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Poverty and Inequality Nexus: Illustrations with Nigerian Data

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Abstract:
The main aim of this paper is to explore the link between poverty and inequality. In developing countries, there is a general consensus that high inequality can dampen significantly the impact of economic performance on poverty. In this paper, we propose a new theoretical framework that links poverty and inequality. We also show between and within group inequalities, as well as inequality in income sources, can contribute to total poverty. The methodology of the paper is illustrated using the 2004 Nigerian national living standard survey.

Keywords: Poverty, Inequality

JEL Classification: D63, D64

1 Introduction

There has been a recent upsurge of interest among both policy makers and researchers in the link between poverty and inequality in their static and dynamic forms. Indeed, understanding the contribution of total inequality or its components to total poverty can help design appropriate economic policies to reduce inequality and poverty. The aim of this paper is to propose a new theoretical framework to establish a link between poverty and inequality. Among other things, we decompose the value of poverty indices into contributions of average income and various inequality components.

The usual main components of inequality that are modelled in the literature are the between-groups, the within-groups and income-sources inequalities. To perform a decomposition of total poverty into contributions of such a set of components, we use the Shapley approach[1]. An application of this approach to decompose distributive indices was recently introduced by Shorrocks (1999). A nice property of such an approach is the additivity of the contribution of components and the exactness of the decomposition, by which the residue due to the interaction between components is attributed to each of the components by means of a linear approximation.

While there is a growing consensus concerning the links between average income, inequality and poverty in a static setting, the dynamic link and its optimal path raise another set of issues. Indeed, this "socially" optimal path can shape the temporal governmental interventions in terms of redistribution or investment in the human capital or in the basic infrastructures. Kuznets (1956) indicates that the link between growth in GDP per capita and inequality should take an inverse U shape during economic development. Empirical studies have tended to show that such a U shape cannot be observed for many countries. To assess the contribution of growth and redistribution to the evolution of poverty, Datt and Ravallion (1992) decompose the observed variation in poverty into growth and redistribution components. A technical improvement to this method was proposed by Kakwani (1997) and Shorrocks (1999). Even if the decomposition proposed in this paper is static, its application in a dynamic setting is straightforward since this decomposition can be used to explain the observed variation in total poverty by variations in the contribution of components.

The plan of this paper is as follows. In the next section, we review the static and dynamic links between poverty and inequality. In the third section, we per-

[1] See the Annex for an outline of the Shapley approach.

form the decomposition of total poverty into components of average income and between and within groups inequalities. In the fourth section, we decompose total poverty into average-income and inequality-in-income-sources components. In section five, we illustrate the methodology using the 2004 Nigerian living standard survey. Finally, some concluding remarks are made in section six.

2 The Link Between Poverty and Inequality

This section reviews the link between poverty and inequality under both static and dynamic settings.

2.1 The static link between poverty and inequality

Under a static setting, the two main components of poverty are the average standard of living and shape of the relative distribution (or inequality). An increase in average income is linked negatively with poverty whereas an increase in inequality increases poverty. The temporal evolution of these two components, that are growth and redistribution components, determines the observed variation in poverty. What can be the link between the evolution of these two components and poverty? We try to answer this question in the following subsection.

2.2 The dynamic link between poverty and inequality

Kuznets (1956) suggests that the link between economic growth, represented by the growth in GDP per capita, and inequality takes an inverted U shape during the development period of a country. This postulate is based on the steps of development that he posited:

- I: The primary sector (agriculture) represents the main part in the structure of the economic activity. This phase is characterized by a quasi uniform distribution of income and a low level of inequality.
- II: The emergence of the secondary sector (industry) with higher level of productivity compared to the primary sector. This implies an increase in between-group inequality as well as in total inequality.
- III: Introduction of new technologies in the primary sector partly eliminates the difference in productivity and incomes. Therefore, total inequality is reduced.

The experience during the last few decades shows that the tertiary sector represents an important part of a country's economy. In the last two decades, many researchers have found little signs of a systematic relationship between growth and inequality [2]. However, the main aim of Kuznets theory on the link between disparities in the productivity of economic sectors and inequality continues to be relevant even if the complete U shape cannot be observed empirically for many countries [3]. The ambiguity concerning the link between growth and inequality can be explained by the lower correlation between them. Inequality is linked to the disparity in the productivity of economic sectors rather than economic growth. This disparity can be higher in economic crisis or economic expansion periods. During recession periods, some sectors are more affected by economic shocks than others. This can explain the increase in inequality in developing countries even if the economic growth rate decreases. During expansion periods however, some economic sectors perform better than others. This boosts economic growth but worsens the income distribution.

Datt and Ravallion (1992) decompose the observed variation in poverty into growth and redistribution. This method was improved by Kakwani (1997) and Shorrocks (1999) to deal with the non attributed residue. While the Datt and Ravallion approach explores how the growth in average income affects total poverty, earlier work on pro-poor growth focuses more on the nature of this impact at different segments of the distribution [4].

