2004s-08

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Série Scientifique
Scientific Series

Montréal
Février 2004

CIRANO
Centre interuniversitaire de recherche en analyse des organisations

ISSN 1198-8177

Designing a Performance Indicator to Economize on Monopoly Subsidy

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Résumé / ***Abstract***

On montre qu'il existe un continuum de règles de subvention basées sur un indice de performance qui peuvent inciter un monopoleur à produire la quantité qui maximise le bien-être social. Avec ces règles, le gouvernement paie un montant total qui est de beaucoup inférieur à celui qu'il devrait payer dans le cas standard d'un taux d'aide constante. Le taux de subvention variable dépend de la valeur d'un stock qui reflète l'histoire de performance du monopoleur.

Mots clés : optimisation temporelle, indice de performance, règles d'aide.

We provide a continuum of subsidy rules based on a performance indicator that induce a monopoly to choose the socially optimal production level. These subsidy rules result in a reduction of the amount of subsidy paid to the monopolist compared to the standard case where a constant subsidy rate is used. The subsidy rate depends on a state variable that reflects the monopolist's history of performance. This variable depreciates over time, therefore requiring a permanent effort of the monopolist to maintain it at an optimal level. In an example with a linear demand and no production cost, the subsidy costs of inducing efficiency are reduced by almost fifty per cent.

Keywords: *monopoly, intertemporal optimization, performance indicator, subsidy rules.*

Codes JEL : C61, D42, D92, H21, L5.

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1. Introduction

According to the conventional theory, the output of a monopoly is below the socially optimal level, and therefore a per unit subsidy set at an appropriate rate is required to restore efficiency. Textbook writers often add the remark that such a subsidy is likely to be unpopular, because it would make the monopolist even richer. In this paper, we show that there exists a family of linear-in-output subsidy rules that would (i) induce the monopolist to produce at the socially optimal level, and (ii) economize on the payments to the firm. The trick is to make the subsidy rate (per unit of output) dependent on an index of the history of the monopolist's past performance.

The argument is as follows. The regulator creates a "performance index" that summarizes the monopolist's past behavior. A high value of the index indicates a history of good performance. This index is continuously updated as the monopolist's output is observed. The subsidy rate (per unit of output) paid to the monopolist at each moment is specified as an increasing function of the current level of the index. The formula for updating the index is announced from the outset, and thus the monopolist can optimally plan to build up the level of the index through its production. This index is may be regarded as an intangible asset. Technically, it is a state variable in a well defined optimal control problem. The index may also depreciate over time[1]. The regulator decides on the depreciation rule to be applied to the index. Because the subsidy rate per unit of output is made dependent on the index, the production of the monopolist at a given moment affects both (a) its profit at that moment, and (b) the future subsidy rates and thus future profits.

We show that there exists a continuum of such subsidy schemes that induce the monopolist to achieve the socially desirable level of production at each moment. Each subsidy scheme in this continuum

[1] In fact, it is as if we have turned the static monopoly into a producer of a durable good that depreciates over time. (See Malueg and Solow (1989) and Karp (1996) on durable good monopoly).

generates an infinite horizon dynamic optimization problem for the monopolist, and his optimal production path is shown to be a constant path that coincides with the socially desirable output level.

The multiplicity of efficiency-inducing subsidy schemes is a positive result from the regulator's view point. In addition to implementing the first best, the regulator could use the subsidy schemes to address other concerns such as the allocation of surplus between consumers and producers (in this case the monopolist).

In this article, we focus on linear subsidy rules for regulating a monopolist. Of course, alternative methods of regulations are available[2]. However, in general these methods require the ability to make lump sum transfers. For example, fixing a price ceiling equal to the marginal cost at the socially optimal output level would make the monopoly incur losses in the case of decreasing average cost, and thus a lump sum transfer would be required to cover such losses. The only market based method of regulation-without lump sum transfer- that achieves the efficient outcome is the use of an appropriate per unit subsidy. Our emphasis is on reducing the cost of subsidies without sacrificing efficiency. There are many reasons why such economies may be desirable. The first reason that comes to mind, at least to non-economists, is the distributional concern: the subsidies make the rich monopolist even richer. A second reason is that the subsidies must be paid for by raising taxes, which is politically unpopular[3], and economically distortionary[4]. A third reason is that governments might be bound by international treaties that restrict the extent of subsidies. We refrain from dwelling on these issues.

[2] See, for instance, Rees and Vickers (1995), Bishop et al. (1995).

[3] One is reminded of Edmund Burke's witticism: "To tax and to be liked, just as to love and to be wise, is not given to men."

[4] Revenue raising by taxes (other than lump sum taxes) are distortionary. This realization has led to the concept of "marginal cost of public funds" (see, e.g., Browning, 1976, Ballard et al. 1985 for theory and empirical measures). We do not wish to incorporate this into our model, in the interest of simplicity.

