Série Scientifique
Scientific Series

N° 95s-20

PREDICTIVE TESTS FOR STRUCTURAL CHANGE WITH UNKNOWN BREAKPOINT

Eric Ghysels, Alain Guay, Alastair Hall

Montréal
Mars 1995

CIRANO

ISSN 1198-8177

Predictive Tests for Structural Change with Unknown Breakpoint*

Eric Ghysels[†], *Alain Guay*[‡], *Alastair Hall*[+]

Abstract / Résumé

This paper considers predictive tests for structural change in models estimated via Generalized Method of Moments. Our analysis extends earlier work by Ghysels and Hall (1990a) by allowing for the instability to occur at an unknown point in the sample. We analyze various statistics based on continuous mappings of the sequence of predictive tests calculated for a set of possible breakpoints in the sample. The limiting distribution of these statistics is derived under both the null hypothesis and local alternatives. Percentiles are reported for the distribution under the null. A side product of our analysis is that we can illuminate the power properties of the predictive test and also compare its properties to those of the Wald, LR and LM tests for parameter variation. We study those power properties both via local asymptotic analysis and Monte Carlo.

Cette étude généralise la procédure proposée par Ghysels et Hall (1990a) pour tester le changement structurel pour des modèles estimés par la méthode de moments généralisée. Nous ne supposons plus le point de rupture comme étant connu et proposons plusieurs statistiques prédictives avec changement structurel inconnu. Comme les distributions asymptotiques sont non standard, nous fournissons les valeurs critiques. Finalement, nous étudions la puissance des tests et faisons des comparaisons avec des tests du type Wald, LM et LR.

Key Words: moment conditions, structural change, GMM.
Mots-clés : conditions de moments, changement structurel, méthode des moments généralisée.

---

* The first author would like to acknowledge the financial support of the Social Sciences and Humanities Research Council of Canada and the Fonds FCAR. We would like to thank J. Galbraith and D. Pappell for comments on the paper.

[†] Centre de recherche et développement en économique (C.R.D.E.), Université de Montréal, and CIRANO.

[‡] Bank of Canada.

[+] North Carolina State University.

1. INTRODUCTION

There is a perennial interest in testing whether parametric econometric models are invariant through time. The vast literature on testing for structural change has paid most attention to the linear regression model while only a handful of tests are available for nonlinear dynamic single and multiple equation models. Advances in econometric theory over the last decade have cleared the horizon to address the more challenging task of testing for structural change in dynamic nonlinear models. Andrews and Fair (1988) considered the problem of testing parameter constancy when the sample can be split at some known breakpoint into two subsamples governed by parameter values equal under the null but different under the alternative. They proposed Wald, likelihood ratio-type (LR) and Lagrange multiplier-type (LM) tests and showed that under some weak regularity conditions such tests have standard asymptotic distributions. These developments were made in the context of the Generalized Method of Moments (GMM) estimator which in its generic form covers a large class of estimators for a wide variety of nonlinear dynamic models.[1] Ghysels and Hall (1990a) proposed a predictive test for structural stability. In this approach, parameter estimates from a first subsample are used to evaluate moment conditions in the second subsample. The essential idea behind such tests is that the predicted moment conditions should be statistically insignificantly different from zero when there is no structural change. The null and alternative hypotheses of these tests are formulated in terms of the structural stability of the moment conditions, rather than the parameter variation employed by Andrews and Fair (1988), and so have different power properties to Wald, LR and LM tests. Intuition suggests that neither type of test dominates in all situations and so it is of interest to apply both in applications.

One drawback with all these tests is that they assume the breakpoint is known. While in some cases this may be reasonable, such as exploring the impact of specific economic events like the 1973 oil shock, in many cases one may wish to test for structural stability over all points in the sample. Andrews (1993) proposed a procedure for testing parameter stability when the breakpoint is unknown. His strategy is to consider the Wald tests, say, for a set of possible breakpoints and base inference on the supremum of these tests. Andrews shows that this "Sup-Wald" statistic converges to

---

1 See Hansen (1982), Gallant and White (1988) and the recent surveys by Hall (1993), Newey (1993) and Ogaki (1993) for detailed discussion of GMM estimation. It should, parenthetically, be noted that the LR-type test is only appropriate under more restrictive conditions which are not satisfied in many GMM applications.

the supremum square of a standardized tied down Bessel process under the null hypothesis of parameter constancy.[2] This extension takes the statistical theory outside the conventional framework in which the Wald test is asymptotically optimal because a nuisance parameter, the breakpoint, is not present under the null hypothesis. Therefore the Sup-Wald statistic has no known optimality properties. Andrews and Ploberger (1994) and Sowell (1994) have considered the construction of optimal tests for parameter variation which leads to tests of an average exponential form.

In this paper, we adopt a similar approach to developing predictive tests for structural change at an unknown breakpoint. Using results from Andrews (1993) and Sowell (1993) it is shown that under the null hypothesis the predictive test converges to the sum of the square of a standardized tied down Bessel process and the square of a standardized Bessel process. This structure reflects a decomposition of the statistic into a test for parameter variation and a test of the stability of the overidentifying restrictions. This enables us to clarify the relationship between the predictive and Wald tests. We also derive the distribution of the predictive test under local alternatives which helps to further illuminate its properties. These results are used to characterize the asymptotic behavior of a sup-predictive test and versions of the statistic based on the average exponential form analysed by Andrews and Ploberger (1994) and Sowell (1994). In the special case where the number of moment conditions equals the number of parameters, the various predictive tests proposed in this paper are asymptotically equivalent to the analogous Wald tests. In this case the percentiles of the limiting distributions can be obtained from Andrews (1993) and Andrews and Ploberger (1994). We present percentiles of the limiting distributions for situations in which the number of moment conditions exceeds the number of parameters.

The paper is organized as follows : Section 2 contains the main theoretical results. Section 3 presents the asymptotic local power of the tests. Section 4 covers a simulation study of finite sample properties. All proofs are relegated to a mathematical appendix.

---

2 A similar result applies for the Sup-LR and Sup-LM tests. In the remainder of this introduction all discussion of the Wald test, or functions thereof, similarly applies to the LR and LM tests.

2. TESTS STATISTICS AND THEIR ASYMPTOTIC DISTRIBUTION

In this section, we propose predictive tests with unknown breakpoint and discuss their asymptotic distribution. The details of the proof and the required regularity conditions appear in the Appendix. We consider the class of GMM estimators which subsumes many standard estimators such as quasi-maximum likelihood, certain semi-parametric procedures, as well as least squares and IV procedures. In a general context, the GMM estimator is based on a set of moment conditions :

(2.1) $$E[f(x_t, \theta_o)] = 0,$$

where $f(\cdot)$ is a $(q \times 1)$ vector of continuous differentiable functions of (x_t, θ_o) with $f(\cdot) \in \mathbb{R}^q$; x_t is a $(s \times 1)$ vector of random variables; θ_o is a $(p \times 1)$ parameter vector contained in $\Theta \subset \mathbb{R}^p$. This specification follows the usual practice of assuming that the moment condition is valid throughout the whole sample. If this assumption is invalid then the model is said to be structurally unstable. There are various ways in which one could characterize such structural instability. Andrews (1993) considers the situation in which the parameter vector at which the moment conditions are satisfied is indexed by t, θ_t say. This approach allows a wide variety of models against which it is difficult to design a single test. Consequently, he focuses attention on two homogenous subsamples, i.e. $\theta_t = \theta_1$ for $t = 1, 2, ...\pi T$ and $\theta_t = \theta_2$ for $t = \pi T + 1, ..., T$ where $\pi \in \Pi \subset (0, 1)$. The resulting tests are designed to have power against the explicit alternative of a single breakpoint although, as shown by Andrews (1993), the tests have power against a much wider class of alternatives. The tests we consider are similarly designed to detect situations in which there is a single breakpoint in the sample. However our characterization of structural instablity is different. The predictive tests proposed by Ghysels and Hall (1990a) are formulated in terms of changes in the moment conditions without necessarily attributing such changes to the parameter vector. To present the null and alternative hypotheses of the predictive tests we need the following notation. Let $f_1(\cdot)$ denote $f(\cdot)$ for the observations $T_1(\pi) = \{t = 1, 2, ..., \pi T\}$ and $f_2(\cdot)$ denote $f(\cdot)$ for the remaining observations $T_2(\pi) = \{t = \pi T + 1, ..., T\}$. The null and alternative hypotheses are then:

(2.2a) $$H_0 : E[f_1(x_t, \theta_o)] = E[f_2(x_t, \theta_o)] = 0$$

(2.2b) $\quad H_1 : E[f_1(x_t, \theta_o)] = 0$, but $E[f_2(x_t, \theta_o)] \neq 0$

The idea behind the predictive test is based on evaluating the moment conditions for the observations in the second subsample, $T_2(\pi)$, at the parameter estimators based on only the first subsample, $T_1(\pi)$. If the null hypothesis is correct then these estimated moment conditions should be approximately zero. When the breakpoint π is known, then one can use the test proposed by Ghysels and Hall (1990a). In the remainder of this section we consider the generalization of this test to the case where the breakpoint is unknown. To proceed with the presentation of the tests, let us first present the required GMM estimators :

Definition 2.1 : The set of GMM estimators $\{\theta_T(\pi)\}$ is a sequence of random vectors such that :

$$\hat{\theta}_T(\pi) = \operatorname{argmin} \, (\pi T)^{-1} \sum_{t=1}^{\pi T} f(x_t, \theta)' \, \hat{W}_T \, (\pi T)^{-1} \sum_{t=1}^{\pi T} f(x_t, \theta),$$

where $\hat{W}_T$ is a random symmetric matrix which may depend on π. Following Hansen (1982), the optimal weighting matrix W_T is defined to be the inverse of :

$$(2.3) \qquad \Omega_0 = \lim_{T \to \infty} \operatorname{Var}\left[\frac{1}{\sqrt{T}} \sum_{t=1}^{T} f(x_t, \theta_0)\right].$$

This matrix can be consistently estimated by a variety of procedures, see inter alia Gallant (1987), Newey and West (1987), Andrews and Monahan (1992). Whenever the covariance estimator involves data from the first subsample, we denote it by $\hat{S}_1(\pi)$. Likewise, when data starting with observation $\pi T + 1$ are used, we shall denote the estimator $\hat{S}_2(\pi)$. Equations (A.3) and (A.4) in the Appendix provide generic formula for both estimators. We now proceed with the definition of the predictive test statistics as a function of the unknown breakpoint π :

$$(2.4)\ PR_T(\pi) = \left[(T - \pi T)^{-1/2} \sum_{\pi T+1}^{T} f_2(x_t, \hat{\theta}_T(\pi))\right]' \hat{V}_2^{-1}(\pi) \left[(T - \pi T)^{-1/2} \sum_{\pi T+1}^{T} f_2(x_t, \hat{\theta}_T(\pi))\right],$$

where $\hat{V}_2(\pi)$ is a consistent estimator of :

$$V_2(\pi) = S_2(\pi) + dF_2(\pi)\ [F_1(\pi)'\ S_1^{-1}(\pi)\ F_1(\pi)]^{-1}\ F_2(\pi)',$$

and $d = \frac{1 - \pi}{\pi}$ while the matrices $F_i(\pi)$ appear in the Appendix as equation (A.2). A first theorem establishes weak convergence of the $PR_T(\pi)$ process indexed by π.

Theorem 2.1 : Under the null hypothesis H_0 in (2.2a) and Assumptions A.1 through A.13 of the Appendix, the process PR indexed by π for a given set Π whose closure lies in (0,1) satisfies :

(2.5) $\quad PR_T(\pi) \Rightarrow BBH(\pi) + BMH(\pi)$

where

$$BBH(\pi) = \frac{[BH_1(\pi) - \pi BH_1(1)]'[BH_1(\pi) - \pi BH_1(1)]}{\pi(1 - \pi)},$$

$$BMH(\pi) = \frac{[BH_2(1) - BH_2(\pi)]'[BH_2(1) - BH_2(\pi)]}{(1 - \pi)}.$$

and BH_i are vectors of independent Brownian motions of dimension p when i = 1 and q - p when i = 2.

Hence, the asymptotic distribution is a squared p-dimensional standardized tied-down Bessel process plus the square of a q - p dimensional standardized Bessel process. When the dimension of the orthogonality conditions equals the dimension of θ, the asymptotic distribution is only a function of a squared standardized tied-down Bessel process. In this case, the predictive test based on $PR_T(\pi)$ has exactly the same asymptotic distribution as the Wald, LR and LM tests developed by Andrews (1993). If q > p, the extra term corresponds to a test of the stability of the overidentifying restrictions.

The result in Theorem 2.1 enables us to proceed with the formulation of several test statistics. The first statistic is the sup-predictive test

$$\text{(2.6)} \qquad SupPR_T = \sup_{\pi \in \Pi} PR_T(\pi)$$

This approach amounts to basing inference on the breakpoint which maximizes the evidence against structural stability. Andrews (1993) proposed a similar approach based on the Wald, Lagrange Multiplier and Likelihood Ratio type tests. While this is an intuitively reasonable strategy for testing for structural instability, there is no formal justification in the sense that the tests have no known optimality properties; e.g. see Andrews and Ploberger (1994). To derive an optimal test one must specify a probability distribution on both the breakpoint and the change in the moment conditions which indicate the relative importance of various departures from the null hypothesis. Andrews and Ploberger (1994) have addressed this issue in the context of maximum likelihood estimation and Sowell (1994) generalizes this analysis to GMM estimators. In this paper, we consider two continuous mappings of the predictive tests which are motivated by Andrews and Ploberger (1994) and Sowell (1994). These are :

$$\text{(2.7)} \qquad PR_T^{av} = \int_{\Pi} PR_T(\pi) dJ(\pi)$$

$$\text{(2.8)} \qquad ExpPR_T = \log\{ \int_{\Pi} \exp[0.5 PR_T(\pi)] dJ(\pi)\}$$

where $J(\pi)$ is the probability density function specified for π. The PR_T^{av} statistic represents the average predictive test over π and is anticipated to be powerful for alternatives close to the null; whereas $ExpPR_T$ is anticipated to be powerful against distant alternatives. The distributions of these statistics are presented in Theorem 2.2.

Theorem 2.2 : Under the conditions of Theorem 2.1, we have

$$SupPR_T \Rightarrow \sup_{\pi \in \Pi} \{BBH(\pi) + BMH(\pi)\},$$

$$PR_T^{av} \Rightarrow \int_{\Pi} [BBH(\pi) + BMH(\pi)] dJ(\pi),$$

$$ExpPR_T \Rightarrow \log\{ \int_{\Pi} \exp[(BBH(\pi) + BMH(\pi)) / 2] dJ(\pi)\}.$$

In the case where $p = q$ these distributions are the same as those derived by Andrews (1993) and Andrews and Ploberger (1994) for the analogous Wald, LM and LR based tests. Therefore percentiles for our tests when $p = q$ can be obtained from the appropriate tables in these earlier papers. We tabulate the distributions in Theorem 2.2 for the case where $q > p$; for the PR_T^{av} and $ExpPR_T$ statistics we follow Andrews and Ploberger (1994) and set $J(\pi)$ equal to the uniform distribution on Π. The tables with these critical values are relegated to the appendix. All calculations were performed using GAUSS 3.0 with 10,000 replications.

3. ASYMPTOTIC LOCAL POWER

The predictive tests discussed in the previous section are designed to test for a single breakpoint at which the value of the moment condition changes. However, they have power against a variety of other alternatives as well. In this section we develop the formal asymptotic arguments to support this claim.