2.3 Poverty indices and inequality

As mentioned earlier, average income and the level of inequality are the two factors that determine the level of poverty. When incomes are equally distributed, poverty indices depend on the difference between the poverty line and the average income. Generally, poverty indices can be decomposed as follows:

$$P(y, z) = E_{\mu} + E_{\pi} \tag{1}$$

[2]For this, seeDeininger and Squire (1998), Fields (1989) and Ravallion and Chen (1997).

[3]Deininger and Squire (1998) uses data set of higher quality containing 682 observations on the Gini index for 108 countries. These authors conclude that there exists no support for the Kuznets hypothesis of inverted U-shaped curve. When tested on a country-by-country basis, they found that 90 percent of the countries investigated did not validate the Kuznets hypothesis.

[4]See Ravallion and Datt (2002), Ravallion and Chen (2003), Kakwani, Khandker, and Son (2003) and Son (2004)

where y represents the vector of incomes, z is the poverty line, E_μ is the contribution of average income (μ) with perfect equality and E_π is the contribution of total inequality (π) with the observed average income. Formally, we can re-write the contribution of average income as:

$$E_\mu|_{\pi=0} = \begin{cases} 0, & \text{when} \quad \mu \geq z \\ P(\mu, z), & \text{when} \quad \mu < z. \end{cases} \tag{2}$$

Equation (2) indicates that the quasi perfect equality is not sufficient to eliminate poverty when the average income is very low. In the case when average income is close to the poverty line, any rise in inequality implies a significant increase in poverty. In the other case when average income is relatively high (in developed countries for example), one can observe that the best economic performance periods were accomplished frequently by an increase in inequality. Moreover, this situation can also be Pareto optimal in a dynamic way, where the wellbeing of each household is improved or at the limit, does not worsen[5].

One can note that, even if poverty indices are not sensitive to inequality within the poor group, like the headcount and poverty gap indices, they continue to be sensitive to the inequality between the poor and non poor groups. In addition, one can note that, with the focus axiom that most poverty indices obey, they are not sensitive to the inequality within the non poor group.

2.4 Gini index Lorenz curve and poverty

The Lorenz curve is a useful tool to represent the overall inequality. As shown by Datt and Ravallion (1992), the link between the headcount, noted by H, and the Lorenz curve is:

$$L'(H) = \frac{z}{\mu} \tag{3}$$

The link between the average poverty gap, denoted by $P1$, and inequality represented by the Lorenz curve is:

$$P1 = [z - \mu_p]\, H \tag{4}$$

where μ_p is the average income of the poor group. The link between the severity index, represented by the square of the poverty gaps and the Lorenz curve can be written as:

$$P2 = \int_0^H [z - \mu L'(p)]^2\, dp \tag{5}$$

[5] See Feldstein (1998).

One of the most popular inequality indices is the Gini index. Since the group-income overlap does not exist between the poor and non poor group, the Gini index is easily decomposable across poor and non-poor groups and the residue due to the overlap equals to zero[6]. This decomposition takes the following form:

$$I = \phi_p \psi_p I_p + \phi_{np} \psi_{np} I_{np} + \tilde{I} \tag{6}$$

where I is the Gini index, ϕ_g and ψ_g are the population and income shares for the group g respectively and $\tilde{I}$ is the Gini index where within group inequality is eliminated, i.e., each household has the average income of its group. Based on this, the link between headcount index and the between group inequality is [7]:

$$H = \mu \tilde{I} \left(\frac{1}{\mu - \mu_p} \right) \tag{7}$$

Starting from the last equation, we find that the component between group inequality can be expressed as follows:

$$\tilde{I} = H - L(H) \tag{8}$$

where $L(H)$ is the level of the Lorenz curve when the percentile $p = H$. Thus, the between inequality, measured by the Gini index, equals to the deficit share of the poor group. More this deficit is lower, more is lower the inequality between the poor and the non poor groups. For the poverty gap index, the link can be expressed as [8]:

$$P1 = \mu \tilde{I} \left(\frac{z - \mu_p}{\mu - \mu_p} \right) \tag{9}$$

The link between the Gini index and severity indices of poverty cannot be established directly. This is explained by the different shapes that the distribution can have, with the same level of inequality measured by Gini index.

3 Population Groups, Inequality and Poverty

In this section, we show how the between and the within group inequalities contribute to the total poverty. This is an important investigation as it provides answers to the following questions:

[6]See Lamber and Aronson (1993) and Araar (2006)

[7]See the prove in appendix A.

[8]Recall that $P1 = H(z - \mu_p)$.

- What is the contribution of regional disparities to the total poverty?
- What is the contribution of the within group inequality of a given group (urban area for example) to total poverty?

The form of decomposition, that we propose to answer these questions, takes the following form:

$$P(y,z) = E_{\mu} + E_B + \sum_{g=1}^{G} E_W^g \tag{10}$$

where E_B is the contribution of the between group inequality and E_W^g is the contribution of inequality within the group g.

