C·I·R·P·É·E

Centre Interuniversitaire sur le Risque,
les Politiques Économiques et l'Emploi

Cahier de recherche/Working Paper **07-42**

Uncertain Bequest Needs and Long-Term Insurance Contracts

Wenan Fei
Claude Fluet
Harris Schlesinger

Novembre/November 2007

Fei: Hartford Life Insurance
Fluet: Université du Québec à Montréal and CIRPÉE
fluet.claude-denys@uqam.ca
Schlesinger: University of Alabama

Abstract: We examine how long-term life insurance contracts can be designed to incorporate uncertain future bequest needs. An individual who buys a life insurance contract early in life is often uncertain about the make up of his or her future family, much less their financial needs. Ideally, the individual would like to insure the risk of having high future bequest needs; but since bequest motives are typically unverifiable, a contract directly insuring these needs is not feasible. We derive two equivalent long-term life insurance contracts that are incentive compatible and achieve a higher welfare level than the naïve strategy of delaying the purchase of insurance until after one's bequest needs are known. We also examine the welfare effects of such contracts and we show how third-party financial products, although beneficial to the individual in the short run, can be welfare decreasing over one's lifetime.

Résumé: Nous analysons le design de contrats de long terme d'assurance décès lorsque les besoins futurs d'assurance sont incertains. Au moment de la souscription d'un contrat de long terme, l'acheteur n'a souvent qu'une idée imprécise de sa situation familiale future ou de ses besoins financiers. Idéalement, il aimerait pouvoir s'assurer contre le risque de se trouver ultérieurement dans une situation de besoins élevés; ses besoins ou ses préférences étant invérifiables, un tel contrat n'est toutefois pas possible. Nous étudions deux formes équivalentes de contrat de long terme avec options et satisfaisant à des contraintes d'auto-sélection en fonction des besoins au moment où ceux-ci deviennent connus. Ces contrats sont préférables à la stratégie naïve consistant à attendre de connaître ses besoins avant d'acheter une protection d'assurance décès par le biais d'un contrat de court terme. Nous montrons aussi que des produits financiers récemment offerts sur le marché, permettant le rachat par des tiers des couvertures d'assurance décès, auront pour effet de perturber les arrangements de long terme et se traduiront par une perte de bien-être sur le cycle de vie.

1 Introduction

People purchase life insurance to protect their dependents against financial losses caused by their deaths. In the life insurance market, most contracts extend many years into the future. Such prevalence of long-term contracts is partly due to the premium risk or "insurability risk." In particular, a person's health status may deteriorate, which makes short-term life insurance no longer affordable in the future. In the extreme, a person's health could deteriorate to such an extent that no life insurance is available. A long-term insurance contract with a front-loaded premium schedule, in a certain sense, also provides insurance against this "insurability risk." However, even without such insurability risk, a long-term contract can be beneficial. In particular, we show how such arrangements can improve welfare by partially insuring the risk of having a high bequest demand in the future.

Although it may be advantageous from an insurability standpoint to arrange for life insurance early, the need for life insurance many years later depends on the future demographic structure of the household and may not be known in advance. The impact of one's death often depends on the number of children in the household and the financial condition of other family members, as well the future preferences of these family members, as examined by Lewis (1989). Absent the insurability risk, it would at first appear to be optimal to purchase life insurance contracts later in life, when bequest needs are better known. Of course, another possibility is to purchase short-term contracts and to adjust the insurance level as needed at a later date, as in Polborn et al. (2006). If the status of one's health is private information, this runs into the problem of renewability risk.[1] However, even without the

---

[1] See Pauly et al. (1995). However, if this change in insurability is observable, it might be possible, at least in theory, to insure it directly in a manner similar to Cochrane (1995). For commitment problems associated with long-term contracting when changes in insurability are unobservable risk see Hendel and Lizzeri (2003).

insurability risk, a short-term purchasing strategy for life insurance is not optimal.

Intuitively, although delaying the purchase of life insurance can help individuals to determine the appropriate level of insurance, in concordance with their known bequest demand, one must still pay the extra insurance premium if one's demand is high. That is, one must plan for the possibility of needing to spend more on insurance premia in the future. Note that this form of "premium risk" has nothing to do with the insurability risk. Here the risk is on the budget required to finance the required amount of life insurance; not on whether or not the premium rate is higher.

In this paper, we consider the design of a long-term life insurance contract that also can help to mitigate the risk of possibly having a high bequest need in the future. Our model is similar to that of Polborn et al. (2006), except that we do not consider the insurability risk. With no insurability risk, but with a risk of demand type, the insurance premium per unit of coverage will not change. Hence one can always buy more life insurance later at the same price. Polborn et. al. (2006) also consider this case, but they conclude that there is no benefit to purchasing insurance earlier.

However, a long-term contract can also help to mitigate the risk of bequest type. Although this risk introduces no price risk *per se*, it does require that individuals with a high-bequest demand spend a higher share of their wealth on life insurance. Thus, a high-bequest demand leads to less consumption than a low-bequest demand, if an individual does not die early. We show long-term contracts can partially hedge this future consumption risk. This is accomplished by effectively transferring some wealth in future states where one's bequest needs are low to states for which bequest needs are higher. Since bequest needs are not likely to be easily verifiable, the contract cannot just pay a transfer to anyone who claims to have high bequests needs. Hence, the long-term contract is written with particular options, and the exercise of

2

these options occurs via self selection.

