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From Chess to Catastrophe : Psychology, Politics and the Genesis of von Neumann's Game Theory

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Abstract

> "[S]ometimes he would stand apart, deep in thought, his brown eyes staring into space, his lips moving silently and rapidly, and at such times no one ventured to disturb him".[1]

> "On what you write about politics and psychology, I agree with much, but, in parts, not completely.
>
> I too believe that the psychological variable described by you, where resentment is the primary motive ... is an oft-occurring and important psychological mechanism. But neither is it permissible to forget entirely the other variable: selfishness, wrapped in a veneer of principles and ethics . . ."[2]

Introduction

Most readers of this journal will have heard of von Neumann and Morgenstern's 1944 *Theory of Games and Economic Behavior*, the publication of which can be said to have marked the arrival in 20th century science of the metaphor of the "game".[3] Whether in economics, political science, sociology or even evolutionary biology, the adoption of that metaphor meant the adoption of a fundamental structuring image: that of the game, played by rational, self-interested agents, be they individuals, political actors or even biological organisms, engaging strategically with each other in accordance with a set of rules and in pursuit of certain aims.

In economics, that book provided the point of departure for the work of John Nash, recently celebrated in the book and film,"A Beautiful Mind".[4] Game theory ultimately reshaped what is taught to all students of economics, and underpinned the rise of the culture of laboratory experimentation now sweeping the discipline. In political science and even sociology, the game metaphor has been adopted, with notions of strategic self-interest and the formation of alliances and coalitions becoming part of normal science in these fields. No less quickly, and in a domain where assumptions about "rationality" and "intention" is even more contested than when applied to humans, evolutionary biologists began to draw on game theory in modelling the "game of life", viz., the interaction and evolution of species.

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[1] Israel Halperin, "The Extraordinary Inspiration of John von Neumann", in *Proceedings of Symposia in Pure Mathematics*, Vol. 50: *The Legacy of John von Neumann,* eds. James Glimm, John Impagliazzo and Isadore Singer (Providence: American Mathematical Society, 1990), pp. 15-17 on p. 16.

[2] John von Neumann to Rudolf Ortvay, May 13, 1940, in Nagy, Ferenc (1987), *Neumann János és a "Magyar Titok", A Dokumentumok Tükrében*, (John von Neumann and the "Hungarian Secret") Budapest: Országos Müszaki Információs Központ és Könyvtár. All translation of letters from this volume was done by Mr. Andrew Szirti.

[3] John von Neumann and Oskar Morgenstern, *Theory of Games and Economic Behavior* (Princeton: Princeton University Press, 1947, 1944). Princeton University Press have recently issued a 60th anniversary edition: John von Neumann and Oskar Morgenstern, *Theory of Games and Economic Behavior*, 60th anniversary edition, with an introduction by H. Kuhn and an afterword by A. Rubinstein (Princeton: Princeton University Press, 2004).

[4] Sylvia Nasar, *A Beautiful Mind: a biography of John Forbes Nash, Jr.* (New York: Simon & Schuster, 1998).

For all its impact, however, the *Theory of Games* has been little-read. Among postwar readers, the mathematicians found it cumbersome, and the economists found it inaccessible. For the former, it was plodding mathematics; for the latter, only Morgenstern's didactic introduction was readable. Even today, while the technical aspects no longer constitute a barrier, the book remains a rebarbative and tedious read. One might say that the *Theory of Games* has become a classic, which is to say that it is referred to by many but read by few.

A similar haziness enshrouds game theory's principal architect, von Neumann himself. Though his name be entirely familiar to us and his contributions to mathematics, quantum mechanics, computing and economics universally acknowledged, he has remained something of an inscrutable figure - anecdotes concerning his legendary mental abilities notwithstanding. Of game theory's creator as a living, breathing person, with human foibles, we know relatively little.[5]

We do know that he was only twenty-three years old when, in December 1926, he presented his minimax theorem in a mathematics seminar at the University of Göttingen. We also know, but have not sufficiently emphasized, that, with that paper's publication in 1928,[6] von Neumann essentially *dropped* game theory, scarcely touching it again for over a decade.[7] His return to the subject at the end of the 1930's is typically attributed to his encounter with Oskar Morgenstern, an astute critic of economic theory, freshly emigrated from Vienna, and particularly interested in questions of expectations and economic interaction. However, while the Viennese economist was certainly a

5 Steve Heims, *John von Neumann and Norbert Wiener: From Mathematics to the Technologies of Life and Death* (Cambridge, Mass.: MIT Press, 1980) and Norman Macrae, *John von Neumann: The Scientific Genius who pioneered the Modern Computer, Game Theory, Nuclear Deterrence and Much More* (New York: Pantheon Books Macrae, 1992) provide varying degrees of insight into von Neumann's character. The two treatments balance each other, Heims being particularly harsh on the Cold War hawk, and Macrae gushing in his praise of "Johnny's" scientific genius and anti-communism. Neither is particularly enlightening on von Neumann's creation of game theory. In Philip Mirowski, *Machine Dreams: Economics becomes a cyborg science (*New York & Cambridge: Cambridge University Press, 2002) the treatment of von Neumann is heavily influenced by the later work on computing and automata. The short essays by Marina von Neumann-Whitman, Françoise Ulam, Peter Lax and others in *Proceedings,* eds. Glimm *et al* (cit. n.1) are balanced and informative, as are the discussions in William Aspray, Péter Horváth, Dénes Nagy, Edward Teller, Nicholas Vonneumann and Eugene P. Wigner "Discussion: John von Neumann – A Case Study of Scientific Creativity", *Annals of the History of Computing*, 1989, *11*:165 -169.

6 John von Neumann, "Zur Theorie der Gesellschaftsspiele", *Mathematische Annalen*, *100:*pp. 295-320; trans. by S. Bargmann as "On the Theory of Games of Strategy", *Contributions to the theory of games*, Vol. 4, Eds. Albert Tucker and R. Duncan Luce (Princeton: Princeton U. Press, 1959), pp. 13-42.

7 This fact is rarely emphasized, in part because of von Neumann's interim work on a model of equilibrium economic growth, presented at Princeton in 1932 and published in Vienna in 1937 by mathematician Karl Menger (see John von Neumann, "Über ein ökonomisches Gleichungssystem, und eine Verallgemenierung des Brouwerschen Fixpunktsatzes" *Ergebnisse eines mathematischen Kolloquims*, 1937, *8*:73-83, trans. as "A Model of General Economic Equilibrium", *Rev. Econ. Stud*., 1945, *13*:1-9). Because the growth model involves the minimax principle, it provides an illusion of continuity between the 1928 paper and the *Theory of Games* (see, for example, Mary Ann Dimand and Robert W. Dimand, *The History of Game Theory, Volume I: From the beginnings to 1945* (London and New York: Routledge, 1996, p.144). But this is an illusion only, for beyond the use of the minimax principle, von Neumann's growth model shares nothing with game theory in terms of source of inspiration or object of scientific study. Von Neumann was prompted to send his 1932 paper to Menger in Vienna when he saw the 1934 work in general equilibrium by Abraham Wald, which had been developed and presented under the auspices of Menger's Mathematical Colloquium. Von Neumann knew these Viennese mathematicians well and was an occasional visitor to that city when travelling to and from Budapest in the 1930's. On Menger and the Viennese, see Robert J. Leonard "Ethics and the Excluded Middle: Karl Menger and social science in interwar Vienna", *Isis*,1998, *89*, pp. 1-26

critical catalyst and interlocutor, new evidence now suggests that matters on von Neumann's side were more complex than previously believed.

What follows is intended to be both an "archaeology" of the metaphor of the game, and an occasionally intimate portrayal of John von Neumann as creator of game theory. The central thesis can be put simply: game theory saw the light of day as part of the rich Central European discussions of the psychology and mathematics of chess and other games at the beginning of the 20th century, and was transformed into a way of thinking about interactions in society, by von Neumann primarily, in the political tumult of the late1930's. While our narrative is centered upon von Neumann, it forays widely in the realms of chess, Hungarian mathematics and politics, in order to provide essential contextual background against which his individual creativity can be understood. The first part of the paper explores the interlinked worlds of Hungarian Jewry and Hungarian mathematics, both of which are critical to any understanding of von Neumann: these were the people that shaped him and to which he remained loyal throughout, and the growth of von Neumann's interest in the social order was intricately bound up in the changing fortunes of those communities. The next part carries us into a realm with which many of those Hungarians were familiar: the world of chess in the early 20th century. Von Neumann was initially led to the mathematics of games in a context in which *Schach* and the exploits of the chessmasters were the object of lively discussion amongst intellectuals of all stripes, be they philosophers, psychologists, mathematicians or even novelists. It was here that chess took hold of the cultivated imagination, and here that we begin to see hints of the emergence of the metaphor of the "game" as a way of thinking about society. This tendency is captured in the writings of a remarkable figure, familiar to many in our story, the German Jewish chessmaster and mathematician, Emanuel Lasker. The third part of our essay takes us closer still to von Neumann, and shows why, after a long hiatus, which included emigration to the U.S. and becoming established at Princeton, he was drawn back to game theory in the late 1930's. A key stimulus here was the political upheaval of a period: for the Hungarian Jewish community in Budapest, and the von Neumann family in particular, the close of the 1930's brought with it upheaval, exile, psychological malaise and suicide. Drawing on newly-translated correspondence, we show how, at this time, when confronted with them in most immediate fashion, von Neumann turned his attention to questions of human behaviour and the social order. In going from the study of parlor games to the creation of an embryonic social science, von Neumann was truly swept along by history.

Not only do the social changes of the 1930's appear to have provided a goad for von Neumann, but our examination of them sheds light on a fundamental analytical feature of the *Theory of Games*, its analysis of coalitions. Von Neumann saw game theory as an attempt to capture the essence of stable social arrangements, and shifts from one social order to another. He interpreted different equilibria in games as the expression of different forms of social organization, supported by varying ethical norms, or "standards of behavior". By considering von Neumann's background and his place in the political drama that culminated in the destruction of Hungarian Jewry, we see why he gave game theory the shape and flavour that he did.

Part I: The Hungarians

"Deeply rooted, yet alien"

A characteristic emphasized in many histories of the Jews of Hungary is the degree to which, beginning in the mid-19th century, they achieved integration into Hungarian society.[8] A Jewish community had been present in Hungary since the 10th century, its numbers growing at the end of the 11th with the arrival of refugees escaping pogroms. The first Jewish law in the history of Hungary was passed when King Béla put the community under his protection, with taxes being paid to the court. The second half of the 19th century saw a large wave of refugees from further pogroms in Russia and the eastern part of the Monarchy, so that, between 1840 and 1890, the Jewish proportion of the Hungarian population rose from 2% to almost 5%. The emancipation of Hungary's Jews began in 1849, with the law passed that year forming the basis for a more substantial law in 1867. This was the year of the *Ausgleich*, or Compromise, when the Hapsburg Monarchy, in the face of nationalist pressure, grant greater autonomy to Hungary, marking the beginning of a flourishing period for the country. Law XVII of that year, on the "emancipation of the inhabitants of the Israelite faith of the country", allowed Jews to hold various commercial licenses, practice certain professions and enter parts of the public service. Thus Hungary's efflorescence was accompanied by the assimilation of many Jews into the economic and cultural life of the country.

In December 1868, in Pest, the First National Israelite Congress created an organization of Jewish congregations. Because of disagreement over observance of the Jewish codex (*Sculchan Aruch*), there occurred a split, with the liberal majority forming the Neolog community and the traditional group the Orthodox one. In 1895, the Hungarian parliament accepted the Jewish religion as "bevett vallás", i.e., recognized by the laws of the country, with the congregations and associated cultural institutions receiving support from the state and municipalities. From the 1870's onwards, assimilation was greatest amongst the less religiously strict Neolog Jews, amongst whom it became quite common, for example, to educate children at non-Jewish schools, to change one's surname in favour of a more Hungarian-sounding one, and even go so far as to choose Christian baptism. By the late 19th century, quite a few prominent Jewish businessmen and professionals were awarded titles of nobility for their services to the Austro-Hungarian Empire. Many adopted the mores and aristocratic lifestyle of the Hungarian nobility and intermarried with their families. Indeed, from emancipation until the dissolution of Austro-Hungary with World War I, this liberal project of assimilation saw the emergence of a tacit alliance between the assimilated Jews, who represented the country's economic, financial and industrial interests, and the Magyars, or indigenous Hungarians, whose semi-feudal aristocracy tended to be dominated by landowners, army officers and higher civil servants.

[8] On the history of the Hungarian Jews, see Randolph L. Braham, *The Politics of Genocide: the Holocaust in Hungary*, Vol. I. (New York: Columbia University Press, 1981), Nathaniel Katzburg, *Hungary and the Jews: Policy and Legislation, 1920-1943* (Ramat-Gan: Bar-Ilan University Press, 1981), Ralph Patai, *The Jews of Hungary: history, culture, psychology* (Detroit: Wayne State University Press, 1996) and Kinga Frojimovic, Géza Komoróczy, Viktória Pusztai and Andrea Strbik, *Jewish Budapest, Monuments, Rites, History* (Budapest: Central European University Press, 1999).

Miksa (Max) Neumann, father of Janós (John), was one such assimilated Jew.[9] He was relatively unknown until the mid-1890's, when he began working at the *Jelzáloghitel* (or Mortgage) Bank. There he did well, in part because Kalman Szell, head of the bank and Neumann's personal protector, became prime minister of Hungary in 1899. In 1913, Neumann acquired a title of nobility from the emperor Franz Joseph, becoming von Margittai Neumann. In time, as historian William McCagg puts it, Max von Neumann became "as redolent of new wealth as the new baron Henrik Ohrenstein or as József Lukács, the philosopher's father, his colleagues in the banking community".[10]

The von Neumann family were part of a merchant and financial Jewish community that saw itself as patriotically Hungarian. Thus John von Neumann was educated, not at Hebrew school, but at the Lutheran Gymnasium, along with other Jews including Jené (Eugene) Wigner, later a physicist, and Vilmos (William) Fellner, who became an economist. Like later physicists Tódor (Theodore) von Kármán and Ede (Edward) Teller at the Minta Gymnasium, these assimilated Jews were conscious of their cultural inheritance, yet felt themselves to be Hungarian through and through.[11] Thus there was observance of rites on special occasions, and the shared allusions and language of Central European Jewish humour, but the Orthodox traditions of Talmudic scholarship and devotion were not part of their lives. With successive generations of assimilation, their consciousness of being different quietly faded into the background.[12] The first hints that this might no longer be possible

[9] Max Neumann (1870 – 1923) had arrived in Budapest at age 10 from Pecs in the Southwest. Trained as a lawyer, he married Margit Kann, daughter of Jacob Kann. The Kann's were a wealthy Jewish family, whose fortune had been made selling agricultural equipment and hardware to Hungary's large farms. The Kann-Heller firm was located on the ground floor of 62 Vaczi Boulevard (later renamed Bajcsy-Zsilinszky St.), with the rest of the building being divided into apartments. The Hellers occupied the second floor and the Kanns the remaining two floors. The top floor, 18-room apartment went to Max and Margit, to whom John von Neumann was born. See Macrae, *John von Neumann* (cit. n.5), pp. 37-46.

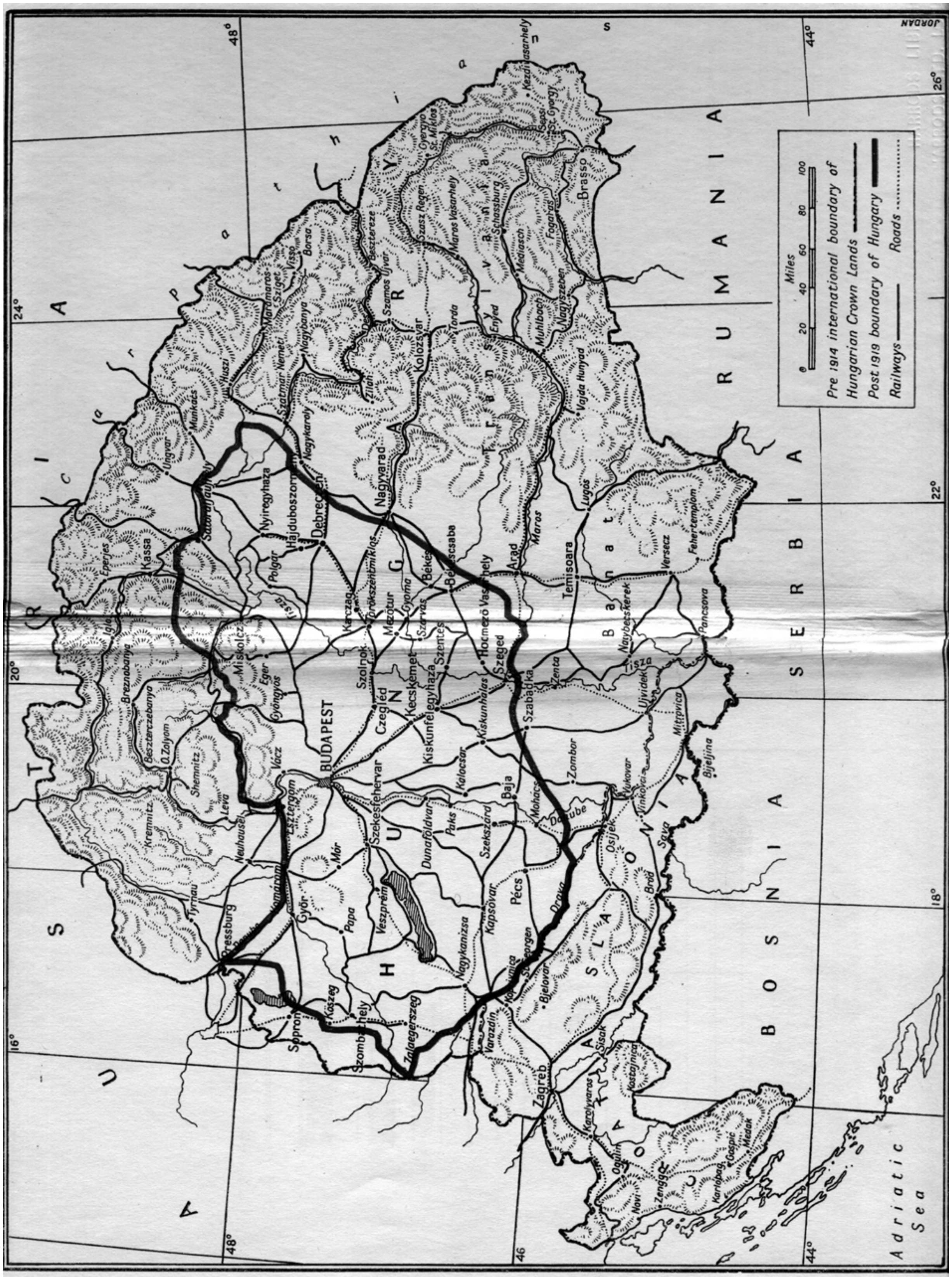
[10] William O. Jr McCagg, *Jewish Nobles and Geniuses in Modern Hungary* (Boulder: East European Quarterly, 1972), on p.69. John Harsanyi, Hungarian game theorist of the next generation, and recent Nobel laureate in economics, while he did not know von Neumann personally, knew very well the society he came from. It was characteristic of von Neumann, said Harsanyi, "that that he always used the title "von" and was sort of offended if somebody didn't use it. Not only that ... but if the sentence starts with von Neumann ... then you don't capitalize the "von", because you don't change the original spelling. And von Neumann insisted even on that... This, of course, was just a very minor human weakness... Of course, this was not uncommon in the Austro-Hungarian monarchy: that Jewish people who were either rich or famous, or both, would try, first of all, to become Christian, and try to acquire aristocratic titles or at least titles of the nobility, and he was obviously influenced by this" (Harsanyi interview with author, April 16, 1992, Berkeley, California).

[11] The Minta had been founded by Mór von Kármán, Tódór's father, and was affiliated with the University of Budapest.

[12] Another assimilated Jew from the same background as the von Neumann's was banker's son and Communist, György (George) Lukács (1885-1971): "The Leopoldstadt families were completely indifferent to all religious matters. Religion only interested us as a matter of family convention, since it played a certain role at weddings and other ceremonies … we all regarded the Jewish faith with complete indifference" György Lukács, *Record of a life* (London: Verso, 1983), p. 26. "At the Protestant Gymnasium I attended, children from Leopoldstadt played the role of the aristocracy. So I was regarded as a Leopoldstadt aristocrat, not as a Jew. Hence the problems of the Jews never came to the surface. I always realized that I was a Jew, but it never had a significant influence on my development" Lukács, *Record of a life,* p. 29. Von Neumann's brother, Nicholas, recalls their other brother, Michael, questioning the family's ambiguous religious stance, to which Max von Neumann replied that it was simply a matter of tradition. Stan Ulam, a Polish Jew and close friend of von Neumann, recalled that the tradition of talmudistic Judaic scholarship was "quite conspicuously absent from von Neumann's makeup" , but he remembered his indulging in Jewish jokes and banter.

emerged at the end of World War I, when John von Neumann was in his teens, and old Hungary was broken up.

"The goys have the following theorem...". Stan M. Ulam, *Adventures of a Mathematician* (New York: Charles Scribner's Sons, 1976), on p. 97.


Miles
Pre 1914 international boundary of Hungarian Crown Lands
Post 1919 boundary of Hungary
Railways
Roads

In 1920, the Treaty of Trianon saw the dismantling of the Austro-Hungarian empire, with Hungary required to sacrifice no less than two-thirds of its lands, and, with them, one-third of ethnic Hungarians, to the successor states, Czechoslovakia, Rumania, Yugoslavia and Austria. The result was a greatly reduced "rump Hungary", and the birth of revisionist ambitions to regain the lost territories.

After Trianon, the position of the Hungarian Jews began to change. With the disappearance of other large ethnic groups with the surrendered regions, the integrated Jews lost part of their political function in the Jewish-Magyar alliance. The numbers of Jews in the country were swelled by further immigrants from the east, many of them Orthodox, keen, as they were, to remain within Hungary. With the loss of the territories, there was also a rise in the proportion of positions held by Jews in the business, legal and medical professions. The result was a sharpening of focus on Hungarian Jewry, with all the usual contradictions inherent in such scapegoating. Thus, even if most assimilated Jews were opposed to Bela Kun’s short-lived Communist insurgency in the Summer of 1919 - including the von Neumanns who left for a holiday home on the Adriatic - the fact that a majority of Kun’s revolutionary Commissars were Jewish intellectuals contributed to the popular image of the “Jewish Bolshevik”. When Admiral Nikolas Horthy regained power and cracked down on Kun’s supporters in the White Terror of 1920, a great many of those killed, tortured or forced to flee were Jewish. At the same time, the visible Jewish presence in commercial life, coupled with the extravagant display of riches of a few, served to reinforce the popular perception of enormous Jewish wealth: the country’s capital was nicknamed "Judapest". It was in this context that Horthy’s Hungary, in 1920, passed the first piece of anti-Semitic legislation in 20th century Europe. Ostensibly designed to control university registrations in general, the key clause of Law 1920: XXV, the *Numerus Clausus*, was one intended to restrict Jewish access to higher education, and thus the professions, to a level corresponding to their proportion of the population.

While their merchant and banking parents had flourished during the Golden Age that followed emancipation and the “Ausgleich”, the more highly-educated generation of Jewish youth that matured around the time of World War I were to be less settled. Not only was Hungary already small and limited in terms of opportunities in science and education; but there was also the added discriminatory element. Thus, by the time the 1920 law was passed, many young Hungarian Jews had in fact already begun looking abroad for opportunities. For example, mathematician Gyorgy (George) Pólya completed a doctorate in Budapest before heading off, in 1911, to Vienna, and then to Göttingen, Paris and Zurich. He would eventually go to Stanford. Chemist Michael Polanyi, having already studied in Germany, left Budapest in 1919 to return to Karlsruhe. He would eventually settle in England. He was helped by Theodore von Kármán, then already professor of aerodynamics in Aachen and a source of guidance to many émigrés. Von Kármán himself would later end up at California Institute of Technology. Polanyi, in turn, helped several others find jobs, including physicists Leo Szilard and Imre Brody.[13] When the law of 1920 was passed, John von Neumann became one of several thousand Hungarian Jewish students who went abroad to study in Austria, Czechoslovakia, Germany, Italy and Switzerland. In contrast to the desultory university environment they were leaving behind, these students were enthusiastically received abroad, especially by the university mandarins in the tolerant climate of Weimar Germany.

[13] See Tibor Frank, “Networking, Cohorting, Bonding: Michal Polanyi in Exile”, *Polanyiana*, 2001, *10*:108-126.

The Mathematicians

Amongst the scientific fields in which assimilated Jewry played an important role in the late 19th century Europe, mathematics stands out. Von Neumann was always particularly proud of the Hungarian mathematical community from which he came, and if I dwell a while on a group that will appear foreign to many readers, it is because they enter the warp and weft of von Neumann's work on games, from chess right through to catastrophe.

Prior to the *Ausgleich*, Hungary's mathematicians were few, the best known of them being the Bolyais, father and son.[14] Parkas Bolyai had studied at Göttingen, where he was a fellow student of Carl Friedrich Gauss, making original contributions in Euclidean geometry. He wrote an important textbook and taught at the Reformed College in Marosvasarhely, Transylvania. His son Janos also worked on the problem of parallels, based on Euclid's Fifth Postulate, and was one of the independent creators, along with Gauss, of non-Euclidean, "hyperbolic" geometry. However, it was not until after his death in 1860, that Janos Bolyai's work received international attention, with Felix Klein and Henri Poincaré both drawing upon him. At the University of Texas, a longtime stronghold in American mathematics, C.B. Halsted was particularly active in translating Bolyai and promoting international recognition of his work..

Parkas Bolyai (1775-1856)
|
Janos Bolyai (1802-1860)
|
Gyula König (1849-1913)   Jósef Kürschák (1864-1933)
|
Lipót Fejér (1880-1959)   Frigyes Riesz (1880-1956)   Dénes König (1884-1944)
|
Alfred Haar (1885-1933)   Rudolf Ortvay (1885-1945)
Bela Kjerekárto (1898-1946)   G. Szego (1895-1985)
|
John von Neumann (1903-1957)   László Kalmár (1905-1976)   Rózsa Péter (1905-1977)
|
Paul Turán (1910-1976)   Paul Erdós (1913-1996)

In the generation after Bolyai, several names stand out, both for their scientific work and their role in the eventual cultivation of a national mathematical culture. Gyula König completed his doctorate at Heidelberg in 1870, working in the area of elliptic functions. He joined the newly formed Technical University of Budapest in 1874, where he worked in algebra, number theory, geometry and set theory. An influential teacher and administrator, he also founded a short-lived journal,

[14] On the world of Hungarian mathematics, see Reuben Hersh and Vera John-Steiner "A Visit to Hungarian Mathematics", *The Mathematical Intelligencer*, 1993, *15*:13-26.

Müegyetemi Lapok.[15] Also at the Technical University, and member of the Hungarian Academy, József Kürschák worked in the fields of geometry, calculus of variations and linear algebra. In 1912, drawing upon König's work, Kürschák established the theory of valuations, which allowed notions of convergence and limits to be used in the theory of abstract fields, the work for which he is most remembered.

1894 was a pivotal year in the development of Hungarian mathematics, for this was the year in which Baron Eötvös Loránd (1848-1919), physicist, and founder of the Mathematical and Physical Society of Hungary, became Minister of Education. Like his politician father of the same name, Eötvös epitomized the Magyar liberalism of the late 19th century, under which the Hungarian Jews eagerly sought assimilation and became thoroughly attached to Hungary. Determined to encourage the development of the country's scientific culture, Eötvös supported the establishment of not a college for the training of mathematics teachers, but both the Eötvös Competition in mathematics for secondary school students and the *Kozeposkolai Mathematicai Lapok*, or "*KöMaL*" for short, a monthly *Mathematics Journal for Secondary Schools*. The contribution to Hungarian mathematical culture of these two institutions, the competition and the magazine, is universally acknowledged.[16]

Promoted by Gyula König in particular, the Eötvös Competition was an annual examination intended to identify students of ability. The best gymnasium students were groomed months in advance, in preparation for the great day when, in a closed room, they faced a series of written questions of increasing degrees of difficulty. Winning it conferred great prestige on both the student and his teachers. Von Kármán (1881-1963), who in his day won the prize, said that the toughest questions demanded true creativity and were intended to signal the potential for a mathematical career. A compilation of the problems was published in 1929 by Jozsef Kürschák, and later translated, in 1961, as the *Hungarian Problem Book*.[17] Over the years, in addition to von Kármán, the winners of the Eötvös Prize included Lipót Fejér, Gyula König's mathematician son Dénes, Alfred Haar, Edward Teller, Marcel Riesz, Gabor Szego, Laszl Redei, László Kalmár and John Harsányi. Von Neumann did not compete as he was out of the country the year he would have been eligible.

The mathematics magazine *KöMaL*, was founded in 1894 by Gyór schoolteacher Daniel Arány, in order "to give a wealth of examples to students and teachers". Each issue contained general mathematical discussion, a set of problems of varying degrees of difficulty, and the readers' most creative or elegant solutions to the questions of the previous issue. Eagerly awaited in the postbox by many Hungarian students, it brought prestige to those who were successful, and contributed, like

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[15] See the website in the history of mathematics at the University of St. Andrews, Scotland (www-history-mcs.st-andrews.ac.uk/history/Mathematicians.html). Each of the site's biographical entries is a synthesis of several sources, all cited and, in the present case, many of them in Hungarian. As for König, the story is told of his dramatic claim in 1904 to have proved that the continuum hypothesis was false. At the International Congress of Mathematicians, at Heidelberg, where the claim was announced, all other conference sessions were cancelled so that König could be given complete attention. The proof was soon found by Ernst Zermelo to contain an error.

[16] See, for example, Tibor Radó, "On Mathematical Life in Hungary", *American Mathematical Monthly*, 1932, 37:85-90 and Hersh and John-Steiner, "A Visit" (cit. n.14).

[17] See József Kürschàk, *Hungarian problem book : Based on the Eötvös competitions, 1894-[1928]*, Rev. and edited by G. Hajós, G. Neukomm and J. Surányi, trans. by Elvira Rapaport (New York: Random House, 1963).

the Eötvös Prize, to the cultivation of a general interest in mathematics among the Hungarian young.