Following Andrews (1993) we adopt a very general specification for the sequence of local alternatives to our H_0. In this section we assume

Assumption 3.1 : The moment conditions satisfy

$$\sup_{\pi\in\Pi} \| T^{-1/2} \sum_{t=1}^{\pi T} f(x_t, \theta_o) - \mu_1(\pi) \| = o_p(1),$$

$$\sup_{\pi\in\Pi} \| T^{-1/2} \sum_{t=\pi t+1}^{\pi T} f(x_t, \theta_o) - \mu_2(\pi) \| = o_p(1).$$

Notice that this sequence of alternatives allows for violation of the moment conditions in both subsamples. If we put more structure on the problem and assume that $E[f(x_t,\theta_0)] = \eta(t/T)/T^{1/2}$ then

$$\mu_1(\pi) = \int_o^{\pi} \eta(s)ds, \qquad \mu_2(\pi) = \int_{\pi}^{1} \eta(s)ds$$

as discussed in Andrews (1993). It is of interest to specialize these results further to the case of a single breakpoint at unknown time π_0. Suppose the value of the moment conditions is $\eta \neq 0$ for $t \geq \pi_0 T$. This can be captured within our framework by putting $\eta(s) = \eta 1(\pi \geq \pi_0)$ from which it follows that $\mu_1(\pi) = \max(\pi - \pi_0, 0)\eta$ and $\mu_2(\pi) = [1 - \max(\pi, \pi_0)]\eta$.

We now present the limiting distribution of $PR_T(\pi)$ under the class of alternatives in assumption 3.1.

Theorem 3.1 : Under assumptions 3.1 and A.1-A.13 given in the appendix, we have

$$PR_T(\pi) \Rightarrow J_p^*(\pi)' J_p^*(\pi) + K_{q-p}^*(\pi)' K_{q-p}^*(\pi)$$

where

$$J_p^*(\pi) = \frac{[BH_1(\pi) - \pi BH_1(1)]}{[\pi(1-\pi)]^{-1/2}} - \left[\frac{(1-\pi)}{\pi}\right]^{1/2} H_1 S^{-1/2}\mu_1(\pi) + \left[\frac{\pi}{(1-\pi)}\right]^{1/2} H_1 S^{-1/2}\mu_2(\pi)$$

$$K_{q-p}^* = \frac{BH_2(1) - BH_2(\pi) + H_2 S^{-1/2}\mu_2(\pi)}{(1-\pi)^{1/2}}$$

and $H' = [H_1' \; H_2']$ is a matrix whose columns form a set of orthonormal vectors with H_1, H_2 of dimensions $p \times q$, $(q - p) \times q$ respectively which is defined in the Appendix.

Notice that in the case where $p = q$ the distribution in Theorem 3.1 reduces to the one presented in Andrews (1993) Theorem 4. It is interesting to note that the first component of the predictive test, which can be viewed as testing the constancy of the parameters over the sample, is sensitive to structural instability in $f_1(\cdot)$ and $f_2(\cdot)$. Whereas the second component, which tests the constancy of the overidentifying restrictions, is sensitive to instability in $f_2(\cdot)$ alone.

Before examining the properties of the tests proposed in this paper, it is interesting to use Theorem 3.1 to learn about the power of the predictive test when the breakpoint is known. If π is fixed then $PR_T(\pi)$ has a χ^2_q with noncentrality parameter equal to

$$(3.1) \qquad \text{constant}[J^*_p(\pi)]^2 + \text{constant}[K^*_{q-p}(\pi)]^2$$

where constant[$\cdot$] denotes the nonrandom part of the the random vector in the brackets. We use this result to examine the power of the test in two situations. First consider the case where the instability is driven by parameter variation. Following Andrews (1993), we assume $E[f(x, \theta + \eta(t/T)/T^{1/2})] = 0$ and so

$$\mu_1(\pi) = -F \int_0^\pi \eta(s)ds, \qquad u_2(\pi) = -F \int_\pi^1 \eta(s)ds$$

Substituting these representations into (3.1) and noting that $H_2 S^{-1/2} F = 0$,[3] it can be shown that the predictive test has exactly the same noncentrality parameter as the Wald, LR and LM tests proposed by Andrews and Fair (1988). However the predictive test has q - p more degrees of freedom and so is less powerful if $q > p$. This is intuitively reasonable. The Wald, LR and LM tests are designed to have power against parameter variation and under this alternative there is no additional information in the overidentifying restrictions. We now turn to the situation where there is a single breakpoint π_0 and examine the power of the test against structural instability either before or after the break. For this discussion assume the correct breakpoint is chosen. If the moment condition (2.1) is only invalid before the break, i.e. prior to $\pi_0 T$, then the noncentrality parameter of the test is

$$ncp_1 = \left[\frac{1-\pi_0}{\pi_0}\right] \eta' S^{-1/2} H_1' H_1 S^{-1/2} \eta$$

3 This follows from equation (A.7) in the appendix because the columns of H form an orthonormal set.

Whereas if the moment condition (2.1) is only invalid after the break the noncentrality parameter is

$$ncp_2 = \left[\frac{\pi_0}{1-\pi_0}\right] \eta' S^{-1/2} H_1' H_1 S^{-1/2} \eta + \eta' S^{-1/2} H_2' H_2 S^{-1/2} \eta$$

The relative magnitudes of these two noncentrality parameters depends on π_0 and the moments of various functions of the data. However, note that if $\pi_0 = 0.5$, then $ncp_2 \geq ncp_1$. In other words, the predictive test has more power against structural instability after the break under these conditions.[4] We also observe that ncp_1 equals the noncentrality parameter of the Wald, LR and LM tests. Therefore if $q > p$ the predictive test is less powerful than the other tests when the moment conditions are only invalid prior to the breakpoint. This follows because instability in the first subsample only affects the test via the parameter estimator $\hat{\theta}_T(\pi_0)$. However if the moment conditions are invalid in the second subsample alone then the predictive test can be more powerful. This is illustrated in Table 3.1. For simplicity, we consider the case where $H = I_q$, $S^{-1/2} \eta = \varepsilon 1_q$ where 1_q is a $q \times 1$ vector of ones. In this case, the predictive test converges to a $\chi^2_q(q\varepsilon^2)$ distribution and the Wald, LR and LM tests converge to a $\chi^2_p(p\varepsilon^2)$ distribution, where $\chi^2_a(b)$ is a χ^2 distribution with a degree of freedom and noncentrality parameter b. From Table 3.1, it is clear that the predictive test can be much more powerful asymptotically. For example, if $q = 10$, $p = 1$ and $\varepsilon = 1.5$, then the predictive test has power equal to .93, while the other three tests only have power equal to .32. The Monte Carlo results reported in the next section will reinforce this finding. Taken together, these properties of the predictive test suggest it is desirable to perform the tests in two ways : using the parameter estimators of the first subsample to evaluate the moment conditions in the second subsample and using the parameter estimators of the second subsample to evaluate the moment conditions of the first subsample. The construction of the latter test is analogous to that of the former and its distribution is easily deduced from Theorem 3.1; in particular note this second predictive test has the same distribution under the null hypothesis.

---

4 Ghysels and Hall (1990a) show that the predictive test has the same power against structural instability either before or after the break. This can be reconciled with the results in this paper because Ghysels and Hall (1990a) concentrate on instability caused by parameter variation alone in which case the second term in ncp_2 is 0.