In the next section, we set up the basic model. We then examine a first-best world in which bequest type is verifiable. We examine the optimal insurance contract, which also provides protection again the risk of having a high bequest need in this setting. Next, we derive two equivalent long-term life insurance contracts for the case where bequest type is unverifiable. These contracts are incentive compatible and achieve a higher welfare level than the naïve strategy of delaying the purchase of insurance until after one's bequest needs are known. These second-best contracts are also compared to the first-best case. We conclude by explaining how some relatively new third party financial products, especially so-called "life settlement" contracts, can upset this long-term contract arrangement.

2   The Model

We develop a simple three-period model of life-insurance purchases when individuals are uncertain about their bequest preferences. A person with initial wealth w_0 at date $t = 0$ learns of his preferences for bequest at date $t = 1$. The individual faces a probability q of death at date $t = 2$. With probability $1 - q$, the individual lives to consume another period. To keep the model simple and to focus on bequest needs, q is non-random and, thus, there is no insurability risk. For similar reasons, we further assume that the interest rate for borrowing or lending is zero.[2]

Denote by w^d and w^l the individual's final wealth in the states of death

---

[2]Obviously, we are simplifying the basic insurance decision to a great extent. For example, we do not consider future income, much less the fact that it might be risky. Likewise, we do not consider intermediate consumption in our model. See, for example, Campbell (1980). For a survey of many theoretical life insurance issues, see Villeneuve (2000).

and survival respectively. Let i refer to the individual's type with respect to preferences for bequest at $t = 1$. The expected utility of final wealth is then

$$qv_i(w^d) + (1-q)u(w^l),$$

where $v_i(w^d)$ is the utility of leaving wealth w^d to dependents at $t = 2$ and $u(w^l)$ is the utility of wealth w^l in the state of living. Both functions are increasing and strictly concave. Moreover, $v_i'(w) > u'(w)$ for all w, implying a demand for life insurance. Taken together, $v_i(w^d)$ and $u(w^l)$ can be viewed as a state-dependent value function for the utility derived from the optimal consumption and savings strategies, given the individual's wealth in each state at the beginning of date $t = 2$ and taking implicitly into account the future labor income that a surviving individual would earn.

Bequest needs are initially uncertain. At $t = 0$, the individual does not know his bequest utility function, which can be either $v_B(\cdot)$ with probability π or $v_A(\cdot)$ with probability $1 - \pi$. We assume that $v_B'(w) > v_A'(w)$ for all w, so that type B is the high-bequest type. An individual's type, once learned, is private information, but insurers know the proportion of types in the population. Any amount of life insurance coverage can be purchased at any time before $t = 2$. Let L be the death benefit purchased.[3] The life insurance premium is assumed to be actuarially fair, so the premium for the amount of coverage L is qL.

As a preliminary step, we examine an individual's demand for life insurance when coverage is purchased at $t = 1$, after the individual has learned his type. We then show that, from the perspective of $t = 0$, the individual would like to insure against the risk of being a high-bequest type. However,

---

[3]We ignore any savings component built into many life insurance contracts. In this sense, we can regard L as the pure death-protection benefit that is paid in the event of an early death at date $t = 2$. More simply, we can view the insurance as a type of term life insurance product that only pays a benefit if death is at date $t = 2$.

insurance against such a risk cannot be bought directly, since one's type is unverifiable.

3   Bequest type is verifiable

Here we consider two insurance strategies. The first is simply to wait until bequest type is known before buying insurance. Even in a world with no insurability risk, the individual has a risk as to how much the total expenditure on insurance will be. In an ideal world, where bequest type is verifiable, this risk can be insured.

The naïve strategy

The simplest strategy for buying life insurance is to wait until $t = 1$ and to purchase coverage after learning one's type. It is useful to characterize the demand for coverage as a function of some arbitrary wealth w at date $t = 1$. Obviously, if nothing has been done before this date, then $w = w_0$.

For an individual with bequest type i and wealth w at date $t = 1$, the life-insurance objective is to

$$\max_{L_i} q v_i(w - qL_i + L_i) + (1 - q)u(w - qL_i), \quad i = A, B.$$

The optimal coverage $L_i^*(w)$ satisfies the first-order condition

$$v_i'(w - qL_i^*(w) + L_i^*(w)) - u'(w - qL_i^*(w)) = 0, \quad i = A, B. \qquad (1)$$

Risk aversion ensures that the second-order condition is satisfied. It is easily checked that $L_B^*(w) > L_A^*(w)$, i.e., B is indeed the high-bequest type.