As the above suggests, schoolteachers of mathematics such as Arány, and also László Rátz and Mikhail Fekete of Budapest, played a critical role in Hungary. While mathematical education was of the first order, university positions were few, with the result that many mathematicians of fine ability found themselves teaching at secondary level or providing private tutorials to Budapest gymnasium students. There, they noticed and groomed young talented pupils and, in this small community in which everyone knew everyone else, guided them onwards towards their university colleagues. Von Neumann's mathematics teacher at the Lutheran Gymnasium, Lászlo Rátz, played a mentoring role that is stressed by many commentators on this period. In addition to from Rátz, von Neumann appears to have received private classes from Gabor Szegö and, later, Michael Fekete.[18]

If the Eötvös Prize and *KöMaL* are the first two factors often cited in discussions of the Hungarian mathematical phenomenon, the third one was a person: Lipót Fejér, probably the most influential figure in the generation following König and Kürschák.[19] Like Max von Neumann, his friend, he was born in Pécs, his surname at birth being Weiss. He distinguished himself in his contributions to *KöMaL* and became known to Lászlo Rátz. Winning the Eötvös Prize in 1897, he studied mathematics and physics at the University of Budapest until 1902, spending a year at the University of Berlin. During this time, he changed his name to the less Jewish, more Hungarian-sounding, Fejér – which also means "white" in Hungarian. Following a doctoral thesis at Budapest in 1902 on Fourier series, he taught in that city for three years, spending some time at Göttingen and Paris. Then, following several years on the university faculty in Koloszvár, Fejér won an appointment in 1911 to a chair in Budapest, where he would spend the rest of his career. He is remembered for his work on Fourier series, the theory of general trigonometric series, and the theory of functions of a complex variable, as well as for some contributions to theoretical physics and differential equations.

Budapest graph theorist Paul Turán would later credit Fejér with the entire creation of a coherent mathematical school. Another wrote that "a whole culture developed around this man. His lectures were considered the experience of a lifetime, but his influence outside the classroom was even more significant".[20] One of the legends surrounding Fejér concerns Poincaré's 1905 visit to Budapest, to accept the Bolyai prize. Greeted upon his arrival by various ministers and high-ranking officials, Poincaré looked around and asked: "Where is Fejér?". "Who is Fejér?", the ministers replied. "Fejér", said Poincaré, "is the greatest Hungarian mathematician, one of the world's greatest mathematicians". Within a year, Fejér was appointed to the professorship at Koloszvár. On a later occasion, in 1911, Fejér's candidacy for the above-mentioned chair at Budapest was apparently opposed by faculty anti-semites, aware that his name had been Weiss. One asked cynically whether Fejér was related to the distinguished university theologian Father Ignatius Fejér, to which Eötvös,

[18] See Lax, "Remembering John von Neumann" (cit. n.5) and also the discussion with Eugene Wigner in Aspray et al, "Discussion" (cit. n.5).

[19] On Fejér, see Paul Turán , "Fejér Lipót mathematikai munkásseaga", *Mat. Lapok, I*:160-170, translated as "Leopold Fejér's mathematical work", in *Collected Papers*, 1949, *I*:474-481, and Paul Turán, "Fejér Lipót, 1880-1959", *Mat. Lapok*, 1960, *12*:8-18, translated as "Leopold Fejér (1880-1959). His life and work", in *Collected Papers*, *2*:1204-1212.

[20] G.L. Alexanderson, et al, "Obituary of George Pólya", *Bulletin of the London Mathematical Society*, 1987,*19*:559-608 quoted in Hersh and John-Steiner, "A Visit" (cit. n.14), p.?

then a faculty member, immediately replied: "Yes. Illegitimate son". The appointment went through without difficulty.

Agnes Berger, who later became professor of statistics at Columbia University, remembers Fejér's teaching:

> "Fejér gave very short, beautiful lectures. They lasted less than an hour. You sat there for a long time before he came. When he came in, he would be in a sort of frenzy. He was very ugly-looking when you first examined him, but he had a very lively face with a lot of expression and grimaces. The lecture was thought out in very great detail, with a dramatic denouement. It was a show"[21]

If contact between Hungarian student and teacher tended to be formal and distant, the relationship with Fejér was different. Apparently, he would sit in coffee-houses frequented by the mathematicians, such as the Erzsébet café in Buda, or the Mignon in Pest, regaling his students with stories about mathematics and mathematicians he had known. A regular dinner guest at the Max von Neumann household, Fejér enjoyed the friendship of creative people of all sorts, including Endré Ady, the revered Hungarian poet. Beyond von Neumann, Fejér had a lasting influence on many younger Hungarian mathematicians, including George Pólya, Marcel Reisz, Gábor Szegô, Paul Erdós, Paul Turán, László Kalmár and Rozsa Péter. Nonetheless, Turán intimates that the events of 1919-1923, the Kun Revolution and Horthy's White Terror - "those times" - weighed heavily on Fejér, that he was not quite the man afterwards that he had been before. Although he was certainly less active after the Second World War, until his death in 1959, Fejér continued to enjoy an international reputation as one of the two recognized leaders of the Hungarian school of analysis. The other was his friend and close collaborator, Frigyes Riesz.

Before discussing Riesz, we should mention the Franz Joséf University at Kolozsvár, the trials and tribulations of which are relevant here. Prior to the First World War, the most important universities in Hungary were at Budapest and in this Transylvanian town, the latter being the country's second city and home of several administrative offices. After Trianon, however, in 1921, when Transylvania was handed over to Rumania, and Kolozsvár renamed Cluj, that university found itself without a Hungarian home.[22] The entire faculty moved temporarily to Budapest, before being transferred permanently to Szeged, a provincial garrison town of 120,000 in the south of the country, lying less than 10 miles from the triple border with the hostile Yugoslavia and Rumania.

Szeged was presided over by Riesz. Born in Gyór, like Arány, he studied at Zurich Polytechnic, Budapest and Göttingen, before completing a doctorate in the Hungarian capital.[23] After several years' school-teaching he took Fejér's post at Koloszvár, in 1911, when the latter left for Budapest. Amongst the contributions for which Riesz is remembered are his representation theorem on the

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[21] Quoted in Hersh and John-Steiner, "A Visit" (cit. n.14), p.18.

[22] Cluj, which the Germans would briefly name Regensburg, was the birthplace of Abraham Wald (1902- 1950), who became a contributor to mathematical economics in the interwar period, well known to von Neumann and the Viennese economics community surrounding Karl Menger and Karl Schlesinger.

[23] See Frigyes Riesz, "Obituary", *Acta Scientiarum Mathematicarum Szeged*, *7*:1-3, Edgar R. Lorch, "Szeged in 1934", (edited by Reuben Hersh) *American Mathematical Monthly*, 1993, *100*:219-230.

general linear functional on the space of continuous functions; his work on the theory of compact linear operators; his reconstruction of the Lebesgue integral without measure theory; and famous Riesz-Fisher theorem, which was a central result on abstract Hilbert space and was essential to proving the equivalence between Schrödinger's wave mechanics and Heisenberg's matrix mechanics.[24]

A quiet man, Riesz apparently gave the impression of aloofness, but he was approachable to those who knew him. His assistant, Tibor Rádo, would later recall Riesz's extreme perfectionism, a characteristic that saw him, and thus poor Rádo, rework his writings many times, over several years, before presenting them for publication. One of the visitors that Riesz attracted from abroad in the 1930's was American mathematician Edgar Lorch. A Columbia PhD, Lorch was finishing a postdoctoral stay at Harvard with Marshall Stone, when, on health grounds, no less, he decided to turn down a prospectively gruelling position as assistant to von Neumann, who was by then at the Institute for Advanced Study, in Princeton. Lorch went to Szeged instead:"If John von Neumann was the acknowledged genius of modern mathematics, Frederick Riesz was the dean of functional analysts. He was not well known to the world at large, but the cognoscenti had the highest respect for him".[25]

Also in the Szeged group were Alfred Haar and mathematical physicist Rudolf Ortvay; of the latter, in particular, we shall have more to say later. In the mid-Twenties, they were joined by topologist Bela Kjerekárto, and two new assistants, István Lipka and Laszló Kálmár. Together, the Szeged mathematicians formed the János Bolyai Mathematical Institute,[26] and they established the *Acta Scientiarum Mathematicarum Szeged*, or *Acta Szeged* for short, which published articles in the international languages and quickly became a mathematics journal of international reputation.

[24] Riesz's brother, Marcel, was also a mathematician of repute. Part of the Hungarian diaspora of the period, he made his career in Stockholm, Sweden.

[25] Lorch, "Szeged" (cit. n.23), p. 222. This article provides a nice portrayal of Szeged in the 1930's. Lorch and Riesz collaborated on one paper during that year, on a problem in transformations in Hilbert space to which von Neumann and Stone had already contributed.

[26] Bolyai Institute, "A Short History of the Bolyai Institute" (no date), available at server.math.u-szeged.hu/general/bolyhist.htm.

Bolyai Institute of Mathematics, Szeged, 1928.
Back: Frigyes Riesz, Béla Kerékjártó, Alfréd Haar, Dénes König, Rudolf Ortvay
Middle: József Kürschák, Garrett Birkhoff, O. D. Kellogg (both visiting from U.S.), Lipót Fejér
Front: Tibor Rádo, István Lipka, László Kalmár, Pál Szász
(from Nagy 1987, with permission)

Amongst the younger mathematicians closer in age to von Neumann were Dénes König, László Kalmár and Rozsa Péter. The first of these, son of mathematician Gyula König, studied at Budapest and Göttingen, obtaining his doctorate in 1907, then becoming a teacher at his father's institution, the Budapest Technische Hochschule. His work represented an important stream in Hungarian mathematical research, that of discrete mathematics, which includes graph theory, combinatorics and number theory.[27] König lectured on graph theory and published a foundational book on it in 1936.[28] As for Kalmár, he was born in Kaposvár to the south of Lake Balaton and raised in Budapest. An orphan by the time he entered the University of Budapest, he was a student of Kürschák and Fejér, specializing in the field of logic. After a stay at Göttingen, he took a position at Szeged, initially serving as assistant to both Haar and Riesz.

Finally we can mention Rózsa Péter, one of the very few women mathematicians of the period. Born Rózsa Politzer, she began studying chemistry at Loránd Eötvös University in Budapest, but switched to mathematics after attending lectures by Fejér.[29] Like Kalmár, to whom she was close,

[27] On König, see Tibor Gallai, "Dénes König: A Biographical Sketch" , in Dénés König, *Theorie der endlichen und unendlichen Graphen.* (Leipzig,1936), trans. by Richard McCoart as *Theory of Finite and Infinite Graphs* (Boston: Birkhäuser, 1986), pp. 423-426.

[28] See König, *Theorie der endlichen* (cit. n.27). In the area of discrete mathematics, König's successors in the next generation were Paul Turán and Paul Erdós.

[29] On Péter, see Edie Morris and Leon Harkleroad, "Rózsa Péter: Recursive Function Theory's Founding Mother", *The Mathematical Intelligencer*, 1990, *12*:59-64.

she graduated in 1927, specializing in number theory, but, as a Jew and a woman, she was doubly handicapped in obtaining a post as secondary school teacher. Depressed by the discovery that some of her theorems had already been proved by foreign mathematicians, Politzer actually abandoned mathematics, concentrating her energies on poetry and translation. It was Kalmár who encouraged her to return to the fold at the beginning of the 1930's, pointing to Gödel's recent results on incompleteness, which Politzer was then apparently able to reach using different methods. This led her to explore, in their own right, the recursive functions that had served as an important tool in Gödel's work, and she began presenting results in 1932, publishing several papers and eventually joining the editorial board of the *Journal of Symbolic Logic* in 1937. Despite her changing her name during this decade from Politzer to the more Hungarian, and less Jewish-sounding, Péter, she remained without a post for a long time, making a living as a private tutor.

In this small community, von Neumann was quickly recognized as a prodigy. As a Gymnasium student, he caught the attention of Rátz, received tutoring in university-level mathematics from Mikhail Fekete, and then enrolled at the University of Budapest, where he worked on set theory under the guidance of Fejér. Although registered there, he worked largely *in absentia*, part of the Hungarian student exodus, taking a parallel degree in chemical engineering at Zurich and then studying mathematics in Berlin. By 1926, when he went to Göttingen as International Education Board post-doctoral fellow under David Hilbert, von Neumann was well-known to the German mathematicians.

His 1928 paper on games may be situated at the confluence of two distinct streams of the mathematical literature on the subject, both of which were influential on him. The first concerns the game of chess, and runs from the 1913 article by erstwhile Göttingen mathematician, Ernest Zermelo, through further contributions by von Neumann's Hungarian friends, König and Kálmár. The second series, by the older and geographically distant French mathematician, Emile Borel, concerns, not chess, but what we shall call for the moment general 2-person games.

The papers on chess by Zermelo and the Hungarians may be regarded as the manifestation in mathematics of an extraordinarily rich "conversation" about this game, occurring in the early part of the century and involving a range of people, from players and grandmasters to psychologists, mathematicians and novelists. As a point of entry to it, we can take Zermelo's oblique remark, about which we shall have more to say later, that his paper was an attempt to treat chess "in a mathematically objective manner, without having to make reference to more subjective-psychological notions such as the 'perfect player' and similar ideas".[30] Behind that perfunctory aside lies the fascinating world of chess in early 20th century Europe.

[30] Ernst Zermelo, "Über eine Anwendung der Mengenlehre auf die Theorie des Schachspiels", *Proc. Fifth Congress Mathematicians*, Cambridge 1912, (Cambridge University Press, 1913), pp. 501-504, translated as "On an Application of Set Theory to the Theory of the Game of Chess" in Ulrich Schwalbe and Paul Walker "Zermelo and the Early History of Game Theory", *Games and Economic Behavior*, 2001, *34*:123-137, on p.?

Part II: Chess, Psychology and the Science of Struggle

Emanuel Lasker

At the turn of the last century, *Schach* enjoyed great visibility in many parts of Continental Europe. The game was important in England France, Germany and Russia and particularly so in the countries of the Austro-Hungarian Empire. Amongst the Jewish communities in those countries, it commanded particular interest. From London to Moscow, the grandmasters enjoyed great visibility and prestige, and the game was played in the chess cafés of the capitals, such as Paris's famous Café de la Régence. Against a background of high tournament drama, chessmasters wrote manuals on strategy; psychologists investigated the thought processes required in the game; and mathematicians wondered whether so human an activity could be made amenable to formal treatment. Others speculated philosophically about the relationship of chess to life in general, and the game was source of inspiration for several writers, including Vladimir Nabokov, author of *The Defence* in 1929, and Viennese exile Stefan Zweig, whose *Schachnovelle* was the last thing he wrote in Brazil before his suicide in 1941.[31]

Looming large over the work of Zermelo and the Hungarians, and indeed the entire game at this time, is the giant figure of Emanuel Lasker (1868-1941), world chess champion for an unprecedented 24 years from 1897 to 1921. Trained as a mathematician, his mentors included Hilbert and Max Noether, and he completed a PhD in mathematics at Erlangen in 1902 on the theory of vector spaces. Noether's mathematician daughter Emmy would later develop Lasker's algebraic work further.

From a family of modest means, Lasker actually interrupted his mathematical studies to play chess for money, and he took the world title in the process. Admired by Albert Einstein, Lasker was regarded as the player who introduced psychological considerations into the game of chess. In this, he stood in particular contrast to previous world champion, Wilhelm Steinitz, and German champion, Siegbert Tarrasch, both of whom advocated a highly logical approach, and the idea that, for every position, there existed a theoretically optimal move, independent of the character of one's opponent.[32] This opposition between the formal/logical and intuitive/psychological approaches to the game runs like a red thread through chess discussions of this period.[33]

[31] Stefan Zweig, *The Royal Game and Other Stories* (New York: Harmony Books 1981) orig. *Schachnovelle*, written in late 1941, early 1942, trans. as *The Royal Game* (New York: Viking Press, 1944).

[32] On Lasker, see Jacques Hannak, *Emanuel Lasker: The Life of a Chess Master* (New York: Simon and Schuster, 1959), an English translation by Heinrich Fraenkel of the 1942 biography in German. This book tends to be criticized for its hagiographic treatment of its subject. Recently, Lasker has been the focus of renewed attention, with the formation of a *Lasker Gesellschaft* in Germany. See Ulrich Sieg and Michael Dreyer (eds.), *Emanuel Lasker: Schach, Philosophie und Wissenschaft* (Berlin: Philo, 2001).

[33] Matters are further complicated in Lasker's case by his advocating one kind of chessplay in his theoretical writings but playing another when at the board himself.

Emanuel Lasker

Of Lasker, British chess champion and commentator Gerald Abrahams later wrote:

> "If he had a style . . . it is revealed in a desire for an unbalanced game; a different type of imbalance from that sought by Alekhine, and possibly a greater strain on playing power… In the battles he fought he was conscious of the truth that there need not be "a best move".
>
> The consequence is that Lasker played a type of chess that is difficult to describe. His vision was very great. . . Consequently he was always dissatisfied. . . and frequently sought to unbalance the game because of the possibilities that he saw – the battle after the skirmish, the course of the war beyond the battle"[34]

Lasker was also a prolific author and his chess writings were unique in their richness. If there exists an embryonic attempt to develop a "science of struggle" of relevance to the social realm, it lies in Lasker's 1907, *Kampf*, an 80-page pamphlet, the short chapters of which bear titles such as "Strategy", "The Work Principle", "The Economy Principle" and "Equilibrium and Dominance". Here, Lasker uses his experience in chess as a point of departure to analyse the place of struggle in various realms, and economic ideas are the thread binding it all together. "What is struggle and victory?", asks Lasker. "Do they obey laws which reason is able to capture and establish? What are these laws? – That is the problem".[35]

Lasker called his science of struggle Machology, after "Machee", the classical Greek term for a fight. The notion of struggle is understood broadly, encompassing any form of struggle against resistance, thus being applicable to the efforts of not only living entities but also plants, nations, races, even languages. Any struggle involves several centres of activity: termed "strata". Thus in a war, the strata are soldiers, the canon, the fleet, etc. Each stratem, in turn, is made up of "jonts". For example, in a marine conflict, a battleship is composed of captain, sailors, equipment etc.

The most important objection against any attempt to attempt a science of struggle, Lasker felt, was the infinite number of events in a struggle, and the uncertainty surrounding them. Suffice it to look at chess, he says, to prove the numerous ways in which a struggle can develop from a certain position. One would think it impossible to establish a law amidst such multiplicity. But even in the

[34] Gerald Abrahams, *Not Only Chess (*London: George Allen & Unwin, 1974), on p. 146.

[35] Emanuel Lasker, *Kampf* (New York: Lasker's Publishing Co., 1907), reprinted in 2001 by Berlin-Brandenburg: Potsdam, with foreword by Lothar Schmidt, on p. 11.

game of chess, he says, and even at an ordinary level, it is evident that the choice of moves is very much limited by their need to be useful. Amongst chess masters, there is even greater reduction of possibilities, and as the game progresses the number of possibilities diminishes more and more.

Throughout *Kampf*, Lasker makes many references to the economic realm and to value, and gives central place to the figure of *homo economicus*. Just as the best merchant is one whose buying and selling is conducted to his best monetary advantage, so too can one speak of a best way to engage in a struggle. For example, a military general striving to achieve a certain objective will do so in such a way as keep his losses in military value as small as possible. In the domain of struggle, says Lasker, the parallel to money in commercial life may be called "energy". *Macheeiden*, or perfect strategists, will be infinitely economical with the energy of the struggle at their disposal.[36] "Let's look at the struggle of a merchant", he writes:

> "The social utility of his activity or of his goods, the work they save society, represent his strata capable of attack. The announcement [i.e., advertisement], in any form, constitutes an army of very flexible strata, which serve for attack and which are sent forward to points of great enemy pressure. His money and credit are his flexibility. Book-keeping is the wall. The enemy is his [competitive] better. Other enemy strata are tasks to be done, such as taking orders, distribution and receiving cash.
>
> The machee-ide solves them following the principle of economy which will be treated later on, and the field of struggle which is determined by the consuming and buying society, its legislation, and the purchasing power of money".[37]

Lasker then goes on to describe what he calls the principle of work (*Principe der Arbeit*) or the value principle (*Werthprincip*). When strata are involved in a struggle, aiming to achieve an objective, they do "struggle work". This is no more difficult to measure, says Lasker, than the work in mechanical, thermal or electric or other forms of power. In a war, for example, the work, or value, of a run of bullets is its expected number of hits. In chess, it is the capture of pieces, the domination of fields of escape of the king, etc.

> "The genius of the strategist", Lasker continues, "lies simply in his ability to accomplish as much work as possible with his army of strata. . . The capacity of a group of strata to perform will be called its "value".
>
> ... The perfect strategist will obtain more work from a group of strata the higher its value. One can prove this exactly. Suppose that at the beginning of a struggle the strategist has the choice of two groups of strata, A and B, which he can include in his army. Because A and B can only be of an advantage to the strategist insofar as they help him with the resolution of his tasks, so the strategist will choose without doubt the group of strata with the greater performance capacity. The strategist cannot be wrong because he can calculate in advance

[36] Even atoms, says Lasker, to the extent that they obey Gauss's principle of least resistance and other minimal principles, are probably *Macheeiden*. Similarly, "instinct" is a *Macheeide*, since plants and animals instinctively behave economically when faced with attacks. A species makes an effort only when facing some resistance: when the resistance disappears, the organ used to combat it will be directed by nature towards some other purpose. Likewise, the energy consumption for a change in lifestyle is, and always has been, infinitely economical and the principle of development of all life is necessarily deduced from that.

[37] Lasker, *Kampf* (cit. n.35) pp.27-28.

> (*im Voraus*) the optimal path (*eumachische Bah*n) of the upcoming struggle. This is why he will really take great advantage of the group of strata with the highest capacity to perform" .[38]

Lasker continues with a verbal account of what would later be described as the problem of optimal force allocation. To derive the greatest utility from an army, one must give a small task to strata of little worth, because then the duty to perform a task diminishes the flexibility of a stratem and therefore any other work you could get out of it. The strata with the highest utility will be most prone to an attack of the enemy. So they have to be placed and protected in a way such that it would cost the enemy enormous efforts to force them to retreat or to eliminate them. Any manoeuvre costs an effort and has to be compensated by an increase of utility.

According to Lasker, the "Principle of Economy" applies to all areas of creation, artistic and scientific. Good work in any of these areas is that which achieves the most with the means at one's disposal:

> "The perfect strategist is by nature infinitely economical with the energy at his disposition. One who longs to come as close as possible to being a perfect strategist will therefore critically examine all actions or manoeuvres, even if a way to obtain an advantage with little effort is clearly available...
>
> The perfect strategist therefore is completely free of panic-like emotions of fear. He is always objective. This statement is so obvious that it sounds banal. It is nonetheless very rarely taken into account."[39]

The principle of economy applies to a struggle between opponents, but also to the realms of art and science:

> "A man full of the creative impulse struggles with an idea which imperatively demands to be artistically expressed or being examined by him with scientific rigour. The artist masters the technical means of his art – words, colours, sounds, building materials – and he wants to create a work, which puts the feelings in a certain motion. The scientist sees a puzzle and wants to make it understandable. The field of struggle is the emotional or spiritual life of society. If they – the artist like the scientist - embody the idea in all its dimensions, but do so with the most economical means, which is to say "eumachisch", then they create a work of art or make a scientific advance. Every lack of economy is felt to be ugly. Every unmotivated or superfluous effort is ugly. And every absolutely economical creation is, in terms of beauty (or scientifically), of lasting importance".[40]

Lasker then explores notions of "equilibrium" and "dominance" arising from the above. If an army A is sent to contain another army, B, then an equilibrium is reached when A, acting as a perfect strategist and obeying the principle of economy, devotes just the right amount of resources to the task. Too much implies resources are being wasted; too little means the task will not be accomplished. Between these two extremes, can be found the number of jonts which will allow him

[38] *Ibid.,* pp.30-31.
[39] *Ibid.,* pp.35-36.
[40] *Ibid.,* pp. 36-38.

to accomplish the task in a economic manner. Any disturbance of such an equilibrium, he says, will lead to strategic dominance of one player over another.

Then, in a striking section, Lasker introduces probability into the picture:

> "I beg your pardon, dear reader. Maybe I should have explained from the beginning how I see the definition of chance, luck and misfortune. It is true, even a perfect strategist can be the victim of bad luck, despite all his precautions. That which can happen has, from time to time, to become a reality. But even a random event has to obey the laws of probability...
> Therefore, *in a struggle where chance plays a role, the perfect strategist A will consider all these random events and their probabilities in order to find a solution for the given task, so that he contains B, in the sense that the danger of losing is as big for B as for A. And when either of these undertakes a manoeuvre the advantage of which represents, according to probability, a particular value, then he will suffer, for all of the reasons mentioned above, a loss whose expected value, according to probability, equals the expected advantage. In other words, when the struggle between A and B, being a strategic equilibrium, is repeated often, then the conquered values of A will be as great as those of B, as long as B behaves in a perfectly strategic way*. Otherwise, they will be greater. If A, on the other hand, has predominance, then, under these conditions, the values that A has won will always exceed those of B, no matter if B maneouvres strategically or not".[41]

In 1908, the year after the publication of *Kampf*, Lasker beat Tarrasch in the World Championship in Germany, bringing an end to the great rivalry between them. Lasker was now unquestionably chess's dominant figure, known the world over. In 1909, in St. Petersburg, he took first place against the great Polish player Akiba Rubinstein.[42]

Not only does Lasker personify the cultural importance of chess during this time, but in his speculative writings on perfect strategy, equilibrium and the science of struggle, he reached out to draw connections between chess and social life. His was the intuition concerning the links between the struggle at the chessboard and that of *homo economicus* in society. His sensibility to these matters was no doubt heightened, not only by his poverty, but by his experience as a Jewish would-be mathematician unable to find a place in academia, scrambling to live by the game. And struggle he would, till his impoverished end in New York in 1941.

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[41] *Ibid.,* p. 46 (emphasis added).

[42] Rubinstein was one of the more enigmatic figures in the chess world of the turn of the century. He suffered from a nervous psychological disorder termed anthrophobia, or fear of men and society, source of lifelong trouble for a player of such a public game. As his condition worsened over the years, he would complain about his tournament concentration being disrupted by a fly – which no-one else in the room could see. He became reticent about receiving visitors to his home, and his wife would welcome guests with a greeting, designed no doubt put them at ease: "Do not stay long, for if you do he will leave by way of the window". Rubinstein was partial inspiration for Luzhin, the protagonist in Nabokov's *The Defense*, readers of which will remember the brilliant final passage in which Luzhin, rendered suicidal, chooses precisely defenestration – the "icy air" gushing into his mouth. See Vladimir Nabokov, *The Defense*, (New York: G. P. Putnam's Sons, 1964). Orig. *Zaschchita Luzhina*, (Berlin: Slovo, 1930).

Mathematics and the Endgame

It was to discussions of this kind that Lasker's contemporary and fellow mathematician Ernest Zermelo (1871-1953) was referring when he said he wanted to consider chess without any reference to subjective-psychological considerations. A keen chessplayer himself, Zermelo knew Lasker. Both were students in mathematics and they shared Hilbert as teacher.[43] At Hilbert's Göttingen, where Zermelo worked from 1897 till 1910, there was considerable interest in chess. Indeed, that university was also home to the oldest surviving handwritten document on the game, the *Göttingen Manuscript*, a Latin treatise on chess problems and openings, written by Portugese player, Lucena, in the late 15th century.

Zermelo studied mathematics, physics and philosophy at Berlin, Halle and Freiburg, and his teachers included Frobenius, Max Planck, Lothar Schmidt and Edmund Husserl. His dissertation on the calculus of variations was completed at the University of Berlin in 1894. After two years there as Planck's assistant, Zermelo went to Göttingen, where he completed his *Habilitation*, and was appointed *Dozent* in 1899. Beginning in 1902, he started to publish on set theory, which was then an important field at Göttingen, with Russell's paradoxes appearing in 1903. Zermelo's 1904 proof that every set can be well-ordered was celebrated, earning him a professorial appointment at Göttingen a year later. It was also controversial work, relying, as it did, on the axiom of choice, which was contested by Intuitionist mathematicians. In 1908, Zermelo produced an axiomatics of set theory, which, improved by Fraenkel in the early 1920's, would become a widely accepted system.

In 1912, having taken a chair at Zurich two previously, Zermelo presented his "On an Application of Set Theory to the Theory of the Game of Chess", to the International Congress of Mathematicians at Cambridge.[44]

> "The following considerations are independent of the special rules of the game of Chess and are valid in principle just as well for all similar games of reason, in which two opponents play against each other with the exclusion of chance events; for the sake of determinateness they shall be exemplified by Chess as the best known of all games of this kind. Also they do not deal with any method of practical play, but only with the answer to the question: can the value of an arbitrary position, which could possibly occur during the play of a game, as well as the best possible move for one of the playing parties be determined or at least defined in a mathematically objective manner...?"[45]

Assuming that only a finite number of positions are possible (in the sense that the number of squares and the number of pieces are both limited), and without assuming any stopping rules (thereby implicitly allowing for infinite sequences of moves), Zermelo asks two questions. First:

[43] On Zermelo, see Sanford L. Segal, *Mathematicians under the Nazis* (Princeton University Press, 2003), pp. 467-469.

[44] See Zermelo, "Über eine Anwendung" (cit. n.30). Schwalbe and Walker, "Zermelo" (cit. n.30) provide an English translation of the Zermelo paper, with an extensive introduction, and clarify the subsequent contributions by Dénes König, "Über eine Schlussweise aus dem Endlichen ins Unendliche" (Trans. "On a Method of Conclusion from the Finite to the Infinite"), *Acta Szeged,* 1927, *III*:121-130 and Lazsló Kalmár "Zur Theorie der abstrakten Spiele" (Trans. "On the Theory of Abstract Games"), *Acta Szeged.,* 1928/29, *V*:65-85 in relation to Zermelo's.

[45] Zermelo, "Über eine Anwendung" (cit. n.30), on p. ?

what does it mean for a player to be in a "winning position", and is it possible to define this mathematically? Second: if a player is in a winning position, is it possible to determine the number of moves necessary to ensure the win?

A player is in a winning position, Zermelo shows, if and only if a particular set is non-empty, namely the set containing all the sequences of moves that guarantee a player can win independently of how the opponent plays. Were this set empty, then the best the player could hope for would be a draw. Thus, Zermelo defines a different set, containing all sequences of moves that would allow the player to postpone his loss indefinitely, thereby implying a draw. If this set is empty, then this is the same as implying that the opponent can force a win. As for the second question, Zermelo answers it, employing a proof by contradiction, and showing that the number of moves in which a player in a winning position is able to force a win can never exceed the number of positions in the game. Were White, say, able to win in a number of moves greater than the number of positions, then at least one of the "winning positions" would have had to appear twice, in which case White could have adopted his winning moves when the winning position appeared the first time round, rather than wait till the second.