Table 3.1 : Probability a $\chi^2_k(k\varepsilon^2)$ random variables exceeds the 95th percentile of the χ^2_k distribution

	k									
	1	2	3	4	5	6	7	8	9	10
ε = .5	.08	.09	.10	.11	.11	.12	.13	.13	.14	.14
ε = 1.0	.17	.22	.27	.32	.36	.40	.44	.48	.51	.53
ε = 1.5	.32	.46	.57	.66	.74	.80	.84	.88	.91	.93
ε = 2.0	.51	.71	.84	.91	.95	.97	.99	.99	1.00	1.00
ε = 2.5	.71	.90	.96	.99	1.00	1.00	1.00	1.00	1.00	1.00

One can derive the limiting distributions of the $SupPR_T$, PR_T^{av} and $ExpPR_T$ by applying the appropriate continuous mapping to the distribution in Theorem 3.1; for brevity exact formula for these limiting distributions are omitted. However, we do explicitly consider the power properties of these predictive based tests. For this discussion we restrict attention to the class of alternatives

(3.2) $$E[f(x_t, \theta_o)] = \xi\, \eta(t/T)/T^{1/2}$$

Corollary 3.1 : Under the conditions of Theorem 3.1, equation (3.1) holds with $\eta(\cdot)$ not equal to a constant vector almost everywhere on Π then

$$\lim_{\xi\to\infty} \lim_{T\to\infty} P[SupPR_T > c_{sup}(\alpha)] = 1,$$

$$\lim_{\xi\to\infty} \lim_{T\to\infty} P[PR_T^{av} > c_{av}(\alpha)] = 1,$$

$$\lim_{\xi\to\infty} \lim_{T\to\infty} P[ExpPR_T > c_{exp}(\alpha)] = 1,$$

where $c_{sup}(\alpha)$, $c_{av}(\alpha)$ and $c_{exp}(\alpha)$ are the 100(1-α) percentiles of the limiting distributions of the $SupPR_T$, PR_T^{av} and $ExpPR_T$ tests given in Theorem 2.2.

Therefore all three tests have nontrivial power against alternatives for which the expectation of the moment condition is not constant over the sample. This result follows from Theorem 3.1 and Corollary 2 of Andrews (1993).

4. FINITE SAMPLE PROPERTIES - A SIMULATION STUDY

We now turn our attention to the finite sample properties of the exponential and supremum predictive tests. The design of the simulation study will emphasize the difference between testing for structural change through moment conditions versus through parameters as in Andrews (1993). We present the simulation design first and discuss the results thereafter.

Consider a data series x_t with a sample of size T available. The data are generated by the following equation :

$$(4.1) \qquad x_t = \theta x_{t-1} + \varepsilon_t + \alpha_{2t}\, \varepsilon_{t-2},$$

where ε_t is i.i.d. N(0,1). We will be interested in comparing two different scenarios : (A) the data generating process is AR(1), i.e., $\alpha_{2t} = 0\ \forall t$ and (B) the data generating process is AR(1) for half the sample and, for the remainder of the sample, it is fixed parameter ARMA(1,2), with a zero restriction on the first lag of the MA polynomial and

$$(4.2) \qquad \alpha_{2t} = \begin{cases} 0 & \text{if } t \leq 1/2\ T \\ \overline{\alpha}_2 \neq 0 & \text{if } t > 1/2\ T \end{cases}.$$

On first appearance, we are in a typical situation of structural change, in this case involving the MA parameter α_{2t}. However, the next element will emphasize the differences which may occur between testing for structural change through moment conditions and parameter estimation. Namely, the econometrician estimates only the parameter θ as being the "parameter of interest". Such a situation is indeed not uncommon. For instance, many applications of GMM involving Euler equations entail estimation only of a small set of parameters which usually have an economic interpretation but do not fully describe the DGP. Our setup of an ARMA(1,2) process with only the estimation of the AR parameter is a simplified example of this commonly encountered situation. The estimator for the parameter θ is based on the following moment function :

$$(4.3) \qquad f(x_t, \theta) = (x_t - \theta x_{t-1})(x_{t-1}, x_{t-2})'.$$

Under the null hypothesis, which is assumed to be scenario (A), the lagged dependent variables x_{t-1} and x_{t-2} are valid instruments. It should also be noted that one has a situation of one overidentifying moment condition in (4.3). From the discussion in sections 2 and 3, we know that the Wald, LM and LR tests on the one hand and predictive tests on the other will have different power properties. Under scenario (B), which is chosen here as a specific class of alternatives, neither x_{t-1} nor x_{t-2} are valid instruments for half of the sample. The LR, LM and Wald tests for structural change discussed in Andrews (1993) and Andrews and Ploberger (1994) will be based on statistics involving parameter estimates of θ over the entire sample or subsamples. In our design, θ will be estimated consistently during part of the sample only. One should observe though that the parameter θ actually never changes. Instead, the validity of the orthogonality conditions are affected through the design of the DGP. In particular, using the notation of the previous section :

$$E[f_1(x_t, \theta)] = 0 \quad \text{for } t = 1, \ldots, 1/2\, T,$$

$$E[f_2(x_t, \theta)] \neq 0 \quad \text{for } t = 1/2\, T + 1, \ldots, T.$$

This design stresses in a simple way the differences between structural change tests proposed here and those considered by Andrews, and Andrews and Ploberger. Obviously, the latter tests will have power because of the inconsistent estimation of θ during part of the sample which will be viewed as a structural change.

In Table 4.1, we report results from a Monte Carlo study involving a total of eight test statistics for twelve parameter settings in equations (4.1) and (4.2) and two samples sizes T = 100 and T = 200. For the autoregressive parameter, we took values $\theta = 0$, 0.5 and 0.9. Size properties of the statistics were simulated by setting $\bar{\alpha}_2 = 0$ in (4.2). The power properties were examined with nonzero values of $\bar{\alpha}_2$. They were set equal to 0.5 as well as -0.5 and -0.9. For the Wald, LR and LM-type tests, we considered both the supremum version appearing in (2.6) and the exponential one appearing in (2.8) (replacing in both formula PR by the applicable test statistics). Likewise, for the predictive tests we also reported both versions. The figures reported in Table 4.1 are based on 1,000 simulations with $\pi \in [.15, .85]$.

The top panel of Table 4.1 reports the size properties in small samples since in all cases $\bar{\alpha}_2 = 0$. There are no important size distortions, sometimes some of the tests are undersized but this seems only to be a minor problem. Let us turn our attention to power properties. They clearly confirm the calculations reported in Table 3.1 where it was shown that the local asymptotic power of the predictive type tests can be remarkable better. I noted, with say $\theta = 0$, i.e., no autoregressive part and $\bar{\alpha}_2 = -0.5$ or -0.9, we notice that the Wald, LR and LM tests have power in the range of 10% to 20% with T = 200. For the predictive test, it is between 80% and 100%. With $\theta = 0$ and $\bar{\alpha}_2 = 0.5$ the difference is not so dramatic, yet it is still up to 45%. When the AR coefficient increases the advantage in power of the predictive test reduces, although it remains the most powerful test for this particular setup regardless of the parameter settings and sample sizes.

Table 4.1 : Size and Power properties of Supremum and Exponential Tests for Structural Change with Unknown Breakpoint (5% Critical Value)

$$x_t = \theta x_{t-1} + \varepsilon_t \quad t \leq 1/2\ T$$

$$x_t = \theta x_{t-1} + \varepsilon_t + \overline{\alpha}_2\, \varepsilon_{t-2} \quad t > 1/2\ T$$