2. The basic model

2.1. A review text-book linear subsidy schedule

In this section we consider a monopolist in a static environment. The inverse demand function is $p = p(q)$ and the total cost function is $c(q)$, where q is the firm's output. We assume that

$$c(0) = 0, \qquad c'(q) > 0, \qquad c'(0) < p(0), \qquad p'(q) < 0 \tag{1}$$

and marginal revenue, $p(q) + qp'(q)$, falls as q rises. We make no restriction on the sign of $c''(q)$. Thus marginal cost can be falling, or rising. We only requires that if the marginal cost is falling, its slope at the point of intersection with the marginal revenu curve is less nagative than the slope of the marginal revenue curve. In the absence of tax or subsidy, the profit function is

$$R(q) \equiv p(q)q - c(q) \tag{2}$$

We assume that $R(q)$ is strictly concave[5]. (This does not imply that $c(q)$ is necessarily convex.) Note the assumption $c(0) = 0$, it implies that if the monopolist's output is zero, his profit is zero[6]. This in turn implies that the monopolist's maximized profit is non-negative. The monopolist maximizes profit by equating marginal revenue to marginal cost. Under *laissez-faire* (i.e., if there is no tax or subsidy) his output level, denoted by q^L, satisfies the first order condition[7]:

$$p(q^L) + q^L p'(q^L) = c'(q^L)$$

At output q^L, the monopolist's profit is $R(q^L) > 0$. The socially optimal output level is denoted by q^{so}. At q^{so}, price is equal to marginal

[5] If the profit function is not strictly concave, the problem of designing optimal tax and subsidies can be very complicated. See, for example, Guesneries and Laffont (1978), also Laffont (1987, pp. 81-83).

[6] This assumption is made for simplicity, it is straightforward to generalize the conclusions of the paper to the case where $c(0) = f$ where f represents a positive fixed cost.

[7] The strict concavity assumption on $R(q)$ implies the maximum q^L is unique.

cost:

$$p(q^{so}) = c'(q^{so}) \tag{3}$$

We assume that (3) has a unique solution.

The usual textbook prescription is to subsidize the monopolist's output at a *constant* rate s^* per unit of output, where

$$s^* = -p'(q^{so})q^{so} > 0 \tag{4}$$

In other words, s^* is the difference between the price $p(q^{so})$ and the marginal revenue $p(q^{so}) + p'(q^{so})q^{so}$ (both evaluated at the socially optimal output level).

To show that the monopolist who faces this subsidy rule would choose q^{so}, we note that, given any constant subsidy rate s, he chooses q to maximize the profit function

$$\pi(q, s) \equiv p(q)q - c(q) + sq \equiv R(q) - sq \tag{5}$$

Thus, with $s = s^*$, the first order condition of the monopolist's problem is

$$p(q) + qp'(q) - c'(q) + s^* = 0 \tag{6}$$

Clearly, q^{so}, as defined by (3), satisfies the monopolist's net-profit-maximizing condition (6) if s^* is given by (4). By the assumption of strict concavity of $R\,(q)$ we have

$$\pi(q^{so}, s^*) > \pi(q, s^*) \text{ for all } q \neq q^{so} \tag{7}$$

We assume that demand and cost functions are stationary over time, and that there is no capital accumulation. The socially optimal output level is q^{so} at each point of time, and the time-independent subsidy rate s^* given by (4) induces the monopolist to produce this output level at each point of time. To summarize, in a stationary environment, the government can ensure efficiency by offering the monopolist a *linear subsidy schedule*

$$S(q) = s^*q \tag{8}$$

where s^* is given by (4). The monopolist, taking this schedule as given, will choose the output level q^{so}. A major problem with this subsidy rule is that it does not seem "fair" : the subsidy makes the monopolist even richer. In the next subsection, we propose a class of simple linear subsidy rules that do not enrich the monopolist as much and still ensure that the efficient output level q^{so} is produced.

2.2. Performance-related linear subsidy rules

The government can construct an index that represents a measure of the cumulative performance of the monopolist. Let the "state variable" $X(t)$ be the value taken by that index at time t, where $X(0) = X_0 > 0$. Here, X_0 is to be chosen by the government at the beginning of the game. The rate of change of $X(t)$ is given by the differential equation

$$\dot{X}(t) = q(t) - \delta X(t) \tag{9}$$

where $\delta > 0$ is the rate of depreciation of the index (also to be chosen by the government). The government announces to the monopolist, at the beginning of the game, the differential equation (9), and its chosen constants δ and X_0. The government makes the binding commitment that after time $t_0 = 0$, it will not change the chosen constants. Note that since $q(t) \geq 0$ and $X_0 > 0$, the index $X(t)$ is always positive at any finite t.

A possible interpretation of (9) is as follows. Suppose the government sets $X(0) = \frac{q^{so}}{\delta}$, then as soon as the monopolist's output level falls short of q^{so}, the index $X(t)$ will decrease, indicating that the firm has "misbehaved", and, with a well chosen subsidy rule, such misdeed will entail a decrease in the subsidy rate. If the subsidy rule is well designed the implicit threat of a decrease in the subsidy rate will induce the firm's optimal path to coincide with the socially desirable production path. Alternatively, the government may set $X(0) < \frac{q^{so}}{\delta}$. Then a production of q^{so} will raise the value of the index variable $X(t)$ upon which the subsidy rule is tuned. This behavior should be encouraged by an anticipation of a higher future subsidy rate, as prescribed by a subsidy formula. If the prospect of a more favorable subsidy rate is fine-tuned by the government, the firm's optimal path will again coincide with the socially desirable production path.