Zermelo's analysis of chess was purely formal, an attempt to say something minimal about the game, without any consideration of the tactical and psychological features of the game that made it interesting to play. It was neither intended to be, nor was, of any value to the chess-player. As Zermelo noted, closing the paper, the question of whether the game's starting position could guarantee a win for one of the players remained open, and answering it would imply that chess would lose its game-like character. Yet, there was enough in the paper to attract the interest of the Hungarian mathematicians in the mid-1920's, after World War I was over. By this time, the psychological dimensions of the game were commanding greater interest than ever. [46]

The "Strangest States of Mind"

The 1920's opened with Lasker handing over the title to Capablanca, in 1921, though he continued to be a dominant figure in international chess. The decade was also marked by the appearance of the Hypermodern Movement, at the instigation of Richard Reti, a Hungarian trained in mathematics and physics at the University of Vienna, his compatriot Gyula Breyer, and Svelly Tartakower and Akiba Rubinstein. Responding to declarations that the possibilities of chess had been exhausted, they broke with the Classical style, personified by Tarrasch, and created a new approach to the game, introducing ideas so radical that, in the eyes of many players, they bordered on the irrational. Some Hypermodernists saw their approach as the chess manifestation of the French Surrealist spirit of Marcel Duchamps, himself another *Schach* fanatic. The strategic essentials of hypermodernism were laid out by Reti, in his 1922 *Modern Ideas in Chess*,[47] and by Nimzovich, in his 1925 *My*

[46] As for Zermelo himself, during the war, with no sign of his lung ailment improving, he resigned his chair at Zurich and left academia, moving to Germany's Black Forest where he taught private classes for a decade. In 1926, he was given an honorary position at Freiburg. Later, his 1935 refusal to give the Hitler salute would provoke a controversy at that university, causing him to withdraw from all teaching activity.

[47] See Richard Réti, *Modern Ideas in Chess,* trans. by John Hart (London: G. Bell and Sons, Ltd, 1923). See also Reti's "Do "New Ideas" Stand Up in Practice?", which was published in Russian in the *Chessplayer's Calendar* and then in the October 1987 issue of the *Chess Bulletin*, and recently translated by R. Tekel and M. Shibut in the Sept./Oct. 1993 issue of the *Virginia Chess Newsletter*. Reti burst onto the international scene toward the end of the war, sharing 1st

System: (1) attacking the centre squares from far away with knights and bishops, instead of occupying them with pawns and pieces in the usual manner, (2) blockading isolated pawns with knights, and (3) deliberate overprotection.[48]

The middle of the decade also saw the appearance of *Lasker's Manual of Chess*.[49] Like no other chess book of the period, it stood in particular contrast to Tarrasch's more conventional *The Game of Chess*, published not long afterwards.[50] Although very much a manual, Lasker's book, like *Kampf*, is replete with reflections on the interrelations between chess, mathematics, economics and social life. For example, the proposition that "The Plus of a Rook suffices to win the game", is an opportunity for Lasker to compare mathematics and chess, and discuss the importance of the *ceteris paribus* condition. His discussion of many propositions concerning the strength of various pieces and the ability to force a win in various situations is imbued with mathematical language: "This demonstration is mathematical",[51] "The question is one of pure mathematics…".[52] Lasker's discussion of the "Exchange-Value of the Pieces", combines economics and the *ceteris paribus* condition, and, as in *Kampf*, his discussion of the aesthetic effect in chess is based on notions of economy.

Psychological considerations loom repeatedly in Lasker's treatment. A combination is born in the player's mind, he says, surviving among many jostling thoughts, true and false, sound and unsound, and achieves victory over its rivals when it is transformed into a movement on the board.

> "Does a Chess-master really cogitate as just outlined? Presumably so, but with detours and repetitions. However, it matters not by what process he conceives and idea; the important point to understand is that an idea takes hold of the master and obsesses him. The master, in the grasp of an idea, sees that idea suggested and almost embodied on the board".[53]

In a closing chapter, "Final Reflections on Education in Chess", Lasker continues his speculations on the embryonic science of struggle, reaching out from the chessboard to the realm of social interaction.

> "It is easy to mould the theory of Steinitz into mathematical symbols, by expressing a kind of Chess, the rules and regulations of which are expressed by mathematical symbols. . . In such a game, the question whether thorough analysis would confirm the theory of Steinitz or not,

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prize at Budapest in 1918, coming first at Rotterdam and Amsterdam in 1919, and Vienna in 1920, and winning an important international tournament in Gothenburg in the same year. In 1924, he caused a storm in New York by becoming the first person to beat Capablanca in ten years, shaking the latter sufficiently to cause him to lose against Lasker later in the tournament. He died in Prague in 1929, shortly before his masterpiece, *Masters of the Chessboard*, went to press.

48 Aron Nimzovich, *My System,* ed.by Fred Reinfeld, (New York: David McKay,1974, orig. 1925)

49 Emanuel Lasker, *Lasker's Manual of Chess*, (New York: Dover 1976) (originally *Lehrbuch des Schachspiels*, 1926. First English translation,1927).

50 Siegfried Tarrasch, *The Game of Chess: A systematic text-book for beginners and more experienced players* (Philadelphia : D. McKay, 1940).

51 Lasker, *Lasker's Manual* (cit. n.49), on p. 15.

52 *Ibid*., p. 22.

53 *Ibid*., p. 114.

> presumably could be quickly solved, because the power of modern mathematics is exceedingly great.
>
> The instant that this solution is worked out, humanity stands before the gate of an immense new science which prophetic philosophers have called the mathematics or the physics of contest".[54]

Such knowledge, he says, could transform political life. It could even lead to outlawing war, because it would provide an alternative way to settle disputes. The mathematics of chess would not eliminate contests of life, which are necessary to the functioning of society, but it would couch such problems "in precise terms and point to a solution":

> "The science of contest will progress irresistibly, as soon as its first modest success has been scored.
>
> It is desirable that institutes to further these ends should be erected. Such institutes would have to work upon a mass of material already extant: theory of mathematical games, of organisation, of the conduct of business, of dispute, or negotiation: they would have to breed teachers capable of elevating the multitude from its terrible dilettantism in matters of contest; they would have to produce books on instruction. . .
>
> Such an institute should be founded by every people who want to make themselves fit for a sturdier future and at the same time to aid the progress and the happiness of all humankind."[55]

The cogitations of the chessmaster's mind were very topical in 1925, when Lasker published his *Manual*. That year saw the appearance of a Russian silent film, *Shakmatnaya goryachka*, or "Chess Fever", in which the mental stability of the protagonist is threatened when he tries to play the game against himself.[56] The film has cameo appearances of Reti and others, who were participating that year in a big international tournament in Moscow, at which Lasker came second after the Russian Bogoljubow. The occasion of that tournament was also used by psychologists at Moscow's Psychotechnics Institute for an important experimental study of the game. Taking a group of the participating chessmasters, researchers Djakow, Petrowski and Rudik subjected them to tests in an attempt to determine what exactly it was that mentally distinguished the good player from the common mortal.[57]

At that point, the only comparable study was French psychologist Alfred Binet's 1894 *Psychologie des Grands Calculateurs et Joueurs d'Échecs*, which dealt with blind chess.[58] Many observers had

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54 *Ibid.,* p. 340.

55 *Ibid*., pp. 340-341.

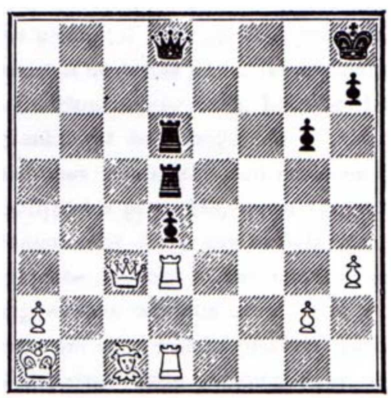
56 Again, a similar theme is exploited in Nabokov's *The Defense* (cit. n.42), where Luzhin, denied the possibility of playing by his respectable family-in-law, resorts to playing in his head. Soon, he sees his own life as one large chess game, in which every social encounter is interpreted in terms of a move or counter-move. His descent into madness follows.

57 See Djakow, Petrowksi and Rudik, *Psychologie des Schachspiels* (Berlin and Leipzig: Walter de Gruyter & Co.,1927).

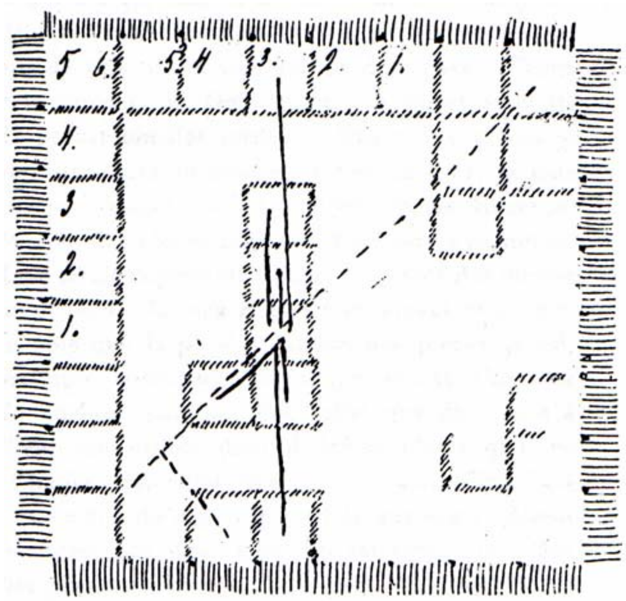
58 Alfred Binet, *Psychologie des Grands Calculateurs et Joueurs d'Échecs* (Paris, Genève: Slatkin, 1894), republished in 1981 with an introduction by François Le Lionnais. Binet (1857-1911) was a initially student of Jean Martin Charcot (1825-1893) at the Hôpital Salpêtrière, Paris, where he worked on hypnosis. Breaking with Charcot in 1890, he moved

been astounded in 1859 by American player Paul Morphy, the youthful Mozart of the game, who, on top of his remarkable performance in normal chess, had fascinated audiences in London and Paris by playing simultaneous blind games. In the interim, the record number of simultaneous blind games had been pushed steadily upwards. Such performances, which lasted for hours, demanded extraordinary stamina on the part of the "blind" player. Curious as to how they did it, Binet conducted a study by means of a questionnaire distributed to a small number of players, including Tarrasch in Germany and Blackburne in England. The study focused on the importance of *visual* representation in the player's mind. From the responses of ten players concerning the way they thought and reasoned during blind play, Binet concluded in the importance of three factors: experience (*érudition*), imagination (*imagination*) and memory (*mémoire*). Imagination involved the capacity to visualize a position, the ability to see the chessboard clearly in one's mind, something that was emphasized by all but three of the players. Memory, too, involved visual representation, and this is where Binet saw the psychological originality of his study.

One player, Sittenfeld, when asked to reproduce what he saw in his mind when he played, drew a picture. Here is the actual position on the board, followed by Sittenfeld's rendition of his mental image of same.



to the Sorbonne's laboratory of Physiological Psychology, directed by Henri Beaunis, where he began working on the study of cognitive processes. In 1895, with Beaunis, he founded the first French psychology journal, *L'Année psychologique*, and then became director of the psychology laboratory. Unreceptive towards the laboratory methods employed in German psychological work, Binet preferred the use of questionnaires and interviews to that of laboratory instruments and measurement apparatus. In 1905, he opened a laboratory for the study of children and pedagogy. His *L'Étude expérimentale de l'intelligence*, (Paris: Schleicher frères et cie., 1903) was a personality study of his own two daughters. Appointed to a ministerial commission to study retarded school children, he developed with Theodore Simon the Binet-Simon intelligence test for which he remains best known. See Theta H. Wolf, *Alfred Binet* (Chicago: University of Chicago Press, 1973), and Alfred Binet, *The experimental psychology of Alfred Binet: selected Papers*, edited by Robert H. Pollack and Margaret W. Brenner, trans. by Frances K. Zetland and Claire Ellis (New York: Spring Publishing Co., 1969).



Apart from the conclusions drawn from the study, Binet's book contains many interesting reflections and asides on the game itself, as seen by this psychologist – no chessplayer himself – towards the close of the 19th century. Insisting on the analogy between chess and mathematics, Binet notes that many well-known historical figures, including Voltaire, Rousseau and J.S. Mill, had been chessplayers, but that women tended not to excel in the game, as it required physical vigour and a taste for combat. Also, chessplayers tended to be rather vain: those remaining passive in victory and defeat, such as Morphy, were exceptional. Up to the end of the 18th century, Binet notes, chess tended to be dominated by Latin players: the Italians, Spanish and Portugese. These were then bypassed by the Germans, Slavs, Anglo-Saxons and, especially, Jews. Showing a table classifying prominent players by country, religion and race, Binet noted that of sixty-two players, eighteen were either Polish or Hungarian Jews. Furthermore, almost all the strong Jewish players were professional, "which shows clearly the seriousness of the race".[59]

In their study, the Russian psychologists of post-revolutionary Moscow referred to Binet's work, but more important to them by 1925 was the appearance in the interim of psychoanalysis and attempts to apply it to chess. By the mid-1920's, psychoanalysis had fallen afoul of the Soviet authorities, and Djakow et al were clearly concerned to retrieve the game from the clutches of the psychoanalysts and show why chess, notwithstanding its individualistic and combative elements, could serve important social and educational functions in the new Russia.[60]

The psychoanalytical study Djakow et al had in mind was most probably Alexander Guerbstman's (1925) *Psichoanaliz sacmatnoj igri* [Psychoanalysis of chess. An interpretative essay], which,

[59] Binet, *Psychologie* (cit. n.58), p. 222.

[60] On the history of psychoanalysis in the Soviet Union, see Alexander Etkind, *Eros of the Impossible. The History of Psychoanalysis in Russia*, trans. by Noah and Maria Rubens (Boulder, Colorado: Westview Press, 1997) and Martin A. Miller, *Freud and the Bolsheviks: Psychoanalysis in imperial Russia and the Soviet Union* (New Haven, CT: Yale University Press, 1998). While neither of these books discuss chess *per se*, they do confirm the Bolshevik state's rejection of psychoanalytic methods by 1925. See, for example, Etkind op cit, p.179ff.

unfortunately, has not been translated.[61] However, the thrust of the psychoanalytical approach to chess can be distilled from several other contributions of the period, including the landmark paper on the subject by Freud's British disciple, Ernest Jones.[62]

In "The Problem of Paul Morphy: A Contribution to the Psychology of Chess", which he read before the British Psycho-Analytical Society in November 1930, Jones put it clearly: "the unconscious motive actuating the players is not the mere love of pugnacity characteristic of all competitive games, but the grimmer one of father-murder".[63] The appeal of chess lay in its ability to gratify hostile Oedipal impulses. To checkmate the King was to render him immobile and sterile, the symbolic expression of the player's desire to overcome the father in an acceptable way, aided by the mother (Queen). The mathematical quality of the game, said Jones, the "exquisite purity and exactness of the right moves", the "unrelenting pressure" and then "merciless *dénouement*", all combined to give the game a particular anal-sadistic nature: "The sense of overwhelming mastery on the one side matches that of unescapable helplessness on the other".[64] Jones recalled some of Paul Morphy's qualities: his ability to play impassively from morning till midnight for several days running with no signs of fatigue. On his famous European trip, when he played eight opponents blindfold at the Café de la Régence, it took seven hours before the first of them was beaten, and another three before the match ended, throughout all of which Morphy neither ate nor drank. At seven the next morning, he promptly called his secretary and dictated to him every move in all of the games, discussing the possible consequences of hundreds of hypothetical variations. Where Binet had found evidence of remarkable visual memory, Jones saw "a very exceptional level of sublimation, for a psychological situation of such a degree of freedom can only mean that there is no risk of its stimulating any unconscious conflict or guilt".[65]

Noting that Morphy's stellar success in chess had begun just a year after the shock of his father's sudden death, Jones surmised "that his brilliant effort of sublimation was, like Shakespeare's *Hamlet* and Freud's *Traumdeutung*, a reaction to this critical event".[66] Jones pursues a psychoanalytical reading of Morphy's European performance, including his vain three-month effort to lure to a challenge the British champion, Staunton, who had become, for Morphy, the "supreme father *imago*". That Staunton took to criticizing Morphy in the press as a monetary adventurer, all the while refusing to play him, accentuated the frustration felt by the young American. To Jones, Morphy's case was illustrative of the connection between genius and mental instability. The artistic

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[61] See Jacques Berchtold (ed.), *Echiquiers d'encre. Le jeu d'échecs et les lettres (XIXe-Xxe siècles),* prologue de George Steiner (Genève: Droz, 1998) especially the editor's introductory essay.

[62] See Ernest Jones, "The Problem of Paul Morphy: A Contribution to the Psychology of Chess", *International Journal of Psycho-Analysis*, January 1931, reprinted in Ernest Jones *Essays in Applied Psychoanalysis*, Vol. 1 (New York: International Universities Press, 1964), pp. 165-196. See also the essay on the Hamlet figure in chess by another disciple of Freud, the Swiss pastor, Oskar Pfister: "Ein Hamlet am Schachbrett", *Psychoanalytische Bewegung*, (1931), pp. 217-222. This paper appeared in the *Psychoanalytische Bewegung* alongside the German translation of Jones' paper "Das Problem Paul Morphy" . See also Isador Coriat "The Unconscious Motives of Interest in Chess", *The Psychoanalytic Review*, 1941, *28:*pp. 30 – 36 and Joan Fleming and Samuel Strong, "Observations on the use of chess in the therapy of an adolescent boy", *The Psychoanalytic Review,* 1943, *30*:pp. 399 – 416.

[63] Jones, "The Problem" (cit. n.62), p. 168.

[64] *Ibid.,* p. 170.

[65] *Ibid.,* p. 173.

[66] *Ibid.,* p. 180.

conscience was characterized by rigour, sincerity and purity, but the psychical integrity of the artist was vulnerable to any of these being disturbed. Morphy's chess-playing ability reflected his capacity for sublimation of parricidal and homosexual impulses, Jones felt, all of which served a defensive function for him. When Staunton persistently refused to accept Morphy's challenge, this sublimation broke down, the defensive function failed, and Morphy could no longer use his talent as a means of guarding against overwhelming id impulses. Stripped bare, the player collapsed.

The Russian psychologists, Djakow et al, rejected as forced and one-sided the attempt to explain chess in Freudian terms, emphasizing, rather, the social dimensions of the game. Chess was a struggle, they said, frequently invoking Lasker. It provided gratification in the activity of playing, not merely in the resulting victory. The game was an "expression of social life, its specific spirit the expression of social desires in the specific form of social activity".[67] It satisfied the desire for friendly company, for public display of strength. If there were biological roots to the game, they were to be found, not in the Oedipal interpretation, but in the "much larger biosocial foundations of life, in which struggle is a fundamental law. To reduce every struggle to a struggle for a woman would mean adopting an extremely one-sided analytical approach. . . The essential figure, the King, has an all-too-clear historical origin to permit any attempt whatsoever to explain it using the sexual desires of individual psychology".[68]

The energies and emotions flowing through chess stemmed, they felt, not from individualistic desires but from "much deeper and more general instincts of great social significance, such as the instinct of activity, of creativity, to display one's power and superiority, social acceptance, and the instinct of struggle or of competition as a basis of personal and social life".[69] Unlike gymnastics and physical exercise, chess was a synthesis of functions, capturing complete episodes of life itself, an activity in which the personality could dissolve yet which offered diverse satisfactions. Chess left no room for chance; success in it depended on intensive solitary work. The rhythm of the game gave rise to "a rich alternation of the strangest states of mind".[70]

In a section titled "Game theories", Djakow et al describe how chess provides pleasure by facilitating the flight from daily effort and work. They are also keen, in Moscow in the mid-1920's, not to insist too much on competition between individuals. "We cannot but share the view of Dr. Em. Lasker that chess is a struggle and that every human being feels the desire to fight – in sports, at the card table, while playing boardgames. . . . But in our opinion the moment of struggle is only one aspect of this phenomenon. Besides that, or perhaps even prior to that, is the moment of solitude, of isolation, of plunging into an entirely different world, which is filled with the purely intellectual struggle".[71] The game was characterized by psychological tension. It required creativity, action, real impulses of will. This was what distinguished it from enjoying, for example, a work of art. In an age when the individual was increasingly subordinate to machines and technology, the authors felt, games allowed for the relief of monotony, the rupture of routine. Unlike engaging with inert material, playing chess meant encountering a flesh and blood adversary

[67] Djakow, *Psychologie* (cit. n. 57), p.16.
[68] *Ibid.,* p.17.
[69] *Ibid.,* p.18.
[70] *Ibid.,* p. 9.
[71] *Ibid.,* p. 9.

and adapting to his movements. "Here, rarely is something foreseeable for sure. Chess is the highest level of such struggle of "foresight". At every moment two ideas, two intentions, collide with each other. Therefore, no advance plan exists; the plan and its realisation emerge during the game. One would see enormous insight (*übersicht*) to foresee just a little bit. . . One needs to invent something new all the time".[72] The game thus produced a heightening of psychic tone (*Tonus der Psyche*) and elevated self-feeling (*Selbstgefühl*).

Unlike in the Binet study, the identities of the international chessmasters participating in the Moscow experiments remain unknown. They were subjected to a range of tests, intended to examine the functions of memory, attention, higher intellectual processes, imaginative power and intellectual character. All tests were conceived with a view to determining the psychological make-up of a good player.[73] In one of the memory tests, for example, the player was shown, for a minute, a chessboard with coloured counters. He then had two minutes to reconstruct the configuration, winning points for correct positioning and correct colours. In another, he was briefly shown the following endgame (in which White can force a win in three moves) and asked afterwards to reconstitute it as exactly as possible.

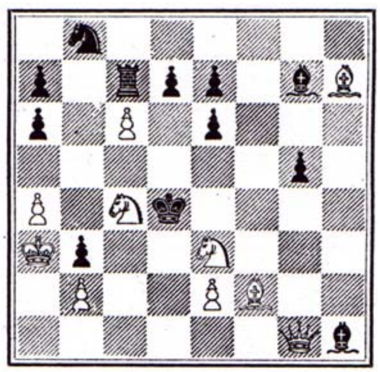

Figure 5, Endgame from Djakow et al (1927)

The attention tests involved examining various shapes and then reproducing them in the correct order. Intellectual processes were gauged with reference to combinatorics and intellectual function. For example, to a board with two "queens", the player had to add five more in such a way that no queen could take any other. Or the player was shown ten numerical equations and had to say, in a limited time interval, whether they were right or wrong. When it came to measuring the player's powers of imagination and psychological type, the Russians employed Rohrschach tests. Players were shown ten images of random ink blots, and their responses were interpreted as providing information about power of imagination, willpower and other attributes.

The results obtained were varied. In terms of general memory of numbers, the players were no better than normal people. In retaining geometrical forms, they were inferior. As regards attention, players were average in their ability to reproduce in the proper order the shapes shown on cards, but

[72] *Ibid.,* p. 14.

[73] It is unclear whether or not a control group was used. The results in the study indicate that 12 subjects were used. Adriaan De Groot, *Thought and Choice in Chess* (The Hague and Paris: Mouton & Co., 1965) originally published as *Het Denken van de Schaker*, (Amsterdam: North-Holland, 1946) says that the study failed to consider the performance of non-chessplayers (p.10).

they were superior when it came to the range and dynamics of attention. For example, when shown eight boards with three pieces in sequence, they showed a strong ability to reconstruct the movement of the pieces throughout the sequence. The thought processes of the players were not particularly fast, but the purely logical side of their reasoning was more developed than in non-chessplayers.

Asking what attributes were required to become a chessmaster, the Russians come up with a portrait, the “psychogram”. It included physical strength, strong nerves, self-control, a perceptive type of psyche, a high level of intellectual development, concrete thinking ability (which was not the same as logical or mathematical thinking), objective thinking, a very strong chess memory (which was not the same thing as psychological memory), disciplined will, emotive and affective discipline, and awareness of one’s strengths. Intellectual development, objective thinking and chess memory could be encouraged, the authors said; the rest were innate.

Djakow et al close their monograph by quoting the Hungarian master, Réti, who insisted that it was not a player’s ability to think 10 or 20 moves ahead that was important:

> “Chessplayers, who ask me from time to time, how many moves in advance I calculate, are astonished to hear me answer: usually none. A bit of mathematics will show us that it is impossible and even useless to foresee an exact sequence of moves. When you try to calculate in advance 3 moves of White and Black, the number of variations mounts already to 36 = 729; to calculate this is thus practically impossible. . . . Every chessplayer – the weakest like the strongest – has consciously or unconsciously well-defined principles through which he is guided in the choice of his moves”[74].

If Binet, in Paris in 1894, had applauded his subjects’ refined and dignified use of geometric visual memory, in Moscow in the mid-1920’s, Djakow and colleagues saw in chess an opportunity to shape the body politic. The game of chess was striking proof of the possibility of unlimited development of single sides of the human psyche when one had sufficient drive and interest. The “dialectics of chess” showed that it provided an objective measure of our own reason; the game “*deprives us of the possibility and the right of appealing to something higher with even more authority. It destroys in the case of defeat our last hope of self-justification*. Such is the tragedy of chess”.[75] The characteristics associated with good chessplay are good for society: “*From its essence as well as the history of its evolution, chess merits without doubt becoming a game of the people*”.[76]

The infinite chessboard

Among von Neumann’s Hungarian teachers, there was a long-standing interest in the mathematics of games. In 1905, in the columns of *KöMaL* itself, the secondary school mathematics magazine, a short paper by one Jószef Weisz, “On the Determination of Game Differences”, dealt with a game that was not one of pure chance. Throughout the 1920’s, *KöMäL* founder, Daniel Arány, published

74 Djakow, *Psychologie* (cit n.57) on p. 59.

75 *Ibid.,* p. 60 (emphasis in original).

76 *Ibid.,* p. 61 (emphasis in original).

papers examining how the probabilities of winning in games of pure chance varied with the number of players.[77] The Eötvös Competition of 1926, conceived by Dénes König, contained one question concerning the solution to a system of two equations, the answer to which was equivalent to proving that, on an infinite chessboard, any square can be reached by a knight, in a sequence of appropriate moves.[78] The intersection between parlour games and mathematics was thus familiar territory to the Hungarians, and it is easy to understand why König himself, Lászlo Kalmár and von Neumann took a special interest in Zermelo's work as they passed through Göttingen in the 1920's.[79] There, the shadow of Lasker and the international excitement surrounding the game would have combined to make the subject particularly topical.

König and Kalmár each sought to refine Zermelo's 1913 paper, and they each cite von Neumann's guidance. In his 1927 paper, "On a Method of Conclusion from the Finite to the Infinite" (cit. n.44), König, at von Neumann's suggestion, applied to chess a lemma from set theory in order to prove the conjecture that the number of moves within which a player in a winning position can force a win is finite. In order to do this, König invokes the use of an infinite board, but with the usual 32 pieces. He also addressed two respects in which Zermelo's earlier proof was incomplete, the most important of which was that it hadn't been proved that a player in a winning position was *always* able to force a win in a number of moves less than the number of positions in the game. In his proof of this, König again cites discussions with von Neumann. As for Kalmár's 1928/29 paper, "On the Theory of Abstract Games" (cit. n.44), he too cites discussions with von Neumann, and, generalizing the work of Zermelo and König, shows that if it is possible in a game to force a win, then this can be done without the recurrence of any position.

This was part of the rich background against which von Neumann came to game theory. Chess was an enormously fertile source of speculation, whether concerning the game's psychological features, its qualities as a "non-psychological" mathematical object, or its potential as a source of insights into social interation more generally. As we shall see, von Neumann's paper was infused with the Laskerian rhetoric of "struggle", "balance" and "equilibrium", and loyal to Zermelo's ambition to excise psychology from the mathematics of games.

Before considering this, however, we need to take account of what was another influence on von Neumann - even if he was always reluctant to acknowledge the fact. This was the work of Émile Borel, the French mathematician, many years the Hungarian's senior and long interested in the relationship between games and mathematics. Unlike the Göttingen people, however, Borel was

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[77] See József Weisz, "Játékkülömbözetek Meghatározásáról", *KöMaL,* April 1905, pp. 185-6; Daniel Arány, "Note sur "Le troisième problème de jeu", *Acta Scientiarum Mathematicarum* (Acta Universitatis Szegediensis), 1924, *2*:pp. 39-42;"Verallgemeinerung des problems der Spieldauer für de fall von drei Spielern", *Mathematikai és Physikai Lapok,* 1927, *34*: pp. 96-105 (in Hungarian). Reviewer: D. König, Budapest; "Sur la Généralisation du Problème de la Durée du Jeu pour Trois Joueurs", *International Congress of Mathematicians*, Bologna, 1928, pp. 73-75; "Considerations sur le problème de la durée du jeu", *Tohoku Mathematical Journal,* 1929, *30* :pp. 157-181; "Note sur le "Seconde problème de la durée de jeu dans le cas de trois joueurs"", *Association francaise Avancement Science,* 1929, *53*:pp. 33-35;"Le problème des parcours", *Tohoku Mathematical Journal*, 1933, *37*:pp. 17-22;"Le problème des parcours", *Assocation française pour l'Avancement des Sciences*, 1933, pp. 20-23. I thank Laszló Filep for drawing the Arány and Weisz papers to my attention during my visit to the University of Nyrieghaza.

[78] See Kürschák, *Hungarian Problem* (cit n. 17), pp. 104-106.

[79] In 1926, König was aged 42, Kalmár 21, and von Neumann 23.

not interested in chess. Also, unlike von Neumann, he was an accomplished player of bridge and other card games. These facts shaped Borel's perspective on games and they help us to understand differences of perspective between him and the Hungarians, including the greater emphasis placed by Borel on the "psychology" of the gameplayer.

In praise of gambling

By the mid-1920's, Borel was in his mid-fifties and sat at the pinnacle of French mathematics, holding a chair at the Sorbonne in probability, an area of mathematics incidentally that was less well-developed in Hungary. His career path had provided a model of French educational achievement.[80] Following an 1894 doctoral thesis on the theory of functions, Borel had made several important contributions, including work on the theories of measure and of divergent series and an elementary proof of Picard's Theorem, which mathematicians had apparently been seeking for over seventeen years. His work in this early period culminated in the beginning of a series on the theory of functions, which he edited and to which he himself contributed five volumes. Under his directorship, some fifty volumes of these "Borel Tracts" would subsequently appear.