Substituting for the optimal amount of coverage yields the date 1 optimal expected utility

$$V_i(w) \equiv q v_i(w - qL_i^*(w) + L_i^*(w)) + (1 - q)u(w - qL_i^*(w)), \quad i = A, B.$$

Here $V_i(w)$ is the value function for a person of type i at date $t = 1$, who has wealth w at that date. Viewed from date $t = 0$ and treating bequest type as a random variable, $V_i(w)$ is a state-dependent utility function exhibiting risk aversion in each state of the world. To see this, apply the envelope theorem and use (1) to obtain

$$V_i'(w) = u'(w - qL_i^*(w)), \quad i = A, B. \tag{2}$$

Since $L_B^*(w) > L_A^*(w)$, it follows that $V_B'(w) > V_A'(w)$. Differentiating a second time yields

$$V_i''(w) = u''(w - qL_i^*(w))\left(1 - qL_i^{*\prime}(w)\right) < 0.$$

The sign follows from

$$1 - qL_i^{*\prime}(w) = \frac{v_i''}{(1-q)v_i'' + qu''} > 0, \tag{3}$$

where the expression is obtained by total differentiation of (1).

From (3), it is also easily verified that

$$1 - qL_i^{*\prime}(w) + L_i^{*\prime}(w) = \frac{u''}{(1-q)v_i'' + qu''} > 0. \tag{4}$$

Thus, bequest and net wealth in the survival state are normal goods, i.e., $w_i^l \equiv w - qL_i^*(w)$ and $w_i^d = w - qL_i^*(w) + L_i^*(w)$ are strictly increasing in the date 1 wealth w.

Insurance against bequest type

An individual who decides to wait until date $t = 1$ to purchase life insurance knows that he will purchase either $L_A^*(w_0)$ or $L_B^*(w_0)$, depending on his bequest needs. At date $t = 0$, his expected utility is therefore $(1-\pi)V_A(w_0) + \pi V_B(w_0)$. Since $V_B'(w_0) > V_A'(w_0)$, transferring wealth at a fair price from the low to the high marginal utility state increases expected utility. Put

differently, the individual would like to insure against the risk of being a high-bequest type.

Suppose for now, contrary to our earlier assumption, that bequest types are verifiable. A contract could then be written at date $t = 0$ that pays some amount Q at date $t = 1$ if the person turns out to be type B. The fair premium for such a contract is πQ paid at date $t = 0$. The date 1 wealth is now either $w_A = w_0 - \pi Q$ or $w_B = w_0 - \pi Q + Q$ depending on the individual's realized bequest type, where $(1 - \pi)w_A + \pi w_B = w_0$.

It is a simple dynamic programming problem to maximize the expectation of the value function

$$\max_Q (1 - \pi)V_A(w_0 - \pi Q) + \pi V_B(w_0 - \pi Q + Q).$$

The optimal Q^* satisfies the first-order condition

$$V_B'(w_0 - \pi Q^* + Q^*) - V_A'(w_0 - \pi Q^*) = 0. \tag{5}$$

It follows trivially that $Q^* > 0$, so that $w_B^* > w_0 > w_A^*$.

The life insurance purchased is then $L_A^*(w_A^*)$ if needs are low and $L_B^*(w_B^*)$ if they are high. The possibility of insuring against bequest needs yields a solution characterized by

$$u'(w_A^{l*}) = u'(w_B^{l*}) = v_A'(w_A^{d*}) = v_B'(w_B^{d*}), \tag{6}$$

where

$$w_i^{l*} = w_i^* - qL_i^*(w_i^*), \ w_i^{d*} = w_i^* - qL_i^*(w_i^*) + L_i^*(w_i^*), \quad i = A, B.$$

We essentially have a complete contingent claims market and equate marginal utility in all four possible states of the world. This is achieved by combining two types of insurance products: one insures against a premature death and the other insures the uncertain bequest needs. Coverage against the risk of being the high bequest type, equivalently the transfer of wealth from state

7

A to state B individuals is $w_B^* - w_A^* = Q^* = q(L_B^*(w_B) - L_A^*(w_A))$, the difference in the life insurance premia. We will refer to this set of contracts as the first-best solution.

Comparison

It is instructive to compare this first-best solution with the naïve strategy used when preference risks are not insurable. Using the first-best strategy, wealth in the survival state is now equalized across bequest types. Moreover, because of the wealth transfer and since bequests are normal goods, bequests are now larger in the high-bequest state and smaller in the low-bequest state, i.e., $w_B^{d*} > w_0 - qL_B^*(w_0) + L_B^*(w_0)$ and $w_A^{d*} < w_0 - qL_A^*(w_0) + L_A^*(w_0)$. Thus, the possibility of insuring against preference risks allows the bequest amount to more closely reflect needs.

```
-- Fig.  1 and 2 about here --
```

Figures 1 and 2 provide a state-space representation of the consumer's problem at date $t = 1$, when bequest type is known but one's date of death is still uncertain. In figure 1 preference risks are not insured. The negatively sloped straight line is the budget constraint arising from the insurer's zero profit condition, i.e.,

$$(1 - q)w_i^l + qw_i^d = w_0.$$

Indifference curves (iso-expected-utility) for both bequest types are shown. For bequest type i, the marginal rate of substitution between wealth in the surviving state and wealth in the death state is

$$-\left(\frac{dw_i^l}{dw_i^d}\right) = \frac{qv_i'(w^d)}{(1 - q)u'(w^l)}.$$

For each type, the equilibrium occurs at the tangency point with the budget line, implying $v_i'(w_i^d) = u'(w_i^l)$. Since type B has steeper indifference curves,

$w_B^d > w_A^d$ and $w_B^l < w_A^l$, leading to equilibria such as the contingent claims E_A and E_B in figure 1.[4]

Figure 2 illustrates the case where preference risks can be insured. The date 1 budget constraint is then

$$(1 - q)w_i^l + qw_i^d = w_i, \quad i = A, B.$$

The equilibrium contingent claims in this case are E_A^* and E_B^* characterized by the condition $w_B^{l*} = w_A^{l*}$. Moreover, w_A^{d*} is smaller and w_B^{d*} larger than the corresponding amounts in the uninsured case.