By the term "marginal contribution" of a component, we refer to the variation in poverty index generated by removing such component. To estimate, at the margin, the contribution of the within group inequality to the total poverty, we compare between poverty index with the observed household incomes and that would occur if the within group inequality is removed. For this, we use a vector of income where each household has the average income of its group, noted by μ_g. Formally, we have that:

$$M_W = P(y) - P(\mu_g) \tag{11}$$

To eliminate the intergroup inequality and to estimate the contribution at the margin of the intragroup inequality to total poverty, we will use a vector of income where each household is given its income scaled by the ratio μ/μ_g. With this new income vector, the average income of each group equals to μ.

$$M_B = P(y) - P(y.\mu/\mu_g) \tag{12}$$

Even if this procedure gives us an idea on the contribution of each factor, this approach overestimate their contributions such that:

$$E_{\pi} < M_W + M_B \tag{13}$$

To avoid this drawback, we use the Shapley approach, by keeping the same rules for eliminating each of the between and within group factors[9].

$$E_{\pi} = E_W + E_B \tag{14}$$

[9]See the appendix B for more details on this approach.

where

$$
\begin{aligned}
E_B &= 0.5\,[P(y) - P(y(\mu/\mu_g)) + P(\mu_g) - P(\mu)] &(15)\\
E_W &= 0.5\,[P(y) - P(\mu_g) + P(y(\mu/\mu_g)) - P(\mu)] &(16)
\end{aligned}
$$

For the additive class of poverty indices, the within group contribution can be easily decomposed across groups as follows:

$$
E_W = \sum_{g=1}^{G} E_W^g \qquad (17)
$$

such that:

$$
E_W^g = 0.5\phi_g\,[P_g(y) - P_g(y(\mu/\mu_g)) + P_g(\mu_g) - P_g(\mu)] \qquad (18)
$$

where ϕ_g is the population share of group g. If poverty index is not additively separable in population groups, one can perform a second stage decomposition, with the Shapley approach, analogously to what was proposed in Araar (2006) for the decomposition of the Gini index.

4 Inequalities in Income Sources and Poverty

It is also interesting to estimate the contributions of inequalities in income sources to total poverty. First, we assume that the sum of K income sources equals the total income and the amount of income source k, is denoted by s_k. The contribution of the inequality of income source k at the margin, is the difference between the observed total poverty and that when the inequality of this component is eliminated. Formally, we replace s_k by μ_k for each household if the component k is eliminated.

$$
M_k = E_\pi - P(y^* = \sum_j s_{j \neq k} + \mu_k) \qquad (19)
$$

Here we find that:

$$
E_\pi < \sum_k M_k \qquad (20)
$$

for this, we suggest to use the Shapley approach with the same rule for removing inequalities in income sources.

5 Illustration Using Nigeria National Survey

The survey data used in this study was collected by Nigeria's National Bureau of Statistics (NBS) formerly known as the Federal Office of Statistics (FOS). They were based on National Living Standard Survey (NLSS) of households that was carried out between September 2003 and August 2004. The sample design is a two-stage stratified sampling. At the first stage, clusters of 120 housing units called Enumeration Area (EA) were randomly selected from each State and the Federal Capital Territory (FCT, Abuja). The second stage involved random selection of 5 housing units from the selected EAs. A total of 600 households were randomly chosen in each of the States and the FCT, summing up to 22,200 households in all (FOS, 2003). However, some households did not fully complete the questionnaires. Out of the 22,200 households that were targeted, only 19,158 completed the survey.

In this application, two main standard of living are adopted. The first is the total expenditure per capita and the second is the per capita income. The latter is useful to perform the decomposition of poverty by components of inequalities in income sources. It should be noted that there is no official absolute poverty line in Nigeria. Usually, the relative approach is adopted to estimate the poverty line in Nigerian studies (poverty line equals to two third of average standard of living). In this study, we additionally use the World Bank poverty line, that is US$1 per day by adult equivalent.

In this application, we analyse the contribution of the regional disparities (between group inequality) and inequality within each of the six geopolitical zones to the total inequality and poverty[10]. Moreover, we estimate the contribution of the observed inequalities in income sources to total inequality and poverty.

Figure (1), illustrates the density curves for the six geopolitical zones in Nigeria. It is obvious from the comparisons between density curves, that the *north east* and *north center* are the poorest zones. Furthermore, the well-being of households living in *south east* and *south west* is better than the well-being of the other households. Similarly, these conclusions are confirmed by comparing between FGT curves ($\alpha = 0$), as presented in figure (2). In table (1), we decompose the FGT index by the six zones. Results of this decomposition indicate that more than 64% of total poverty is attributed to the population group that live in northern zones, while this group represents about 53% of total population.