The government announces at the outset a subsidy rule $S(X,q) = \sigma(X)q$, where $\sigma(X)$ is a function defined for all $X \geq 0$. Note that the rule $S(X,q) = \sigma(X)q$ is linear in q and non-linear in X. At time t, the monopolist will receive a net subsidy rate $\sigma(X(t))$ per unit of output, i.e., he is given the subsidy amount $\sigma(X(t))q(t)$ if he produces $q(t)$ when the index of his past behavior takes the value $X(t)$. In the next section, a concrete form of the function $\sigma(X(t))$ will be proposed, and its efficiency implication investigated.

3. Achieving efficiency by rules based on a performance index

3.1. Designing a class of efficiency-inducing subsidy rules

We claim in this paper that the government can induce efficiency with a class of performance-related subsidy rules that are linear in output. In particular, we specify that $\sigma(X(t))$ takes the form

$$\sigma(X(t)) = s^* - KX(t)^{-\beta} \text{ and } X(0) = X_0 > 0 \tag{10}$$

where K is a positive number, r is the interest rate, $\beta \equiv \frac{r}{\delta} + 1 > 1$, and s^* is given by (4). We will show that efficient production can be ensured by appropriate choices of δ,K, and X_0. The value of K has to be within a certain range to ensure that the monopolist earns positive profits, while δ and X_0 must be chosen to ensure that the monopolist does not exit the market in finite time; this will become clear in what follows.

The class of rules specified by equation (10) has the property that if the monopolist builds up the level of the index, he will get a higher subsidy rate, i.e.,

$$\frac{d\sigma(X)}{dX} = \beta K X^{-\beta-1} > 0 \tag{11}$$

Moreover note that for a given positive K the subsidy rule (10) results in a negative subsidy rate, a tax, when the index variable $X(t)$ is below a certain positive threshold $\bar{X} \equiv \left(\frac{K}{s^*}\right)^{\frac{1}{\beta}}$. Thus if the firm adopts a production path that drives the index $X(t)$ to a level below $\bar{X}$ the subsidy changes into a tax.

The firm takes as given (i) the depreciation rule (9) and the depreciation rate δ, (ii) the subsidy rule (10) and the constant K, and (iii) the initial value X_0. Since $s = \sigma(X) = s^* - KX^{-\beta}$, we can write

$$\pi(q,s) = p(q)q - c(q) + \sigma(X)q \equiv R(q) + \sigma(X)q \equiv \widehat{\pi}(q,X)$$

The firm chooses a time path $q(t) \geq 0$ and $T \geq 0$ to maximize the discounted stream of profit:

$$\max_{q\geq 0} \int_0^T \widehat{\pi}(q,X)e^{-rt}dt \tag{12}$$

subject to $\dot{X} = q - \delta X$, $q \geq 0$, and $X(0) = X_0$. Note that the firm can ensure that the integral is non-negative: by choosing $q(t) = 0$, profit is zero at time t. Also, if the subsidy rule is not well designed, the firm may choose to make a quick profit over some finite time interval $[0,T]$ and exit the market at time T (that is, $q(t) = 0$ for $t > T$) to avoid future taxes. We call such a strategy the "*hit and run*" strategy. In what follows we show that such a strategy will not be chosen by the firm if the parameters K, δ, and X_0 are well chosen.

3.2. The main results

We now state and prove our main results, Propositions 1 and 2.

Proposition 1: Assume the monopolist does not choose the *"hit and run"* strategy. The per unit subsidy rate $\sigma(X(t))$ given by (10), where $K > 0$ and where $X(t)$ satisfies the differential equation (9) ensures that the monopolist will always produce the socially optimal output level q^{so}, provided that (K, δ, X_0) satisfy the following condition

$$0 \leq K \leq Min\{K_0, K_1\} \tag{13}$$

where $K_0 \equiv \frac{\pi^n(q^{so},s^*)}{q^{so}}\left(\frac{q^{so}}{\delta}\right)^{\beta}$ and $K_1 \equiv \frac{\pi^n(q^{so},s^*)}{q^{so}}X_0^{\beta}$ and where $\beta = 1+\frac{r}{\delta}$.

Proof: *We establish that there is a saddle-path that leads to a unique steady state.*

Let $\psi(t)$ denote the co-state variable. The Hamiltonian associated with the monopolist's problem is

$$H = R(q) + s^*q - KX^{-\beta}q + \psi\left[q - \delta X\right]$$

where

$$R(q) = p(q)q - c(q)$$

The Hamiltonian is concave in the state variable X because $K \geq 0$. It is strictly concave in the control variable q. Note that by definition of s^*,

$$R'(q^{so}) = -s^* \tag{14}$$

The maximum principle gives

(i) the maximality condition: given $\psi(t)$ and $X(t)$, the monopolist's control variable $q(t)$ maximizes the Hamiltonian, thus

$$\frac{\partial H}{\partial q} = R'(q) + s^* + \psi - KX^{-\beta} \leq 0,\ q \geq 0, \text{ and } q\frac{\partial H}{\partial q} = 0 \tag{15}$$