Of course, direct insurance against bequest type is not feasible if one's bequest type is unverifiable. An individual purchasing such a policy would always want to claim that he is the high bequest type in order to receive the indemnity Q^*. This is obvious from figure 2. Rather than staying at E_A^*, a type-A individual is better off claiming he is B and moving to the higher budget line.

4   Bequest type is unverifiable

We now turn our attention to the case where bequest type is private information and show how we can improve upon the naïve strategy of waiting until date $t = 1$ to purchase insurance. Note that it does not matter whether or not type is verifiable by the insurer to implement the naïve strategy.

Long-term life insurance contracts are purchased at date $t = 0$, before individuals know their bequest preferences. Many extant life insurance contracts often include provisions that allow for changes to the contract at some future date, at the option of the insured. One such type of provision is an opting out opportunity: the insured can trade-in his policy at a later date

---

[4]See Karni (1985) for a general treatment of models using such state-dependent preferences.

at some pre-specified buy-back price. Alternatively, the contract can include an option for the purchase of additional coverage at some pre-specified rate. We show that such long-term insurance contracts can improve the individual's welfare even though bequest types are non-verifiable. In particular, a well designed policy allows wealth to effectively be transferred from type-A individuals to type-B individuals.

Opting out contracts

We consider a contract with a sell back option. We define the contract by the triplet (P, L, K), where P denotes the premium paid at $t = 0$, L denotes the death benefit and K is the price at which the policy can be traded in (i.e. sold back to the insurer) at date $t = 1$. With insurers earning zero profit, if only type-A individuals sell back their policies, such a contract will effectively transfer the amount $qL - K$ from type-A individuals to type-B individuals at date $t = 1$. In essence, the insurer sells the original coverage L at a subsidized price. The insurer finances this subsidy by buying back the policy at an unfair price from the low-bequest types, who then subsequently purchase a lower level of coverage.

Such an arrangement works if the following three incentive-compatibility conditions are satisfied:

(a) type-A individuals choose to sell back their policy at $t = 1$ and buy a new short-term policy on the "spot" market at actuarially fair prices[5]:

$$V_A(w_0 - P + K) \geq qv_A(w_0 - P + L) + (1 - q)u(w_0 - P). \qquad (7)$$

(b) type-B individuals prefer keeping their policy at $t = 1$:

$$qv_B(w_0 - P + L) + (1 - q)u(w_0 - P) \geq V_B(w_0 - P + K). \qquad (8)$$

(c) From the perspective of date $t = 0$, the arrangement dominates the strategy of waiting until $t = 1$ to buy insurance:

$$U \geq (1 - \pi)V_A(w_0) + \pi V_B(w_0), \qquad (9)$$

where U denotes the expected utility provided by the long-term contract

$$U \equiv (1-\pi)V_A(w_0-P+K)+\pi\left[qv_B(w_0 - P + L) + (1 - q)u(w_0 - P)\right]. \quad (10)$$

In addition, the contract must yield a non-negative profit:

$$P \geq \pi qL + (1 - \pi)K. \qquad (11)$$

It is easily seen that the set of contracts that satisfy the above constraints is not empty. In particular, consider the contract defined by $L = L_B^*(w_0)$ and $P = K = qL_B^*(w_0)$, where $L_B^*(w_0)$ is the optimal death benefit for type B under the naïve strategy. The non-negative profit condition and (8) are then satisfied as equalities, and (7) is satisfied as a strict inequality. Clearly, this arrangement yields the same outcome as the naïve strategy described in figure 1, implying that (9) is then satisfied as an equality.

In a competitive market, insurers are led to offer the best contract subject to profits being non negative. The equilibrium contract is therefore the one that maximizes U defined as in (10) subject to the non-negative profit condition and the incentive compatibility conditions. Since it is a maximum, the optimal contract is at least as good as the naïve strategy, i.e., the constraint (9) is trivially satisfied. Also, given $K \leq P$, it is easily checked that (9) implies (8). Thus, the only relevant constraints are (7), which is the opting out condition for type A, and the non-negative profit condition (11).