Figure 3 shows clearly how the poor and the non poor groups, as well as, the

[10]These geopolitical zones cut across the three main agro-ecological zones of the country.

interaction between them contribute to the total inequality, measured by the Gini index (review again equation (6)). The contribution of the within poor group to the total inequality is relatively lower than that of the non poor group. The major part of this inequality is explained by the inequality between the poor and the non poor groups.

In table 2, we decompose the Gini index by the six Nigerian geopolitical zones. The contribution of the within group inequality to total poverty is less than that of the between group inequality[11]. The highest level of the overlap component indicates also that the level of identification of groups, based on these six geopolitical zones, is low. One can recall here that the group identification by a given indicator, like the household income, is high when one can identify population groups only by using this indicator.

The decomposition of the FGT index by average per capita expenditures and inequality components across zones is presented in tables (3) and (4) for $\alpha = 0$ and $\alpha = 1$ respectively. The World Bank poverty line earlier mentioned is about twice the average per capita expenditures. This means that the contribution of the average standard of living is not nil. For the headcount index, inequality contributes positively to the total poverty if poverty line is lower than the average standard of living and contributes negatively if the poverty line exceeds this average.

While the between group inequality contributes more to the total inequality measured by the Gini index, its contribution to total poverty is very low. This paradoxical result can be explained partially by the focus axiom that poverty indices obey. This means that poverty indices are not sensitive to incomes that are higher than the poverty line. Indeed, when the within group inequality is removed and when the average income of each group is greater than the poverty line, the Gini index is not nil but the poverty index is equal to zero. With respect to the within group inequality components, one can remark that the northern zones contribute more than the southern zones.

In figures 4 and 5, we display the magnitude of the contribution of each component according to the poverty line when the parameter $\alpha = 0$ and $\alpha = 1$. When the poverty line is below the average per capita expenditures, the contribution of this average is nil. For the headcount index, the contribution of inequality components is greater than zero when poverty line is below the average standard of living.

[11] The residue component is explained by the overlap group income. According to Araar (2006), this component can be attributed to the between group component.

Nigeria presents an interesting case for the view of our development application. One can remark easily that the relative poverty line is very low (far from the World Bank poverty line). The policy implications of this would have far reaching effects. For instance, when the country is very poor and the poverty line is higher than the average standard of living, the explanatory power of inequality on poverty will be very low. In this case, the best policy option to fight poverty is to boost the economy by increasing per capita GDP. On the other hand, if the average standard of living is relatively higher than the poverty line, redistributive policies are appropriate for quick poverty alleviation.

To show how inequality of each income source contributes to the total poverty, we use the per capita income as the household standard of living. Since some households did not report their income sources, observations that contains missing values were omitted and 17764 observations were used for this application. Inequalities of components *Employment income* and *Non farm business income* contribute more to total poverty than inequality in *Agricultural income*. Results of this decomposition can guide and inspire policy makers to formulate workable poverty reduction policies. For instance, a progressive income tax structure and introduction of subsidy program for some goods that are largely consumed by poor households will result in a sharp reduction of total poverty in Nigeria.

6 Conclusion

Explaining the persistence of poverty in developing countries continue to interest researchers and policy makers. Based on distributive analysis, two main factors can explain the level of poverty. The first is the average standard of living, which reflects the level of development of the country, while the second is the shape of the distribution of income. This paper is devoted to explore the link between inequality and poverty. In this paper, some theoretical developments were provided to estimate the contribution of inequality components to total poverty. Inequality components can be the between and within group inequalities or inequality in income sources. We have illustrated theoretical frameworks developed using the Nigerian Living Standard Survey for the year 2004. The main lesson drawn from these results is that redistributive policies cannot be the main tool to fight poverty when the country is very poor. Hence, the best option is to improve the general economic performance.

Figure 1: Density functions according to the Nigerian geopolitical zones
Nigeria (2004)

