>52%</td>
<td>128%</td>
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<tr>
<td>Optimal contract, $b=0.1$, $a=10$</td>
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<td>-33%</td>
<td>37%</td>
<td>78%</td>
</tr>
<tr>
<td>Optimal contract, $b=0.5$, $a=-5$</td>
<td>-61%</td>
<td>-36%</td>
<td>46%</td>
<td>101%</td>
</tr>
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<td>-35%</td>
<td>42%</td>
<td>90%</td>
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<td>38%</td>
<td>81%</td>
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<td>32%</td>
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<td>-32%</td>
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