11

Second-best arrangement

Under the above arrangement, type A's wealth at date $t = 1$, after exercising his option to sell back his policy, is $w_A = w_0 - P + K$. This type then purchases the optimal death benefit $L_A^*(w_A)$ in the date 1 market. This yields the final contingent wealth levels $w_A^l = w_A - qL_A^*(w_A)$ and $w_A^d = w_A - qL_A^*(w_A) + L_A^*(w_A)$. Type B does not opt out and thus the final contingent wealth allocation follows directly from the long-term insurance contract, i.e., $w_B^l = w_0 - P$ and $w_B^d = w_0 - P + L$. The date 1 value of this allocation is

$$w_B = qw_B^d + (1 - q)w_B^l = w_0 - P + qL.$$

The implied wealth transfer from type-A to type-B individuals is therefore $w_B - w_A = qL - K$.

Written in terms of w_A, w_B^l and w_B^d, the second-best arrangement solves the following program:

$$\max_{w_A, w_B^l w_B^d} U = (1 - \pi)V_A(w_A) + \pi \left[qv_B(w_B^d) + (1 - q)u(w_B^l) \right]$$

subject to

$$V_A(w_A) \geq qv_A(w_B^d) + (1 - q)u(w_B^l) \tag{12}$$

and

$$(1 - \pi)w_A + \pi \left[qw_B^d + (1 - q)w_B^l \right] \leq w_0. \tag{13}$$

The first inequality is type A's incentive compatibility constraint; the second follows from the insurer's non-negative profit condition.

It is straightforward to characterize the main features of the solution to the above problem. The resource constraint (13) is obviously binding. We show first that the self-selection condition (12) must be binding as well. Suppose, to the contrary, that the optimal solution maximizes U subject to (13) only. This is readily seen to yield a solution

$$V_A'(w_A) = v_B'(w_B^d) = u'(w_B^l).$$

12

Substituting from (1) and (2), we then have

$$u'(w_A^l) = v_A'(w_A^d) = v_B'(w_B^d) = u'(w_B^l),$$

which corresponds to the first-best allocation represented in figure 2. However, as is clear from the figure, type A strictly prefers E_B^* to E_A^*, implying that type A would not opt out, i.e., (12) is not satisfied.

Secondly, the naïve strategy is not a solution. As discussed above, (12) holds as a strict inequality under the naïve strategy. Since this condition must bind, the naïve strategy does not solve the problem. However, since it nevertheless satisfies the constraints, it must be the case be that the individual is strictly better off under the long-term contract. The following proposition summarizes these results.

Proposition 1 *Long-term opting-out contracts make individuals strictly better off than the naïve strategy, but they remain second-best compared to the (complete-information) case where bequest needs are directly insurable.*

Levels of coverage

We next examine how the levels of coverage differ under the various insurance arrangements. The Lagrangian of the second-best program is

$$\mathcal{L} = (1-\pi)V_A(w_A) + \pi \left[qv_B(w_B^d) + (1-q)u(w_B^l) \right]$$
$$+ \mu \left[V_A(w_A) - qv_A(w_B^d) - (1-q)u(w_B^l) \right]$$
$$+ \lambda \left[w_0 - (1-\pi)w_A - \pi \left(qw_B^d + (1-q)w_B^l \right) \right],$$

with positive multipliers μ and λ. Together with (12) and (13) holding as equalities, the solution satisfies the first-order conditions

$$\frac{\partial \mathcal{L}}{\partial w_A} = (1-\pi+\mu)V_A'(w_A) - \lambda(1-\pi) = 0, \tag{14}$$

$$\frac{\partial \mathcal{L}}{\partial w_B^d} = q\left(\pi v_B'(w_B^d) - \mu v_A'(w_B^d) - \lambda \pi\right) = 0, \tag{15}$$

$$\frac{\partial \mathcal{L}}{\partial w_B^l} = (1-q)\left[(\pi - \mu)\, u'(w_B^l) - \lambda \pi\right] = 0. \tag{16}$$

Denote the solution by $(\widehat{w}_A, \widehat{w}_B^d, \widehat{w}_B^l)$. The date-1 value of type B's allocation is $\widehat{w}_B \equiv q\widehat{w}_B^d + (1-q)\widehat{w}_B^l$. Type A's allocation is $\widehat{w}_A^d$ and $\widehat{w}_A^l$ satisfying

$$v_A'(\widehat{w}_A^d) = u'(\widehat{w}_A^l) = V_A'(\widehat{w}_A). \tag{17}$$

An illustration is given in figure 3. We derive two results. First, the wealth transfer from type-A to type-B individuals in the long-term arrangement is smaller than in the first-best contract. Secondly, the second-best contract provides a greater death benefit than type B would wish if he could purchase freely on the basis of his contractually defined date 1 wealth.

```
-- Fig.  3 about here --
```

Wealth transfer. We show that the subsidy from type-A to type-B individuals is lower in the second-best solution vis-à-vis the first-best one: $\widehat{w}_B - \widehat{w}_A < w_B^* - w_A^*$. Suppose, to the contrary, that

$$\widehat{w}_B - \widehat{w}_A \geq w_B^* - w_A^*. \tag{18}$$

From the zero-profit condition, the state wealth levels satisfy

$$(1-\pi)\widehat{w}_A + \pi\widehat{w}_B = (1-\pi)w_A^* + \pi w_B^* = w_0.$$

Hence, (18) implies $\widehat{w}_A \leq w_A^*$ and $\widehat{w}_B \geq w_B^*$. A's date-1 budget line in the second-best arrangement is then below the first-best one represented in figure 2, while B's budget line would be above the one in figure 2.