θ	$\overline{\alpha}_2$	T	Wald		LR		LM		PR	
			Sup	Exp	Sup	Exp	Sup	Exp	Sup	Exp
					Size Properties					
0	0	100	3.5	4.6	2.8	4.1	2.2	3.9	1.5	3.6
		200	3.8	5.2	3.5	4.7	3.3	4.3	2.7	4.5
0.5	0	100	3.5	5.0	3.3	4.6	2.5	4.3	2.5	3.5
		200	4.1	5.0	3.9	5.0	3.3	4.6	2.6	4.4
0.9	0	100	4.8	4.6	4.2	4.1	3.4	3.8	3.2	3.6
		200	4.3	5.4	3.6	4.8	3.4	4.3	3.0	4.5
					Power Properties					
0	0.5	100	11.7	13.6	11.0	13.3	10.6	11.9	2.3	21.0
		200	13.7	14.9	12.7	14.0	14.1	15.0	61.8	83.2
	−0.5	100	0.9	0.8	1.0	0.9	10.6	0.6	13.8	40.6
		200	1.4	1.7	1.4	1.5	1.3	1.5	81.0	92.4
	−0.9	100	0.5	1.0	0.7	1.1	0.5	0.7	44.7	79.7
		200	0.4	1.1	0.7	1.1	0.6	1.0	98.7	100.0
0.5	0.5	100	8.2	11.6	8.4	11.6	7.8	9.9	1.1	7.2
		200	11.8	16.2	11.8	16.2	12.9	15.7	18.7	39.7
	−0.5	100	6.5	8.7	7.2	8.9	6.7	7.7	18.6	38.6
		200	11.0	14.9	11.6	15.9	10.4	13.8	76.5	88.6
	−0.9	100	7.6	12.7	8.7	14.1	6.4	10.9	47.6	77.4
		200	17.9	25.9	20.1	27.9	15.6	21.8	98.7	99.7
0.9	0.5	100	1.9	3.1	2.4	3.1	1.5	1.3	0.5	1.0
		200	3.5	4.7	3.7	5.0	2.7	2.4	1.6	4.1
	−0.5	100	27.6	30.9	28.3	31.2	27.9	27.6	31.2	35.0
		200	55.9	58.7	56.7	60.0	55.0	54.5	68.1	69.8
	−0.9	100	65.9	72.8	68.0	74.4	63.8	69.2	78.4	86.7
		200	94.9	96.4	95.6	97.0	94.4	95.6	99.9	100.0

Note : In all computations $\pi \in [.15, .85]$.

APPENDIX

In this appendix, we describe the set of regularity conditions used to derive the asymptotic distribution of the tests. Next we present the proofs of Theorems 2.1 and 3.1, followed by tables with critical values of the asymptotic distribution.

1. Regularity conditions

Assumption A.1 : The estimator is based on the argument (2.1) where f is a R^q-valued function of orthogonality conditions.

Assumption A.2 : The true parameter vector θ_o is an element of the parameter space $\Theta \subset R^p$.

Assumption A.3 : (Θ, σ) is a separable metric space.

Assumption A.4 : The function $f(x_t, \theta)$ is Borel measurable for each $\theta \in \Theta$.

We now list a set of regularity condition to obtain weak convergence of the GMM estimators $\hat{\theta}_T(\pi)$, indexed by π and defined in Section 2, to a function of Brownian motions. The main distributional results will hold under two alternative assumptions regarding the stochastic process x_t. Following Hansen (1982), one can impose stationarity and ergodicity conditions or else, as in Gallant and White (1988) and Andrews (1993), one can also consider a setup with conditions on a triangular array of random variables x_{Tt}. Assumptions A.5 through A.13 are taken from Andrews (1993), who provides a complete discussion.

Assumption A.5 : The process x_t is stationary and ergodic.

Assumption A.6 : The $\{x_{Tt} : \leq T, T \geq 1\}$ is a triangular array of X-value random vectors that is L^o-near epoch dependence on a strong mixing base $\{y_{Tt} : t = ..., 0, 1, ...; T \geq 1\}$, where X is a Borel subset of R^k, and $\{\frac{1}{T} \Sigma_1^T x_{Tt} : T \geq 1\}$ is tight on X.[5]

---

5 For a definition of L^p-near epoch dependence and tightness, see Andrews (1993, p. 830). For a presentation of the concept of near epoch dependence, we refer the reader to Gallant and White (1988) chaps. 3 and 4.

Assumption A.7 : For some $r \geq 2$, $f(x_{Tt}, \theta) : t \leq T, T \geq 1$ is a triangular array of R^p-valued random vectors that is L^2-near epoch dependence of size $-1/2$ on a strong mixing base $\{y_{Tt} : t = \ldots, 0, 1, \ldots; T \geq 1\}$ of size $-r/(r-2)$ and $\sup_{t \leq T, T \geq 1} E \| f(x_{Tt}, \theta) \|^r < \infty$.

Now, we defined the following matrices indexed by π :

(A.1) $$\bar{f}_1(\pi) = \frac{1}{\pi T} \Sigma_1^{T\pi} f(x_t, \hat{\theta}(\pi) \text{ and } \bar{f}_2(\pi) = \frac{1}{T - \pi T} \sum_{\pi T+1}^{T} f(x_t, \hat{\theta}(\pi)),$$

(A.2) $$\hat{F}_1(\pi) = \frac{1}{\pi T} \Sigma_1^{T\pi} \frac{\delta}{\delta \theta'} f_1(x_t, \hat{\theta}(\pi)) \text{ and } \hat{F}_2(\pi) = \frac{1}{T - T\pi} \sum_{\pi T+1}^{T} \frac{\delta}{\delta \theta'} f_2(x_t, \hat{\theta}(\pi)),$$

(A.3) $$\hat{S}_1(\pi) = \frac{1}{\pi T} \sum_{k=-\ell(\pi T)}^{\ell(\pi T)} \Sigma_1^{T\pi} \omega(\pi T, k)(f(x_t, \theta(\pi)) - \bar{f}_1(\pi))(f(x_{t-k}, \theta(\pi)) - \bar{f}_1(\pi))',$$

(A.4) $$\hat{S}_2(\pi) = \frac{1}{T - T\pi} \sum_{k=-\ell(\pi T)}^{\ell(\pi T)} \sum_{T\pi+1}^{T} \omega(T - \pi T, k)(f(x_t, \theta(\pi)) - \bar{f}_2(\pi))$$

$$(f(x_{t-k}, \theta(\pi)) - \bar{f}_2(\pi))',$$

where $\hat{\theta}(\pi) \equiv \hat{\theta}_T(\pi)$ defined in section 2.

Assumption A.8 : Var $\left(\frac{1}{\sqrt{T}} \Sigma_1^{T\pi} f(x_{Tt}, \theta)\right) \to \pi S$, $\forall \pi \in [0,1]$ for some positive qxq matrix S.

Assumption A.9 : $\sup_{\pi \in \Pi} \| \hat{\theta}(\pi) - \theta_0 \| \xrightarrow{p} 0$ for some θ_0 in the interior of Θ.

Assumption A.10 : $\sup_{\pi \in \Pi} \| \hat{W}(\pi) - W(\pi) \| \xrightarrow{p} 0$ for some qxq matrices $W(\pi)$ for which $\sup_{\pi \in \Pi} \| W(\pi) \| < \infty$.

We define :

$$\text{(A.5)} \qquad F = \lim_{T \to \infty} \frac{1}{T} \sum_{1}^{T} E \frac{\delta}{\delta\theta'} f(x_{Tt}, \theta_0) \in R^{q \times p}.$$

Assumption A.11 : $F(\pi)' S^{-1}(\pi) F(\pi)$ is nonsingular $\forall \pi \in \Pi$ and has eigenvalues bounded away from zero.

Assumption A.12 : $f(x_{Tt}, \theta)$ is partially differentiable in $\theta \in \Theta_o$ $\forall x \in X$, where Θ_o is some neighborhood of θ_o, $\frac{\delta}{\delta\theta'} f(x_{Tt}, \theta)$ is continuous in (x, θ) on $X \times \Theta_o$, and $\limsup_{T \to \infty} \frac{1}{T} \Sigma_1^T E \sup_{\theta \in \Theta_o} | \frac{\delta}{\delta\theta'} f(x_{Tt}, \theta) |^{1+\epsilon} < \infty$ for some $\epsilon > 0$.

Assumption A.13 : $\lim_{T \to \infty} \frac{1}{T} \Sigma_1^{T\pi} E \frac{\delta}{\delta\theta'} f(x_{Tt}, \theta)$ exists uniformly over $\pi \in \Pi$ equals πF. $\forall \pi \in \Pi$.