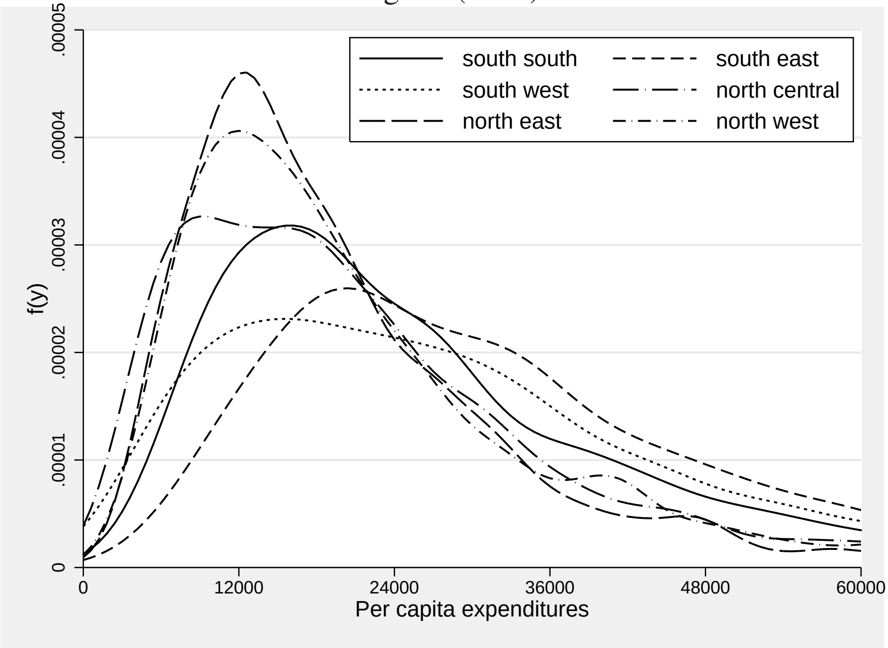

Figure 2: FGT curves (α=0) according to the Nigerian geopolitical zones
Nigeria (2004)

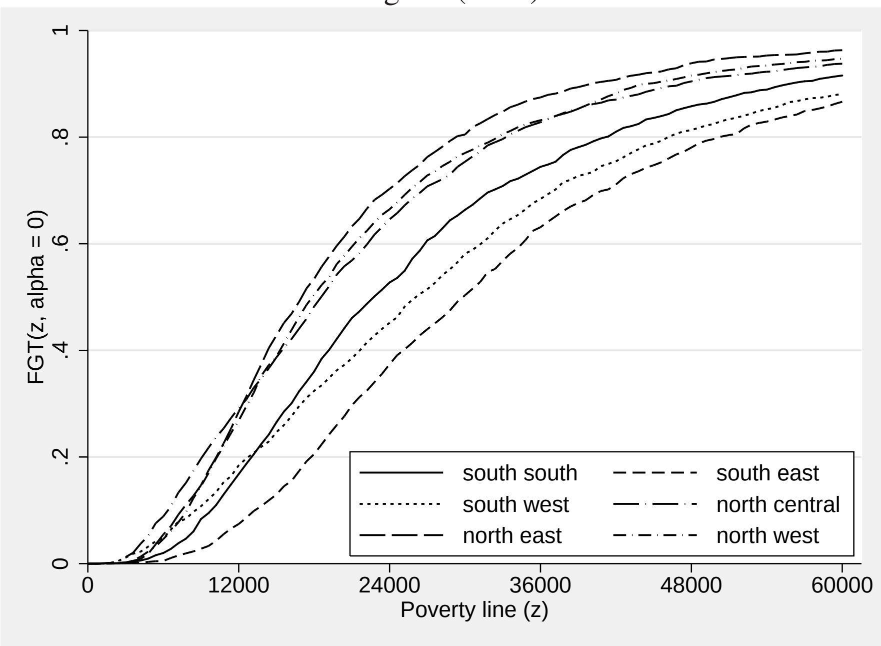

Figure 3: Lorenz curve, Gini index and poverty
Nigeria (2004)