During the first decade of the 20th century, his interests shifted towards probability theory, and he also became something of a popularizer of mathematics and science, in 1906 founding the *Revue du Mois*, a magazine to which he contributed articles of scientific, philosophical and sociological interest. He also edited a series of popular books, including one about flight, *l'Aviation* and another about the role of chance in everyday life, *le Hasard*. After World War I, he entered public life, becoming a member of Parliament for 12 years, all the while continuing to write in mathematics. In 1921, he began to edit and contribute to the monumental series of monographs, *Traité du Calcul des Probabilités et de ses Applications.*[81]

For much of his career, Borel's home in Paris was site of an important *salon*, where a group of French scientists, intellectuals and public figures would gather. They included physicists Jan Perrin and Pierre and Marie Curie, writer Charles Péguy, politicians Paul Painlevé and Léon Blum, and poet Paul Valéry. Borel also knew psychologist Alfred Binet and was familiar with some of his work. Indeed, one of the first things he did in the newly-founded *Revue du Mois* was to challenge a study by the psychologist which claimed to show that intelligence was correlated with the quality of subjects' handwriting.[82]

[80] See Collingwood, E.F. "Émile Borel", Journal of the London Mathematical Society, 1959, *34*: 488-512; Maurice Fréchet, *La Vie et l'Oeuvre d'Émile Borel* (Genève: L'Enseignement mathématique, 1965).

[81] Émile Borel and Paul Painlevé, *L'Aviation* (Paris: Alcan, 1910); Émile Borel, *Le Hasard* (Paris: Alcan, 1914); Émile Borel, *Traité du calcul des probabilités et de ses applications* (Paris: Gauthier-Villars, 1938).

[82] See Alfred Binet, *La Graphologie. Les Révélations de l'écriture d'après un contrôle scientifique*, Introduction de Serge Nicolas. (Paris: l'Harmattan, 2004, orig.1906). Borel apparently countered Binet by conducting a similar experiment with the same writings, but in typewritten form, to conclude that it was the *content*, not the graphological quality, that was correlated with intelligence. See Émile Borel, "Le calcul des probabilités et la méthode des majorités", *Année psychologique*, 1908, *14* :125-151, reprinted in *Oeuvres de Émile Borel*, 4 vols., Vol. 2 (Paris: Centre National de Recherche Scientifique, 1972), pp. 1005-1031, which discusses another experiment by Binet, designed to investigate how photographic views of children's hands affected surmises as to their gender and intelligence. This article was published in Binet's own journal, *L'Année Psychologique*.

Familiarity with Binet notwithstanding, Borel's interest in the mathematics of games did not derive from an interest in chess. In this, he appears to have been influenced by Henri Poincaré's claim that chess was not a proper mathematical object, because it could be played only on a chessboard of 8x8 = 64 squares. Poincaré said that in chess the value 8 was essential, with no possibility of generalization to a board of n^2 squares. Even on König's infinite chessboard, so to speak, there were still only 16 pieces per team. On the other hand, said Borel, by Poincaré's criterion, cardgames were indeed mathematical objects, since they were played with 4*n* cards, with *n* typically varying between 8 and 13 but there being nothing, in principle, preventing *n* from assuming any integer value.[83] It was thus as a player of bridge and cardgames that Borel the probabilist began to analyse games of strategy, and this worldly experience as a player coloured his view of the power of mathematics in this domain.

If there is no evidence that Borel had read Emanuel Lasker's speculations on the science of struggle, there are striking similarities in certain places. For one, if Lasker had suggested in *Kampf* that the perfect strategist was one who took account of any existing randomness in the effectivness of his actions, Borel went a step further, recommending the deliberate use of probabilistic play. This he did in a series of notes written throughout the 1920's, establishing the notion of a strategy and the principle of random play, and investigating the range of 2-person games in which the latter could be employed profitably.

Borel asks us to consider a game "in which the winnings depend on both chance and the skill of the players", unlike such games as dice where skill does not influence the outcome.[84] Defining a "method of play" as "a code that determines for every possible circumstance . . . . what the person should do", Borel asks "whether it is possible to determine a method of play better than all others". Considering the particular case where the number of strategies is 3, Borel shows that each player can choose probabilities that ensure an even chance of victory.[85]

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[83] See Borel, *Traité* (cit. n. 81), p. 39.

[84] Émile Borel, "La théorie du jeu et les équations intégrales à noyau symétrique", *Comptes Rendus de l'Académie des Sciences*, 1921 (December 19), *173*:1304-1308, translated in Maurice Fréchet, "Emile Borel, Initiator of the Theory of Psychological Games and its Application", *Econometrica*, 1953, *21*: 95-127. Three of Borel's five notes appeared in the Académie's *Comptes Rendus*. As some of Borel's observations are repeated from one paper to the next, we consider the three most important: the aforementioned (1921); Borel, "Sur les jeux où l'hasard se combine avec l'habileté des joueurs", *Comptes Rendus de l'Académie des Sciences*, 1924, *178 :* 24-25; and Borel, "Sur le système de formes linéaires à déterminant symétrique gauche et la théorie générale du jeu", in "Algèbre et calcul des probabilités", *Comptes Rendus de l'Académie des Sciences*, 1927, *184 :* 52-53

[85] Borel, "La théorie du jeu" (cit. n. 84), on p.97. He considers a game with two players *A* and *B*, who choose strategy ("method") C_i and C_k, respectively. Each has the same set of *n* strategies available. Given the strategies chosen, the entries in the matrix represent not payoffs but A's probability of winning the game. The numbers a_{ik} and a_{ki} are contained between -1/2 and +1/2, and satisfy $a_{ik} + a_{ki} = 0$. Also, $a_{ii} = 0$. His examples are confined to games that are symmetric, and fair, in the sense that the expectation for each player is zero.

		Player B		
		C_1	C_2	C_n
	C_1	$1/2+a_{11}$	$1/2+a_{12}$	..... $1/2+a_{1n}$
Player A	C_2	$1/2+a_{21}$	$1/2+a_{22}$	..... $1/2+a_{2n}$
	:			
	:			
	C_n	$1/2+a_{n1}$	$1/2+a_{n2}$	..... $1/2+a_{nn}$

In Borel (1924), he extends this analysis, slightly modified, to the case of 5 strategies, i.e., $n = 5$, where he shows that "nothing essentially new happens compared to the case where there are three manners of playing", i.e., each player can ensure an expected payoff of zero, and Borel wonders whether this is likely to hold for n arbitrarily large.[86] This, he conjectures, is improbable. Three years later, however, in another note presented to the Académie, he reports that what has held for 3 and 5 strategies seems also to hold for 7, and that it would thus "be interesting either to demonstrate that it is unsolvable in general or to give a particular solution".[87]

In various places in these papers of the 1920's, Borel considers applications. Considering the finite game "Paper, Scissors, Stone", he shows in detail how the calculation of the optimal mixed strategies depends on relative payoffs. For example, if the payoff to A for a particular strategy is relatively large, then the probability attached to it in the optimal mixed strategy will be correspondingly low: otherwise, B could gain by anticipating A's emphasis on the favored strategy.[88] In a manner reminiscent of Lasker, Borel notes that the "problems of probability and analysis that one might raise concerning the art of war or of economic and financial speculation, are not without analogy to the problems concerning games".[89]

In a 1924 review of John Maynard Keynes' (1921) *A Treatise on Probability*, Borel was quite explicit about the possibility of a new science. The context was the discussion of subjective and objective probability, Keynes' book having proposed a thoroughly subjective interpretation. Borel felt that there were cases where the subjective evaluation of probabilities posed serious problems. In particular, there were situations where the very attempt to make a probability judgement altered the probability one was trying to evaluate: for example, in betting on the result of an election, where the size of bets placed can influence the probability of a candidate's success. In such circumstances, says Borel, it is difficult to ascribe a precise number to the probability of success attributed by a gambler to a particular candidate:

Players are assumed to automatically cast aside "bad" strategies, i.e., methods of play which guarantee a probability of winning of less than half. Having done this, the question is how the remaining strategies might be employed in the best manner possible. Borel suggests that a player can act "in an advantageous manner by varying his play", i.e., C_k is played with probability x_k by A and y_k by B, where $\sum_1^n x_k = 1 = \sum_1^n y_k$. Given this, A's expectation of winning is $\sum_1^n \sum 1/2 x_i y_k = 1/2 + a$, where $a = \sum_1^n \sum_1^n a_{ik} x_i y_k$ and B's probability of winning is thus $1/2 - a$.

[86] Borel, "Sur les jeux où l'hasard..." (cit. n.84), on p.114.

[87] Borel, "Sur le système de formes linéaires…" (cit. n.84), on p.117. Borel's analysis is confined to games with odd numbers of strategies because of his use of determinants of skew symmetric matrices to calculate the optimal mixed strategy.

[88] Borel also introduces the infinite game, where strategies are drawn from a continuum, and shows how the continuous analogue of player A's expected payoff may be expressed as a Stieltjes integral:

$$a = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(C_A, C_B)\, d\o_A(C_A)\, d\o_B(C_B)$$

where $f(\,)$ is the function relating A's payoff to the strategies chosen, and $\o_A(\,)$ and $\o_B(\,)$ are A's and B's respective cumulative distribution functions over strategy space. In Borel's example, each player chooses three real numbers summing to 1, the winner being the one with two choices of greater value than the opponent's. This is what was later to become known as a game on the unit square, and was taken up in more detail in 1938 by Borel's student Jean Ville. See Ville, "Sur la théorie générale des jeux où intervient l'habileté des joueurs", 1938, in Borel, *Traité* (cit. n.81), pp. 105-113.

[89] Borel, "La théorie du jeu" (cit. n.84), p.10.

> "The problem can be put in a form that is both simple and yet which remains complex enough to preserve its entire difficulty, if we consider the game of poker, where each player bets on his hand against that of the opponent. If the opponent proposes a large bet, this tends to indicate that he has a very good hand, unless he is bluffing; thus, the very fact that the bet is made alters the probability of the judgment on which the bet is based.
>
> The further study of certain games will perhaps lead to the creation of a new chapter in the theory of probability, a theory the origins of which go back to the study of the simplest games of chance; it will be a new science, where psychology will be no less useful than mathematics"[90]

This insistence on the importance of psychology would become a hallmark of Borel's, and it distinguished him from von Neumann. Constantly, he reminds his reader of the limited extent to which matters of human psychology can be clarified by the use of mathematics. Games, he says, like problems of war and economics, are in reality highly complex, so that mathematical calculation can be at best a supplement to strategic cunning. The only advice the mathematician can give the player, "in the absence of psychological information", is to vary his play in such a way that the probabilities remain invisible to his opponent. Repeatedly, Borel insists on the limitations of mathematical analysis in aiding the comprehension of such games. If the player of strategic games must be a good master of combinations, he says, it is no less true that he must be a good psychologist too. Although theoretical solutions may require a player to play probabilistically, the challenge for the player of real games, says Borel, himself an accomplished bridge player, is to be able to discern the way in which the opponent is playing - the probabilities he is using, so to speak. This requires psychological skill, which is why treatises on card games such as bridge are quite inadequate to the task of teaching superior play. The same problems arise in the "art of war", where, again, "knowledge of the psychology of the adversary" is necessary. Borel continued, long thereafter, to insist on the practical limitations of the mathematics, given the psychological complexity of real games. Indeed, as we shall see, his most eloquent expression of it was provoked by his encounter with the contribution of von Neumann.

The latter entered the picture in May 1928, when he sent a note to Borel in Paris providing an answer to the French mathematician's question concerning the existence of a "best" way to play in the general 2-person, zero-sum case. Von Neumann claimed to have been working independently on the matter and to have proved a theorem two years previously.[91]

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90 Émile Borel, "A propos d'un traité des probabilités", *Revue philosophique*, 1924, *98* : 321-326, reprinted in *Oeuvres de Émile Borel* (cit. n.82), Vol. 4, pp. 2169-2184. Borel goes on to say that this new theory will add to older theories without modifying them. "The theory of value and the law of supply and demand are not changed by a fact such as the following: I am unable to distinguish true from false diamonds and yet would like to buy jewellery today; before the display of a seemingly honest shop, I notice a jewel marked 500 francs and I decide not to buy it because I believe it to be a forgery and the price appears too high; nonetheless, I enter the shop, and the jeweller, after a moment, offers me the jewel I had noticed; I then notice that it is, in fact, marked 5000 francs; I thus conclude that the stones are real, and I decide to buy".

91 John von Neumann, "Calcul des Probabilités – Sur la théorie des jeux" presented by E. Borel, *Comptes Rendus Hebdomadaires des Séances de l'Academie des Sciences*, Tome 186, No. 25, Lundi 18 juin 1928, pp. 1689-1691.

From Struggle to Equilibrium

"[O]ther mathematicians prove things they know, Neumann proves what he wants to prove"[92]

In August 1927, by then the youngest *Privatdozent* ever appointed at the University of Berlin, von Neumann wrote to the International Education Board to report on his stay, the previous year, at Göttingen. He gave the game theory paper somewhat short shrift, putting it as almost an afterthought amongst his other papers on formal logic and the axiomatic theory of sets, the theory of groups, and the mathematical foundations of quantum mechanics: "In addition to this I published . . . another concerning a question in the theory of games".[93] The commentator who wishes to discuss that paper faces a double challenge: first, understanding it; second, doing justice to it without testing the patience of the reader. Perhaps we can elicit the reader's generosity if we remind him that von Neumann wrote the paper when he was a mere 23-year old.

The first thing that strikes the reader of the paper is its generality. Citing chess, baccarat, roulette and poker as examples, von Neumann goes beyond the concerns of Zermelo, the Hungarians or Borel, to lay out a theory of the generic strategic game. He writes: "n players S_1, S_2, . . . S_n are playing a given game of strategy, G. How must one of the participants, S_m, play in order to achieve a most advantageous result?". The problem, he says, is well known, and "there is hardly a situation in daily life into which this problem does not enter". Yet, the meaning of this question is not unambiguous. For, as soon as there is more than one player, the fate of each "depends not only his own actions but also on those of others, and their behavior is motivated by the same selfish interests as the behavior of the first player. We feel that the situation is inherently circular". Therefore, we must formulate the problem clearly. "What, exactly, is a game of strategy?" he asks. "A great many different things come under this heading, anything from roulette to chess, from baccarat to bridge. And after all, any event - given the external conditions and the participants in the situation (provided the latter are acting of their own free will) - may be regarded as a game of strategy if one looks at the effect it has on the participants. What elements do all these things have in common?".[94]

He then gives a rather loose definition of a game of strategy, as a series of events, each of which may have a finite number of distinct results. In the some cases, the outcome depends on chance, i.e., where the probabilities are known; in others, they depend on the free choices of the players. For each event, it is known which player affects the outcome, and what information he has with respect to the previous decisions of other players. When the outcome of all events is known, then

92 Rózsa Péter, *Játék a Végtelennel*. 1945, trans. Z. P. Dienes as *Playing with Infinity : mathematical explorations and excursions* (New York: Dover Publications, 1976, [1961c]) on p. 246. Péter also continues, parenthetically: "(He is reputed to have said at a Congress in Bologna that the formalization of metamathematics was not interesting, but that he would do the whole thing himself for a box of chocolates)". *Ibid*.

93 Letter von Neumann to Trowbridge, August 9, 1927 (date of receipt), I.E.B.R.. Series 1, Subseries 3, Box 55, Folder 896, Von Neumann Papers Library of Congress (hereafter VNLC).

94 Von Neumann, "Zur Theorie der Gesellschaftsspiele", *Mathematische Annalen*, 1928, *100*: 295-320; translated by S. Bargmann as "On the Theory of Games of Strategy" , in *Contributions to the theory of games*, Vol. 4, ed. Albert Tucker and R. Duncan Luce. (Princeton, N.J.: Princeton Univ. Press, 1959), pp. 13-42, on p. 13. Here, in a footnote, von Neumann writes that this is the main problem of "classical economics: how is the absolutely selfish 'homo economicus' going to act under given external circumstances?" (p.13, fn. 2).

the payments amongst players can be calculated. Von Neumann then makes precise the above definition, designating the following the "rules of the game":[95]

> (α) The number of events depending on chance (i.e., "draws") is z, and the number depending on decisions (i.e., "steps") is s. Let $E_1, E_2, \ldots E_z$, indicate the sequence of "draws", and $F_1, F_2, \ldots F_s$, be the sequence of "steps".
>
> (β) Let M_μ and N_ν ($\mu = 1, 2, \ldots, z$; $\nu = 1, 2, \ldots, s$) indicate the number of possible results of each "draw" and "step", respectively.
>
> (γ) For each "draw", the probabilities $\alpha_\mu^{(1)}, \alpha_\mu^{(2)}, \ldots, \alpha_\mu^{(M_\mu)}$ of the different results 1, 2, . . ., M_μ must be given. The usual rules concerning summation and non-negativity of probabilities are given.
>
> (δ) For every "step", F_ν, the player, S_m, whose step it is, must be specified, as must the draws and steps whose outcome he knows at the time he acts on F_ν. In an implicit reference to the papers by König and Kalmár, von Neumann also rules out cycles in the game: the game must always "go forward".
>
> (ε) Finally, n real-valued functions $f_1, f_2, \ldots, f_n$ must be given, each depending on the set of values of the z + s variables representing the results of all the "draws" and "steps" in the game. It holds identically that $f_1 + f_2 + \ldots + f_n = 0$.

There still remains the problem, von Neumann points out, of making precise the expression "S_m tries to achieve a result as advantageous as possible", given that S_m alone is in no position to choose the value of f_m. If f_m depended on S_m's choices and chance draws only, then he would be in a position to calculate his expected gain and play accordingly, which is the problem treated in the "theory of games of chance" such as roulette.[96] Here, however, the main emphasis is not on chance but on the fact "(so typical of all social happenings!) that each player influences the results of all other players, even though he is only interested in his own".[97]

The paper then continues with some "General Simplifications", in which all strategic games are brought into a "much simpler normal form" so as to reduce the arbitrary complexity of the games discussed above. In the simplest form, there will be only one "draw", z = 1, and the number of "steps" will be the same as the number of players, s = n. Furthermore, the n "steps" will be taken simultaneously: each player will move in ignorance of how the others move, and without knowing the results of the "draw". Therefore, each of the n players chooses a number $1, 2, \ldots N_m$, without knowing the choices of the others. A "draw" then takes place in which their numbers appear with the probabilities $a_1, a_2, \ldots a_M$. Player S_i thus wins $f_i(x, y_1, y_2, \ldots y_n)$.

[95] The reader uninterested in the anatomy of von Neumann's paper and theorem can safely gloss over the following paragraphs.

[96] *Ibid*, p. 16.

[97] *Ibid*, p.17.

For each player, therefore, the choice amongst the entire range of possible sequences of moves has been collapsed to the choice of a number, to one "step". This may seem somewhat strange, says von Neumann, but it is in fact reasonable to assume that, before the play, each player "knows how to answer the following question: What will be the outcome of the $\nu_{\kappa}^{(m)}$ -th "step", provided the results of all "draws" and "steps" "earlier" than $\nu_{\kappa}^{(m)}$ are available?".[98] In other words, the player knows beforehand how he is going to act in a precisely defined situation: he enters the play with a theory worked out in detail. Even if this may not be the case for a particular player, adds von Neumann, "it is clear that such an assumption will certainly not spoil his chances".[99] With that, he gives a formal definition of "strategy": a decision corresponding to each of the possible finite number of sequences of "draws" and "steps". Player Sm, therefore, has only a finite number of strategies available to him: $S_1^{(m)}$, $S_2^{(m)}$. . . $S_{\Sigma m}^{(m)}$.

The next simplifying step is to remove altogether the random part of the game, the "draw", and replace the results for each player, originally indicated by the function f(.), by their expected values, now indicated by the function g(.). We now have an even more schematized and simplified basic strategic game:

> "Each of the players S_1, S_2, . . . S_n chooses a number, S_m choosing one of the numbers 1, 2, . . . , Σm (m = 1, 2, . . .n). Each player must make his decision without being informed about the choices of the other participants. After having made their choices x_1, x_2, . . ., xn (x_m = 1, 2, . . ., Σm, m = 1, 2, . . ., n) the players receive the following amounts respectively: g_1 (x_1, x_2, . . ., xn), g_2 (x_1, x_2, . . ., xn), . . ., g_n (x_1, x_2, . . ., xn) (where identically $g_1 + g_2 + . . . + g_n = 0$)."[100]

The rules of the game, he says, have now been reduced to a form containing only the essential characteristics of the game, without loss of generality. "Nothing is left of a 'game of chance'", and "everything takes place as if each of the players has his eye on the expected value only".[101] We are now ready to pursue the matter of how to play.

Section 2 considers "The Case n = 2", the 2-person case which takes up the bulk of the paper, suggesting that von Neumann added the theoretical structure of the general game after proving the central theorem. In the 2-person case, the players independently choose amongst the numbers 1, 2, . . ., Σ_1 and 1, 2, . . ., Σ_2 respectively, and then receive the sums g(x, y) and -g(x, y) respectively. The Laskerian rhetoric of "struggle" is present throughout. "It is easy to picture the forces struggling with each other in such a two-person game", writes von Neumann. "The value of g(x, y) is being tugged at from two sides, by S_1 who wants to maximize it, and by S_2 who wants to minimize it. S_1 controls the variable x, S_2 controls the variable y. What will happen?"[102]

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98 *Ibid*, p.18.
99 *Ibid*, p.18.
100 *Ibid*, p.20.
101 *Ibid*, p. 21.
102 (p. 21).

By choosing x appropriately, S_1 can guarantee himself at least $\mathrm{Max}_x\ \mathrm{Min}_y\ g(x, y)$, irrespective of what S_2 does. Similarly S_2 can ensure, irrespective of S_1, that g(x, y) reaches no more than $\mathrm{Min}_y\ \mathrm{Max}_x\ g(x, y)$. Is it the case that $\mathrm{Max}_x\ \mathrm{Min}_y\ g(x, y) = \mathrm{Min}_y\ \mathrm{Max}_x\ g(x, y) = M$? Although, in general, $\mathrm{Max}_x\ \mathrm{Min}_y\ g(x, y) \leq \mathrm{Min}_y\ \mathrm{Max}_x\ g(x, y)$, it is not generally true that the equality holds. Thus it does not hold, for example, in the simplest example (Tossing Pennies) where

$$\Sigma_1 = \Sigma_2 = 2, \qquad \begin{array}{ll} g(1, 1) = 1, & g(1, 2) = -1 \\ g(2, 1) = -1, & g(2, 2) = 1 \end{array}$$

where MaxMin = -1 and MinMax = 1. Nor does it hold in the game of "Morra", or "Paper, Stone, Scissors".

If, however, rather than choosing a strategy directly, from 1, 2, . . ., Σ_1, player S_1 instead specifies Σ_1 probabilities $\zeta_1, \zeta_2, \ldots, \zeta_{\Sigma 1}$ and draws his strategy "from an urn containing these numbers with these probabilities", then equality of the two expressions can be ensured. This may look like a restriction of the player's free will, von Neumann says, but it is not. If the player really wants to choose a particular strategy, he can attach to it a probability of 1, but in choosing probabilistically in general he can protect himself against his adversary's "finding him out".[103] Not even S_1 himself knows what he is going to choose! And S_2 can do likewise, choosing Σ_2 probabilities $\eta_1, \eta_2, \ldots, \eta_{\Sigma 2}$ and proceeding similarly.

Letting $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_{\Sigma 1})$ and $\eta = (\eta_1, \eta_2, \ldots, \eta_{\Sigma 2})$, S_1's expected value becomes:

$$h\ (\zeta, \eta) = \Sigma\ \Sigma\ g\ (p, q)\ \zeta_p \eta_q$$

and S_2's is - h (ζ, η). The considerations applied earlier to g(.) can now be applied to h(.). In particular, is it the case that $\mathrm{Max}_\zeta\ \mathrm{Min}_\eta\ h\ (\zeta, \eta) = \mathrm{Min}_\eta\ \mathrm{Max}_\zeta\ h\ (\zeta, \eta)$?

Let ϑ be the set of all ζ for which $\mathrm{Min}_\eta\ h\ (\zeta, \eta)$ assumes its maximal value M, and let B be the set of all η for which $\mathrm{Max}_\zeta\ h\ (\zeta, \eta)$ assumes its minimal value M. Given S_1's and S_2's choice of probability distributions ζ and η respectively, the play has the value M and -M to the respective players. In a "fair" game, M = -M = 0: this holds for Tossing Pennies and Morra, mentioned above. In a "symmetric" game, the players have the same roles, i.e., interchanging ζ and η, we get h (ζ, η) = - h (η, ζ). Thus, even though the explicitly probabilistic aspects of the game were earlier eliminated by introducing expected values and discarding 'draws', chance has now reappeared spontaneously, in the need for each player to apply a probability distribution to his set of strategies.[104]

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103 *Ibid*, p.23.

104 This allows von Neumann to make what is likely an allusion to quantum mechanics, saying that "the 'statistical' element . . . is such an intrinsic part of the game itself (if not of the world) that there is no need to introduce it artificially by way of the rules of the game: even if the formal rules contain no trace of it, it still will assert itself" (p.26).

Von Neumann then proceeds to the mathematically central part of the paper, the 6-page proof that $\mathrm{Max}_\zeta \mathrm{Min}_\eta h(\zeta, \eta) = \mathrm{Min}_\eta \mathrm{Max}_\zeta h(\zeta, \eta)$, i.e., the Minimax theorem. This time, I shall spare the reader the details. Suffice it to say that it involves a painstaking argument based on the lower- and upper-semi continuity of the functions bounding the two elements of the saddlepoint, and an implicit application of Brouwer's Fixed Point Theorem.[105] With his theorem proved, von Neumann has shown that, in the generic 2-person, zero-sum game, there exists an optimal way to play, possibly requiring a player to choose randomly amongst the set of strategies, and ensuring that, on average, victories and defeats counterbalanced each other.

Von Neumann's paper to this point is a *tour de force*. With remarkable clarity, he establishes his theme, stripping away the complexity of actual 2-person, zero-sum, game situations - the elimination of 'draws', the creation of the concept of 'strategy', the suggestion of probabilistic play - establishing the basic axiomatic elements. Step by step, the reader is led to the point where each player has to make one move, i.e., choose a mixed strategy. Yet, when it comes to the actual proof that there exists a saddlepoint, the tone distinctly changes, and one is struck, not by the simplifying clarity, but rather the sheer bulldozing power with which von Neumann pursues the proof. It is an early testament to the remark, often made subsequently of him, that elegance in proof frequently gave way to something resembling brute force, in which he showed no fear of pursuing tangential arguments and taking the difficult, sometimes contorted, route. It quickly becomes difficult for even the mathematically qualified reader to retain the thread: individual passages are clear, but the reasoning linking them is not always, and the argument builds relentlessly. One finishes, satisfied somehow that von Neumann has indeed accomplished what he set out to do, but equally aware of the difficulty involved in following him. One begins to sympathize with his only doctoral student's description of him: "a magician, a magician in the sense that he took what was given and simply forced the conclusions logically out of it, whether it was algebra, geometry, or whatever. He had some way of forcing out the results that made him different from the rest of the people".[106] Or, as Rózsa Péter put it, not entirely reassuringly: von Neumann proved what he wanted to prove.[107] Both of those remarks are also a reminder that we should not confuse the finished paper with the creative process of which it was the final, tidy expression.

Von Neumann shared Zermelo's resistance to psychologizing: the existence of an equilibrium, he said, showed that "it makes no difference which of the two players is the better psychologist, the game is so insensitive that the result is always the same".[108] Later in the paper, he promises a publication which will contain numerical examples of such two-person games as Baccarat, and a

[105] See Harold Kuhn and Albert Tucker,"John von Neumann's Work in the Theory of Games and Mathematical Economics", *American Mathematical Society Bulletin*, 1958, *64*, Part 2:100-22.

[106] Halperin Interview, The Princeton Mathematics Community in the 1930's, Transcript Number 18 (PMC18). See also Paul Halmos, "The Legend of John von Neumann", *American Mathematical Monthly*,1973, *80*: 382-394.

[107] Similarly, of von Neumann's work on lattice theory, Harvard's Garrett Birkhoff wrote: "a truly remarkable feat of logical analysis and ingenuity . . . Anyone wishing to get an unforgettable impression of the razor edge of von Neumann's mind, need merely try to pursue this chain of exact reasoning for himself - realizing that often five pages of it were written down before breakfast . . .". Garrett Birkhoff, "Von Neumann and Lattice Theory", *Bulletin of the American Mathematical Society,* 1958, *64*:51-52, quoted in Heims, *John von Neumann* (cit. n.5), p. 171. Arguably the foremost American mathematician of the interwar period, Birkhoff became friend of von Neumann after the latter moved to the U.S.

[108] Von Neumann, "Zur Theorie der Gesellschaftspiele" (cit. n.94), p. 23.

simplified Poker, the "agreement of the results [of which] with the well-known rules of thumb of the games (e.g., proof of the necessity to 'bluff' in poker) may be regarded as an empirical corroboration of the results of our theory".[109] There was nothing mysterious about bluffing: it was simply rational play. Nowhere does von Neumann discuss the prosaic difficulties of how a player might actually know the correct mixed strategy or how the psychological evaluation of one's opponent might affect one's approach to a game. With existence proved, the game had been collapsed, reduced to its essential skeleton, and any psychological complications consigned to the periphery.

With the tension of the central proof dissipated entirely, von Neumann draws the paper to a close with preliminary considerations of the 3-person, zero-sum game. Here, certain complications are essential to the matter, and cannot be overlooked. Can we find, for each of the players, the value of the game, w_1, w_2, w_3?, asks von Neumann. For these values to be satisfactory, it must be the case that no two players are able to together secure a value exceeding the sum of their individual values. Letting $M_{1,2}$ be the amount that the coalition of players S_1 and S_2 can secure, we must have:

$w_1 + w_2 \geq M_{1,2}$ $\qquad$ $w_1 + w_3 \geq M_{1,3}$ $\qquad$ $w_2 + w_3 \geq M_{2,3}$

$w_1 + w_2 + w_3 = 0$

This is possible if and only if $M_{1,2} + M_{1,3} + M_{2,3} \geq 0$

But there are many games for which this is not true, i.e., in which it is impossible to provide individual values w_i, because two players can collaborate and "rob" the third, thereby doing better in coalition than they could have individually. If S_1 succeeds in entering a coalition, he can expect to receive 1/2 ($M_{1,2} + M_{1,3} - M_{2,3}$); if he fails, he will receive - $M_{2,3}$. Because the rules of the game have nothing to say about which coalition will be formed, von Neumann suggests that we regard the probability that S_1 enters a coalition as 2/3. His basic expected value, v_1, is thus 1/3 ($M_{1,2} + M_{1,3} - 2M_{2,3}$). In general, the 3-person, zero-sum game can be divided into two types. In the first, $D = M_{1,2} + M_{1,3} + M_{2,3} = 0$, and S_1's basic value is - $M_{2,3}$. This type, like the 2-person game, is strictly-determined. In the second, $D > 0$, and S_1's basic value is - $M_{2,3}$ + 1/3 D. In this type of game, the multiple possibilities of coalition make the 3-person game qualitatively different from the 2-person one: it is symmetric, but not strictly determined. The outcome of any such negotiations will depend on factors on which the coalitional values can shed little light *a priori*. Here, as von Neumann puts it, the "actual game strategy of the individual player recedes into the background", and a new element enters, which is "entirely foreign to the stereotyped and well-balanced two-person game: struggle".[110] He concludes with the suggestion that a similar approach could be taken to games of 4 and, ultimately, any number of players, the result of which would be a "satisfactory general theory" of all such games. But he stopped there, dropping game theory for what would turn out to be over a decade. If he was brought back to it, it was, as we shall see, when prompted by precisely questions of social struggle.