A's allocation satisfies (17). Since bequest and survival wealth are normal goods, (18) therefore implies $\widehat{w}_A^l \leq w_A^{l*}$ and $\widehat{w}_A^d \leq w_A^{d*}$. Consider now B's

14

allocation. This is given by the intersection of A's indifference curve through $(\widehat{w}_A^d, \widehat{w}_A^l)$ and B's budget line. Obviously, the foregoing implies $\widehat{w}_B^d > w_B^{d*}$. Combining both these results yields

$$v_A'(\widehat{w}_A^d) \geq v_A'(w_A^{d*}) = v_B'(w_B^{d*}) > v_B'(\widehat{w}_B^d), \qquad (19)$$

where the equality follows from the optimality conditions for a first best.

We now turn to the restrictions imposed by the first-order conditions. From (14) and (15) it follows that

$$v_B'(\widehat{w}_B^d) = \left(\frac{1 - \pi + \mu}{1 - \pi}\right) V_A'(\widehat{w}_A) + \left(\frac{\mu}{\pi}\right) v_A'(\widehat{w}_B^d). \qquad (20)$$

Substituting for $V_A'(\widehat{w}_A) = v_A'(\widehat{w}_A^d)$ from (17), (20) implies $v_B'(\widehat{w}_B^d) > v_A'(\widehat{w}_A^d)$ which contradicts (19). The wealth transfer must therefore be strictly smaller in the second-best arrangement.

Distortion. Here we show that the B-type is forced to "overinsure," which can be interpreted as a type of signalling cost in the second-best setting. This distortion is represented by a point such as $\widehat{E}_B$ in figure 3. As drawn, the long-term contract provides a larger bequest (and correspondingly smaller survival wealth) than type B would wish to puchase voluntarily on the basis of the post-transfer wealth level $\widehat{w}_B$. In other words, $L > L_B^*(\widehat{w}_B)$ or equivalently $\widehat{w}_B^d > \widehat{w}_B - qL_B^*(\widehat{w}_B) + L_B^*(\widehat{w}_B)$. This is a necessary feature of the second-best arrangement. The intuition is that this "distortion" facilitates the transfer of wealth from state A to state B, by making it more costly for type A not to opt out of the initial contract.

To see this more formally, substituting from (15) and (16) yields

$$v_B'(\widehat{w}_B^d) - u'(\widehat{w}_B^l) = \frac{\mu}{\pi}\left(v_A'(\widehat{w}_B^d) - u'(\widehat{w}_B^l)\right).$$

No "distortion" would require that the left-hand side is zero, which in turn implies $v_A'(\widehat{w}_B^d) = v_B'(\widehat{w}_B^d)$, a contradiction. Moreover, B's indifference curve

15

through $\widehat{E}_B$ cannot be steeper than the fair-odds line. Otherwise, a pair (w_B^d, w_B^l) could be chosen on the same fair-odds line below A's indifference curve through $\widehat{E}_B$ that satisfies A's incentive compatibility constraint and is strictly preferred by B. Hence, we must have L larger than $L_B^*(\widehat{w}_B)$ as claimed.

The next proposition summarizes our results.

Proposition 2 *Under the second-best long-term contract*
(i) the wealth transfer between type A and type B is smaller than the first-best transfer and
(ii) high-bequest types are over-insured relative to the coverage they would like to have at the contractually defined wealth level.

The foregoing results imply that type-A's bequest and survival wealth are greater than in the first best, while type-B's survival wealth is smaller. However, how type-B's bequest compares with the first-best level is ambiguous. There are two opposing effects so to speak. On the hand, because of the distortion, B's equilibrium bequest is larger than he would wish at the wealth level $\widehat{w}_B$. On the other hand, his wealth level is lower than the first-best w_B^*. When conditional wealth levels do not differ too much from the first best, it may therefore be that B leaves a larger bequest. In this case, bequest by both types are greater than in the first best.

It is interesting to note that at date 1, an individual who turns out to be type A will be better off with the second-best contract than with the first-best one. This is to be expected, since the subsidy is lower under the second-best arrangement. In other words, at date 0 the individual would prefer the extra protection afforded by a larger subsidy. However, an individual who eventually turns out to be of a low-bequest type will be happier if the subsidy is smaller when date 1 arrives.

Opting in contracts

An alternative to the opting out arrangement is to offer a contract with an option to purchase additional coverage at date $t = 1$. Such an "opt-in" contract is defined by the vector $(\overline{P}, \overline{L}, S, k)$ where $\overline{P}$ is the premium paid at $t = 0$ for coverage $\overline{L}$ and S is the optional additional coverage that the individual can purchase at date $t = 1$ for an additional premium k. If $\overline{P} > q\overline{L}$ and $k < qS$, then wealth will be transferred from type-A individuals to type-B individuals, provided of course only type B exercises the option to purchase additional coverage. Here, the original coverage $\overline{L}$ is sold at an unfair premium. The non-negative profit constraint under this opting-in arrangement is

$$\overline{P} \geq q\overline{L} + \pi(qS - k). \tag{21}$$

The above contract is equivalent to selling both types an initial contract with death benefit $(\overline{L} + S)$ for a premium of $(\overline{P} + k)$. The type-B individual "opts in" by maintaining this package at date $t = 1$. The type-A individual refuses to "opt in" at date $t = 1$ by obtaining a refund of the extra premium k, and reducing the death total benefit by an amount S. As before, effectively only the A-type's incentive compatibility constraint matters, with the type-A individual being just indifferent between opting in or not opting in.