2. Proof of Theorem 2.1

We denote $\hat{\theta}_1(\pi)$ as the estimator of θ_o for the first πT observations and $\hat{\theta}_2(\pi)$, the estimator for the remaining subsample $(\pi T + 1, \ldots, T)$. From Theorem 1 of Andrews (1993) :

$$\sqrt{T}(\hat{\theta}_1(\pi) - \theta_o) \Rightarrow -\frac{1}{\pi}((F'(\pi)\, S^{-1}(\pi) F(\pi))^{-1}\, F(\pi)'\, S^{-1/2}(\pi)\, B(\pi),$$

$$\sqrt{T}(\hat{\theta}_2(\pi) - \theta_o) \Rightarrow -\frac{1}{(1-\pi)}\, ((F'(\pi)\, S^{-1}(\pi)\, F(\pi)))^{-1}\, F(\pi)'\, S^{-1/2}(\pi)\, (B(1) - B(\pi)),$$

where $B(\pi)$ is a p-vector of independent Brownian motions on [0,1] and $\Rightarrow$ denotes weak convergence as defined by Pollard (1984).

A mean value expansion for the functional f_2 evaluated at $\hat{\theta}_1$ yields :

$$\text{(A.6)} \qquad \frac{1}{\sqrt{(T - \pi T)}} \sum_{\pi T+1}^{T} f_2(\hat{\theta}_1) = \frac{1}{\sqrt{(T - \pi T)}} \sum_{\pi T+1}^{T} f_2(\theta_o) +$$

$$\left[\frac{1}{(T - \pi T)} \sum_{\pi T+1}^{T} \frac{\delta}{\delta\theta'} f_2(\bar{\theta})\right] d^{1/2}\sqrt{T\pi}(\hat{\theta}_1 - \theta_0).$$

In premultiplying by $\sqrt{1-\pi}\, S^{-1/2}$, we obtain that :

$$\frac{1}{\sqrt{T}} S^{-1/2} \sum_{\pi T+1}^{T} f_2(\hat{\theta}_1) \Rightarrow B(1) - B(\pi) - \left[\frac{(1 - \pi)}{\pi}\right] S^{-1/2} F(F' S^{-1} F)^{-1} F' S^{-1/2} B(\pi).$$

We now decompose the matrix of the projection spanned by $S^{-1/2}F'$ like Sowell (1993) namely :

$$\text{(A.7)} \qquad S^{-1/2} F(F' S^{-1} F)^{-1} F' S^{-1/2} = H'\Lambda H,$$

where $H H' = I_q$ and

$$\Lambda = \begin{bmatrix} Ip & 0 \\ 0 & 0 \end{bmatrix}.$$

It follows that :

$$\frac{1}{\sqrt{T}} S^{-1/2} \sum_{\pi T+1}^{T} f_2(\hat{\theta}_1) \Rightarrow B(1) - B(\pi) - \left[\frac{(1 - \pi)}{\pi}\right] H'\Lambda H B(\pi),$$

and premultiplying by H yields :

$$\frac{1}{\sqrt{T}} HS^{-1/2} \sum_{T\pi+1}^{T} f_2(\hat{\theta}_1) \Rightarrow \begin{bmatrix} -(BH_1(\pi) - \pi BH_1(1))/\pi \\ BH_2(1) - BH_2(\pi) \end{bmatrix},$$

where BH_1 and BH_2 are vectors of independent Brownian motions of dimensions p and q − p respectively.

Since, H et S are full rank matrix, then PR(π) equals :

$$\text{(A.8)}\quad \left[\frac{1}{\sqrt{(T-\pi T)}} HS^{-1/2} \sum_{\pi T+1}^{T} f_2(\hat{\theta}_1)\right]' [HS^{-1/2}\hat{V}S^{-1/2}H']^{-1}\left[\frac{1}{\sqrt{(T-T\pi)}} HS^{-1/2} \sum_{\pi T+1}^{T} f_2(\hat{\theta}_1)\right].$$

some algebra yields that :

$$\text{(A.9)}\qquad [HS^{-1/2}\hat{V}S^{-1/2}H']^{-1} = \begin{bmatrix} \pi I_p & 0 \\ 0 & I_{(q-p)} \end{bmatrix},$$

then, we obtain the desired result :

$$PR(\pi) \Rightarrow \frac{[BH_1(\pi)-\pi BH_1(1)]'[BH_1(\pi)-\pi BH_1(1)]}{\pi(1-\pi)} +$$

$$\frac{[BH_2(1)-BH_2(\pi)]\;[BH_2(1)-BH_2(\pi)]}{(1-\pi)}.$$

3. Proof of Theorem 3.1

From (A.6) it follows that

$$T^{-1/2}HS^{-1/2}\sum_{\pi T}^{T} f_2(\hat{\theta}_1) \Rightarrow HB(1) - HB(\pi) + HS^{-1/2}\mu_2(\pi) - dHH'\Lambda H[B(\pi) + S^{-1/2}\mu_1(\pi)]$$

The result then follows directly from (A.7), (A.8), $HH'\Lambda H=H_1$ and the symmetry of the distribution of $BH_1(\pi) - \pi\, BH_1(1)$.

Table A.1 : Critical Values of Supremum Test for $\pi \in (.20, .80)$

dim q − p	dim p: 1	2	3	4	5	6	7	8
1	11.02	13.67	15.97	18.10	19.79	21.71	23.51	25.33
	12.68	15.41	17.87	20.07	21.93	23.79	25.70	27.69
	16.19	19.53	22.15	24.33	26.10	28.32	30.71	33.87
2	13.48	15.63	17.80	19.78	21.65	23.54	25.20	27.06
	15.31	17.58	19.76	21.97	23.90	25.76	27.43	29.42
	19.47	21.43	24.19	26.34	28.10	30.24	31.96	34.33
3	15.35	17.47	19.51	21.53	23.42	24.98	26.93	28.58
	17.28	19.50	21.61	23.53	25.65	27.22	29.33	30.91
	21.64	23.68	26.36	28.12	30.78	32.09	33.78	36.26
4	17.28	19.26	21.26	23.29	25.14	26.66	28.15	29.78
	19.37	21.30	23.48	25.54	27.34	29.28	30.45	32.28
	23.75	25.86	28.16	30.35	32.39	34.28	35.53	37.91
5	19.09	20.97	23.06	24.67	26.32	28.12	29.85	31.33
	21.26	23.08	25.54	27.00	28.99	30.43	32.27	33.77
	25.94	27.82	30.95	31.83	33.03	35.25	36.85	38.56
6	20.78	22.86	24.42	26.27	28.03	29.95	31.34	32.86
	22.93	25.16	26.68	28.58	30.45	32.46	33.81	35.56
	27.41	30.28	31.46	33.50	35.68	37.78	39.26	40.63
7	22.56	24.50	26.26	28.03	29.71	31.29	32.75	34.50
	24.72	26.80	28.60	30.44	32.15	33.79	35.36	37.23
	29.77	31.41	33.40	35.24	37.64	39.42	40.48	42.70
8	24.00	26.00	27.92	29.52	30.81	32.54	34.53	35.81
	26.32	28.20	30.33	32.09	33.32	35.21	37.16	38.71
	31.22	33.18	35.27	36.90	38.23	40.78	42.21	44.18

Note : First figure is 10%, followed by 5% and 1%.