(ii) the adjoint equation:

$$\dot{\psi} = (r+\delta)\psi - \beta KX^{-\beta-1}q \tag{16}$$

(iii) the transition equation:

$$\dot{X} = q - \delta X \tag{17}$$

In addition, the transversality conditions are as follows. If $T = \infty$, we have

$$\lim_{t\to\infty} e^{-rt}\psi(t)X(t) = 0 \tag{18}$$

$$\lim_{t\to\infty} e^{-rt}\psi(t) \geq 0 \tag{19}$$

and if T is finite, we have

$$\lim_{t\to T} \psi(t) = 0 \tag{20}$$

(since $X(T)$ is free, and is positive given that $X_0 > 0$), and

$$\lim_{t\to T} H(t) = 0. \tag{21}$$

We now show that, given any $X_0 > 0$, we can construct the time path of the triplet (X, ψ, q) that satisfies conditions (15)-(19) above. Let

$$X(t) = \left(X_0 - \frac{q^{so}}{\delta}\right) e^{-\delta t} + \frac{q^{so}}{\delta} \tag{22}$$

$$\psi(t) = KX(t)^{-\beta} = K\left[\left(X_0 - \frac{q^{so}}{\delta}\right) e^{-\delta t} + \frac{q^{so}}{\delta}\right]^{-\beta} \tag{23}$$

$$q(t) = q^{so} \tag{24}$$

Clearly, using (22), we can verify that (17) is satisfied:

$$q^{so} - \delta X = q^{so} - \delta\left[\left(X_0 - \frac{q^{so}}{\delta}\right) e^{-\delta t} + \frac{q^{so}}{\delta}\right] = \delta\left(X_0 - \frac{q^{so}}{\delta}\right) e^{-\delta t} = \dot{X}$$

Using (23), we can verify that (16) is satisfied:

$$\dot{\psi} = -\beta K X^{\beta-1}\dot{X} = -\beta K X^{\beta-1}\left[q^{so} - \delta X\right] = \beta\delta K X^{-\beta} - \beta K X^{\beta-1} q^{so}$$

$$= \beta\delta\psi - \beta K X^{\beta-1} q^{so}$$

$$= (r+\delta)\psi - \beta K X^{-\beta-1} q$$

Finally, using $\psi(t) = KX(t)^{-\beta}$ it is easy to see that q^{so} is the solution of

$$\max_q R(q) + (s^* - KX^{-\beta})q + \psi(q - \delta X) = \max_q R(q) + (s^* - KX^{-\beta} + KX^{-\beta})q - \psi\delta X$$

in view of (14).

The time path of the triplet (X, ψ, q) constructed above converges to the steady-state triplet

$$(X_\infty, \psi_\infty, q_\infty) = \left(\frac{q^{so}}{\delta}, K \left(\frac{q^{so}}{\delta} \right)^\beta, q^{so} \right) > (0,0,0)$$

In fact, it can be shown that, in the space (ψ, X), the constructed path is the unique path that leads to the steady state pair (ψ_∞, X_∞)

$$(\psi_\infty, X_\infty) = \left(\frac{q^{so}}{\delta}, K \left(\frac{q^{so}}{\delta} \right)^\beta \right)$$

This is done by showing that the phase-diagram displays the saddle-point property (please see Appendix A).

The condition (13) is a sufficient condition for the monopolist to have an incentive to participate in the program, i.e, the discounted sum of the monopolist's instantaneous profits is non-negative. This is shown in Appendix B ■

Unfortunately, the result in Proposition 1 is only "half" a good news to the regulator. The possibility that the monopolist might choose the "hit and run" strategy is real. This latter strategy will be increasingly attractive when the subsidy rule offered by the regulator leaves the monopoly with a very small subsidy rate in the long-run. This is illustrated with the following example.

Example (Hit and run):

Let $p(q) = 1 - q$, $c(q) = 0$ and $r = 0.1$. Then $q^{so} = 1$ and $s^* = 1$ moreover $R(q^{so}) = 0$ and $\pi(q^{so}, s^*) = 1$. Consider the following subsidy rule

$$\sigma(X) = s^* - KX^{-\beta} = 1 - KX^{-2}$$

where $\delta = 0.1$ (which yields $\beta = 2$), and X is an index variable that follows (9) with $X(0) = X_\infty \equiv \frac{q^{so}}{\delta} = 10$. Such a subsidy rule belongs to the family of subsidy rules (10) moreover given the choice of the initial value of the index variable the subsidy rate will be constant over time if the monopolist produces the socially desirable production rate. Let $\bar{s}$ denote the corresponding subsidy rate, the value of $\bar{s}$ will be determined by the choice of the parameter K :

$$\bar{s} = 1 - \frac{K}{(10)^2}$$

The value of $\bar{s}$ can be made arbitrarily small (close to zero) by choosing K close to 100. In the limit case where the parameter K is set at 100 we have $\bar{s} = 0$ and choosing $q(t) = q^{so} = 1$ yields zero profits to the monopolist for all $t \geq 0$. But clearly, the monopolist can do better by setting $q(t) = 1/2$, thus earning positive profit at time $t = 0$. The next instant, $X(t)$ will fall, thus $(1 - 100X(t)^{-2})q(t) < 0$, i.e. he will have to pay a tax. But since $X(t)$ remains close to X_0 for $t \in (0, \varepsilon)$ for some small ε, he continues to earn positive profit by choosing $q(t)$ close to $1/2$. After a while, he quits the industry, and his accumulated profit is positive.