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109 *Ibid*, p. 42.
110 *Ibid*, p. 38.

As for Lasker, in the meantime, he had emerged the victor at the New York tournament of 1924 at which Reti shook Capablanca. Then, after the Moscow tournament of 1925, he gave up competitive chess. Having long given up any hope of pursuing an academic career in mathematics, he returned to Berlin in 1927, learning to play Go and Bridge, becoming an international player and Life Master in the latter. That New York competition was used as an example by Zermelo in a 1928 paper he wrote employing maximum likelihood methods to develop a rating system for chess players.[111] He sent the paper to Lasker, who showed great interest and returned the compliment by sending him a copy of his own "The Philosophy of the Unattainable", the essay in which Lasker criticizes Einstein's theory of relativity.[112] He had gotten to know Albert Einstein in 1927, with whom he became friendly, engaging him in conversations on long walks through Berlin. Over a decade later, when Lasker's biography was written, it was Einstein who would write the Foreword.[113]

The "Futile Search for a Perfect Formula"

Having presented von Neumann's minimax note to the Paris Académie in 1928, Borel appears to have ignored the main paper for a number of years. Not until 1936, when giving a talk at a mathematical congress in Oslo was he reminded of it by someone in the audience, which prompted Borel to say that he hadn't had the time to study it carefully.[114] And, indeed, the best that can be said about Borel in relationship to von Neumann in the 1930's is that he would never embrace the Hungarian's contribution. On the few occasions he was to write about it, it would be to add qualifications as to its usefulness for the consideration of real games.

The culmination of Borel's work in this period was his 1938 volume on the analysis of games of chance, based on university courses given in 1936-37, and written up by his student Jean Ville. The recent Depression, on which Borel had written economic articles in the early 1930's, allowed him to draw connections between parlour games and the economic world:

> "[Economic] phenomena are caused, on the one hand, by material causes, which have concrete manifestations, such as the valuation of existing stocks, and, on the other hand, by causes dependent on the human will. Economic theories that take account only of causes of the first kind give rise to developments that are interesting, but of practically little value. And economists may be reprimanded in a manner similar to the way meteorologists are criticized: just as the latter are excellent at scientifically explaining yesterday's weather rather than forecasting that of next week, economists are better at producing theories of something that has just happened than they are at prescribing measures to be taken to ensure that tomorrow's

111 See Ernst Zermelo, "Die Berechnung der Turnier – Ergebnisse als ein Maximumproblem der Wahrscheinlichkeitsrechnung", *Mathematische Zeitschrift*, 1928, *29*: 46-460.

112 See Lasker to Zermelo, date ?, Zermelo archives, University of Freiburg. My thanks to Professor Volker Peckhaus of Paderborn University for drawing my attention to this exchange.

113 See Hannak, *Emanuel Lasker* (cit. n.32).

114 Émile Borel, "Quelques remarques sur l'application du calcul des probabilités aux jeux de hasard", *Congrès international de mathématiciens*, Oslo, 1936, *2*:187-190, reprinted in Borel, Oeuvres?. There, he continues, possibly with the slightest hint of displeasure: «von Neumann cited … a note I had published in 1927 in the *Comptes rendus de l'Académie des Sciences* de Paris, but was unaware of the third edition of my *Éléments de la théorie des probabilités* (Paris, Hermann, 1924), a work in which I develop, for certain symmetric games, in particular the Japanese game of paper, stone and scissors, considerations quite analagous to his (he calls this game baccara du bagne)» on p. 1173.

> economic life remains normal. In order to treat economic questions satisfactorily, room must be made for probability and psychology: the study of games of chance, and of psychology will provide a useful basis for such inquiry". [115]

Later in the book, he returns to the parallel between games and problems of strategy or economics, analysing the altogether Laskerian problems of two enemies allocating their opposing forces across several battlefields, two merchants deciding how to apply discounts in different markets, or two entrepreneurs bidding for the same set of contracts.[116]

In that 1938 volume, the von Neumann proof is the subject of a note, not by Borel, but by his student Jean Ville, in which the latter provides an elementary proof of the minmax theorem, all of which is quite congruent with Borel's having prodded Ville to take a look at "that theorem", following the Oslo meeting.[117] Following Ville's proof, Borel offers some telling remarks in commentary. "It appears esential for me to indicate, however, to prevent all misunderstanding, that the practical applications of this theorem to the actual playing of games of chance is, for a long time, unlikely to become a reality". Actual games are exceedingly complicated, he says, and even if one could simplify a game to the point where such calculations were possible, the advantage of playing according to von Neumann's prescription would be had only on average, after a great many rounds. Even if one could draw on experienced players to locate reasonable strategies, the number of variables remaining was still so great as to make the task of writing the equations "absolutely insurmountable".[118] Games are interesting, says Borel, precisely because they perpetually evolve. Sometimes, they even move in cycles. No sooner has agreement been reached concerning a good way to play than players take advantage of that consensus by introducing novel approaches, only to later find themselves returning to older ways of playing. Even where ideal play involved the use of probabilities, says Borel, it was very difficult not to follow some regularity when actually playing. In bridge, for example, probabilistic play intended to defeat one's opponent may well mislead one's partner also! "All these remarks", Borel concludes, "are obvious to anyone with some experience in games. Perhaps they will make clear, to those uninterested in games, how enjoyable games are as leisurely distraction, at the same time showing to those who would wish to turn games into an occupation, how futile is the search for a perfect formula which is forever likely to elude us".[119]

Whether this was intended as gentle put-down or not, it does speak to the gulf separating Borel from von Neumann. The latter, however, was unaware of it all - and he probably wouldn't have cared anyway. With the 1928 publication, he had moved onto other things, and put game theory, Borel and all that completely aside. Indeed, he wouldn't learn of the existence of Borel's 1938 book until early 1941 or 1942, when it was brought to his attention by a Viennese economist, Oskar Morgenstern, who stumbled across it by accident in the library of the Institute for Advanced Study at Princeton.[120] By that point, however, a great deal of water had flowed under the bridge for von

[115] Borel, *Traité* (cit. n.81), pp. X-XI.

[116] *Ibid*., pp. 86-87.

[117] *Ibid.*, pp. 105-113.

[118] *Ibid*, p. 115

[119] *Ibid.*, p. 117.

[120] In a priority debate raised later in the 1950's by Borel's protégé, Maurice Fréchet, von Neumann replied, quite starchily, that his own work was independent and that Borel's work had come to his attention only as he was writing up

Neumann. If Lasker, Borel and the young Hungarian had all variously hinted at the possible connections between games and social interaction, only the latter would fully pursue the idea, and that, as we shall see, was intimately bound up in his personal experience of the political events of the 1930's.

Part III: Mathematics and the Social Order

From Berlin to Princeton

In the climate of the late 1920's, von Neumann knew that his chances of obtaining a chair in mathematics in Germany or Hungary were negligible. Although his family had nominally converted to Christianity on the death of the father in 1923, socially the young von Neumann was still perceived as Jewish, and that in a Germany where many Dozents were competing for promotion. Stan Ulam also remembered him speaking of the worsening political situation, which made him doubt that intellectual life could be pursued comfortably. Thus, von Neumann readily accepted when, at the beginning of the 1930's, Oswald Veblen, Princeton mathematician and occasional visitor to Göttingen, arranged to have him six months per year at Princeton. For the next two years, von Neumann commuted from Berlin to Princeton, by cruise-liner, first-class as always, to a professorship in the Mathematics Department, shared with his fellow Hungarian, mathematical physicist Eugene Wigner.

Princeton's strength in mathematics in the 1930's resulted from its having two centres of gravity: the university's Department of Mathematics and the nearby, but independent, Institute for Advanced Study. The latter had been officially incorporated in 1930 through a large endowment by supermarket millionaires, Louis Bamberger and his sister Mrs. Caroline Bamberger Fuld. Also involved in the inception was the Institute's first director, Abraham Flexner.[121] It was decided to locate the Institute at Princeton University because of the its excellent library and the quality of its mathematics department, in whose building, Fine Hall, the Institute was first located. The first full faculty member, secured by Flexner in 1932, was Albert Einstein, then keen to leave Germany and being courted by universities the world over. By the time the Institute opened its doors in the fall of 1932, Oswald Veblen, von Neumann and James Alexander were on the faculty, they having transferred from Princeton's mathematics department. Einstein physically arrived in 1933. The Institute, which at the outset had only a School of Mathematics, paid lavish salaries, averaging twice those of Princeton professors. In 1933, it moved to a new building, constructed on a site bought for the purpose, just south of Princeton campus, where it has remained to this day.

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his own paper. There was nothing worth reporting in game theory, he had felt, until the theorem had been proved. Be that as it may, the careful reader of Borel and von Neumann is hardput not to conclude that, in 1926, the latter *must* have had some inkling – even if only through hearsay at Göttingen – of how Borel had framed the problem, of what he was trying to do, and the fact that he had worked on poker, scissors-paper-stone and such games.

[121] Flexner had been author of the famous "Flexner Report", a slamming indictment of the medical college system, and in particular of its then large number of "quack" colleges, where well-paying students could quickly become medical doctors with less than a minimal amount of medical training. Following the report, a number of these colleges were forced to close down, and Flexner became quite famous. He was writing another book, comparing the American, English and German university systems, when he was approached by Bamberger and Fuld. On the Institute see Ed Regis's, *Who got Einstein's office?: Eccentricity and Genius at the Institute for Advanced Study*, (New York: Addison Wesley, 1988).

Princeton Mathematics was known for its strength in toplogy and algebra, two relatively young fields, important in the growth of American mathematics. Veblen was a leader in combinatorial topology, a field in which Alexander and Solomon Lefschetz also worked. Other mathematicians included Bob Robertson, who would become a friend of von Neumann, Luther Eisenhart, the logician Alonzo Church, Marston Morse, Carl Siegel, Albert Tucker and statistician Sam Wilks. During the 1930's, it was one of the finest departments in the U.S., in time training the Milnor's, Nash's, Kuhn's and Shapley's of the postwar generation – to mention just a few with a connection to game theory. In addition to the quality of faculty, the social occupation of space seems to have been important. For all those in reminiscence about Princeton mathematics point to the importance of the Fine Hall Common Room, where afternoon tea and the playing of games made for a certain *esprit de corps*, quite unlike the mathematics departments at Columbia or Harvard, where nothing comparable existed.[122] Von Neumann was a stalwart at Princeton teas.

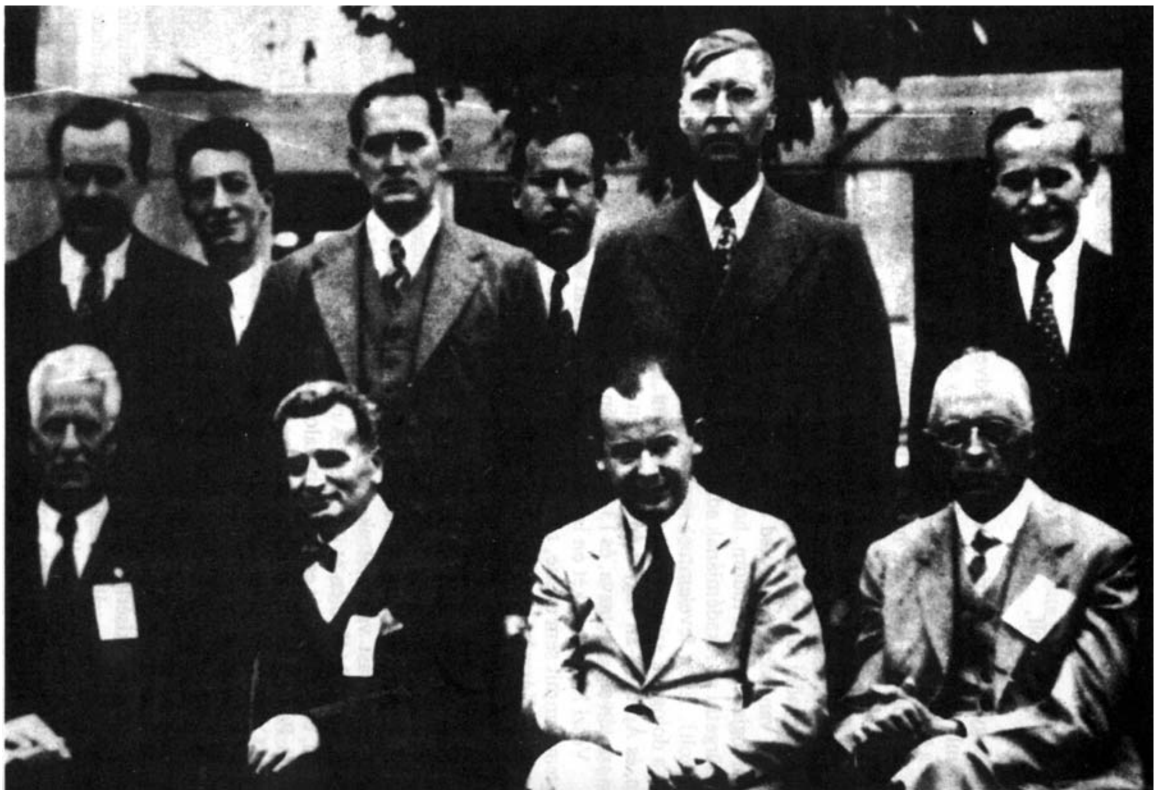

Figure 6, Princeton mathematicians (from Nágy)

From the beginning, von Neumann took greatly to life in the U.S.. Temperamentally, the country suited him, and, although he would always dress formally – including when on horseback and on the beach – he seems to have appreciated the freshness of life in the States. The Depression Era was kind to him and his first wife, Mariette Kovèsi, herself from a prominent family in Budapest. They soon moved into a large house in Princeton with domestic staff, and when not working hard, von Neumann "played hard", throwing famous parties, with copious cocktails and caviar imported from Russia. He drove a powerful car, dangerously some say. Indeed, he crashed cars several times, on

122 With tea at 4.30pm every afternoon – an institution begun by Anglophile Veblen - the Common Room allowed professors and graduate students to mingle. Games were played: cards, chess, *Kriegspiel* (a form of blind chess), invented challenges involving stacking the chess pieces on top of each other. Some students, such as Merrill Flood, relied on all-night poker sessions to supplement their income.

both continents, so that, at Princeton, a particular curve in the road became known to acquaintances as "von Neumann Corner". Although he embraced American life generously, he returned to Hungary virtually every summer, fleeing the heat and humidity of summertime Princeton, just as his teachers such as Fejér continued to flee that of Budapest.

He had no reason to regret his decision to leave Germany. In mid-March 1933, a few weeks after the Reichstag fire, and days after the sweeping Nazi election victory under Hitler, von Neumann wrote from Budapest to Flexner back at the Institute. His Summer plans were not yet fixed, he wrote, in his still imperfect English. He had hoped to spend the Summer lecturing in Berlin, but the "newer german developments" (sic) had thrown this into question. He didn't think conditions would improve.[123] Flexner wrote back expressing his concern, saying that another German friend had written to him about what was happening to the Education Ministry under Hitler. "The whole thing seems to be the act of mad men", said Flexner, "I cannot believe that it will endure".[124] A week or so later, in April, the German "Restoration of Civil Service Act" was passed, effectively allowing the Nazi government to dismiss academics for reasons relating to politics or race. It marked the beginning of the systematic release of Jewish faculty members from the German universities. Writing to von Neumann, Flexner condemned the German government's madness, and its destruction of the Göttingen faculty in particular.[125] From Budapest, he wrote to Oswald Veblen with great interest and in detail about American economic affairs and the "decision to inflate". His comments on European politics are laced with irony:

> "There is not much happening here, excepted that people begin to be extremely proud in Hungary, about the ability of this country, to run into revolutions and counter-revolutions in a much smoother and more civilized way, than Germany. The news from Germany are bad: heaven knows what the summer term 1933 will look like. The next programme-number of Hitler will probably be the annihilation of the conservative-monarchistic - ("Deutsch National" = Hügenberg) – party.
>
> You have probably heard that Courant, Born, Bernstein have lost their chairs, and J. Franck gave it up voluntarily. From a letter from Courant I learned 6 weeks ago (which is a very long time-interval now in Germany), that Weyl had a nervous break-down in January, went to Berlin to a sanatorium, but that he will lecture in Summer.
>
> I did not hear anything about changes or expulsions in Berlin, but it seems that the "purification" of universities has only reached till now – Frankfurt, Göttingen, Marbürg, Jena, Halle, Kiel, Königsburg – and the other 20 will certainly follow.
>
> I am glad to learn from your letter that these things received the full attention and appretiation [sic] in America which they deserve. It is really a shame that something like that could happen in the 20th century.[126]

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123 John von Neumann to Abraham Flexner, March 18, 1933, Faculty Files, John von Neumann, Folder "1933-35", Von Neumann Papers, Institute for Advanced Study (hereafter VNIAS).

124 Abraham Flexner to John von Neumann, March 30, 1933, Faculty Files, John von Neumann, Folder "1933-35", VNIAS

125 Flexner to von Neumann, May 6, 1933, Faculty Files, John von Neumann, Folder « 1933-1935 », VNIAS. On Göttingen after 1933, see Segal, *Mathematicians* (cit. n.43).

126 VN (Budapest) to Veblen, April 30, 1933, Veblen Papers, Library of Congress (hereafter VLC), Box 15, Folder 1.

Von Neumann chose not to go back to Berlin. After a leisurely summer, with weekends spent on Lake Balaton and in the Hungarian countryside, he returned permanently to the Institute at Princeton, and never set foot in Germany again.[127]

Lasker, too, left Germany that year, his property confiscated by the Nazis. He drifted for a number of years, staying in the Netherlands and then England, and then accepting a research post at Moscow's Institute of Mathematics. There, Gerald Abrahams says, the great man was somewhat inert, being content to throw out mathematical ideas but not pursue them seriously. Abrahams fails to emphasize that Lasker was by then over 65 years of age.

Into Disequilibrium

It was the beginning of a difficult period for many, von Neumann included. Amongst his Hungarian correspondents, an important mentor was the above-mentioned Rudolf Ortvay, a physicist eighteen years his senior. Born in 1885 in Miskolc, in the northeast of the country, Ortvay too studied at Göttingen and, following a period at Koloszvár and Szeged, moved to Budapest in 1928, where he ran the Institute for Theoretical Physics. He was also a friend of the von Neumann family and, like Lipót Fejér, a frequent guest at their dinner table in the 1920's. He had followed the young von Neumann's career from the beginning, and he maintained a revealing correspondence with him all through the 1930's.[128]

[127] William Aspray in *John von Neumann and the Origins of Modern Computing* (Cambridge, Mass.: MIT Press, 1990) indicates that von Neumann visited Germany, Austria, Hungary, Italy, and France in 1930 and 1931; Germany and Hungary in 1932; Hungary and Italy in 1933; Hungary alone in 1934; England and Hungary in 1935; and France in 1936; and Hungary alone in 1937 and 1938 (p. 256, n. 35).

[128] The von Neumann-Ortvay letters, written in Hungarian, are located in the Library of the Hungarian Academy of Sciences in Budapest and in the von Neumann papers at the Library of Congress, with copies in the Stan Ulam papers at the American Philosophical Society in Philadelphia. There are 60 of them, running from May 9, 1928 to February 16, 1941. Most of them have been reproduced in Hungarian in Ferenc Nagy, *Neumann János és a "Magyar Titok", A Dokumentumok Tükrében* (Budapest: Országos Müszaki Információs Központ és Könyvtár,1987), the title of which may be translated as John von Neumann and the «Hungarian Secret». With a few exceptions, which are indicated, all the following quotations in the present paper are based on translations of the letters found in Nagy, *op cit*.

Figure 7, Rudolf Ortvay in the 1930's
(from Ferenc Nagy with permission).

In October 1933, von Neumann wrote to Ortvay describing Einstein's arrival the previous day at the Institute, the German physicist having been slipped off the boat and spirited away so as to avoid the reception committee awaiting him on the dock. "What is new in Budapest?", von Neumann continued. "How is the mood in general, and especially with regard to the German situation?".[129]
By then, Ortvay had become quite pessimistic about European politics, and his remarkable letters from here on were at once a lament for cultural decline, a meditation on the place of the scientist in society, and an inquiry into the vagaries of the human spirit. Together with von Neumann's replies, the letters speak about the two personalities in their time, quietly and with depth. From 1934 onward, while continuing his mathematical work on the spectral theory of Hilbert space, ergodic theory, rings of operators and Haar measure, von Neumann was increasingly preoccupied by politics, entering into the finest detail in his letters. Faced with a relatively emotional Ortvay, he tended to maintain a certain detachment:

> "What you write about the uncertainty of the future of European civilization is regrettably plausible. There is one consolation in it, but even this isn't an excessively certain conclusion: the war demoralised principally the countries that lost, and in history after a lost war experimentation with a state structure of tyranny or dictatorship, and the rise of a romantic, irrational nationalism, is neither a new nor rare phenomenon. Naturally references to historical analysis are especially arid and hopeless, since if these could be trusted, then new wars could not be avoided"[130]

[129] Von Neumann to Ortvay, October, no exact date, 1934.

[130] And he then went on to discuss Roosevelt's foreign exchange policy and negotiations with Congress over his spending plans. Von Neumann to Ortvay, January 26, 1934.

In November 1934, von Neumann wrote of the many new doctoral students arriving at the Institute, amongst whom there were two German Rockefeller fellows: "Whether they are genuine Nazis, I don't know, they are fairly discreet… How do you now judge Central Europe?" he continued, where "the situation ... seems to be so tense that in the end there will be trouble! There are so many uncertain and easily misunderstood circumstances in the European 'balance' that there may exist a government that jumps into an adventure".[131] England and Italy were equally indecisive, or, rather, hypocritical, he said, and the weight of Russia was just as incalculable as it had been in 1914. Later that month, he grew sombre: "The European political situation appears to be quite dark even from here…; to wit, here the people have already accepted that the lesson was for nought, and that in Europe there shall be a war in the next decade. The conclusion is likely to be premature yet, but without doubt it is more plausible than its opposite".[132]

It was at this time that Polish mathematician, Stan Ulam, entered von Neumann's life. They met in Warsaw in 1934, when the latter was returning from a Moscow conference with Birkhoff and Marshall Stone. Years later, Ulam would remember meeting him on the platform: "The first thing that struck me about him were his eyes – brown, large, vivacious, and full of expression. His head was impressively large. He had a sort of waddling walk… At once I found him congenial. His habit of intermingling funny remarks, jokes, and paradoxical anecdotes or observations of people into his conversation, made him far from remote or forbidding".[133] The two hit it off immediately. Both were what Ulam describes as third or fourth-generation wealthy Jews, comfortable with each other, and they were also linked through mutual acquaintances, for Ulam's widowed aunt had married Árpád Plesch, one of the richest Jews in Budapest, who was, of course, well known to the von Neumann's. When Ulam moved to the States in 193?, he was von Neumann's assistant at the Institute, before being appointed Junior Fellow at Harvard, through the intervention of Garrett Birkhoff.

By January 1936, von Neumann was writing about the effect of Mussolini's Italy on the European situation, and, within a few months, was predicting a cataclysm: "Here Europe is judged darkly, as with every affair that is distant and complicated. But even I cannot bring myself to tranquility. The danger of war appears to be truly great, even if the catastrophe does not take place this year. I hope that from near by, the picture is not this desolate. How do you judge it?".[134] Throughout the mid-1930's, he devoted his main efforts to continuous geometry, carrying on this work even when on holiday in Ontario with the Flexners.[135] Domestically, von Neumann's political sympathies appear

[131] Von Neumann to Ortvay, November 2, 1934.

[132] Von Neumann to Ortvay, November 28, 1934.

[133] Ulam, *Adventures* (cit. n.12), p. 67.

[134] Von Neumann to Ortvay, April 1, 1936.

[135] Von Neumann was staying with the Flexners at their holiday home in Magnetawan, Ontario, when he wrote to Ulam congratulating him on his appointment to the Harvard Society of Fellows. "I need not tell you how glad I am that this has happened and I hope very that it will only be the beginning of your 'career' on this side. And I hope that - the distance from Princeton being 'merely' 250 miles - we will see a lot of you in the months and years to come". Perhaps an augury of things to come, von Neumann was holidaying without Mariette, she having chosen to extend her Summer stay in Budapest, being "a better child, relative, and all that", while he had returned to America after only a few weeks. After Magnetawan, he was off to Cambridge for 10 days to join Garrett Birkhoff for the Harvard Tercentary celebrations. Von N to Ulam, July 24, 1936, Ulam Papers, American Philosophical Society, Philadelphia (hereafter SUAPS).

to have been Republican, so that when the Democrats achieved a strong victory later that year, he noted that the economic situation was good and improving but that Roosevelt's victory would increase the likelihood of worker-employer conflict.[136]

In late 1936 or early 1937, he gave a popular talk at Princeton in which, according to the *Science News Letter*, he presented some ideas on what was for him "a mere recreation", his analysis of games and gambling. All of it appears to have referred to the work he had done at Göttingen a decade previously. There was no mention of anything other than 2-person parlour games. He spoke about "stone-paper-scissors", showing that by "making each play the same number of times, but at random, ... your opponent will lose in the long run".[137] Also briefly reported are his comments on the probabilities of making particular plays in both dice and a simplified poker. Amongst those attending the Princeton talk was Merrill Flood, then a graduate student, and later a mathematician at the RAND Coporation:

> "I don't know what the title of the lecture was, but I went because of von Neumann, whom I'd come to know well. He lectured on the minimax theorem, although he didn't call it that. In fact, he didn't tell us that there was such a theorem. He gave us examples of how mixed strategies could be used in games. It made a great impression on me, and I remember going to Kleene and Einstein and half a dozen other people to find out if they had ever heard of that. . . Nobody came up with the idea of mixed strategy among all these bright people. That convinced me that that's a subtle thing"[138]

In late 1937, von Neumann's personal life became complicated. For that Christmas, his wife, Mariette Kovèsi, left him for a long-time Princeton graduate student in physics named Kuper, a regular guest at the von Neumann parties on Wescott Road. Although Stan Ulam would later say that the rupture greatly shook his friend, in his correspondence at least, von Neumann bore it all with equanimity:

> "Many thanks for your letter . . . and particularly for what it contained about my 'domestic' complications. I am really sorry that things went this way - but at least I am not particularly responsible for it. I hope that your optimism is well founded - but since happiness is an eminently empyrical (sic) proposition, the only thing I can to is to wait and see . . .".[139]

Von Neumann's highly punctuated missives jump discretely from one topic to the next, be it personal matters or mathematics, in paragraphs separated by a short dashes. They take the reader

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136 Von N. to Ortvay, November 7, 1936.

137 *Science Letter News*, April 3, 1937, "Princeton Scientist Analyzes Gambling: "You Can't Win", p. 216.

138 Interview with Merrill Flood by Albert Tucker, San Francisco, May 14, 1984, Transcript No. 11 (PMC11) of oral history project *The Princeton Mathematics Community in the 1930's,* deposited in the Seeley Mudd Library, Princeton University. This was Flood's introduction to game theory and, a year later, he returned to it when asked to give a popular lecture to Princeton undergraduates in a bid to recruit mathematics majors. It was around that time that he approached von Neumann who handed him "a 20-page manuscript in his handwriting in Hungarian, which was all he then knew about game theory… I had that paper for a couple of years… I was never able to read the darn thing. I was too reticent to go and persuade von Neumann to give me the time, which he would have done" (ibid). This was probably von Neumann's notes on a simple 2-person poker, which he had also completed in Göttingen.

139 Von Neumann to Ulam, Oct. 4, 1937.

closer to him, yet, for all that, he somehow remains untouchable, standing apart. Throughout, whether the topic was divorce, the greater merits of a combinatorial approach to measure theory, or the turn of events in Europe, there prevails a certain levity. Always moving forward, to the next paragraph, the next theorem, never does von Neumann slow down. Just as his approach to mathematics and science made little room for philosophical or spiritual qualms, so too are they muted in his letters, even in those to Ulam who was probably his closest friend. Certainly, he makes numerous references to being with Ulam - "we will then have a chance of a tête à tête, since I feel now prepared to describe to you the present aspect of the Universe as far as I am concerned" - but to the reader sifting through the record over fifty years later, part of the epistolary von Neumann remains distant.[140]

Emotional detachment notwithstanding, events took their toll on him. As his marital difficulties became intermeshed with political developments in Hungary, he entered a critical period in which saw his normally volcanic output of papers collapsed: to one in 1938, and none the year after.

In early 1938, Ortvay wrote from Budapest of the effect of political interference on science and intellectual life, citing a debate that he and von Neumann had been reading in the columns of *Nature*, a magazine that had been banned from German public libraries. "I needn't say", wrote Ortvay,

> "how anxious I am seeing the tendencies that arise in more and more places against science and that I feel it very important to safeguard the freedom of scientific life... It is unconditionally important that those occupied with science realise the importance of this question and do their duty in their own sphere of action. A healthy scientific public opinion - I believe - is one of the most fundamental conditions of science's freedom and influence. It is a pity that here we could be in for a very poor experience and I regard with astonishment how few disciples of science are of noble opinion. I believe that one principal reason for this is the proletarianisation of the intelligentsia; there are few independent men, few who strive towards culture rather than rapid success; as a consequence, completely uncivilized but productive specialists predominate. The instability of conditions force those who are not go-getter types into this mould. Thus the scientific crisis here is interconnected with the general cultural and moral crisis".[141]

He went on to insist on the need to separate the active promotion of the freedom of science from any particular form of political system or party, especially given the likelihood of increased general instability. "In my view, it would be most important, this inner moral regeneration of scientific life, perhaps even the propagation of an intelligently understood ascetic ideal: renunciation of life's

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[140] Heims, *John von Neumann* (cit. n.5) contrasts von Neumann with Norbert Wiener, portraying the latter as a more intuitive, more human, mathematician, and von Neumann as not only more analytical in his mathematical style, but indeed coldly rational, indeed barely human. In a 1982 review of Heims' book, Harvard mathematician Marshall Stone, a longtime acquaintance of von Neumann, took exception, saying that Heims had portrayed von Neumann in a manner foreign to those who knew him. At the same time, beyond pointing to von Neumann's genuine modesty concerning his own achievements, to the roundabout, rather than clinically engineered, manner in which he became involved in wartime mathematics, and to the lack of documented knowledge of von Neumann's role at Los Alamos and the Atomic Energy Commission, Stone offers little to contradict Heims' portrayal.