Table A.2 : Critical Values of Supremum Test for $\pi \in (.15, .85)$

	dim p							
dim q − p	1	2	3	4	5	6	7	8
	10.09	11.89	14.13	15.90	18.26	19.97	21.59	23.72
1	11.92	13.59	16.03	18.07	20.42	22.08	24.00	26.15
	15.75	17.50	20.06	22.78	25.16	27.20	28.53	31.65
	11.99	14.81	16.71	18.67	20.18	21.95	23.67	25.25
2	13.94	16.79	18.89	20.94	22.59	24.53	26.13	28.33
	17.87	21.67	23.37	25.75	27.27	29.38	31.28	32.76
	14.94	16.80	18.64	20.54	22.16	23.69	25.58	27.10
3	17.10	19.20	21.00	23.10	24.74	26.21	28.17	29.86
	21.91	23.56	26.03	27.85	30.04	31.30	33.51	35.21
	17.26	18.80	20.60	22.60	24.11	25.88	27.19	28.67
4	19.58	21.32	23.07	25.02	26.75	28.67	29.73	31.41
	24.61	26.55	28.48	29.76	31.94	34.51	35.33	37.47
	19.11	20.80	22.64	24.18	25.84	27.63	29.10	30.48
5	21.58	23.55	25.12	26.84	28.33	30.35	31.79	33.36
	27.51	28.96	31.57	32.86	33.64	36.30	36.70	39.09
	21.14	22.93	24.40	26.11	27.66	29.50	30.78	32.22
6	23.64	25.54	26.98	28.71	30.45	32.24	33.43	35.03
	29.24	31.49	32.68	34.05	36.58	38.80	38.84	41.53
	23.19	24.73	26.55	28.02	29.51	31.16	32.41	34.02
7	25.68	27.51	29.49	30.86	32.34	34.00	35.29	37.28
	31.38	33.10	34.38	36.50	38.29	40.11	41.11	43.26
	24.71	26.56	28.27	29.92	30.95	32.39	34.26	35.61
8	27.47	29.40	31.09	33.09	33.59	35.25	37.49	38.66
	32.98	35.23	37.19	39.23	39.49	41.83	43.31	44.87

Note : First figure is 10%, followed by 5% and 1%.

Table A.3 : Critical Values Exp Test (c = ∞) for $\pi \in$ (.20, .80)

dim q − p	dim p: 1	2	3	4	5	6	7	8
	2.60	3.56	4.50	5.37	6.16	6.98	7.86	8.68
1	3.24	4.25	5.33	6.23	7.09	7.88	8.83	9.71
	4.67	5.93	7.09	8.18	9.13	10.07	10.84	12.38
	3.54	4.49	5.39	6.27	7.02	7.85	8.63	9.42
2	4.23	5.28	6.25	7.15	7.96	8.75	9.61	10.62
	5.95	7.11	8.10	9.08	9.91	10.87	11.70	12.87
	4.50	5.32	6.14	6.99	7.84	8.57	9.44	10.16
3	5.29	6.15	7.06	7.96	8.88	9.58	10.49	11.22
	7.19	7.93	9.10	10.11	11.13	11.76	12.55	13.66
	5.32	6.13	7.00	7.88	8.62	9.43	10.09	10.80
4	6.22	7.04	7.90	8.83	9.66	10.59	11.08	11.94
	8.09	9.07	10.17	10.93	11.94	12.94	13.38	14.56
	6.16	6.87	7.79	8.54	9.24	10.58	10.84	11.58
5	7.08	7.94	8.80	9.48	10.36	11.31	11.97	12.69
	9.39	9.99	11.37	11.90	12.69	13.60	14.19	14.92
	6.90	7.80	8.45	9.28	10.10	10.88	11.58	12.26
6	7.88	8.79	9.54	10.38	11.16	12.11	12.67	13.50
	9.91	11.00	11.67	12.56	13.60	14.56	14.96	16.01
	7.75	8.54	9.37	10.12	10.83	11.59	12.25	13.01
7	8.73	9.59	10.53	11.20	11.91	12.76	13.43	14.27
	11.02	11.82	12.67	13.43	14.74	15.49	15.97	16.85
	8.43	9.23	10.04	10.79	11.45	12.20	13.12	13.69
8	9.48	10.33	11.19	12.00	12.53	13.41	14.33	14.91
	11.78	12.76	13.62	14.37	14.86	15.92	16.83	17.62

Note : First figure is 10%, followed by 5% and 1%.

Table A.4 : Critical Values Exp Test ($c = \infty$) for $\pi \in (.15, .85)$

dim q − p	dim p: 1	2	3	4	5	6	7	8
	2.79	3.45	4.35	5.15	6.17	6.92	7.69	8.61
1	3.48	4.17	5.18	6.02	7.14	7.89	8.74	9.69
	5.16	5.80	6.99	8.13	9.37	10.16	10.70	12.38
	3.65	4.76	5.56	6.44	7.08	7.86	8.59	9.43
2	4.32	5.69	6.53	7.35	8.16	8.93	9.76	10.66
	6.01	7.71	8.64	9.48	10.32	11.17	12.11	12.96
	4.93	5.70	6.47	7.34	8.05	8.73	9.56	10.23
3	5.85	6.72	7.45	8.41	9.17	9.78	10.75	11.49
	8.07	8.65	9.80	10.66	11.61	12.24	13.14	13.92
	5.94	6.64	7.38	8.24	8.90	9.74	10.37	11.04
4	6.97	7.76	8.50	9.41	10.16	11.05	11.48	12.31
	9.35	10.22	11.06	11.63	12.60	13.65	14.15	15.19
	6.85	7.50	8.31	9.07	9.73	10.58	11.23	11.92
5	7.93	8.69	9.43	10.22	10.96	11.75	12.53	13.20
	10.45	11.32	12.41	13.06	13.47	14.62	14.75	15.84
	7.70	8.51	9.16	9.88	10.66	11.46	12.06	12.75
6	8.88	9.73	10.33	11.14	11.95	12.76	13.23	14.03
	11.43	12.34	13.05	13.67	14.79	15.55	15.74	17.01
	8.64	9.37	10.17	10.84	11.51	12.25	12.83	13.60
7	9.84	10.54	11.51	12.12	12.81	13.60	14.15	15.11
	12.55	13.36	13.87	14.75	15.52	16.36	16.99	18.17
	9.41	10.18	11.00	11.68	12.25	12.90	13.72	14.34
8	10.68	11.49	12.28	13.16	13.46	14.21	15.19	15.86
	13.29	14.27	15.07	16.03	16.06	17.27	18.08	18.94

Note : First figure is 10%, followed by 5% and 1%.

Table A.5 : Critical Values Average Test (c = 0) for $\pi \in (.20, .80)$

dim q − p	dim p: 1	2	3	4	5	6	7	8
1	3.68	4.98	6.31	7.44	8.69	9.92	11.05	12.32
	4.58	5.88	7.24	8.48	9.87	11.11	12.26	13.58
	6.55	7.89	9.40	10.95	12.11	13.60	15.02	16.51
2	5.10	6.51	7.74	8.89	10.02	11.23	12.39	13.72
	6.09	7.65	8.84	10.09	11.19	12.49	13.64	15.10
	8.37	10.03	11.32	12.82	13.87	15.02	16.35	17.97
3	6.70	7.88	9.00	10.25	11.40	12.47	13.75	14.87
	7.89	9.06	10.16	11.54	12.75	13.85	15.15	16.28
	10.47	11.54	12.95	14.32	15.66	16.86	17.80	19.25
4	8.02	9.14	10.37	11.60	12.71	13.88	15.00	16.10
	9.37	10.47	11.70	13.00	14.22	15.44	16.46	17.60
	12.12	13.50	14.76	15.63	16.95	18.84	19.65	20.86
5	9.40	10.46	11.71	12.80	13.78	15.12	16.24	17.38
	10.81	11.89	13.28	14.23	15.36	17.01	17.78	18.86
	13.94	14.74	16.23	17.66	18.61	20.26	20.87	22.14
6	10.67	11.87	12.91	14.09	15.30	16.40	17.37	18.56
	12.08	13.35	14.37	15.62	16.82	18.14	18.94	20.17
	15.03	16.64	17.54	18.98	20.22	21.35	22.16	23.80
7	12.01	13.08	14.33	15.45	16.53	17.64	18.62	19.88
	13.50	14.58	15.89	17.02	18.11	19.25	20.18	21.58
	16.76	17.91	19.19	20.08	21.68	22.71	24.12	25.12
8	13.18	14.33	15.37	16.59	17.61	18.74	20.02	20.87
	14.73	15.87	17.23	18.35	19.29	20.19	21.72	22.75
	18.32	19.39	20.56	21.71	22.42	23.77	25.19	26.33

Note : First figure is 10%, followed by 5% and 1%.