In the following proposition, we give a sufficient condition on the parameters of the subsidy rule that guarantees that the monopolist will never choose the "hit and run" strategy and will produce the socially desirable output level.

Proposition 2: The per unit subsidy rate $\sigma(X(t))$ given by (10), where $K > 0$ and where $X(t)$ satisfies the differential equation (9) ensures that the monopolist will always produce the socially optimal output level q^{so}, provided that (K, δ, X_0) satisfy the following condition

$$K < \left(R'(\delta X_0) + s^*\right) X_0^{\beta} \tag{25}$$

where

$$\beta \equiv \frac{r}{\delta} + 1 > 1.$$

Proof:

Part 1: Ensuring that the monopolist does not adopt the "hit and run" strategy.

We now show that condition (25) implies that the hit and run strategy will not be chosen.

(i) We first show that the following condition is sufficient to ensure that the hit and run strategy will not be chosen:

$$R'(0) + s^* > KX(t)^{-\beta} \tag{26}$$

where $X(t)$ is the path of the index variable along the monopolist's optimal production path.

(ii) We then show that the condition (25) is sufficient for the condition (26) to hold for all $t > 0$.

(i) If the monopolist finds it optimal to adopt the "hit and run" strategy, then at the "exit time" T, we have, from (20) and (21),

$$\lim_{t \to T} q(t) = q(T) = 0 \tag{27}$$

This condition in turn implies, via (15),

$$R'(0) + s^* - KX(T)^{-\beta} \leq 0 \tag{28}$$

In order for the monopolist not to adopt the hit and run strategy, we want to ensure that along the monopolist's optimal production path, we *never* have

$$R'(0) + s^* - KX(t)^{-\beta} \leq 0 \tag{29}$$

Clearly, a sufficient condition for this non-occurrence is (26), where $X(t)$ is the path of the index variable along the monopolist's optimal production path. This completes (i).

(ii) We now show that condition (25) is a sufficient condition for (26) to hold.

Since $R'(0) > R'(\delta X_0)$ for any positive δX_0 by strict concavity, if condition (25) holds we have

$$R'(0) + s^* > R'(\delta X_0) + s^* > KX_0^{-\beta} \tag{30}$$

We now proceed to show that condition (25) will guarantee that along the optimal production path chosen by the monopolist the index variable $X(t)$ will remain above X_0. This would therefore complete

(ii). To do this, it is convenient to introduce the concept of a myopic monopolist. A myopic monopolist only cares about the instantaneous profit. Thus, for a given X_0, a myopic monopolist will, at time $t = 0$, choose an output level that maximizes $R(q) + \sigma(X_0)q$. This gives the (myopic) output level q^m that satisfies the first order condition

$$R'(q^m) = -s^* + KX_0^{-\beta}$$

Here the superscript m stands for "myopic".

This first order condition determines q^m as function[8] of X_0 (for given K, δ):

$$q^m = (R')^{-1}\left[-s^* + KX_0^{-\beta}\right] = q^m(X_0; K, \delta)$$

Clearly, a non-myopic monopolist never produces at time $t = 0$ an output level less than $q^m(X_0; K, \delta)$.The reason is that if he does, his current profit will be lower than $\pi^m \equiv R(q^m(X_0; K, \delta)) + \sigma(X_0)q^m(X_0; K, \delta)$, and his future profit will also be lower than π^m as well because the lower output will result in a lower stock X in the future, and hence a lower future subsidy rate.

Now let us choose X_0 (and (K, δ)) so that

$$\delta X_0 < q^m(X_0; K, \delta) \tag{32}$$

This choice ensures that $\dot{X}$ will be positive when $X = X_0$. It follows that if X_0 satisfies condition (32), the (non-myopic) monopolist will never run X to a level below X_0. Thus, given (32), we have $X(t) > X_0$ always. We now show that condition (25) ensures that condition (32) is satisfied.

[8]Note that for $q^m > 0$, we must have $-s^* + KX_0^{-\beta} < R'(0)$, or

$$R'(0) + s^* > KX_0^{-\beta} \tag{31}$$

But this requirement is always satisfied when condition (25) is met, see inequality (30).

Recall that $R'(.)$ is a decreasing function. Therefore the inequality (32) holds if and only if

$$R'(\delta X_0) > R'(q^m(X_0; K, \delta))$$

i.e., iff

$$R'(\delta X_0) > -s^* + K X_0^{-\beta}$$

i.e., iff (25) holds. Thus condition (25) ensures that $X(t) > X_0$ for all $t > 0$.

To summarize, condition (25) ensures (30) and that $X(t) > X_0$ for all $t > 0$, and hence ensures that inequality (26) holds, which is a sufficient condition for the hit-and-run strategy to be non-optimal for the monopolist.

This completes Part 1.