[141] Ortvay to von Neumann, January 28, 1938.

manifold pleasures in the interest of spiritual independence. It is not required that everybody should be like this, but if this type of scientist is esteemed, maybe it could be of service to science, like the religious orders to the general culture in similarly stormy weather ".[142] Ortvay probed the possibility of using mathematics to model “spiritual” states, i.e., emotions and attitudes, and this became a thread running through their correspondence:

> "From your two articles about 'continuous geometry' in the Proceedings of the Academy [of Science] I am trying to shape for myself a conception of this topic. Please send me the detailed paper as well, when it appears. These interest me insofar as I believe the increasingly general geometries signify a procedure indicative of the extension of the immediately given conception of space, and perhaps it will even contribute to our being able to describe the acts of spiritual life with adequate concepts. The philosophers continually emphasize that this is a different realm than the spatial world, but cannot characterise it exactly, because the appropriate apparatus does not exist. Perhaps even these will be ripe for scientific discussion".[143]

In March 1938, Ortvay pursued the question. He felt that it was because man's attention was naturally directed towards the external world that he had first attended to the geometry of space, first in Euclidean form, later Riemannian. It was now time to turn inward, he felt, and examine the formation of judgments and attitudes, the existence of "spiritual states". Whether spatial concepts would be adequate to this or whether a different kind of mathematics would be necessary, was still unclear, said Ortvay, but he felt that the generalisation of the concepts in modern set theory and axiomatics would be of use. Ortvay’s interest in modelling attitudes and dispositions was clearly bound up with both his own depressed state and the way in which political developments highlighted the relationship between the individual and the larger culture. “I would very much like if Chamberlain's policies were successful”, he wrote, “since a war would entail unfathomable consequences. But I am not optimistic and feel that developments are not everywhere going in a healthy direction”.[144]

The Hungarian Social Balance

It was, indeed, the beginning of an important period in Hungary. A few days after Ortvay’s letter, in a well-known speech at Gyór, not too far from the Austrian border, the Hungarian prime minister, Kálmán Darányi, outlined his plans for concrete legal measures designed to cope with the “zsidókérdés”, the "Jewish Question":

> “I see the essence of the question in the fact that the Jews living within Hungary play a disproportionately large role in certain branches of the economic life, partly owing to their particular propensities and positions and partly owing to the indifference of the Hungarian race. Their position is also disproportionate in the sense that they live to an overwhelming extent in the cities, and above all in the capital. . . . The planned and legal solution of the question is the basic condition for the establishment of a just situation – a just situation that

142 *Ibid.*

143 *Ibid.*

144 Ortvay to von Neumann, March 2, 1938.

> will either correct or eliminate the aforementioned social disproportions and will diminish Jewry's influence. . . to its proper level"[145]

With the *Anschluss* of Austria a fortnight later, the Gyór Programme became something of a national obsession in Hungary, giving rise to a 3-month parliamentary debate on Bill No. 616, designed to ensure the "more effective protection of the social and economic balance". That the bill was not opposed in the upper house by representatives of the Lutheran, Calvinist and other churches helped psychologically condition the Hungarian people as regards the Jewish Question.

Writing after the *Anschluss*, von Neumann admitted to being even more pessimistic than Ortvay: "I don't think that the catastrophe can be avoided. The arming for war is more intensive than before 1914. The view that (today's) dictators are more peaceful by nature than the (former) monarchies, is firmly belied even by the happenings of the recent past. Thus, since we are not familiar anyway with the 'time' mechanism, in my view the most stark empiricism is justified. What happened in 1914 will happen now a fortiori".[146] It was not a case of proving why it would happen, he said, but why it would not. He was certain that, if there were no other means to ensure an English victory, the U.S.A. would intervene on England's behalf, the latter being essential to U.S. security in the Far East. He was also very interested in how domestic politics in Hungary would be affected by Austria's demise.

Ortvay's pessimism deepened. Even putting aside the danger of a catastrophic war, he said, he judged the whole development of culture very darkly. The "advance of the masses" was a negative feature of early 20th century modernity: the development of the popular press, the "adoration of the automobile and machinery", the excesses of propaganda, mass travel - this was "modern barbarianism, with all its technical superlatives as described so nicely by A. Huxley", and it prevented the emergence of a higher form of life. The problem was not how to further satisfy the masses, said Ortvay, it was, rather, how to keep them under control. The obvious need for a strong moral stance, in scientists given that it could not be expected in politicians, served to underline the importance of emotions and spiritual qualities. Yet never before, wrote Ortvay, had there been so great a gulf between the scientist's technical capacities and his level of culture or moral state. It caused him great anguish daily, he said.[147]

In May, Hungarian Bill No. 616 became Law No. XV, the famous "Balance Law". Its aim was to reduce to 20% the proportion of Jews in the professions and in financial, commercial and industrial enterprises of 10 employees or more. Those to be exempted included war invalids and those who had converted before August 1919 and their descendants. The aims of the law were expected to be achieved within 5 years, through the dismissal of 1,500 Jewish professionals every six months.

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145 Quoted in Braham, *The Politic*s (cit. n.8), on p. 121.

146 Von Neumann to Ortvay, March 17, 1938.

147 Ortvay to von Neumann, April 4, 1938. In the next letter, he reported visiting Germany, for a celebration for the physicist Sommerfeld, his teacher, at which Planck, Heisenberg and others were present. He had visited some of the Nazi architectural sites, *Haus der Deutschen Kunst*, the *Führerhaus*, which, despite their Spenglerian striving for gigantic dimensions, with their huge columns and stone cubes, he said, he preferred to the retrograde modern buildings.

It is important to view clearly this legislative anti-semitism in Hungary of 1938, especially in light of our concern with von Neumann's return to the development of a mathematics of coalitions. The law was a concrete measure indicating a change of the rules of Hungarian life, the instantiation of a new norm, with the moral authority for same being provided by the Church and the Hungarian liberal-conservative leaders of the gentry and the old feudal order. It was a response to the popular perception of injustice as regards Jewish privilege, undertaken in such a way as to dampen the claims of the Hungarian radical Right, the *Nyilas*. The latter, under Szálasi, were clamoring not only for much harsher measures against the Jews, pointing to Germany, but also for significant reforms in the area of land ownership and the franchise. Therefore they represented a genuine threat to the traditional semi-feudal order and were feared. Thus when Darányi appeared to be too close to the popular Right, he was ousted and replaced as Prime Minister by Béla Imrédy. Other attempts to stall the far right included Horthy's forbidding civil servants to join extremist political parties, in April 1938, and the Interior Minister's banning the *Nyilas* Party less than a year later.

This was the Hungary to which von Neumann returned in April 1938, when he fled Princeton for a while. It was the beginning of a difficult interlude for him, newly divorced and travelling on an American passport in his own country, which itself was changing by the month. He wrote to Veblen:

> "I am familiarized by now with the state of mind, the bellyaches and the illusions of this part of the world – such as they are since the annexation of Austria. The last item (illusions) is rather rare, the preceding one not at all... Hungary was well under way of being Nazified by an *internal* process – which surprised me greatly – in March/April. The new government, which was formed in May stopped this process, or slowed it down, but for how long, is not all clear".[148]

In June, he was in Warsaw, for a conference organised by the League of Nations' International Institute for Intellectual Cooperation, in which several physicists including Bohr and Heisenberg took part. He also gave a talk to Ulam's former teachers and colleagues, including the logicians Knaster, Kuratowski and Tarski.

The previous Summer, he and Ulam had returned to Europe by liner, visiting Ulam's Lwów together. Ulam remembered their visiting an Armenian church with frescoes by the Polish religious artist, Jan Henryk Rosen. This time, in the summer of 1938, Ulam travelled down to Hungary to join von Neumann, visiting Budapest and travelling with him through the countryside. They visited von Neumann's teachers Lipót Fejér and Frigyes Riesz at Lillafüred, near Miskolc, where the mathematicians were spending part of their summer.[149] Lying in an attractive forested area in the mountains, about a hundred miles from Budapest, with luxurious castle-like hotels, Lillafüred was then a favourite resort of the Hungarian elite. The Hotel Palota ("Palace Hotel") had been built a decade previously.

148 Von Neumann to Veblen, June 8, 1938, VLC, Box 15, Folder 1 ?, emphasis in original.

149 See Ulam, *Adventures* (cit. n.12), p. 111.

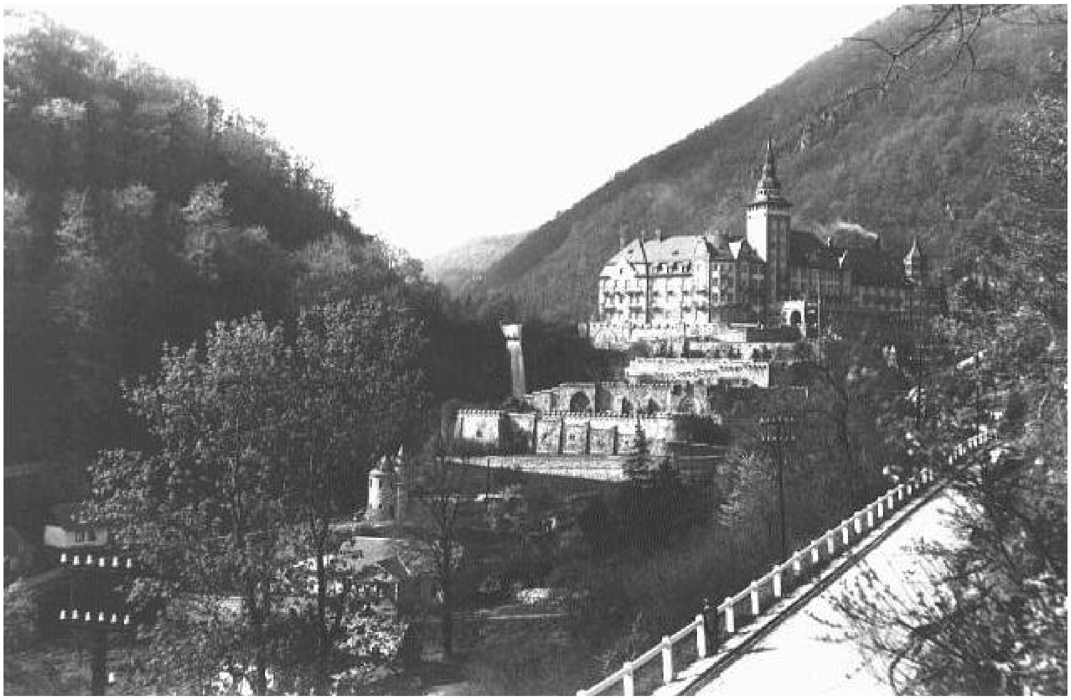

Figure 8, Hôtel Palota, Lillafüred, 1930's

Ulam remembered their walking through the forests of Lillaufüred with Fejér and Riesz, talking about the possibility of war. After that, he returned northwards to Poland, by train through the Carpathian foothills:

> "The whole region on both sides of the Carpathian Mountains, which was part of Hungary, Czechoslovakia, and Poland, was the home of many Jews. Johnny used to say that all the famous Jewish scientists, artists, and writers who emigrated from Hungary around the time of the first World War came, either directly or indirectly, from these little Carpathian communities, moving up to Budapest as their material conditions improved"[150]

When later asked why these Jews were so creative, von Neumann felt that it was "a coincidence of some cultural factors which he could not make precise: an external pressure on the whole society of this part of Central Europe, a feeling of extreme insecurity in the individuals, and the necessity to produce the unusual or else face extinction".[151]

In the Summer of 1938, when Ulam and von Neumann were there, the pressure on the area was real. For several months, Hitler had been dangling before Hungary the promise of the return of Subcarpathian Ruthenia and Slovakia, should the Hungarians cooperate with his plans for the rest of Czechoslovakia. Hungary held back, keen to involve Great Britain along with the 3rd Reich and Italy in settling these East Central European disputes.

150 *Ibid.*.
151 *Ibid*, p. 114.

Exodus

The reason why von Neumann was gravitating around Budapest that year was that his wife-to-be, Klára “Klari” Dán, had left her second husband that February and was now sitting out the 6-month waiting period before divorce proceedings could begin. By late Summer, she had sent von Neumann away from Budapest, claiming that his meddling in these matters only made things worse. Her letters reveal her to have been the ironic and intelligent, but also demanding, person remembered by Ulam years later:

> “I happen to want a man and not a sissy!! A man who knows what he wants, who doesn’t whimper at each little thing, who gets things done, who can behave with people even he does not care for and, especially, I want a man who gives the reassuring feeling that I can depend on him not only in big *evènements* but in minor crises as well
>
> ... This is a desperate cry, please help me, help me to believe in you again, to find the man I was fighting for, the man I thought you to be!!... Darling, take a good advice: Don’t take me for granted!!!”[152]

For a month from late August 1938, such sentiments were typical. The tension surrounding the divorce, exacerbated, it appears, by von Neumann’s manner, left Klari sounding quite desperate at times. Her almost daily letters, many of which were written from the finest hotels and spa resorts in Lucerne, Venice and Montecatini, are pervaded by signs of depression and even hints at suicide. But, soon, political anxieties began to dominate. By late August, Klari could write von Neumann that it had been decided that her sister, Böske, and children should absolutely leave the country, in a matter of days:

> “I don’t know what fate will bring us, things look very dangerous at present and maybe in a few days we shall have such worries we won’t have time to think of this (sic) [divorce] questions anymore. I wish it were not like that, but our wishes don’t seeme (sic) to count for much in this mad world. But should anything serious happen it’s me who is telling you now please don’t get panicy (sic), stay where you are or go to a place where you are safe, I can look after myself and you can’t help me. I always keep a cool head in real danger, but the knowledge that you are running around somewhere to find me would make me jumpy”.[153]

To add to the tensions von Neumann’s ex-wife Mariette and their daughter, Marina, were also present in Budapest, a restricted community in which everyone knew everyone else. Thus, at one point, Klari would berate von Neumann for discussing *their* personal affairs with *his* ex-wife – all of which had been reported to her by *his* mother, Gittus, who, of course, idolized her grandaughter from the first marriage.

Isolated in various hotel rooms, Klari seems to have had few friends in whom she could confide, and she soon began to write about wishing to see the “Fellner’s”. This was Vilmos “Willy” Fellner and his wife Valerie “Vally” Koralek. Like the von Neumann’s and the Dán’s, the Fellner’s were a

[152] Klári (Grand Hotel National, Lucerne) to von Neumann, August 16, 1938, VNLC, Box 1, Folder 7.
[153] Klári (Grand Hotel & La Pace, Montecatini Terme) to vN, Aug. 28, 1938, VNLC, Box 1, Folder 7.

prominent assimilated family, their fortune going back to the 1860's and the beginning of the liberal period during which Hungarian Jewry flourished. Von Neumann and Fellner had attended the same *gymnasium* in Budapest and were students at the same time in chemical engineering at Zurich. Fellner would later say that it was von Neumann and a mutual friend, another Hungarian named Imré Revesz, (later Emery Reves, and confidante of Churchill), who were responsible for sparking his interest in economics. Switching to that field, and accompanied by von Neumann, Fellner transferred to Berlin, where he completed a PhD in economics in 1929. After Berlin, he returned to Budapest, where he was involved in running the family manufacturing business (sugar, alcohol and paper). Like a number of cultivated non-academics, such as Schlesinger and Kaufmann in Vienna, Fellner pursued an active interest in economics, although without publishing anything of note during that period. He and his wife visited the U.S. in 1928 and 1934.

On September 10th, having written the previous day about Budapest being in a "frantic state" with the tension "getting worse every day", Klari met the Fellners.[154] That evening, throughout a film and then a late-night circus cabaret, complete with animals and trapeze artists, Klari and Willy Fellner talked politics till three in the morning. "[E]ven a huge snake fully alive", she wrote to von Neumann, "could not disturb our happy projecting of who is now going to be killed. Well I suppose this is what happens if two full-blooded pessimists meet. Poor Vally again tried to persuade us to watch the show or at least not to use certain names too often as the place was terribly crowded and people seemed rather interested in our opinion".

> "Dearest I'm afraid that we never had more reason to be pessimistic as just now, we [have] got to be ready for the worst happening any minute. I don't see any way to stop it anymore. Most people try to persuade themselves that it won't happen, but this of course is just the instinct of self-preservation...I'm so worried that I don't talk of this (sic) matters with my family anymore. I don't want to know them (sic) how terribly scared I am. I don't know what's awaiting us in the future, but never as long I may live will I forget 1938.
>
> Darling I'm sorry this letter is turning gloomy again, but what is there to do if I can't think of nothing (sic) else. Everything one does seems terribly futile as one does not know what horrors tomorrow will bring. – I'm eager to know what your next plans are... If you go to England I should very much like an objective report from you whether the Jewish question is really getting so bad there as I heard" .[155]

Their petty bickering was now beginning to look silly, Klári admitted. They should really save their strength and nerves for the times ahead. Yet she continued to alternate between chiding him for screaming over the telephone and reassuring him that she was calm. "Johnny dear you must understand this. I shall never ask you to come back... I shall never ask you to risk your life or anything for my sake. But don't expect me to ask you to come back in this dangerous corner [of the world]. I refuse to take over this responsibility. We had a long argument with Will [Fellner] over this question. He thinks that it might be easier for me to get out when you are here. I don't think so, unless we are already married, but that's still a long way of (sic)... I intentionally don't speak of

[154] Klári to vN, Sept. 9, 1938, VNLC, Box 1, Folder 7.
[155] Klari to vN, Sept. 11, 1938, VNLC, Box 1, Folder 7.

politics, it's no use as it changes every second. Father keeps a wonderfully cold (sic) head and we all try to be cheerful for his sake. But I do believe that this is the age of heroes".[156]

Over the telephone from Sweden, von Neumann deliberately appeared optimistic, in an attempt to boost her morale. But in his letters to Veblen, he was less sanguine: "I agree with you, that war at this moment is improbable, since neither side seems to want it just now – but the Sudeten-german (sic)- population seems to be very nearly out of control, so you can never tell. It also seems, as if Messrs. H[itler]. and M[ussolini]. were a little more emotional lately than rational, so you really cannot tell. So we may be much nearer liquidation than it seemed 2 weeks ago. God knows what will happen...".[157] He was on his way back to Copenhagen, he said, where he wanted to see Bohr and talk especially about the latter's ideas on biology. Yet, the following day, he wrote Veblen that his wandering around Scandinavia was beginning to seem futile, that he felt like getting back to Budapest straightaway. "The trouble is, that I don't see what I can do when I get there: My brother cannot leave now, my mother won't leave without him, and Klári cannot leave either during the next weeks. I hope that things won't be quite as bad when you get my letter, as they look now".[158] By late September, Klári was warning von Neumann against discussing politics over the telephone, and the Fellners had announced to her "their future plans".

Back in Budapest in early October, von Neumann could write to Veblen that the Munich Non-Aggression Treaty between Chamberlain and Hitler had provided welcome breathing space. Following that agreement, Imrédy had visited an aggressive Hitler at Berchtesgaden, with no satisfactory conclusion as regards the Czech territories. A month later, however, Hungary's claims were submitted to German-Italian arbitration. This resulted in the First Vienna Award, made official in November, which granted the *Felvidék* in southern Czechoslovakia to Hungary. Imrédy sought re-election that month and formed a new government. All of this provided respite for von Neumann. Married in late November, he and Klari sailed away a fortnight later. The Fellner's had already left for the States. But like the vast majority of Hungarian Jews, those "Magyars of the Israelite faith", von Neumann's and Klari's families clung to Hungary, soothed for the meantime by the Munich outcome.

On human motivations

From Princeton, the von Neumann's watched events unfold in Hungary. If the retrieval of the *Felvidék* had been welcomed by all Hungarians, for whom Trianon had been a injustice, it also brought with it a population of 1 million, including several orthdox Jewish centres, and, into Imrédy's government, an anti-semitic minister Andor Jaross. Thus emerged that contradictory feature of Hungarian politics during this period: territory was regained, satisfying a need shared by all Hungarians, but, with it, came pockets of orthodox Jews, the effect of which was to inflame anti-semitism. By December, Imrédy was promoting a second anti-Jewish bill "Concerning the Restriction of the Participation of the Jews in Public and Economic Life". Then, in a strange twist, Imrédy himself was unable to refute an accusation by the radical Right that there was Jewish blood in *his* own ancestry, which compelled him to resign in February 1939. Horthy swore in Pál Teleki a

156 Klari to vN, Sept. 14, 1936, VNLC, Box 1, Folder 7.

157 VN (Grand Hotel, Lund) to Veblen, Sept. 15, 1938, VLC, Box 15, Folder 1 ?

158 VN (Gand Hotel, Lund) to Veblen, Sept. 16, 1938, VLC, Box 15, Folder 1 ?

few days later. A renowned academic and cartographer, the aristocratic Teleki was tolerant of the Magyarized Jews, but less so of the "Ostjuden". This became more topical an issue with Hungary's acquisition of Subcarpathian Ruthenia in March 1939, which brought with it a substantial Jewish Orthodox population, whose urban politicised intellectuals were left-leaning.[159] This stimulated the parliamentary debates on the second Jewish law, which took place in the first half of 1939. Compared to the law of the previous year, the new bill was more "Nazi" in content, referring not only to the Jewish threat to economy and culture but also to the racial, psychological and spiritual difference of the Jews. Anxiety grew in Budapest. On New Year's day, 1939, Ortvay could write to von Neumann in Princeton that Leo Libermann, an opthalmologist and university professor known by both of them, had just committed suicide. "In the state of the world, one cannot find great joy, I see it as slipping downward… ".[160]

If Ortvay persisted in his psychological probing throughout this time, von Neumann remained at the surface: Ortvay searching for psychological depth, von Neumann resisting it – not entirely unlike the situation with Borel discussed earlier. Yet, at the same time, von Neumann was ready to speak of what he called the pathology of the general situation. It was difficult to write about politics, he admitted, and especially difficult to be sure that his diagnosis was not simply the expression of his own desires - "*Wunschbestimmt*" – but he felt reasonably objective about the matter: the war was inevitable, he said, and the arguments that it was not necessary, or that it would not resolve the problems, were beside the point. "The whole affair", he wrote, "is a pathological process and, viewed clinically, is a plausible stage of further development. It is 'necessary' even emotionally - if it is permissible to use the word 'necessary' in this connection. It will bring the acute problems to a resolution insofar as it will diminish the moral and intellectual weight of the European continent and its vicinity, which, considering the world's structure, is justifed. May God grant that I am mistaken".[161]

In von Neumann's letters, the emphasis shifted subtly from the inevitability of catastrophe to the question of what would follow it. Apologising to Ortvay for not delving into the mathematics of the "spirit", i.e., emotions and attitudes, he dwelt persistently on politics. Steadily, the vocabulary of structure and equilibrium began to infiltrate his prose. He agreed with Ortvay on the inevitability, and futility, of general war.[162] Point by point, he went through the issues. It was naive to hope that any outcome would be useful to the Jewry stranded in Europe. One possibility was an outcome similar to "the Turkish-Armenian affair during the World War" - the genocide of Armenians by the Turkish government - to which Hitler had referred in a recent speech, an outcome which, von Neumann said, was "superfluous to analyse".[163] Even if this did not occur, he said, in

[159] On the Jews of Subcarpathian Ruthenia, see Livia Rothkirchen, "Deep-Rooted Yet Alien: Some Aspects of the History of the Jews in Subcarpathian Ruthenia", *Yad Vashem Studies*, 1978, *12*: 147-191.

[160] Ortvay to von Neumann, Jan. 1, 1939.

[161] Von Neumann to Ortvay, Jan. 26, 1939.

[162] Von Neumann to Ortvay, Feb. 26, 1939

[163] The Armenian catastrophe of 1915, in which the Turks razed Armenian villages and walked over a million people to their deaths in the eastern deserts, occupied an important place in the imagination of many German and Central European intellectuals during the interwar period. This was largely thanks to Franz Werfel's interwar novel, *The Forty Days of Musa-Dagh*, trans. from German by Geoffrey Dunlop (New York: Viking Press, 1934, [1933c]), which described the affair. In *Principles of Topological Psychology* (New York: McGraw-Hill, 1966, [1936c]) a 1936 book

the vanquished countries there would be social chaos and lasting social division between the various sides, making it impossible that a "state of equilibrium could take place". A victory to the Western powers, he said, would be in many respects Pyrrhic, with the rapprochement of the dissatisfied, in the form of a German-Russian coalition, posing a future threat at least as worrying as the present one. The position of the Western powers vis-à-vis their allies and dominions outside Europe would be at least as weak as after 1914-1918. Economically, the U.S. stood to gain little from a war. The wartime boom would be only temporary, with debts incurred never being repaid and the American social structure dangerously loosened. Speculating on the possibility of American imperialism emerging in the event of their victory in war, von Neumann felt that this would be possible "only if the war liquidates Japan too", which wasn't completely out of the question. Although he felt that there was currently little popular support for such a development in the U.S.: it was quite foreign to the ordinary American, he said, and the terminology and symbols of politicians and big business suggested that they looked in other directions to satisfy their ambitions. It all depended on what happened to the British, as the Great War had shown. American support for British power in order to maintain world stability was "a very negative motivation, the avoidance of damage rather than achieving a gain". Even the Roman Empire became imperialistic only in the 2nd century A.D., said von Neumann, when it agreed that that the permanent annexation of the Balkans was 'unavoidable' from the viewpoint of their security. The U.S. might go in such a direction, but, for the moment, all instincts were isolationist. The war, he agreed with Ortvay, would indeed be a terrible cultural loss in Europe - indeed, such a loss was already being incurred - but neither should one exaggerate: when the Romans took over Greek culture, the ancient civilization remained essentially intact for another 300 years.

> "After all this", he concluded, "I believe the war is plausible in spite of all, and with the relatively early participation of the U.S.A. Because it is a pathological procedure, which does not take place because anyone considered it intelligently, that it is in his interest, but because certain abnormal spiritual tensions - which no doubt exist today in the world - search for 'resolution' in this direction. And because from a rational point of view, England and France cannot let one another perish, nor can the U.S.A. let England. Truly I could only hope that Southeast Europe will be left out of it, partly because the possibilities there are very murky, partly because one always hopes for miracles".[164]

During this time, von Neumann periodically apologized for writing so much about what he called the themes of "war and peace", but persisted in it nonetheless. The pessimistic diagnosis, he said, on which they now seemed to agree, was much closer to reality than had been the illusions of last October. He found himself wondering increasingly about the "future of white civilization – or rather that of the civilization of natural science and industry, and the role of Europe and America. Do you find it impossible that here the Greek-Roman analogy would be applicable?". The last 50 years in Europe, he said, were quite similar to the history of the war of the Peloponnese and the period

that Ortvay was reading at the time of his correspondence with von Neumann, Berlin social psychologist Kurt Lewin draws on Werfel's account of the Armenians under siege.

164 Von Neumann to Ortvay, Feb. 26, 1939

immediately after it. Even after Romans took over Greek civilization, he said, Greek lifestyle was prolonged, and even improved, for another 350 years.[165]

Replying at length, Ortvay found the analogy with Greece to be appropriate, but doubted that any unambiguous conclusions could be drawn from it. Western Europe was in decay, as evidenced by its excesses of capitalism and mechanisation, its shallow rationalism, "which consists in the fact that a few easily comprehensible viewpoints are fulfilled to the extreme", and its "excessive cult of the will", which conferred power upon "a very aggressive, half-cultured mass". America, although hampered by the absence of an aristocracy, still showed signs of cultural health and force, and thus bore a responsibility for regeneration – indeed, for the future of humanity. If only a minority there, he implored von Neumann, could substitute for the absent aristocratic class, and set an example for the rest of the population. He realised how non-modern his thinking was – it was, he admitted, as if he were living in Herder's time...

He then clarified his earlier suggestions on modelling the brain: that it must be considered as a complete system, to which individual spiritual processes must be subordinated, just as the quanta were ordered to the system's vibrations. It would be necessary to use general coordinates and consider the discrete states of the brain – which was nothing other than the transfer of the basic ideas in quantum mechanics to the organism. The acceptance of discrete elements had shown itself to be important in the study of heredity, and it was thought today that genes were molecules that changed their structure, jump-like, at a mutation. Although the knowledge of the physical system of the brain was very imperfect, perhaps someone who was knowledgeable in mathematics and physics could construct an independent axiomatics of such a system. This had come to his mind, he told von Neumann, as he leafed through his paper on games. "Approximately that which mathematics gave to a great extent concerning the natural, perceptible materials . . . must be accomplished in the field of spiritual acts, social relations".[166] One approach to this was the question of parallelism, he said, but it was not the only one. Perhaps it was the case that sensations were indeed located in the brain, but spiritual acts such as judgments and the taking of positions were determined independently: "the so-called material world would be but a region of a complete world that includes the spiritual as well... But enough of these very "broad" considerations".[167] He

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165 Von Neumann to Ortvay, March 29, 1939. Von Neumann was well read in the history of antiquity, one of his favourite books being Thucydides' *The Peloponnesian Wars*. In same, he was particularly fond of the Melian dialogues, which are a model of rationalist, *realpolitik* discourse.