Defining

$$P \equiv \overline{P} + k, \quad L \equiv \overline{L} + S \quad \text{and} \quad K \equiv q\overline{L} + k, \tag{22}$$

it is easily seen that the non-negative profit constraint (21) is equivalent to the previous non-negative profit constraint (11), and it will be satisfied once again with an equality at the optimum. We have one additional requirement here, namely that a type-A individual who would receive a refund $q\overline{L} + k$ at date $t = 1$ would purchase an optimal level of insurance coverage $L_A^*(w_0 - \overline{P} + q\overline{L}) = \overline{L}$, if insurance would be available at a fair price. More formally, from the first-

17

order condition for L_A^*, this requires that

$$v_A'(w_0 - \overline{P} + q\overline{L} - qL_A^* + L_A^*) = u'(w_0 - \overline{P} + q\overline{L} - qL_A^*) \qquad (23)$$

be satisfied when $L_A^*(w_0 - \overline{P} + q\overline{L}) = \overline{L}$.

It then follows in a straightforward manner that the same triple (P, L, K) is optimal, which together with (22) and (23) determine the parameters for the optimal opt-in contract: $(\overline{P}, \overline{L}, S, k)$. Thus, the opting-in and opting-out arrangements are effectively identical.[6]

5   Concluding Remarks

This paper has shown how a long-term insurance contract can be designed, within a competitive insurance market, to insure the uncertain future bequest needs of the individual. We derived two equivalent forms for this long-term insurance contract:[7]

(i) It provides a high level of initial coverage at a subsidized (low) price, with an option to sell back the policy at an unfair price (i.e. at a loss to the insured)

or

(ii) It provides a low level of initial coverage at an unfair (high) price, with an option to purchase additional coverage at a subsidized (low) price.

The existence of such contracts in the market place depends crucially on the self selection of types in exercising the various options.   But some

---

[6] One can also verify this directly by writing out the incentive compatibility constraints and then finding the optimal $(\overline{P}, \overline{L}, S, k)$ directly, which together with (23) shows the equivalence of the two types of contract arrangements.

[7] These two forms will not be unique. For example, an intermediate level of insurance could be offered with both "opt in" and "opt out" opportunities to achieve the same final wealth levels.

relatively new innovations in the financial marketplace may have an untoward effect on the development of the long-term contracts we propose. In particular, the market for life settlements poses such an obstacle.

A life settlement contract essentially offers to "buy back" the life insurance policy of an individual.[8] This is effected via a third party paying cash to the insured, in exchange for being named the beneficiary of the life insurance death benefit. Although this seems to eliminate any benefit to the original beneficiary, this will not be the case. In particular, under contract (i), the low-bequest need individual will opt to sell the policy to a life settlement broker, rather than back to the insurer, and receive more money for the policy. The insured can then purchase insurance at a fair price, since there is no insurability risk. Under contract (ii), both bequest types might purchase the additional extra insurance at the low price, with the low-bequest need type individual then immediately selling back the extra coverage in the life settlement market for a profit.

The existence of such markets provides an alternative for the insured that is beneficial *ex post* (i.e., after signing the original long-term contract). Insurance companies had originally protested as these markets developed, claiming that they should have the exclusive right to buy-back (i.e. "settle") contracts that they had written. But others disagree. For example, Doherty and Singer (2002) tout the benefits of life settlement markets to the insurance consumer. Such analysis might be incomplete, however, in that it excludes the fact that *ex ante* (i.e. prior to learning one's bequest type) one would prefer the longer term contracts described in this paper; and the life settlement market might preclude such contracts from ever being offered.

---

[8]A similar arrangement is a viatical settlement, which is exclusively for people who are terminally ill. See Doherty and Schlesinger (2000) and Doherty and Singer (2002). Since the viatical-settlement market depends critically on a large change in the insurability risk, the life settlement market is more appropriate here.

Although the long-term contracts we describe in this paper give the insurer monopoly power *ex post*, a competitive market *ex ante* should ensure that insurers cannot earn undue monoply rents.

Obviously, we simplified the setting of our analysis by assuming away many complicating factors, such as the insurability risk. This allowed our focus to be on the bequest needs and the (non-random) probability of death. Integrating these results into more complex settings is difficult. Hopefully, our paper takes a good first step in this direction.

References

[1] Campbell, R. A. (1980). Demand for life insurance: An application of the economics of uncertainty. *Journal of Finance* 35, 1155-1172.

[2] Cochrane, J. H. (1995). Time-consistent health insurance. *Journal of Political Economy* 103, 447-473.