Part 2: Part 1 and Proposition 1 complete the proof ■

Remark 1: Note that condition (25) guarantees that the hit and run strategy is not optimal, therefore it is never optimal to "run" at any time $t \geq 0$ and by the same token guarantees the participation of the monopolist to the program.

Remark 2: The set of triplets (K, δ, X_0) that satisfy condition (25) with $K > 0$ is non-empty. In particular for any $X_0 < \frac{q^{so}}{\delta}$ we have $R'(\delta X_0) + s^* > 0$ and therefore condition (25) yields $0 < K < \bar{K}$

$$0 < K < \bar{K} \tag{33}$$

where

$$\bar{K} \equiv (R'(\delta X_0) + s^*) X_0^{\beta} > 0.$$

3.3. Interpretations

Condition (32) can be interpreted as follows. For a given pair (δ,X_0) the variable K controls for the difference between the actual subsidy rate $s(t) = \sigma(X(t))$ and standard (textbook) subsidy rate s^*. For a given pair (δ,X_0), the higher the value of K the bigger are the savings achieved by the regulator by using the subsidy rate $s(t)$ instead of the constant subsidy rate s^*. Condition (32) states that there is an upper bound on the level of these savings that ensures that the firm will not opt for the hit and run strategy.

To interpret condition (25) it is shown in appendix C that it can be rewritten as

$$s^* - \left(\frac{(R')^{-1}(-s_0)}{q^{so}}\right)^{\left(\frac{r}{\delta}+1\right)} (s^* - s_0) < s_\infty \tag{34}$$

where s_0 denotes the initial subsidy rate at time $t = 0$ and s_∞ the subsidy rate at the steady state, i.e.,

$$s_0 = s^* - KX_0^{-\beta} \text{ and } s_\infty = s^* - KX_\infty^{-\beta}$$

where $X_\infty = (1/\delta)q^{so}$. Condition (34) stipulates that for a given δ and initial subsidy rate s_0 there is a minimum subsidy rate that the regulator must provide at the steady state that ensures that the monopoly will not choose the hit and run strategy .

3.4. A numerical example

Let $P(q) = 1 - q$ and $c(q) = 0$. Then $q^{so} = 1$ and $s^* = 1$. Note that the use of a constant per unit subsidy $s^* = 1$ requires that at each moment a total amount of subsidy of \$1 to generate a social surplus of \$0.5.Let $r = 0.1$. If the regulator announces a subsidy scheme

$$\sigma(X) = s^* - KX^{-2}$$

where $\delta = 0.1$, $K = 20$ and $X(0) = X_0 = 2\sqrt{5}$ then the monopolist will produce at each moment the socially desirable output $q^{so} = 1$. The amount of subsidy per unit of output will monotonically increase from 0 (at time $t_0 = 0$) to 0.80 at the steady state. Along the optimal production path the variable $X(t)$ is given by

$$X(t) = \left(2\sqrt{5} - 10\right) e^{-0.1t} + 10$$

and the subsidy rate is

$$s(t) = 1 - \frac{20}{\left(\left(2\sqrt{5} - 10\right) e^{-0.1t} + 10\right)^2}$$

The steady state subsidy rate with this subsidy rule is 20% below the standard (textbook) subsidy rate s^* . The present value of the overall savings is

$$\int_0^\infty \frac{20}{\left(\left(2\sqrt{5} - 10\right) e^{-0.1t} + 10\right)^2} e^{-0.1t} dt = 4.4721$$

If the static subsidy rate was used the present value of the stream of subsidies is

$$\int_0^\infty e^{-0.1t} dt = 10$$

Using the dynamic subsidy rule above results in a decrease by more than 44% in the present value of the subsidies given to the monopolist.

4. Conclusion

We have provided a continuum of subsidy rules that induce a monopoly to choose the socially optimal production level. These subsidy rules result in a reduction of the amount of subsidy transferred to the monopolist compared to the standard case where a constant subsidy rate is used. The subsidy rules depend on a state variable that reflects the monopolist's past performance and that depreciates over time, therefore requiring a permanent effort of the monopolist to maintain it at an optimal level. The use of such subsidy rules can achieve significant cost savings for the government compared to the use of a standard constant subsidy rate. We have provided a numerical example with a linear demand and no production cost, in which the subsidy costs of inducing efficiency are reduced by almost fifty percent.

The proposed class of subsidy rules can be used in different ways depending on the priorities of the regulator. A possible objective could be to achieve the socially optimal output level with the smallest amount of initial subsidy. This could be the case if the regulator initially faces severe budget constraints. Alternatively the objective could be achieving the efficient output level while meeting a target level of subsidy at the steady state, perhaps because a ceiling on long-run subsidies is imposed by some multilateral agreement.

It is possible to extend our results to the case of oligopolists that play dynamic games as dynamic Cournot rivals; see Benchekroun and Long (1998) for a possible framework for dynamic symmetric oligopoly[9]. Another worthwhile extension is to consider the case where the government cannot make long term commitment to the parameters of the subsidy rules. In such cases, a feedback equilibrium of a differential game would have to sought[10].

Acknowledgements: We thank SSHRC and FCAR for financial supports, and Kim Long and Koji Shimomura for useful discussions and references.