166 Ortvay to von Neumann, date?

167 Ortvay to von Neumann, April 10, 1939. Having seen what von Neumann had done on games, an area not generally thought amenable to axiomatic treatment, he wondered whether something similar could be accomplished for the operation of the brain, just as he had wondered earlier whether von Neumann's work on continuous geometries could have applications to modelling "spiritual" states. It is not clear whether the game paper in question is von Neumann "Zue Theorie..." (1928) or the unpublished paper on poker in Hungarian, referred to above by Merrill Flood. Regardless, it was not until four years later, when stimulated by McCulloch and Pitts's 1943 "A Logical Calculus of the Ideas Immanent in Nervous Activity", *Bulletin of Mathematical Biophysics*, *5*:115--133, that von Neumann took up Ortvay's suggestions and seriously turn to the operation of the brain (see Aspray, *John von Neumann* (cit. n.127), p. 180). He discussed this in his Hixon Lectures of 1948 and in his 1950 talk to the Cybernetics Group in Atlantic City, which repeats verbatim many of the lines of argument raised by Ortvay a decade previously. See Nagy et al 1989, (cit. n.128).

closed, poignantly, by noting that he was learning English in case conditions ever allowed him to venture out again into the wide world, but that that was unlikely.[168]

In May, 1939, Ortvay turned to several books on set theory and logic, by Fraenkel, Heyting, Carnap and others. In Carnap's *The Logical Syntax of Language*[169] and *The Logical Structure of the World*,[170] he had attempted to analyse the logical structure of language and to construct axiomatics of other branches of knowledge. Ortvay felt that an examination of the logic of living languages would lead to the establishment of interesting types of structures and eventually to a "fertile interaction of the mathematical and humanities' spheres of thoughts". On the other hand, he found Carnap's sketches of the axiomatics of the "sensory realms" to be extremely primitive. Much more empirical research was necessary before the field "becomes ripe for serious axiomatics". The excessive emphasis on sensations did not conform to today's psychology, and physics had shown that precipitate reductionism only led to insipidity. "Much more important is the elaboration of characteristic structures and perhaps their provisional axiomatisation as well".[171] He felt that Heidegger, in *Being and Time*, had shown an intuitive grasp of many characteristic structures but was unable to formulate them at all exactly. Neither did he agree with Carnap's aim of replacing the summary statements of all fields with sensation statements. The question was far from being settled, he felt.

The mention of Carnap raised von Neumann hackles. Yes, he confirmed, Gödel's results meant that there wasn't a complete axiom system, even in mathematics. But, from the mathematical perspective, Carnap's insights were very feeble and naive. He simply did not have, said von Neumann, the objective knowledge minimally required to say something in this area. He may well have persuaded the school philosophers of the value of the philosophy of science, but that was about the extent of it. On the completeness of mathematical axiomatics, for example, he expressed "completely naive, simplistic views with a terribly important 'air'". If the affair were as simple as Carnap imagined it to be, he said, then "there would be no need for fundamental mathematical research - at least from a mathematical point of view!". It was a pity that we had to rely on such a "turbid source", said von Neumann, to learn about such solid topics! He was especially annoyed that Carnap, although always with Gödel's name on the tip of his tongue, "obviously has absolutely no understanding of the meaning of Gödel's results".[172] Did Ortvay really believe that Carnap was saying something new about the structure of language? That what he was doing would be useful in the preparation of "ultimately serious efforts"? In the same breath, von Neumann then turned to a book that he had read previously, by Viennese mathematician Karl Menger: "A few years ago, Menger wrote an axiomatic treatment of the field of human relations (ethics?)". This was Menger's *Moral, Wille und Gestaltung* (*Morality, Decision and Social Organisation*), a book that, quite

168 In his next letter, Ortvay wished that he had somebody to talk to about the recent work in mathematical logic by Gödel, Church and others, as these were areas in which he was a complete layman. He missed having Kalmár in Budapest close by to talk to. (Ortvay to von Neumann, May 3, 1939). This was Laszlo Kalmár, of course, the Budapest colleague who had earlier worked on the mathematics of chess.

169 Rudolf Carnap, *Logische Syntax der Sprache*, 1934, trans. as *The Logical Syntax of Language* (New York: Humanities Carnap, 1937).

170 Rudolf Carnap, *Der Logische Aufbau der Welt* (Leipzig: Felix Meiner Verlag: 1928), trans. as *The Logical Structure of the World: Pseudoproblems in Philosophy* (Berkeley: Univ. California Press, 1967).

171 Ortvay to von Neumann, May 28, 1939.

172 Von Neumann to Ortvay, July 18, 1939.

unbeknownst to von Neumann, had exerted a significant influence on Oskar Morgenstern in Vienna.[173] Von Neumann didn't think much of it: "I found it to be completely 'flat' and say little. It is my feeling that most promising today would be the discussion of some specific psychological fields and maybe that of economics...".[174] Turning to politics, it was clear that, in the present state of transition, no further concessions should be made by the Western powers. Perhaps Chamberlain would do so, but he was no longer in a position to shape his own politics. The experience of the last few months had shown that concessions only evoked further demands.

For over a year, at that point, emboldened by Hitler's advances, Hungary's German-speaking Swabians had been growing vocal in their demands for increased economic and cultural autonomy. By 1939, their organisation, the *Volksdeutsche*, had become an important political presence, providing a direct link with the 3rd Reich. As previously with Czechoslovakia, Germany wanted to have Hungary's support for its designs on Poland. Hungary resisted, Poland being an old ally, but it was also keen to placate Germany, whose support it would need in its own claims on Transylvania, which it wanted to retrieve from Rumania.

The first anti-Jewish law, that of 1938, had met with the surprise, but not opposition, of the Jewish population. To most of them, the idea that Hungary might be turning against them was unthinkable. Thus, when English and French Jewish organisations sought to intervene, they were shunned by the Hungarian Jews themselves, and told that it was an internal matter, with no need for outside interference. When the second anti-Jewish bill was put forward, the realization began to dawn, and there was protest. The Hungarian Jews proclaimed their patriotism, pointing to their sacrifices during the Great War, to their contribution to the economic, cultural and scientific life of the country. This time, they turned to the British Jews for assistance, while Hungary's ecclesiastical leaders spoke in favour of the reforms. In February 1939, Szalassi's followers launched a grenade attack on Jews leaving Budapest's Dohàny St. Synagogue. In May, the second law was enacted. It prohibited Jews from obtaining citizenship (something aimed at recent refugees and those residents in the recently acquired territories), ordered retirement of all Jewish court and prosecution staff by 1940, and primary and secondary teachers by 1943. Reintroducing the 1920 *Numerus Clausus* of 6% limit on admission to universities, it also prohibited Jews as editors or publishers of periodicals or producers or directors of plays or films. Licenses held by Jews for various kinds of businesses were to be withdrawn. Firms of 5 employees or less could have 1 Jew, while those of 9 employees or more could have 2.[175]

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173 Written amidst the civil unrest in Vienna in 1934, Menger's book, *Moral, Wille und Weltgestaltung: Grundlegung zur Logik der Sitten* (Vienna: Julius Springer, 1934), trans. as *Morality, Decision and Social Organisation. Towards a logic of ethics* (Dordrecht: D. Reidel, 1974), was a formal, mathematical exploration of ethical norms and their effect on social stability. Von Neumann appears to have read it when on holiday with Veblen in Florida at Christmas 1934. On Menger, see Robert Leonard, "From Parlor Games to Social Science: von Neumann, Morgenstern and the creation of game theory, 1925-1944", *Journal of Economic Literature*, 1995, *XXXIII*: 730-61 and Robert Leonard, "Ethics and the Excluded Middle: Karl Menger and social science in interwar Vienna", *Isis*, 1998, *89*:pp. 1-26.

174 Von Neumann to Ortvay, July 18 1939.

175 On the economic impact of the Jewish laws on the Hungarian economy, see Kádár and Vági. Gábor Kádár and Zoltán Vági "Rationality or Irrationality? The Annihilation of Hungarian Jews", *The Hungarian Quarterly*, 2004, *XLV*: ?; Gábor Kádár and Zoltán Vági, *Self-Financing Genocide: The Gold Train, the Becher Case and the Wealth of Hungarian Jews* (Budapest & New York: Central European University Press, 2004).

Jewish historian, Ralph Patai, has written about the psychological effect of the laws of 1938 and 1939.

> "It was on Jews whose mentality was formed and informed by such convictions that the blows of the Jewish Laws fell, and the effects were devastating. Even if the laws did not immediately endanger their lives . . . the psychological unpreparedness for being legally cast on the dustheap rendered these laws more hurtful than the attacks on life and limb that had been perpetrated say, in the 1880's after Tiszaeszlár. For one thing, the new situation demanded a total rethinking of their own position in Hungary, something of which most Hungarians Jews were simply incapable"[176]

Patai goes on to describe how that attachment to Hungary left many Jews somewhat paralysed. Many of them shunned Zionism, regarding themselves as patriotic Hungarians, so that even though the 1939 law made express provision for the emigration of Jews from Hungary – subject, of course, to financial restrictions - relatively few resorted to it.

Von Neumann's mother and brothers, and his in-laws, the Dán family, were among those reluctant to leave. Thus, that summer, in July 1939, Klari returned to Budapest from Princeton to try to persuade them to do so. While she was gone, Ulam and von Neumann slipped away for a few days to visit Veblen at his Summer home in Maine. On the way, they "discussed some mathematics as usual, but mostly talked about what was going to happen in Europe. We were both nervous and worried; we examined all possible courses which a war could take, how it could start, when".[177] The next month, Hitler overran Poland. Ulam recalls Polish topologist Witold Hurewicz phoning him to describe the start of war. With his father, sister and many other relatives still in Poland, Ulam felt as if a curtain had fallen on his past life, cutting it off from his future: "This was the period of my life when I was perhaps in the worst state, mentally, nervously, and materially. My world had collapsed . . . There was a terrible anxiety about the fate of all those whom we had left behind – family and friends".[178] That August, after some delays, von Neumann's mother and brother arrived in New York. After further delays, the Dán family, too, left Budapest for the States. Ulam's family did not escape from Poland.

With the second Hungarian law and the invasion of Poland, Ortvay's anguish, too, deepened. "In spite of everything, what we feared for years and sometimes hoped wouldn't take place has finally happened...".[179] His letters from Budapest now ran to several pages, ranging on subjects from axiomatics to God to Freud. He hoped that the European nations would wake up before European culture collapsed entirely. A desirable solution would see, not one side crushed by the other, but an entente, where each recognised the other's virtues, their right to exist, as well as their faults and sins. He spoke of Freud's death in London that year. He had been interested in Freudianism since its inception and had been in contact with several of Freud's followers, he said, with sometimes unpleasant experiences. Freud, he said, had provided the first systematic exploration of the psychology of the subconscious and of repression. Ortvay acknowledged the importance of

[176] Patai, *The Jews of Hungary* (cit. n.8), p. 541.

[177] Ulam, *Adventures* (cit. n.12), p. 115.

[178] *Ibid*, p. 118.

[179] Ortvay to von Neumann, Sept. 26, 1939.

sexuality, but not to the extent suggested by Freud. Drives such as aggression, the will to power, revenge and envy were important, as, in a few people, were higher spiritual emotions. The Freudian view was quite unbalanced, he said, and its success lay in its being drilled into his followers. Through effective propaganda, it took on a political or religious dimension.[180] Yet, looking at the war and the events leading to it, Ortvay felt that he could not deny the great importance of repression and of sharply distinguishing between the causes superficially believed to be important and the underlying mechanisms:

> "I believe that these are economic forces only to a very slight degree; rather they are enormously primitive and brutal passions, and the 'economic' reasons are in many cases only suitable for the purpose of letting modern man hide the real reasons from himself... Nietzsche already saw a great deal here. It is the scientific treatment of the whole sphere of thought that is the mission of the future".[181]

These discussions of rationality and passion, of politics, social structure and equilibrium, saw von Neumann return to the mathematics of games. In November, he was planning a visit to the University of Washington, Seattle, at the invitation of Abraham Taub, where he was to spend part of the following Summer semester as visiting professor of mathematics. In a letter to the department, he suggested possible topics for his lectures, including the theory of games: "I wrote a paper on this subject in the Mathematische Annalen 1928, and I have a lot of unpublished material on poker in particular. These lectures would give a general idea of the problem of defining a rational way of playing".[182] A week later, he returned to Ortvay: "Unproductive as it is to meditate upon political problems, it is hard to resist doing so. Maybe from Hungary the meaning of the European, and particularly East-European, situation's elements are clearer. But from here it makes a fairly complicated and confused impression. In particular, it appears in all likelihood that not 2, but 3 or 4, enemies are facing one another".[183] The European situation, it was clear, was not a 2-person game.

In Poland, one of the first steps taken by the Germans in subjugating the population was the suppression of their intellectuals. Thus began the elimination of a large number of Polish

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[180] It is interesting to speculate whether Ortvay had contact with Sandor Ferenczi, Freud's principal interpreter in Hungary. According to von Neumann's brother, Nicholas, Ferenczi was actually a close relative of the von Neumann family, and, like Ortvay and Fejér, a dinner guest at the von Neumann household during John's youth. Psychoanalysis was a frequent topic of dinner-time conversation. Ferenczi was initially one of Freud's closest disciples, accompanying Freud on his 1909 trip to America, with Ernest Jones and Carl Jung, and, unlike most of Freud's intimates, actually undergoing an analysis with him, in 1914 and 1916. By 1920, however, their relationship was strained, with Ferenczi rejecting Freud's authoritarianism and favouring a more equal, and even emotional, relationship between the analyst and analysand. See Nicholas A. Vonneuman, *John von Neumann as seen by his brother* (Meadowbrook, PA: N.A. Vonneuman, 1987) on p. 36. On Ferenzci's life see Arnold W. Rachman, *Sandor Ferenczi: The psychotherapist of tenderness and passion* (New York: Jason Aronson, 1997), For his impact on the neo-Freudians and humanistic psychologists, see Dassie Hoffman, "Sandor Ferenczi and the Humanistic Psychologists", *mimeo*, (Saybrook Graduate School, New York, 2000).

[181] *Ibid*. Ortvay then continues with several further pages, most of which is devoted to speculations about the possible relationship between the general axiomatic approach and the study of philology.

[182] Von Neumann to Prof. Carpenter, Nov. 29, 1939, VNLC, Container 4, File 3, Personal Correspondence 1939-40.

[183] Von Neumann to Ortvay, Dec. 8, 1939.

mathematicians, Ulam's teachers, many of them Jewish, who were shot or sent to labour camps.[184] "Did you hear anything about the Polish mathematicians?", wrote von Neumann to Ortvay, "Especially about Warsaw? Because *no* news arrives here".[185] Then, the next day, he could write to Ulam that he had news. It was a characteristic von Neumann letter, jumping from one topic to the next, in paragraphs separated by little dashes:

> "It was so good to heare [sic] that you have news of your family. By the way: I heared [sic] indirectly from Poland, and I will repeat it, although it may be no news to you, or you may have more detailed information on the same matter – but perhaps it is new:
>
> Zygmunt wrote from Wilno to Lefschetz. He seem to be well and conditions relatively normal, although the unviersity is being replaced by the Lithuanian university, to be transferred to W. from Kaunas. Until about a week before he wrote (in early October, I think), people in W. could communicate by mail with Russian Poland, in particular with Lwów. It seems that all Lwów mathematicians were well at that time, including Saks who had gone there. (Saks' family stayed in Warsaw). No communication was possible, however, with German Poland, especially with Warsaw. The Russians seem to want to Ukrainize the University of Lwów.
>
> What you write about the measure for all subsets of the continuum, sounds very interesting. When will there be more news about this?
>
> To revert to political matters: Unluckily it seems, that Gödel is stuck in Germany, and won't receive a permission to go abroad.
>
> … I'd love to know your conversation with Tomochichi about those Frogs. I offer, in return, to give you my own, entirely uninteresting and probably erroneous views about the world in general, and the psychoanalytic interpretation of research".[186]

Then, a week before Christmas, 1939, disaster struck the von Neumann household in New Jersey: his father-in-law, Károly, or "Charles", Dán, who had been persuaded into reluctant exile that Autumn, committed suicide. The Weyl's and others at Princeton rallied round the von Neumann's in their difficulty. Veblen's secretary at the Institute kept him up with the news:

> "Mrs. von Neumann came to call on me yesterday afternoon! I hope she did not feel under any kind of compulsion… But it seemed to some satisfaction to her to talk. She looked shrunken, but did become natural in talking of general conditions – in England now for instance. She said she has now no courage to try to dissuade her mother from returning as soon as possible to Hungary; that she had insisted on her parents' coming here as the only best course she could then see. Now she questions whether alternative courses might not

[184] For the names of the dozens of murdered mathematicians, see Kazimierz (Casimir) Kuratowski, "A Half Century of Polish Mathematics", *Fundamenta Mathematicae*, 1945, *XXXIII*: v-ix. See also *Annals of the Polish Mathematical Society*, 1945, *XVIII*: i-iv.

[185] Von Neumann to Ortvay, Dec. 8, 1939, VNLC.

[186] Von Neumann to Ulam, Dec. 9, 1939, SUAP.

> have been better. I told her it seems to me we must in such cases rest on the assurance that we did what seemed best at the time (which we should probably do again in the same conditions, with the same experience). Professor Weyl also has been conscious of this special cause of her depression.
>
> She would like, if her mother were willing, to take her away somewhere for a little change; but unless her mother's re-entry permit into Hungary can be extended, she must be back there - January 22 I think was the date. She also would like Professor von Neumann to get at least a few days rest away from Princeton, "even 20 miles away". But she herself apparently needs it as much as anyone. She says she has been closely confined by her father all fall, conscious of his abnormality, trying to help him, and not wanting to expose his condition to other people"[187]

This event would prompt von Neumann to take his wife with him on his visit to the West coast, where she could find solace in the company of the Fellner's. In March, he confirmed with Seattle that he would give three evening lectures on games. They would cover, he said, "The case of chess; The notion of the "best strategy"; Problems in games of three or more players" - in short, precisely the elements of our earlier story. He would leave in May and drive across the country to Washington state.[188]

Then, another long letter from Ortvay, with most space devoted to politics. In mass movements and war, Ortvay again insisted, rational, utilitarian, considerations played only a secondary role: the fundamental reasons were "primitive passions". This conformed to the Freudian mode of thinking, he said, but the passions were different from Freud's. Anything which challenged our self-worth evoked hate, which, in the case of mass movements, was directed towards destroying the object of the animosity. Even in business, where utilitarian considerations were perhaps strongest, a fundamental force was often the suppression of a competitor, who simply could not be tolerated, and not just for reasons of profit. Passions of this kind, he felt, were at the root of the last war, the present one, and the antisemitic movements as well. In this connection, their woman mathematician friend, Rozsá Péter, was in need of help, having lost her position because of the Jewish law. Was there anything von Neumann could do?[189]

It was around this time that Oskar Morgenstern, who had left Vienna for Princeton in 1938, became a significant presence in von Neumann's life. Even during the mid-1930's, when life in Vienna was becoming increasingly uncomfortable, Morgenstern had begun to eye the Institute for Advanced Study, and part of his motivation in settling at the then sleepy Princeton University was the presence of the Institute nearby. This was how he got to know von Neumann and his colleagues.[190]

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[187] December 27, 1939, Mrs Blake to Veblen, VLC, Box 15, Folder 1

[188] Von Neumann to Prof. Carpenter, March 29, 1940, Container 4, File 3, Personal Correspondence 1939-40, VNLC.

[189] Ortvay to von Neumann, March 30, 1940. Ortvay also wrote, somewhat cryptically, about one Barnóthy, who was likely to be denied a university chair in Budapest. Obstacles would arise because Barnóthy had given up his decoration for heroism from the Great War, apparently for reasons relating to his mixed (gentile-Jewish) marriage.

[190] "If I only had a position at the Institute. Perhaps in time something can be done. I am on good terms with Walt Stewart, amongst others. Now also with Weyl, John von Neumann, Lowe etc. But that doesn't mean I will be able to get something", Morgenstern Diary, Oskar Morgenstern Papers, Duke University (hereafter OMDU) Feb. 15, 1939.

But Morgenstern also felt a natural solidarity with these cultivated exiles as they watched the turn of events in Europe. He first met the Neumann couple in early 1940 at the home of Hermann Weyl, at what appears to have been one of their first outings after their recent bereavement.[191] By March 1940, von Neumann had become “Johnny” to Morgenstern and he was showing an interest in the economist’s concerns for the problems of foresight and decision. They drew closer in April, when the relationship between Morgenstern’s concerns and von Neumann’s work on games began to become clear:

> “We spent nearly four hours on discussion. Maxims of behavior (I understand perfectly what it’s about and how difficult this is), about games, and about foundational questions in mathematics, where he especially talked about Gödel and general scientific theory. I have not had such an interesting evening for a long time”.[192]

Von Neumann, in turn, read and praised some of Morgenstern’s earlier work, and while the mathematician headed off to the West Coast, the economist read Richard von Mises’ (1939) *Kleines Lehrbuch des Positivismus*, and regretted not having turned away sooner from the universalism of Othmar Spann and the idealistic philosophy he had encountered in Vienna.

By mid-May, von Neumann and Klari were driving across the U.S., he having collected her in Chicago, where she had taken a brief holiday. He was to lecture in Seattle from mid-June to end-July and they were spending six weeks travelling across the U.S. In addition, they were going to see Willy and Valérie Fellner, the former having managed to secure a lecturing position in Economics at Berkeley when they fled Hungary in 1938.[193]

From a hotel in Winslow, Arizona, von Neumann wrote to Ortvay: “The travel is quite dreary until the middle of Kansas, but from then on the land is incredibly beautiful and varied – I am really ashamed that for 10 years I have always put it off till “next year”. Furthermore, the most beautiful parts, the Grand Canyon, Northern California and Oregon are still ahead of us”.[194] Political concerns continued to loom large. The letters to and from Ortvay continued. Yes, von Neumann reassured him, he would do what he could for Barnóthy:

> “Naturally, from the perspective of bringing Europeans over here, all that can be said is that the bottom has fallen out of the world – I don’t even dare to think what the disintegration of the Scandinavian countries, the Netherlands, Belgium (and tomorrow and the day after, who knows what else?). But even if these – and other evident political possibilities – make even the slightest degree of success doubtful, I will do everything I can. If I know more, I will write to you.

[191] Although the highly cultivated Weyl was a proponent of Brouwer’s Intuitionist philosophy of mathematics, and a Heidegger scholar, on both of which counts he lay far away from von Neumann, he had been his teacher and now, abroad, the émigré couples were close.

[192] OMDU, April 5, 1940.

[193] On Fellner, see James Marshall, “Fellner, William J.” in Sobel, R. and B.S. Katz eds., *Bibliographical Dictionary of the Council of Economic Advisors* (New York, 1988), Gottfried Haberler, “William Fellner In Memoriam”, in W. Fellner, *Essays in Contemporary Economic Problems, Disinflation* (Washington and London, 1984), pp. 1-6 and Irma Adelman, “Fellner, William John”, *Encyclopaedia of the Social Sciences* (?), p. 301.

[194] Von Neumann to Ortvay, May 13, 1940.

> Concerning Sierpinski, I will try to find some connection. Regrettably, I received your letter after my attendance at the April meeting of the Academy in Washington, which was the best place to make propaganda for this. I would be very happy to receive the promised information from Kerékjartó. Do the possibilities you mentioned still exist?
>
> I heard from a Pole who came to Princeton (R. Smoluchowski, a Warsaw physicist, the son of M. Smoluchowski) that Bielobrzeski is alive; a man having the same name was executed, which gave rise to the misunderstanding. Tarski and Zygmund are in America. Apparently Banach became the dean of Lemberg University".[195]

The scientific discussion continued unabated. Von Neumann agreed with Ortvay that theories that were unduly complicated could not be right. For these reasons, he was especially "horrified" by biochemistry. "I cannot accept", he said, "that a theory of prime importance, which describes processes which everybody believes to be elementary, can be right if it is too complicated, i.e, if it describes these elementary processes as being horribly complex and sophisticated ones". But he could not substantiate this with any detailed factual knowledge, and was convinced that "before proceeding any further, we must find new terminology, new formulas (i.e. models) in all these fields". And they should be relatively simple, for a "bad vision (model) can easily suggest a terribly complicated situation whereas later on, with the aid of a smarter "Ansatz", everything is settled simply".[196] He could not rationalize this with any factual knowledge but he felt it intuitively.

One area where von Neumann sought simplicity was in "politics and psychology". Thus, here, although he agreed with much of what Ortvay had written, he could not go with him entirely. On this, it is worth quoting von Neumann at length:

> "I too believe that the psychological variable described by you, where resentment is the primary attitude, and the "egotistic-", "profit motive" only a secondary and (often not even quite plausible) rationalization – is an oft-occurring and important psychological mechanism. But neither is it permissible to forget entirely the other variable either: selfishness, in a wrapping of principles and ethics... In the present conflict, particularly given the antecedents, I would still find it difficult to believe that the enemies of the Germans are moved by mainly by the first mechanism.
>
> Concerning the practical chances, and the future, and what would be desirable... It is difficult to write about this, since the letter will travel for 3-4 weeks, and this time interval is not "negligible". You know that I do not believe 'compromise' to be either desirable or possible. The survival of the German power in any form signifies, among other things, the rapid liquidation of the European "Vielstaatlerei" [federation]. I don't believe that this would be a factor ensuring equilibrium from a small European nation's point of view. If the allies are victorious, then without doubt they will orient Europe to the "Vielstaatlerei". Viewed from afar, this is a retroactive development, but from the viewpoint of small European nations e.g. the Hungarian nation, it is the only chance at all. To speak of a German counterweight against Russia, I believe, is an impractical daydream.

195 *Ibid*.
196 *Ibid.*

> That the war, in the case of the Western Allies also, even if they are victorious, will result in the extension of state power and the impoverishment of today's economically leading classes is very plausible to me too. But I believe that this has to be interpreted as follows:
>
> If, in physics, it can be shown of a procedure that it is accelerated by all disturbances and entirely independent of the disturbance's nature, and clearly accelerated more the greater the disturbance - then it is usual to assume that the procedure leads to a state of equilibrium. This is most likely true in politics as well. Further, in politics, even more complicated is the fact that if such a procedure is carried out by means of a given political movement, then it soon becomes clear that efforts directed towards combatting this movement serve as at least a good mechanism in the same direction...
>
> I don't believe that cultural wealth would be less in a centralised society than in the old, free economy. Although such a thesis could be defended dialectically, its opposite, I believe, could be defended just as well. Empirically, all that is clear is that the transition is harmful, but this, naturally, is no miracle.
>
> Returning to the purely political theme: I don't see how both sides could acknowledge the other's raison d'être: If the German nation's frame of mind, which evolved during the last ten years, does not end with a very obvious cataclysm, then no one else on this earth has a raison d'être".[197]

Did all of this – this insistence on simplicity, whether in scientific theories or in human motivation, and this projection from physical onto political equilibrium - speak to the "*Wunschbestimmt*" that von Neumann had written of previously? Were they projections of his own desires, signs of his hopes for order, beyond the inevitable cataclysm in Europe, in the same spirit as his earlier reminders that Greek civilisation had remained intact long after the Roman conquest?

At the end of May, Ortvay was able to send the slight reassurance that Barnóthy's situation was not so serious because his wife was considered an Aryan under the Jewish laws. However, he still had no chance of a university chair given the mood of the times. Ortvay and another colleague wanted Békésy in the Tangl chair, something that all serious physicists agreed with, but it was difficult because one of Békésy's grandparents' birth certificates, necessary for deciding whether he was acceptable under the laws, was missing. "Your considerations regarding the world situation, as always, interested me greatly, even where I could not agree with you completely..."

> "On the European situation, you will know much more than I do today by the time this letter reaches you. We are now living in the state of greatest expectation, because a chapter of the war is over and it is entering a really decisive phase. Will the offensive against England really begin and will it produce a decision? I believe it is futile to philosophize about this... I simply hope that European relationships will become ordered and stabilized...".[198]

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197 *Ibid.*

198 Ortvay to von Neumann, May 30, 1940. This letter continues with a further several pages devoted to philosophical considerations of nominalism and realism.

On Stable Coalitions

Leaving the Fellner's, von Neumann and Klari travelled northwards to Seattle to join Abe Taub. The lectures given at the University of Washington were attended by Israel Halperin, a young mathematician at Queen's University in Canada. Halperin had completed a PhD in mathematics at Princeton a few years previously, under von Neumann's supervision, and now, in 1940, was returning there for a few months to be close to him.[199] He followed von Neumann out to the West Coast and back, attended the Seattle lectures, which he later remembered as being on minimax theory and poker applications only. It is clear, however, that discussions with Fellner at Berkeley were important to von Neumann as he turned to a full theory of coalitions, creating the concepts of equilibrium and stability. For he was no sooner back on the East Coast than he wrote to Fellner, clearly in the light of earlier conversations, thanking him for reminding him of a paper by Gerhard Tintner, which he was reinterpreting in the light of game theory.[200]

Now back at Princeton, von Neumann plunged into this work on games. One characteristic of his working practice as a mathematician was his need for an interlocutor, even a passive one, as he worked through constructions and proofs. For his work on game theory in late summer, 1940, that person was Halperin. In the morning, he would go to the house on Westcott Rd., where von Neumann would:

[199] See Halperin Interview with Albert Tucker, May 25, 1984, Princeton University, Princeton Mathematics Community in the 1930's, Transcript Number 18 (PMC18). Halperin was von Neumann's only doctoral student. Unaware that Institute professors were under no obligation to supervise theses, he had approached him, and ended up working on continuous geometries. See also Halperin, "The Extraordinary Inspiration" (cit. n.1).

[200] Von Neumann's letter shows that he had begun working out a concept of "solution" to the 3-person game: "I agree with your interpretation, that his cases with A and B give one imputation each, while his C is a set of imputations, which ought to form one 'solution'. But I disagree with the analysis of Tintner and the authors cited by him, respectively. Here's my objection: The 'model' is: a producer of iron ore, 1, a steel-mill owner, 2, and a steel consumer, 3. Obviously, 1 or 2 or 3 alone are improductive. Moreover, 1 plus 2 (without 3) or 1 plus 3 (without 2) or 2 plus 3 (without 1) are also improductive, while 1 plus 2 plus 3 together are productive. Consequently the (1, 2, 3) coalition gives a positive value, let us call it *a*. In this case it is evident that there is only one 'solution' which consists of all the permissible imputations - that is of all the (x_1, x_2, x_3) imputations where $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$ and $x_1 + x_2 + x_3 = a$. You can resume it 'in Hungarian' that "Nichts gewisses wiess man nicht" ["You know nothing if you don't know for sure"] - but that's what inevitably comes from the theory of games. Well, I guess that this 'solution', in spite of its sheer negativity, is not quite incorrect. I don't see namely why 2 (the mill owner) should have precedence over the others or 2 with 1 (the ore mine) over 3 (the consumer). Any of them is equally dependent on any of the others, whatever the construction be. The fact that in certain relations one is always a buyer and another is always a seller must not be essential, at least not at this level of schematisation. If 2 or 1 is in better position than 3 is, it ought to have some other explanation. It can either be that 2 and 1 have such alternative possibilities that 3 is devoid of (for instance 1 or 2 or 1 plus 2 are not totally improductive) or that 1 and 2 are really one 'player' while 3 is a group of many 'players' that do not necessarily cooperate (or if 1 and 2 and 3 are composite groups each, maybe 1 and 2 are smaller and more rigid groups than 3). If the model could articulate these types of shades, we could look for non-trivial 'solutions'. N.B. I should be very grateful if you proposed some model that seems sensible for you. I know all the solutions with 3 players, and I think I also could find the essential solutions in the case of 4". (Von Neumann to Willi Fellner, Aug 15, 1940, von Neumann Papers, National Technical Information Centre and Library, Budapest, original and translation kindly provided by Mr. Ferenc Nagy). In the light of our discussion of politics and the social order, note von Neumann's broaching the possibility of there being extraneous considerations preventing the formation of particular coalitions, as well as the possibilities that one "player" in the model may be a group of players in reality, or that two separate "players" in the model may be, in fact, one player in reality. In his *Competition among the Few* (New York: Alfred Knopf, 1949), Fellner himself emphasized that the stable set was a concept of very wide application.