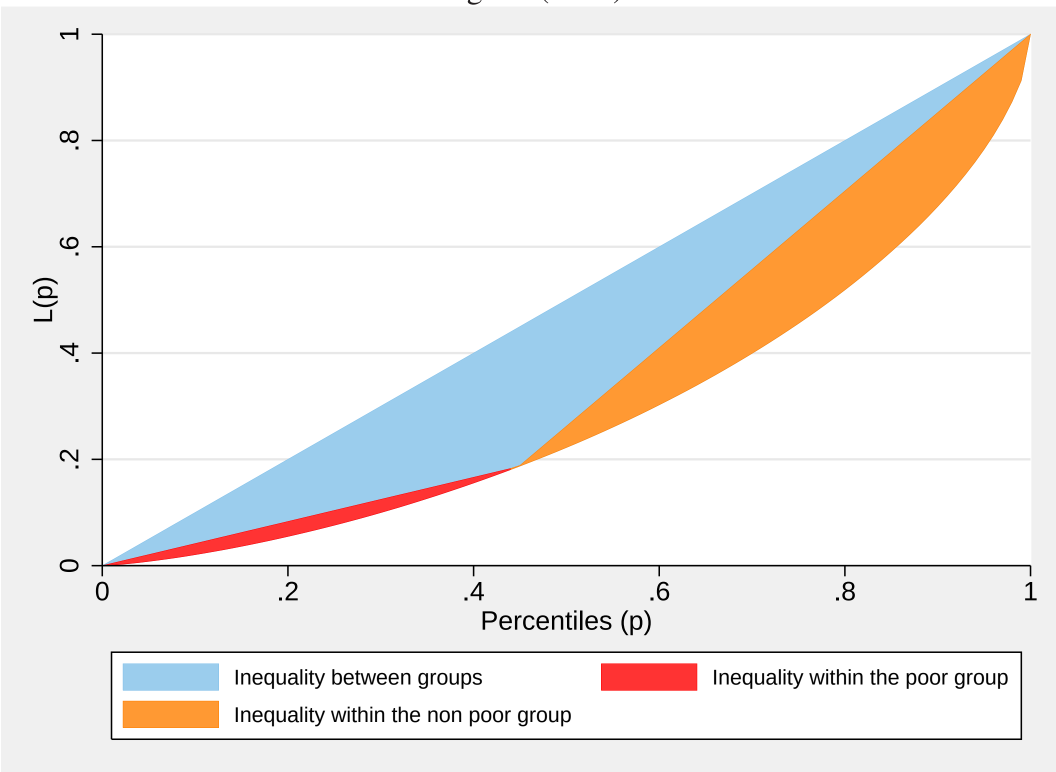

Table 1: Decomposition of the FGT index according to the geopolitical zones.
($\alpha = 0$; z = 19223 NG)

Group	FGT Index	Population Share	Absolute Contribution	Relative Contribution
South south	0.4019	0.1498	0.0602	0.1343
South east	0.2435	0.1208	0.0294	0.0656
South west	0.3485	0.1955	0.0682	0.1520
North central	0.5211	0.1437	0.0749	0.1670
North east	0.5780	0.1336	0.0772	0.1721
North west	0.5401	0.2565	0.1386	0.3090
Total	**0.4484**	**1.0000**	**0.4484**	**1.0000**

Table 2: Decomposition of the Gini index according to the geopolitical zones.

Group	Population Share	Income Share	Gini Index	Absolute Contribution	Relative Contribution
South south	0.1498	0.1562	0.3909	0.0091	0.0217
South east	0.1208	0.1582	0.3694	0.0071	0.0168
South west	0.1955	0.2339	0.4304	0.0197	0.0467
North central	0.1437	0.1229	0.4253	0.0075	0.0178
North east	0.1336	0.1150	0.4471	0.0069	0.0163
North west	0.2565	0.2138	0.3968	0.0218	0.0516
Within group	—	—	—	0.0720	0.1709
Between group	—	—	—	0.0959	0.2275
Overlap (residue)	—	—	—	0.2536	0.6016
Total	**1.0000**	**1.0000**	**—**	**0.4216**	**1.0000**

Figure 4: Contributions of the average expenditures and inequality components to the total poverty (FGT($\alpha = 0$)).

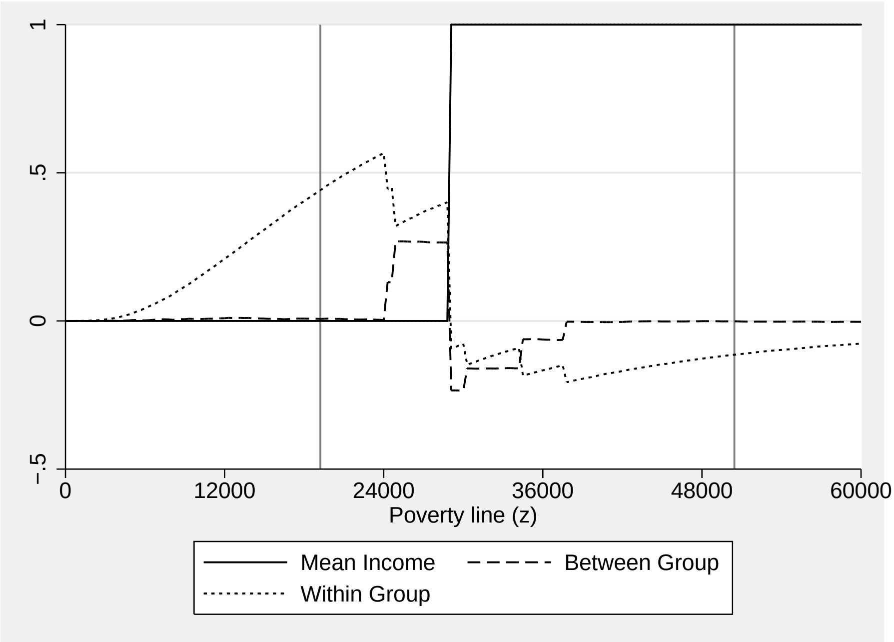

Figure 5: Contributions of the average expenditures and inequality components to the total poverty (FGT($\alpha = 1$)).