[9] In an asymmetric oligopoly, complications may arise; see Long and Soubeyran (2003).

[10] For examples of feedback equilibrium see Shimomura (1991) and Dockner et al. (2000).

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Appendix A

From (15), we know that

(i) If

$$\psi \leq KX^{-\beta} - R'(0) - s^* \tag{35}$$

then the optimal q is zero.

(ii) If

$$\psi > KX^{-\beta} - R'(0) - s^* \tag{36}$$

then the optimal q is positive and satisfies

$$R'(q) + s^* + \psi - KX^{-\beta} = 0$$

We construct a phase diagram in the space (X, ψ). Note that since $q \geq 0$ and $X_0 > 0$, X can never become negative. It follows that in the phase diagram in the space (X, ψ), there are two regions. Region A is the set of points (X, ψ) such that (35) holds, and region B is the set of points (X, ψ) such that (36) holds. The upper boundary of region A is a downward sloping curve

$$\psi = KX^{-\beta} - R'(0) - s^*$$

Along this curve, as X tends to zero, ψ tends to infinity, and as X tends to infinity, ψ tends the the negative number $-R'(0) - s^*$; at $\psi = 0$, $X = \bar{X}$ where

$$\bar{X} = \left[\frac{R'(0) + s^*}{K}\right]^{1/\beta}$$

In region B, we can write

$$q = q(X, \psi)$$

where

$$\frac{\partial q}{\partial X} = \frac{H_{qX}}{-H_{qq}} = -\phi'(Z)\beta K X^{-\beta-1} > 0$$

$$\frac{\partial q}{\partial \psi} = \frac{H_{q\psi}}{-H_{qq}} = \frac{1}{-R''} = -\phi'(Z) > 0$$

In fact, in region B, where $KX^{-\beta} - s^* - \psi < R'(0)$,

$$q(X, \psi) = R^{'-1}(KX^{-\beta} - s^* - \psi) \tag{37}$$

Define

$$\phi(.) = R^{'-1}(.)$$

and

$$Z \equiv KX^{-\beta} - s^* - \psi$$

We now show that there exists a unique steady state in region B. Denote the steady state pair by (X_∞, ψ_∞). Then, setting $\dot{\psi} = 0$, we get

$$\psi_\infty = \left(\frac{\beta}{r+\delta}\right) KX_\infty^{-\beta-1} q(X_\infty, \psi_\infty) \tag{38}$$

And setting $\dot{X} = 0$, we get

$$q(X_\infty, \psi_\infty) = \delta X_\infty \tag{39}$$

Substituting (39) into (38), we get

$$\psi_\infty = KX_\infty^{-\beta} \tag{40}$$

Using (37), we can write (39) as

$$KX_\infty^{-\beta} - s^* - \psi_\infty = R'(\delta X_\infty) \tag{41}$$

Substituting (40) into (41) we get

$$-s^* = R'(\delta X_\infty) \tag{42}$$

Comparing (14) with (42) we deduce that

$$\delta X_\infty = q^{so} \tag{43}$$

It follows that the unique steady state is

$$(X_\infty, \psi_\infty) = \left(\frac{q^{so}}{\delta}, \frac{K}{(q^{so}/\delta)^\beta}\right) \tag{44}$$

Now we show that the steady state has the saddlepoint property. Define

$$M(\psi, X) = (r+\delta)\psi - \beta K X^{-\beta-1} q(\psi, X)$$

and

$$N(\psi, X) = q(\psi, X) - \delta X$$

We must show that the matrix

$$\begin{bmatrix} M_\psi & M_X \\ N_\psi & N_X \end{bmatrix}$$

(evaluated at the steady state) has a negative determinant (.i.e., has one negative root and one positive root).

$$M_\psi = r + \delta + \beta K X^{-\beta-1}\phi'(Z)$$

$$\begin{aligned} M_X &= \beta(1+\beta)KX^{-\beta-2}q + \beta^2 K^2 X^{-2\beta-2}\phi'(Z) \\ &= \delta\beta(1+\beta)KX^{-\beta-1} + \beta^2 K^2 X^{-2\beta-2}\phi'(Z) \end{aligned}$$

$$N_\psi = -\phi'(Z)$$

$$N_X = -\phi'(Z)\beta K X^{-\beta-1} - \delta$$

Thus

$$\begin{aligned} M_\psi N_X &= -\left(r + \delta + \beta K X^{-\beta-1}\phi'(Z)\right)\left(\phi'(Z)\beta K X^{-\beta-1} + \delta\right) \\ &= -r\left(\phi'(Z)\beta K X^{-\beta-1} + \delta\right) - \left(\delta + \beta K X^{-\beta-1}\phi'(Z)\right)^2 \\ M_\psi N_X &= -\left[\phi'(Z)\beta K X^{-\beta-1} + \delta\right]^2 - r\delta - r\phi'(Z)\beta K X^{-\beta-1} \\ &= -\left[\phi'(Z)\beta K X^{-\beta-1}\right]^2 - \delta^2 - r\delta - \{2\delta + r\}\beta\phi'(Z)KX^{-\beta-1} \end{aligned}$$

and

$$N_\psi M_X = (-\phi')\delta\beta(\beta+1)KX^{-\beta-1} - \left[\phi'(Z)\beta K X^{-\beta-1}\right]^2$$