Table A.6 : Critical Values Average Test (c = 0) for $\pi \in (.15, .85)$

	dim p							
dim q − p	1	2	3	4	5	6	7	8
	4.26	5.28	6.65	7.84	9.44	10.69	11.80	13.16
1	5.38	6.32	7.72	9.07	10.81	12.14	13.27	14.69
	7.76	8.61	10.30	12.21	13.73	15.28	16.64	18.40
	5.68	7.46	8.67	9.87	10.99	12.23	13.31	14.80
2	6.73	8.87	10.08	11.32	12.47	13.79	15.05	16.54
	9.35	11.90	13.28	14.46	15.85	16.98	18.48	20.25
	7.84	9.09	10.23	11.54	12.69	13.78	15.08	16.08
3	9.27	10.55	11.68	13.12	14.30	15.39	16.77	17.99
	12.58	13.73	15.35	16.78	18.23	19.09	20.30	21.69
	9.45	10.56	11.75	13.02	14.21	15.44	16.50	17.60
4	11.09	12.33	13.51	14.79	16.09	17.45	18.36	19.49
	14.71	16.22	17.24	18.39	19.72	21.61	22.44	23.60
	11.06	12.16	13.39	14.56	15.45	17.06	18.02	19.20
5	12.91	13.96	15.16	16.24	17.43	19.20	19.90	21.15
	16.76	18.06	19.33	20.48	21.48	23.13	23.84	25.22
	12.54	13.75	14.78	16.06	17.20	18.43	19.38	20.58
6	14.25	15.76	16.64	17.86	19.24	20.56	21.37	22.69
	18.18	19.53	20.85	22.18	23.56	24.72	25.32	27.17
	14.15	15.26	16.47	17.59	18.68	19.76	20.78	22.16
7	16.09	17.24	18.56	19.61	20.74	21.93	22.87	24.35
	20.25	21.53	22.71	23.74	25.25	26.24	27.63	28.77
	15.55	16.68	17.80	19.07	19.97	21.08	22.45	23.23
8	17.52	18.68	20.09	21.21	22.18	23.23	24.71	25.73
	22.01	23.14	24.32	25.53	25.87	27.96	29.23	30.09

Note : First figure is 10%, followed by 5% and 1%.

REFERENCES

Andrews, D.W.K. (1993), "Tests for Parameter Instability and Structural Change with Unknown Change Point," *Econometrica* 61, 821–856.

Andrews, D.W.K. and R. Fair (1988), "Inference in Econometric Models with Structural Change," *Review of Economic Studies* 55, 615–640.

Andrews, D.W.K. and J.C. Monahan (1992), "An Improved Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimator," *Econometrica* 60, 953–966.

Andrews, D.W.K. and W. Ploberger (1994), "Optimal Tests When a Nuisance Parameter is Present Only Under the Alternative," *Econometrica* 62, 1383–1414.

Epstein, L. and S. Zin (1991), "Substitution, Risk Aversion and the Temporal Behavior of Asset Returns," *Journal of Political Economy* 99, 263–286.

Gallant, A.R. and H. White (1988), "A Unified Theory of Estimation and Inference for Nonlinear Dynamic Models," Basil Blackwell, New-York.

Ghysels, E. and A. Guay (1994), "Structural Change Tests with Unknown Breakpoint : A Monte Carlo Investigation," work in progress.

Ghysels, E. and A. Hall (1990a), "A Test for Structural Stability of Euler Conditions Parameters Estimated via the Generalized Method of Moments Estimator," *International Economic Review* 31, 355–364.

Ghysels, E. and A. Hall (1990b), "Are Consumption-Based Intertemporal Capital Assset Pricing Models Structural?," *Journal of Econometrics* 45, 121–139.

Hall, A. (1993), "Some Aspects of Generalized Method of Moments Esstimation," in G.S. Maddala, C.R. Rao and H.D. Vinod (eds.), *Handbook of Statistics* 11, North-Holland, Amsterdam, Chapter 15.

Hansen, L.P. (1982), "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica* 50, 1029–1054.

Kramer, W., W. Ploberger and R. Alt (1988), "Testing for Structural Change in Dynamic Models," *Econometrica* 56, 1355–1369.

Newey, W.K. (1993), "Efficient Estimation of Models with Conditional Moment Restrictions," in G.S. Maddala, C.R. Rao and H.D. Vinod (eds.), *Handbook of Statistics* 11, Econometrics, North-Holland, Amsterdam, 419–454.

Newey, W.K. and K. West (1987), "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica* 55, 703–708.

Ogaki, M. (1993), "Generalized Method of Moments : Econometric Applications." in G.S. Maddala, C.R. Rao and H.D. Vinod (eds.), *Handbook of Statistics* 11, Econometrics, North-Holland, Amsterdam, 455–488.

Sowell, F. (1994), "An Approach to Specification Testing in the Generalized Method of Moments Framework," manuscript, Graduate School of Industrial Administration, Carnegie Mellon University.

Liste des publications au CIRANO

Cahiers CIRANO / *CIRANO Papers* (ISSN 1198-8169)

94c-1 Faire ou faire faire : La perspective de l'économie des organisations / par Michel Patry

94c-2 Commercial Bankruptcy and Financial Reorganization in Canada / par Jocelyn Martel

94c-3 L'importance relative des gouvernements : causes, conséquences, et organisations alternatives / par Claude Montmarquette

95c-1 La réglementation incitative / par Marcel Boyer

Série Scientifique / *Scientific Series* (ISSN 1198-8177)

95s-4 Firm Heterogeneity and Worker Self-Selection Bias Estimated Returns to Seniority / par David N. Margolis

95s-5 An Analysis of the Real Interest Rate Under Regime Shifts / par René Garcia et Pierre Perron

95s-6 Are the Effects of Monetary Policy Asymmetric? / par René Garcia et Huntley Schaller

95s-7 Asymptotic Null Distribution of the Likelihood Ratio Test in Markov Switching Models / par René Garcia

95s-8 An Empirical Analysis of the Canadian Budget Process / par Bryan Campbell et Eric Ghysels

95s-9 Excess Sensitivity and Asymmetries in Consumption: An Empirical Investigation / par René Garcia, Annamaria Lusardi et Serena Ng

95s-10 When Piece Rates Work: More Lessons from the Cotton Mills / par Michael Huberman

95s-11 Innovation Under the Threat of Stricter Environmental Standards / par Olivier Cadot et Bernard Sinclair-Desgagné

95s-12 Regulation and Productivity in the Quebec Manufacturing Sector / par Charles Dufour, Paul Lanoie et Michel Patry

95s-13 Higher Moment Estimators for Linear Regression Models With Errors in the Variables / par Marcel G. Dagenais et Denyse L. Dagenais

95s-14 The Structure of Incentives in a Major Information Systems Outsourcing Contract: The Case of a North American Public Organization / par Benoit Aubert, Michel Patry et Suzanne Rivard

95s-15 Development of Measures to Assess Dimensions of IS Operation Transactions / par Benoit A. Aubert, Suzanne Rivard et Michel Patry

95s-16 On Stable Factor Structures in the Pricing of Risk / par Eric Ghysels

95s-17 What Did Unions Do? An Analysis of Canadian Strike Data, 1901-14 / par Michael Huberman et Denise Young

95s-18 Models and Priors for Multivariate Stochastic Volatility / par Éric Jacquier, Nicholas G. Polson et Peter Rossi

95s-19 Is Seasonal Adjustment a Linear or Nonlinear Data Filtering Process? / par Eric Ghysels, Clive W. J. Granger et Pierre L. Siklos

95s-20 Predictive Tests for Structural Change with Unknown Breakpoint / par Eric Ghysels, Alain Guay et Alastair Hall

95s-21 On Periodic Structures and Testing for Seasonal Unit Roots / par Eric Ghysels, Alastair Hall et Hahn S. Lee