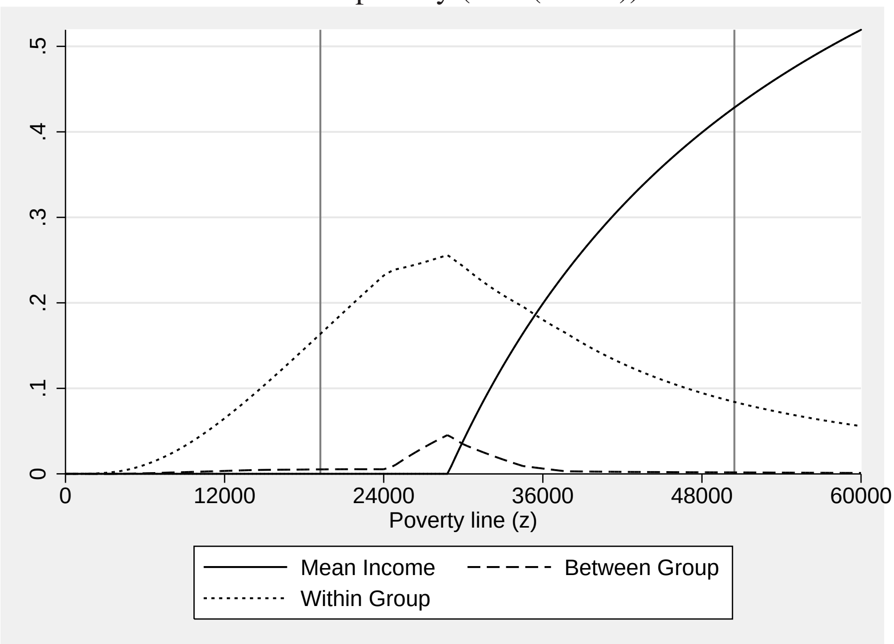

Table 3: Decomposing the FGT index ($\alpha = 0$) by average expenditures and inequality components

	Poverty line $= 2/3\mu^a$		*Poverty line* = *1\$* US^b		
Components	*Absolute Contribution*	*Relative Contribution*	*Absolute Contribution*	*Relative Contribution*	*Population Share*
South south	0.0623	0.1390	-0.0183	-0.0206	0.1498
South east	0.0389	0.0868	-0.0185	-0.0210	0.1208
South west	0.0764	0.1704	-0.0282	-0.0318	0.1955
North central	0.0682	0.1521	-0.0150	-0.0169	0.1437
North east	0.0709	0.1582	-0.0092	-0.0104	0.1336
North west	0.1247	0.2780	-0.0249	-0.0281	0.2565
Within group (Sub-Tot.)	0.4414	0.9844	-0.1140	-0.1289	1.0000
Between group	0.0070	0.0156	-0.0012	-0.0013	—
Average income	0.0000	0.0000	1.0000	1.1302	—
Total	**0.4484**	**1.0000**	**0.8848**	**1.0000**	**1.0000**

^{a}: Poverty line equals to two third of the average per capita expenditures (poverty line = 19223 NG).

^{b}: Poverty line equals to one US dollar. Exchange rate to US\$1 (as of January, 2004) 138.21 (poverty line = 50446 NG).

Table 4: Decomposing the FGT index ($\alpha = 1$) by average expenditures and inequality components

	Poverty line $= 2/3\mu^a$		*Poverty line = 1\$ US*^{b}		
Components	*Absolute Contribution*	*Relative Contribution*	*Absolute Contribution*	*Relative Contribution*	*Population Share*
South south	0.0209	0.1238	0.0114	0.0222	0.1498
South east	0.0118	0.0698	0.0118	0.0230	0.1208
South west	0.0313	0.1851	0.0213	0.0414	0.1955
North central	0.0285	0.1690	0.0102	0.0199	0.1437
North east	0.0258	0.1529	0.0136	0.0265	0.1336
North west	0.0453	0.2684	0.0157	0.0306	0.2565
Within group (Sub-Tot.)	0.1636	0.9690	0.0841	0.1636	1.0000
Between group	0.0052	0.0310	0.0017	0.0033	—
Mean income	0.0000	0.0000	0.4284	0.8331	—
Total	**0.1689**	**1.0000**	**0.5142**	**1.0000**	**1.0000**

^{a}: Poverty line equals to two third of the average per capita expenditures (poverty line = 19223 NG).

^{b}: Poverty line equals to one US dollar. Exchange rate to US\$1 (as of January, 2004) 138.21 (poverty line = 50446 NG).

Table 5: Decomposing the FGT index ($\alpha = 0$) by average income and inequality components

	Poverty line $= 2/3$ *average income* [a]		*Poverty line = 1\$ US* [b]			
Components	*Absolute Contribution*	*Relative Contribution*	*Absolute Contribution*	*Relative Contribution*	*Income Share*	*Concentration Index*
Employment income	0.3517	0.5853	-0.0531	-0.0600	0.3530	0.7533
Agricultural income	-0.0420	-0.0699	-0.0084	-0.0095	0.1999	0.3182
Fish processing income	0.0026	0.0044	-0.0020	-0.0023	0.0237	0.5262
Non farm business income	0.2834	0.4716	-0.0448	-0.0505	0.3441	0.6666
Remittances received	0.0029	0.0047	-0.0015	-0.0017	0.0246	0.5622
All other income	0.0023	0.0038	-0.0046	-0.0052	0.0548	0.5727
Inequality component	0.6008	1.0000	-0.1145	-0.1292		
Average income component	0.0000	0.0000	1.0000	1.1292		
TOTAL	**0.6008**	**1.0000**	**0.8855**	**1.0000**	**1.0000**	

[a]: Poverty line equals to two third of the average income, (poverty line = 15610 NG).