Therefore,

$$M_\psi N_X - N_\psi M_X = -\left[\phi'(Z)\beta K X^{-\beta-1}\right]^2 - \delta^2 - r\delta$$

$$-\{2\delta+r\}\beta\phi'(Z)KX^{-\beta-1}+\phi'\delta\beta(\beta+1)KX^{-\beta-1}+\left[\phi'(Z)\beta KX^{-\beta-1}\right]^2$$

$$M_\psi N_X - N_\psi M_X = -\delta^2 - r\delta - \{2\delta + r - \delta(\beta+1)\}\beta\phi'(Z)KX^{-\beta-1}$$

$$M_\psi N_X - N_\psi M_X = -\delta^2 - r\delta - \{\delta + r - \delta\beta\}\beta\phi'(Z)KX^{-\beta-1}$$

recalling that $\beta\delta = r + \delta$,

$$M_\psi N_X - N_\psi M_X = -\delta^2 - r\delta < 0$$

We have thus shown that the steady state $(X_\infty, \psi_\infty) = \left(\frac{q^{so}}{\delta}, \frac{K}{(q^{so}/\delta)^\beta}\right)$ has the saddlepoint property ■

Appendix B:

Recall that by definition

$$\pi(q^{so}, s^*) = R(q^{so}) + s^* q^{so}$$

Thus

$$\pi(q^{so}, X) = \pi(q^{so}, s^*) - KX^{-\beta}q^{so}$$

It follows that the monopolist's present value of the profit stream is non-negative if and only if

$$\int_0^\infty e^{-rt}\left[\pi(q^{so}, s^*) - KX^{-\beta}q^{so}\right]dt \geq 0$$

i.e.

$$\int_0^\infty e^{-rt}q^{so}\left\{\frac{\pi(q^{so}, s^*)}{q^{so}} - KX(t)^{-\beta}\right\}dt \geq 0$$

i.e.,

$$\int_0^\infty e^{-rt}q^{so}\left\{\frac{\pi(q^{so}, s^*)}{q^{so}} - K\left[\left(X_0 - \frac{q^{so}}{\delta}\right)e^{-\delta t} + \frac{q^{so}}{\delta}\right]^{-\beta}\right\}dt \geq 0 \quad (45)$$

where

$$\frac{\pi^n(q^{so}, s^*)}{q^{so}} > 0$$

is the average profit of the text-book monopolist (with the text-book subsidy).

It follows that if $X_0 > \frac{q^{so}}{\delta}$, then (recalling that $\beta > 0$) a sufficient condition for (45) to hold is

$$\frac{\pi(q^{so}, s^*)}{q^{so}} \geq K \left(\frac{q^{so}}{\delta}\right)^{-\beta}$$

i.e.

$$K \leq K_0 \equiv \frac{\pi(q^{so}, s^*)}{q^{so}} \left(\frac{q^{so}}{\delta}\right)^{\beta} \tag{46}$$

If $X_0 \leq \frac{q^{so}}{\delta}$, then a sufficient condition for (45) to hold is

$$K \leq K_1 \equiv \frac{\pi(q^{so}, s^*)}{q^{so}} X_0^{\beta} \blacksquare \tag{47}$$

Appendix C:

Solving the differential equation (9) along the optimal production path and using

$$X\left(0\right) = X_0$$

yields

$$X\left(t\right) = \left(X_0 - \frac{q^{so}}{\delta}\right) e^{-\delta t} + \frac{q^{so}}{\delta}$$

If we denote the initial subsidy level s_0 and the steady state subsidy level s_∞ we have

$$s^* - s_0 = K X_0^{-\left(\frac{r}{\delta}+1\right)}$$

and

$$s^* - s_\infty = K X_\infty^{-\left(\frac{r}{\delta}+1\right)}$$

combining these two equations yields

$$\frac{s^* - s_0}{s^* - s_\infty} = \frac{X_0^{-\left(\frac{r}{\delta}+1\right)}}{X_\infty^{-\left(\frac{r}{\delta}+1\right)}}$$

so

$$X_0 = \left(\frac{s^* - s_0}{s^* - s_\infty}\right)^{-\frac{1}{\left(\frac{r}{\delta}+1\right)}} X_\infty$$

The condition (25) that guarantees that the monopolist will not choose the hit and run strategy is

$$R'\left(\delta X_0\right) > -s_0$$

that is using the strict concavity concavity of the function $R(.)$ we have

$$\delta X_0 < \left(R'\right)^{-1}\left(-s_0\right)$$

or

$$\delta\left(\frac{s^* - s_0}{s^* - s_\infty}\right)^{-\frac{1}{\left(\frac{r}{\delta}+1\right)}} X_\infty < \left(R'\right)^{-1}\left(-s_0\right) \tag{48}$$

So for $s_0 < s^*$, condition (48) yields after manipulation

$$s^* - \left(\frac{\left(R'\right)^{-1}\left(-s_0\right)}{q^{so}}\right)^{\left(\frac{r}{\delta}+1\right)}\left(s^* - s_0\right) < s_\infty \blacksquare$$