[3] Doherty, N. A. and Schlesinger, H. (2000). Viaticals: A matter of life and death, Unpublished Working Paper, University of Pennsylvania.

[4] Doherty, N. A. and Singer, H. J. (2002). The benefit of a secondary market for life insurance policies, *Wharton Financial Institutions Center Working Paper* No. 02-41.

[5] Hendel, I. and Lizzeri, A. (2003). The role of commitment in dynamic contracts: Evidence from life insurance. *Quarterly Journal of Economics* 118, 299-327.

[6] Karni, E., (1985). *Decision Making Under Uncertainty: The Case of State-Dependent Utility.* Cambridge, MA: Harvard University Press.

[7] Lewis, F. D. (1989). Dependents and the demand for life insurance. *American Economic Review* 79, 452-467.

[8] Pauly, M. V.; Kunreuther, H.; and Hirth, R. (1995). Guaranteed renewability in insurance. *Journal of Risk and Uncertainty* 10, 143-156.

[9] Polborn, M. K., Hoy, M., & Sadanand, A. (2006). Advantageous effects of regulatory adverse selection in the life insurance market. *Economic Journal* 116, 327-354.

[10] Villeneuve, B. (2000). Life insurance, in: G. Dionne, Editor, *Handbook of Insurance,* Boston: Kluwer Academic Publishers.
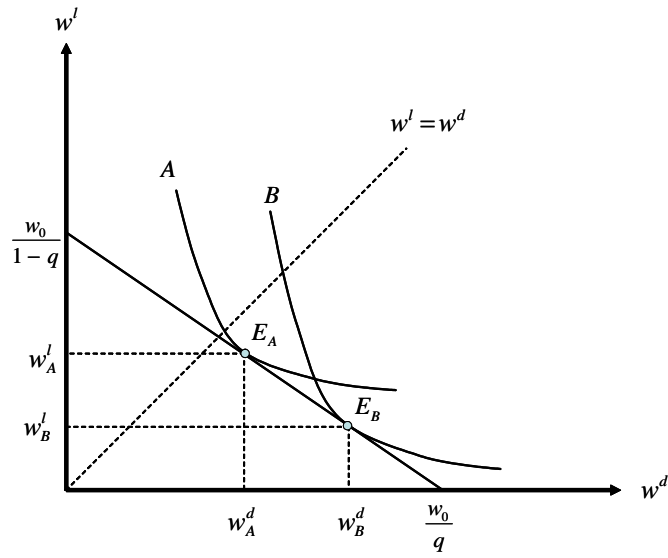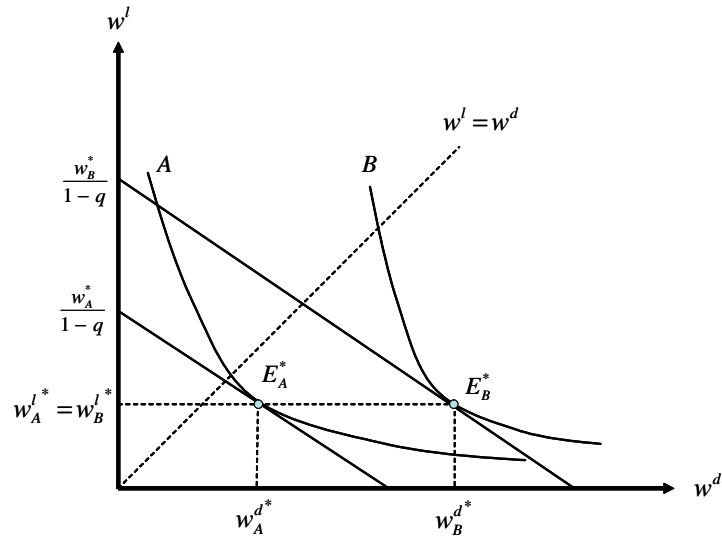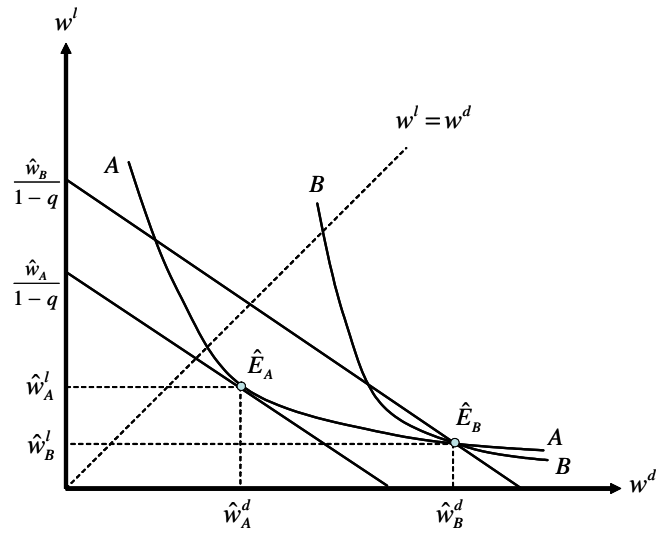
Figue 1: Purchase after type is known

Figure 2: First-best insurance

Figure 3: Second-best long term contracts