> "go over ideas or create them, and fill my head full of this stuff for an hour and a half. Then he would tell me to come back the next morning. . . It was my impression that he wasn't just talking about it, he was doing the work, and that the reason he sent me home after each morning was that he wanted to think alone for a while. . . I realized I was right at the beginning of something very hot, but it wasn't the sort of thing I felt comfortable with". [201]

Morgenstern was, obviously, a more active interlocutor. At this point, he was independently pursuing ideas that had grown out of an earlier paper on the difficulties of assuming perfect foresight in economic theory. His thinking is best displayed in a 1940 draft, never published, entitled "Maxims of Behavior", which drew on the book by Karl Menger mentioned above in an attempt to tackle the problem of modelling interaction between economic agents. Unlike Halperin, or Fellner, given his absence, Morgenstern could engage von Neumann on the economic aspects of the new theory.[202]

Figure 9, Morgenstern and Von Neumann, Sea Girt, New Jersey
(courtesy of Mrs. Dorothy Morgenstern-Thomas)

[201] Halperin Interview with Albert Tucker, May 25, 1984, Princeton University, Princeton Mathematics Community in the 1930's, Transcript Number 18 (PMC18). See also Halperin's remark quoted at the beginning of this paper.

[202] See Leonard, "Ethics" (cit. n.173).

By October, von Neumann had produced an unpublished typed draft "Theory of Games I (General Foundations)".[203] Following a presentation of the 2-person, zero-sum case, he turns to presenting the set function v(S) for the *n*-person game. It shows the value available to a coalition, S, who, by complete internal cooperation, play minimax against their complement. Von Neumann conjectures that this set function, v(S), will be sufficient to determine the strategies to be adopted for the entire game by each of the *n* players. He explicitly acknowledges the difficulties facing such a conjecture: first, v(S) has been defined using a game between two coalitions that is "altogether fictitious ... related to the real *n*-person game only by a theoretical construction"; second, v(S) describes what a given coalition, S, can obtain from their opponents, but "fails to describe how the proceeds of the enterprise are to be divided among the partners... This division, the "apportionment", is indeed directly determined by the individual [payoff] functions. . . while v(S) depends on their sum... only".[204]

> "We now study the special case n = 3 for a clue as to what we should mean by a solution to our problem. Assuming a fully normalised game, v(S) is here uniquely determined by … :
>
> $$v(S) = \begin{matrix} 0 \\ -1 \\ 1 \\ 0 \end{matrix} \qquad \text{for } a(S) = \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix}$$
>
> Clearly then the advantageous strategy is for any two players to form a coalition against the third: by this the set will gain, and the third lose, one unit".[205]

Von Neumann then describes how the apportionments between the three players are determined by the above set function.[206] Each member of the "winning coalition" will receive 1/2. Were either of them to insist on more, the other could profitably deflect to form a coalition with the "defeated" player. Also, no player can improve his chances of entering a winning coalition by offering to accept less than 1/2, for the other two players would compete with each other to join him, thereby eroding away the premium offered.

> "So we see: each of the two members of the "winning" coalition gets 1/2 . . . and the formation of any particular one among the three possible "winning" coalitions cannot be brought about by paying "compensations" and the like. Which "winning" coalition is actually formed, will be due to causes entirely outside the limits of our present discussion".[207]

[203] Von Neumann, "Theory of Games I (General Foundations)", OMDU, File John von Neumann, 1940-1948.

[204] *Ibid*, p.5.

[205] *Ibid*, p. 12.

[206] This function would later become the "characteristic function" in the book by von Neumann and Morgenstern, *Theory of Games*, (cit. n.3).

[207] Von Neumann, "Theory of Games I", p.13.

These causes outside the limits of the present discussion were those sociological or other features, not reflected in the rules of the game, that restricted or promoted the formation of particular coalitions. Here were the limits of the theory. It carried the analysis up to the point where such social influences entered the picture and it showed how they mattered, but could say little about where they came from .

Looking at the three possible outcomes, each of two players against the other:

> "(4.b) None of them "can be considered a solution by itself – it is the system of all three and their relationship to each other, which really constitute a solution.
>
> (4.c) The three apportionments possess together, in particular, a certain "stability" to which we have referred so far only very sketchily. It consists in this, that any strategic course, followed by a majority of the players, will ultimately lead to one of them. Or, that no equilibrium can be found outside of these three apportionments.
>
> (4.d) Again it is conspicuous that this "stablity" is only a characteristic of all three apportionments together. Neither one possesses it alone – each one, taken by itself, could be circumvented if a different coalition pattern should spread to the necessary majority of the players.
>
> We will now proceed to search for an exact formulation of the heuristic principlies which lead us to our solution…
>
> A more precise statement of the intuitive "stability" of the above system of three apportionments may be made in this form: If we had any other possible apportionment, then some group of players would be able and willing to exchange it for one of the three already offered, but within the system of given apportionments we cannot find a group of players who find it both desirable and possible to exchange one scheme for another…"[208]

He then shows why any apportionment other than those in the above solution, would be rejected by a coalition of at least two players, and closes with further "heuristic elaborations" on the solution concept just developed.

> "We see that the "defeated" player finds no one who desires spontaneously to become his partner, and he can offer no positive inducement to anyone to join him, certainly none by offering to concede him more than 1/2 of the proceeds of the their future coalition. Indeed, for the reasons outlined previously, any player would be unwise if he considered entering into a coalition in which he is promised to get more than 1/2. If he did, ulterior developments would be likely to exclude him from all coalitions.

[208] *Ibid*, p.14.

So there is no way to overcome the indifference of either of the two possible partners. We stress: there is on their part no positive motive against the suggested change – just the indifference characteristic of certain types of stability".[209]

In that unpublished draft, von Neumann moves on to the case of the general *n*-person game, developing further notation and terminology and extending the solution concept to it. A coalition is effective for a particular valuation (later called an imputation) if, by forming a coalition, members may find it possible to get as much as the valuation offers them. Thus, it becomes possible to speak of a valuation, α, *dominating* another, β, if there exists a non-empty set, S, effective for α, for which $\alpha_i > \beta_i$ for all members of S. Von Neumann discusses the dominance relation, $>$, noting its similarity to the order relation but also its particular curious properties: lack of completeness, intransitivity, the possibility that both $\alpha > \beta$ and $\beta > \alpha$. For the *n*-person game, the solution can be defined as a collection of valuations, ν, such that (i) for, α, $\beta \in \nu$, $\alpha > \beta$ never holds, (i.e., no imputation in the solution is dominated by any other member imputation) and (ii) for every $\alpha' \notin \nu$ there exists an $\alpha \in \nu$ for which $\alpha > \alpha'$ (i.e., every imputation outside the solution is dominated by at least one imputation inside). He proceeds to discuss the properties, in a manner quite different from that done earlier with the 3-person game. He notes that the definition of a solution has not ruled out the existence of a α' where $\alpha' > \alpha$, i.e., the existence of imputation lying outside the solution which dominates at least one of the member-imputations, and therefore would be preferred by some effective coalition. His defence of the definition of solution in the face of such a possibility is most interesting:

> "If the solution ν, i.e., the system of valuations, is "accepted" by the players 1, . . . n, then it must impress upon their minds the idea that only the valuations $\beta \in \nu$ are "sound" ways of apportionment. An $\alpha \notin \nu$ with $\alpha' > \beta$ will, although preferable to β, fail to attract them, because it is "unsound". [For the 3-person game, he refers here to the earlier explanation of why a player will be averse to accepting more that 1/2 in a coalition]. The view on the "unsoundness" of α' may also be supported by the existence of an $\alpha \in \nu$ with $\alpha > \alpha'$. [i.e., the mere presence in the solution of a third imputation that dominates the "dominating" non-member, α', may be sufficient to deter players from seeking α']. All of these arguments are, of course, circular in a sense, and again dependent on the selection of ν as "standard of behaviour", i.e., as a criterion of soundness. But this sort of circularity is not unfamiliar in everyday considerations dealing with "soundness".
>
> If the players have accepted ν as a "standard of behaviour", then it is necessary, in order to maintain their faith in ν, to be able to discredit with the help of ν any valuation not in ν. Indeed for every outside α' ($\notin \nu$) there must exist an $\alpha \in \nu$ with $\alpha > \alpha'$.
>
> . . . The above considerations make it even more clear that only ν in its entirety is a solution and possesses any kind of stability – but none of its elements individually. The circular character stressed [above] makes it also plausible that several solutions ν may exist for the same game – i.e., several stable "standards of behaviour" in the same

209 *Ibid*, p.15.

factual situation. Each one of these would, of course, be stable and consistent in itself, but conflict with all others".[210]

He then devotes several pages to a graphical illustration of the solutions to the 3-person, zero-sum, normalized game, which he uses to illustrate the distinction between proper and improper solutions, the first being a solution set that is finite, the latter being one that is infinite.

> "The example 7.B also indicates one of the major reasons which lead to improper solutions. There one player – it happens to be 2 – is being discriminated against, for no intrinsic reason, i.e., for no reason suggested by the rules of the game itself, which are perfectly symmetrical. Yet a "stable standard of behavior", i.e., a solution ν can be built up on such a principle. This player has a – rather arbitrary – value assigned to him : $\alpha_2 = b_0$ for all valuations $(\alpha_1, \alpha_2, \alpha_3) \in \nu$. He is excluded from the competitory (sic) part of the game, which takes place betwen the other players exclusively -1 and 3.
>
> This discrimination, however, need not be clearly disadvantageous to the player who is affected. It is disadvantageous if $b_0 = -1$. But we can also choose $b_0 > -1$, as long as $b_0 < 1/2$. At any rate, however, it amounts to an arbitrary segregration of one of the players from the general competitive negotiations for coalitions, an arbitrary assignment of a fixed – uncompetitive - value for this player in all valuations of the solution, and all this causes an indefiniteness of apportionment between the other players".

Von Neumann closes by noting that subsequent discussions will show that there may be other causes of improper solutions, all of which "can be interpreted as expressing some arbitrary restriction on the competitive negotiations for coalitions which does nevertheless permit the definition of a "stable standard of behavior"".[211]

The "stable set" is the central solution concept of the *Theory of Games and Economic Behaviour*, with two-thirds of the book devoted to its exploration in games of 3 players and more. That exploration is enormously ramified and complex, given the combinatorial complexity of certain games, but the importance of social norms in determining equilibrium outcomes remains fundamental throughout. To take a simple example, Chapter VI, Section 33, treats the solutions to the 3-person, zero-sum game. One of the solutions, which contains a continuum of possible imputations, is given by $(c, a, -c-a)$, where $-1 \leq a \leq 1-c$. In words, player 1 gets c, as dictated by the prevailing social norm, and the others bargain over the spoils:

> "The interpretation of this solution consists manifestly of this : One of the players (in this case 1) is being discriminated against by the two others (in this case 2, 3). They assign to him the amount which he gets, c. This amount is the same for all imputations of the solution, i.e., of the accepted standard of behavior. The place in society of player 1 is prescribed by the two other players; he is excluded from all negotiations that may lead to coalitions. Such negotations do go on, however, between the two other players: the

[210] *Ibid*, pp. 17-18.
[211] *Ibid*, pp. 26-27.

> distribution of their share, -*c*, depends entirely upon their bargaining abilities. The solution, i.e. the accepted standard of behavior, imposes absolutely no restriction upon the way in which this share is divided between them – expressed by *a*, -*c*-*a*. This is not surprising. Since the excluded player is absolutely 'tabu', the threat of the partner's desertion is removed from each participant of the coalition.".[212]

And the account continues with a discussion of how the value of *c* captures the nuances of different forms of segregation, from the 100 per cent injurious form, $c = -1$, through a continuous family of less and less injurious ones. Von Neumann now had the beginnings of a theory that allowed for simple selfishness, the influence of social norms and the possibility of many different equilibria.

While von Neumann wrote up the *Theory of Games*, issues of arbitrary restriction remained critical in Hungary. If the Téléki government believed that the laws of 1938 and 1939 were satisfactory in restraining Jewish participation, the Germans did not, accusing the Hungarians of not going far enough. Anxious to preserve Hungarian-German relations, the prime minister, Téléki, in November 1940, endorsed the Tripartite Pact signed by Germany, Italy and Japan. He then visited Hitler in Vienna. The latter, at that point, was considering segregating Europe's Jews by sending them to the French colonies, all of which he discussed with Téléki, who apparently agreed that Europe should be free of the Jewish presence.[213] Having aligned itself with the Axis, Hungary was now no longer neutral. Part of its purpose here lay in its revisionist designs to regain territories lost to Yugoslavia after Trianon. By March 1941, however, Hitler had decided to invade Yugoslavia as well as Greece. Téléki conceded on the use of Hungary for passage of German troops through to Yugoslavia. This, in turn, brought a threat of reprisal from Britain. Under the intense pressure, at the beginning of April, Téléki committed suicide. The Germans attacked Yugoslavia, and the Hungarians followed through, annexing their old territories, including the Délvidék, in the Yugoslavian northwest. Téléki was replaced by his foreign minister, László Bárdossy, whose tenure would show the 1938-'39 bid for stability to have been futile, and prove disastrous for the Jews of Hungary.

Conclusion

In his absorbing account of the dialectic between creation and discovery in mathematics, Ulam protégé and M.I.T. combinatorics specialist, Gian-Carlo Rota, describes the field of mathematics as the ultimate escape from reality.

> "All other escapes . . . are ephemeral by comparison. The mathematician's feeling of triumph, as he forces the world to obey the laws his imagination has freely created, feeds on its own success. The world is permanently changed by the workings of his mind and the certainty that his creations will endure renews his confidence as no other pursuit"[214]

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[212] Von Neumann and Morgenstern, *Theory of Games* (cit n.3), p. 288.

[213] Braham, *The Politics,* (cit. n.8), p. 177.

[214] Gian-Carlo Rota, *Indiscrete Thoughts* (Boston: Birkhäuser, 1997), p. 70. In an unwitting moment of poetry, Rota goes on to illustrate the "monstrosity" of the mathematician's view of the world by comparing him to none other than Nabokov's Luzhin, "who eventually sees all life as subordinate to the game of chess". Rózsa Péter, in the wartime book, written in Budapest, in which she commented on von Neumann's power as mathematician, also said of mathematics: "how essentially human it is . . . it bears on it for ever the stamp of man's handiwork".

Against a background of seismic social change, von Neumann took certain emphases – equilibrium; theoretical simplicity; selfishness as a benchmark; the centrality of social norms – and worked them into an analysis of stable coalition-formation. It is difficult not to see in his efforts an element of perhaps subconsious resistance to the conditions of the time; an almost defiant willingness to see order beyond the disorder, equilibrium beyond the confusion, to seek an inevitable return to normality once the present transition, with its "abnormal spiritual tensions", was over.

Of course, von Neumann's approach was neither necessary nor inevitable. His friend, Ortvay, was also a mathematical physicist, yet his reaction to the upheaval of the period was to begin probing the functioning of the mind and the formation of attitudes. He was prepared to emphasize complicated psychological forces involving repression, aggression and revenge. Von Neumann resisted all such Freudian complications, preferring to emphasize the simpler instinct of rational selfishness - yet rational selfishness that existed in a context of extra-rational social norms, "standards of behaviour" – prejudices or privileges that were there today simply because they had been yesterday.

Reading Morgenstern's introduction to the *Theory of Games*, one could be forgiven for believing that the book was written in response to the inadequacies of the neoclassical theory then dominant in economics. But that is at best a half-truth. Certainly, by responding to Morgenstern's misgivings about perfect foresight and economic interaction, von Neumann's minimax theorem signalled the possibility of a new departure in economic theory, and the mathematician was able to provide some economic applications for situations involving one seller and one or two buyers.[215] But the vast bulk of the book was taken up with an abstract theory of the division of economic gains in large games that allows for the influence of factors broadly described as social or ethical. The heuristics of von Neumann's game theory are rooted in his own experience as a Hungarian, Jewish mathematician.

As the *Theory of Games* was being completed, von Neumann became directly involved in war work and the applied mathematics that went with it. It began with his consulting to the Aberdeen Proving Ground on weapons testing and culminated in his involvement in the Los Alamos Project, where he not only worked on the mathematics of detonation but was directly involved in the inner circle that, choosing Hiroshima and Nagasaki, oversaw the "liquidation" of Japan after all. Perhaps he felt that the pathological tensions abroad in the world could be "nudged" towards resolution.[216]

To the very end, he continued to speak about game theory in terms congruent with our account. In 1953, young Princeton mathematician Harold Kuhn wrote to him, asking him about the possibility of testing the stable set solution using the experimental methods then beginning at the RAND Corporation. Von Neumann replied in the negative:

> "I think that nothing smaller than a complete social system will give a reasonable 'empirical' picture [of the stable set solution]. Here, over relatively long periods of time, one can meaningfully assert that the 'system' has not changed, while the positions of various

[215] See von Neumann and Morgenstern, Theory of Games (cit. n.3), Chap. 11, Sections 61 – 64, pp. 555 – 586.

[216] In time, he became noted for his hard line with the U.S.S.R. Ulam felt that von Neumann's stance depended on too formal a game-theoretic view of the world.

> participants within it may have changed many times. This would seem to me to be the analogue of a single solution and an 'exploration' of the imputations that belong to it. After relatively long times, there occur discontinuous changes, 'revolutions' which produce a different 'system'"[217]

Nor should we be surprised that von Neumann was dismissive of John Nash's 1950 proof of the existence of an equilibrium point in a game without cooperation.[218] To von Neumann, Hungarian Jew and product of Central European society, the formation of coalitions was a *sine qua non* in any theory of social organisation. It is easy to understand why the idea of non-cooperation would have appeared artificial to him, elegance of Nash's proof notwithstanding.

The resistance he showed Nash in 1950, von Neumann maintained to the end. At a Princeton conference in 1955, the year he was diagnosed with bone cancer, he defended, against the criticism of Nash himself, the multiplicity of solutions permitted by the stable set: "[T]his result", he said, "was not surprising in view of the correspondingly enormous variety of observed stable social structures; many differing conventions can endure, existing today for no better reason than that they were here yesterday".[219] Within two years, however, von Neumann was gone, and with him the knowledge of what game theory owed to the demise of *Mitteleuropa*.

Coda

Ortvay's letters from Budapest trickled to a halt in 1941. In January, he was sending three separate copies, to be sure that they reached von Neumann in Princeton. He appealed for help in raising funds for the beleagured Mathematical and Physical Society.[220] At the University, said Ortvay, the Barnóthys and Kalmár were still employed, under no threat for the time being, but their future looked bleak. István Rybár, their chairman, was friendly towards them and had tried to secure them employment elsewhere, but prospective employers wanted to hear nothing of it. Ortvay continued to write about the application of mathematics to the realm of "spiritual" states. He was now reading Kurt Lewin's *Principles of Topological Psychology*,[221] which took set theory, topology and Karl Menger's dimension theory and used them to recast psychological situations. "Even these are very intelligent, and I think correct, although mostly trivial". He felt it likely that the areas most ripe for such mathematical treatment were those where we made sharp distinctions, such as music, or

217 Von Neumann to Harold Kuhn, April 14, 1953, Container 24, File: Kuhn, H.W., Von Neumann Papers, Library of Congress.

218 On von Neumann's dismissive attitude towards the Nash Equilibrium, see Robert Leonard, "Reading Cournot, Reading Nash: the creation and stabilization of the Nash equilibrium", *Economic Journal*, May 1994, *104*: 492-511 and Nasar, *A Beautiful Mind* (cit.n.4).

219 Philip Wolfe, "Report of an informal conference on Recent Developments in the Theory of Games", (Logistics Research Project, Department of Mathematics, Princeton University, January 31 – February 1, 1955), p. 25.

220 The Society had originally been established by Loránd Eötvös and Gyula König, with financial support from the Hungarian Academy and from its members. Von Neumann had remained an ordinary member before becoming an honorary one, through Ortvay's intervention, around 1940.

221 Lewin, *Principles* (cit. n.163).

juridical systems.[222] His last letter, in February 1941, was a brief summary of the previous one, written as though he thought von Neumann had not received it the first time.

That year also saw the publication of Lasker's last book. In 1938, he had left Stalin's Russia for New York, where he tried, with increasing difficulty, to make ends meet. He nonetheless found the time to write *The Community of the Future*, a 300-page book about the establishment of a non-competitive community as a way to absorb the unemployed Jews of Europe.[223] It was an attempt to "try the method of the chess master on a political problem ... that of unemployment".[224] A theme pervading the book is that of the parallel between social life and games. Society was like chess played on an "infinite board".[225] Questions of stability and balance, and ethics and power, were central. The study of chess shed light on the analysis of social power, allowing one to observe, for example, that "The alliance of weak powers is enduring but not that of the strong", or "In a state of balance every piece has political authority proportionate to its intrinsic power".[226] The first of these maxims was an application of the concept of balance, which, said Lasker, lay at the root of every compromise. The second determined the authority due to each force when the game was in a state of balance. Were it not respected, there could be no peace. Although a gulf lay between them in 1940, with Lasker struggling in old age and poverty in New York, and von Neumann secure in academia, they shared a common trauma, and Lasker is strikingly similar to von Neumann in his use of the game as a social metaphor with which to come to terms with the times. In chess, as soon as the state of balance is disturbed, "a tension arises which seeks an outlet". Likewise in society,

> "As long as no new needs or aspirations arise, the old established parties maintain their hold. They have laid down the rules of the game which have stood the test of experience... But at a period of distress which all feel, or of injustice... or of great creations, which set novel problems, the written and the unwritten law become the object of criticism backed by ethical force, and the ensuing struggle is apt to lead to changes in the prevailing mode of life".[227]

Perhaps Albert Einstein had not read the book on the non-competitive community when, a year later, in 1942, he wrote the Foreword to the Lasker's biography. Although he expressed admiration for someone he regarded as a true Renaissance man, Einstein felt that game of chess exercised too strong a hold over Lasker's imagination:

> "I am not a chess expert and therefore not in a position to marvel at the force of mind revealed in hist greatest intellectual accomplishment – in the field of chess. I must even confess that

[222] Ortvay to von Neumann, January 29, 1941. It should be possible to achieve in these areas, Ortvay felt, what had been achieved in the science of heredity, where natural selection became something that could be discussed rigorously once the essentials were properly treated. Here he continued to describe what he saw as the appropriate way to model the functioning of the neural system. As Aspray, *John von Neumann* (cit. n.127) points out, von Neumann, did not respond to Ortvay's suggestions at this point, but would do so in 1955, when he read the McCulloch-Pitts, "A Logical Calculus" (cit. n.167) paper.

[223] Emanuel Lasker, *The Community of the Future* (New York: M. J. Bernin, 1941).

[224] *Ibid.,* p.12.

[225] *Ibid.,* p.66.

[226] *Ibid.,* p.138.

[227] *Ibid.,* pp. 140-141.

> the struggle for power and the competitive spirit expressed in the form of an ingenious game have always been repugnant to me... To my mind, there was a tragic note in his personality...
>
> The enormous psychological tension, without which nobody can be a chess master, was so deeply interwoven with chess that he could never entirely rid himself of the spirit of the game, even when he was occupied with philosophic and human problems. At the same time, it seemed to me that chess was more a profession for him than the real goal of his life. His real yearning seems to be directed towards scientific understanding and the beauty inherent only in logical creation, a beauty so enchanting that nobody who has once caught a glimpse of it can ever escape it"[228]

In 1944, by which time von Neumann and Morgenstern's book had appeared, matters had worsened in Budapest. Because of their services to the State, von Neumann's teachers, Fejér and Riesz, were granted special status and each allowed to spend part of the war in the one of the protected houses in Budapest's "little ghetto", around Pozsonyi and Szent István streets.[229] These houses were under the diplomatic protection of various countries, and it was here that the Swedish diplomat Raoul Wallenberg managed to save many Hungarian Jews. Fejér and Riesz appear to have been housed in the hospital of the Swedish Embassy at 14 – 16 Tátra St.: Riesz early in 1944, when the Jews of Szeged and the provinces were being deported; Fejér later in the year. There, although crowded, in terrible conditions, with up to fifteen in a room, they were at least safe from deportation, and they survived the war.

So, too, albeit with greater precarity, did their student, Paul Turán. Looking back on the time many years later, on the eve of his death, Turán remembered working on mathematics. In September 1940, he had been making a living as private tutor in Budapest when he was called to labour camp service. A friend in Shanghai had recently written him about a problem in graph theory: what is the maximum number of edges in a graph with *n* vertices not containing a complete subgraph with *k* vertices? In the camp, Turán was recognized by the commandant, a Hungarian engineer with mathematical training. The commandant took pity on Turán's weak physique and gave him an easy job, directing visitors to piles of wooden logs of different sizes. In this "serene setting", Turán recalled, he was able to work on the extremal problem in his head and solve it:

> "I cannot properly describe my feelings during the next few days. The pleasure of dealing with a quite unusual type of problem, the beauty of it, the gradual nearing of the solution, and finally the complete solution made these days really ecstatic. The feeling of some intellectual freedom and being, to a certain extent, spiritually free of oppression only added to this ecstasy".[230]

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228 Einstein in Hannak, *Emanuel Lasker* (cit. n.32), p.?. Here, Einstein also defends his interpretation of relativity against Lasker's criticism that, since it was impossible to create a true vacuum, it was illegitimate to make assumptions about the constancy of the speed of light in a vacuum. Einstein emphasized the need to make assumptions, without which no science could advance.

229 See Frojimovics et al (cit. n.8), pp. 402-403.

230 Paul Turán, "A Note of Welcome", *Journal of Graph Theory,* 1977, *1*: 7 – 9 on p.8. Questions of the psychological ambiguity and ambivalence of life in confinement are central to the work of Hungarian writer, Imré Kertész, recipient of the Nobel Prize in Literature in 2002. See his *Fateless*, trans. by Christopher C. Wilson and Katharina M. Wilson

And who was the camp commandant in question? One Joszéf Winkler, erstwhile contributor to *KöMäL* and joint winner of the Eotvös Competition eighteen years previously - in 1926, the year with none other than König's question about the knight's move on the infinite chessboard.

In July 1944, by which time the threat of deportation was real, Turán was working in a brick factory near Budapest. There, all the kilns were connected by rail to all the storage yards, but, at the crossings, the moving trucks tended to jump the tracks. He began to work on the graph-theoretic problem of minimizing the number of crossings in a yard with *m* kilns and *n* storage yards. This time, however, his thinking was stifled by fears for his family. By late 1944, there was no work to do, but Turán and other Jews expected to be deported from one day to the next. He began to think about another problem, concerning the maximal size of subgraph in a graph of given size. He conjectured a solution, for which he had no support other than "the symmetry and some dim feeling of beauty; perhaps the ugly reality was what made me believe in the strong connection of beauty and truth. But this unsuccessful fight gave me strength, hence, when it was necessary, I could act properly".[231]

Others were not so sustained. Dénes König's elder brother, the literary scholar, György, took his life after the German occupation of Budapest on March 19. Then, when the *Nyilas* took over on October 16, König himself, the very one who had introduced von Neumann to the mathematics of chess almost twenty years previously, also committed suicide. Under the Arrow Cross gangs of the *Nyilas*, Hungary entered its darkest period, with Jews being tortured and shot, their bodies dumped into the Danube. At one point, in late December, von Neumann's teacher Fejér and the occupants of the Swedish hospital were marched by night to the river's edge by the Arrow Crossers, but saved by the last minute intervention of an army officer.[232] The forced labour, deportation to the camps and the local attacks saw the destruction of Hungarian Jewry, with 600,000 perishing within a few short months. On January 2, 1945, when the Germans were fleeing and the Russians about to enter Budapest, von Neumann's friend, Ortvay, took his own life, apparently fearing revenge by the "liberators".[233] Neither he, König nor others close to its genesis would get to read the *Theory of Games.*

(Evanston, Ill.: Northwestern University Press, 1992), or *Kaddish for a Child not Born*, translated by C. C. Wilson and K. M. Wilson (Evanston, Ill. : Hydra Books, 1997).

231 *Ibid*, p. 9.

232 See Turán, "Fejér Lipót, 1880-1959" (cit. n.19), p. 1205.

233 In the Ulam papers of the American Philosophical Society, in a document describing "Family Memorabilia of Nicholas A. Vonneuman relevant for John von Neumann biography donated to the A.P.S.", September 15, 1994, Nicholas Vonneuman writes that Ortvay had been able to retain his position at the Physics Department of the University of Budapest by virtue of his having "qualified" ancestors". Whether this implies that Ortvay was Jewish is unclear. Dr. László Filep claims that Ortvay was not. Filep also notes that, being unassociated with the Nyilas, Ortvay's fear of the Russians was unnecessary, but it illustrates well the fear and tension abroad in Hungary at this time. In a letter to his brother, Marcel, in Sweden, written in July 1945, Frigyes Riesz wrote of the König and Ortvay suicides, and of Fejér's sufferings during the war. He also said that Szeged mathematician István Lipka (see our photo above) had been fired the previous day from his university position, having been discovered to have joined the Nazi party as early as 1939. (Riesz, F. to M. Riesz, July 18, 1945, Marcel Riesz Papers, Lund, Sweden). I am grateful to Dr. Filep for providing me with a copy of this letter.

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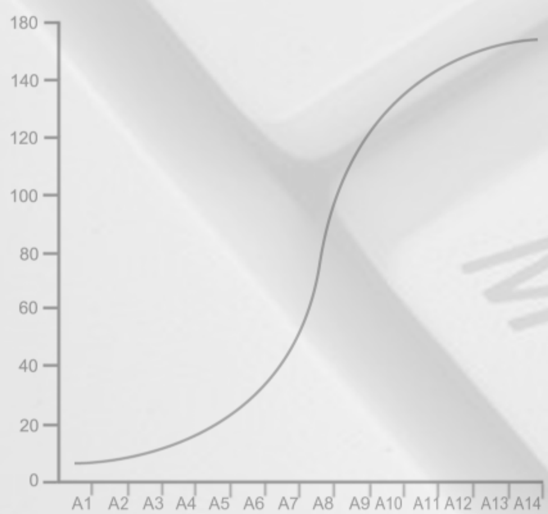

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