[b]: Poverty line equals to one US dollar. Exchange rate to US$1 (as of January, 2004) 138.21 (poverty line = 50446 NG).

Appendix A: *The link between the headcount and the Gini Index*

According to Runciman (1966), the magnitude of relative deprivation is the difference between the desired situation and the actual situation of a person. We define the relative deprivation of household i compared to j as follows [12]:

$$\delta_{i,j} = (y_j - y_i)_+ = \begin{cases} y_j - y_i & if \quad y_i < y_j \\ 0 & otherwise. \end{cases} \tag{A.1}$$

The expected deprivation of household i equals to:

$$\bar{\delta}_i = \frac{\sum\limits_{j=1}^{N} (y_j - y_i)_+}{N} \tag{A.2}$$

The Gini coefficient can be written in the following form:

$$I = \sum_{i=1}^{N} \frac{\bar{\delta}_i}{\mu_y N} = \frac{\bar{\delta}}{\mu} \tag{A.3}$$

For the inequality between the poor and the non poor groups, the expected deprivation of the poor is:

$$\begin{aligned} \bar{\delta}_p &= (1 - H)(\mu_{np} - \mu_p) \\ &= \mu - \mu_p \end{aligned} \tag{A.4}$$

Thus, the between group Gini index equals to:

$$\tilde{I} = \frac{1}{\mu} H\,(\mu - \mu_p) \tag{A.5}$$

This implies also:

$$H = \mu \tilde{I} \left(\frac{1}{\mu - \mu_p} \right) \tag{A.6}$$

[12] See also Yitzhaki (1979) and Hey and Lambert (1980).

Appendix B: *The Shapley Value*

Applied in several scientific domains, the Shapley approach can serve to perform an exact decomposition of the distributive indices[13]. The Shapley value is a solution concept often employed in the theory of cooperative games. Consider a set N of n players that must divide a given surplus among themselves. The players may form coalitions (these are the subsets S of N) that appropriate themselves a part of the surplus and redistribute it between their members. The function v is assumed to determine the coalition force, i.e., which surplus will be divided without resorting to an agreement with the outsider players (the $n - s - 1$ players that are not members of the coalition S). The question to resolve is: How can the surplus be divided between the n players? According to the Shapley approach, introduced by Loyd. (1953), the value or the expected gain of player k, noted by E_k, is shown by the following formula:

$$E_k = \sum_{\substack{s \subset S \\ s \in \{0, n-1\}}} \frac{s!(n-s-1)!}{n!} MV(S,k) \tag{B.1}$$

$$MV(S,k) = (v(S \cup \{k\}) - v(S)) \tag{B.2}$$

The term $MV(S,k)$ is the marginal value that the player k generates after his adhesion to the coalition S. What will then be the expected marginal contribution of player k, according to the different possible coalitions that can be formed and to which the player can adhere? First, the size of the coalition S is limited to: $s \in \{0, 1, ...n-1\}$. Suppose that the n players are randomly ordered and we note the order by σ, such that:

$$\sigma = \left\{ \underbrace{\sigma^1, \sigma^2, \cdots, \sigma^{i-1}}_{s}, \sigma^i, \underbrace{\sigma^{i+1}, \cdots, \sigma^n}_{n-s-1} \right\} \tag{B.3}$$

For each of the possible permutation of the n players, which equals $n!$, the number of times that the same first s players are located in the subset or coalition S is given by the number of possible permutations of the s players in coalition S (that is $s!$). For every permutation in the coalition S, one finds $(n-s-1)!$ permutations for the players that complement the coalition S. The expected marginal value that player k generates after his adhesion to a coalition S is given by the Shapley

[13]See Shorrocks (1999).

value. For every position of the factor k (predetermined cuts of the coalition S), there are several possibilities to form coalitions S from the $n-1$ player (that is the n players without the player k). This number of possibilities is equal to the number of combinations, C_{n-1}^{s}.

How many marginal values would one have to compute to determine the expected marginal contribution of a given factor or player k? Because the order of the players in the coalition S does not affect the contribution of the player k once he has adhered to the coalition, the number of calculations needed for the marginal values is: $\sum_{s=0}^{n-1} C_{n-1}^{s} = 2^{n-1}$. If we do not take into account this simplification, we can write the extended formula of the Shapley Value as follows:

$$E_k = \frac{1}{n!} \sum_{i=1}^{n!} MV(\sigma^i, k) \tag{B.4}$$

where for each order σ of the $n!$ orders, the players k have only one position that determines the coalition to which he can adhere. The term $MV(\sigma^i, k)$ equals the marginal value of adding the player k to its coalition. The properties of the decomposition of this approach are:

- Symmetry, which ensures that the contribution of each factor is independent of its order of appearance on the list of the factors or the sequence.
- Additivity of components.

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