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On Stable Factor Structures in the Pricing of Risk*

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Abstract / Résumé

Much of the research describing the cross-sectional and time series behavior of asset returns can be characterized as a search for the relevant state variables and also a search for the relevant model specification. Ultimately the scope of such efforts is to find a satisfactory and stable asset pricing structure. In this paper we discuss various methods to accomplish this and appraise the success of two recently proposed classes of asset pricing models in tracking predictable patterns in risk and return trade-offs. The two classes are the conditional CAPM and the nonlinear APT. The parameters of both models are estimated via a set of moment conditions using the GMM estimator and the model fit is judged on the basis of the overidentifying restrictions. The fundamental problem is that overidentifying restrictions tests are not designed to diagnose whether a model, provides a stable relationship between the return series and risk factors. We use a set of recently developed tests for structural stability of parameter estimates for the GMM estimator to diagnose which factor structures appear stable through time in the context of the two aforementioned classes of models. In the course of trying to sort out whether there is systematic mispricing we shall also try to determine what type of model looks most promising for further development. In that regard we find the nonlinear APT more satisfactory than the conditional APT and CAPM.

Dans cette étude nous réexaminons les modèles à facteurs qui ont été proposés récemment, c'est à dire le CAPM conditionnel et l'APT non-linéaire. Ces modèles ont été estimés par la méthode des moments généralisée. La diagnostique usuelle pour juger ces modèles est la statistique de suridentification. Le problème fondamental de cette statistique est qu'elle n'a pas de puissance par rapport à des alternatives caractérisés par des variations de paramètres. Évidement, ces variations entraînent des erreurs sur l'évaluation du risque. Nous proposons d'appliquer des tests de changement structurel pour les paramètres et analysons plusieurs modèles du type APT non-linéaire et CAPM conditionnel. Peu de modèles semblent être stable. Nous trouvons que la spécification du APT non-linéaire semble être quand même la plus satisfaisante.

Keywords: structural change, factor models, APT
Mots-clés : changement structurel, modèles à facteurs, APT
JEL: G12, C12, C13, C22

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1 Introduction

Linear factor models such as the unconditional CAPM and the APT have been the cornerstone of theoretical and empirical finance for decades now. Supported by seminal papers, like Sharpe (1964), Lintner (1965), Merton (1973) and Ross (1976), they are the most widely used tool to value the return on risky assets. While the theory maintains a linear and stable relationship between risk factors and returns there is now considerable empirical evidence documenting time variation in market betas and other factor payoffs. This is perhaps not so surprising since the theoretical underpinnings of the unconditional arbitrage-pricing theory reveal that time invariant linear factor structures are only obtained when one imposes strong assumptions on underlying probability distributions and investor's attitudes towards risk[1]. In practice many portfolio managers constantly update and reestimate factor returns and indeed Harvey (1989), Ferson and Harvey (1991, 1993) and Ferson and Korajczyk (1995) find that estimated betas exhibit statistically significant time variation.

Much of the research describing the cross-sectional and time series behavior of asset returns can be characterized as a search for the relevant state variables and also a search for the relevant model specification. In a recent survey Fama (1991) notes "since multifactor models offer at best vague predictions about the variables that are important in returns and expected returns, there is the danger that measured relations between returns and economic factors are spurious, the result of special features of a particular sample". This danger is very real and is the subject of our paper. Ultimately the scope of specification searches is to find a satisfactory and stable asset pricing structure. To allow for time varying risk premia certainly yields more sophisticated asset pricing models, but the search for adequate model specifications is obviously more delicate. In particular, the dynamics of predictable patterns needs to be scrutinized seriously as misspecification could be costly in terms of pricing error. In this paper we discuss various methods to accomplish this and appraise the success of several recently proposed asset pricing models in tracking predictable patterns in risk factor/expected return trade-offs.

Two recently proposed dynamic factor asset pricing models are extremely attractive for two reasons: (1) they accommodate market betas changing through time and (2) they maintain the fundamental and intuitively appealing idea of the CAPM and APT that only a few state variables are needed to explain expected returns. The two models are the conditional CAPM and the nonlinear APT. Ferson (1985), Ferson and Harvey (1991, 1993), Harvey (1991), Ferson

[1]Several general equilibrium developments of the unconditional CAPM and APT have been advanced, see e.g. Huberman (1982), Chamberlain and Rothschild (1983), Ingersoll (1984), Connor (1984), Connor and Korajczyk (1989), among others.

and Korajczyk (1995), Dumas and Solnik (1993) among others discuss and apply the former while the latter is presented in Bansal and Viswanathan (1993) and Bansal, Hsieh and Viswanathan (1993). The conditional CAPM uses the insights of the CAPM, put in a multiperiod context, and exploits the predictable variation in factor loading coefficients. The nonlinear APT, in contrast, is based on the existence of a low-dimensional nonnegative nonlinear pricing kernel which is nonparametrically estimated. In a sense both developments can be related and justified by the results in Hansen and Jaganathan (1991). They show that, for a given set of payoffs, there always exists a unique pricing kernel which is a conditional linear combination of all the payoffs. As the set of all payoffs is typically large one approximates the representation either with a small set of factors in a conditionally linear structure, or else one computes a fixed nonlinear functional pricing kernel also involving only a small set of factors. Both models were successfully used to price international equities, bonds, size-sorted and industry-based portfolios as well as forward currency contracts.

The parameters of the conditional CAPM and the polynomial series expansion of the nonlinear APT are estimated via the generalized method of moments (GMM) procedure discussed in Hansen (1982). The success of the model fit is primarily judged on the basis of GMM-based criteria. In particular, one tests whether the overidentifying restrictions imposed by the model agree with the data[2]. The fundamental problem is that overidentifying restriction tests are not designed to diagnose whether a model, be it a conditional CAPM or nonlinear APT or anything else, provides a stable time invariant relationship between the variables. Technically speaking, one can easily face a situation where a model's overidentifying restrictions are not rejected, while the conditional CAPM or nonlinear APT parameters vary through time. Indeed, the method of moments approach will conceal the time variation as the GMM estimator will converge to some sort of sample average of time parameter variation[3]. Hence, the question whether one has found an asset pricing formula providing a reliable prediction of expected returns as a function of a small number of risk factors is still unresolved. In this paper we propose to apply a set of procedures which are explicitly aimed at testing parameter stability. In fairness to the papers (and authors) quoted on the conditional CAPM, APT and nonlinear APT it should be noted though that they do not exclusively rely on the overidentifying restrictions tests. They also tend to look at the pricing error of the models and conduct other informal

[2] Bansal, Hsieh and Viswanathan (1993) also used the Hansen-Jaganathan distance to gauge the fit of alternative models. Such a criterion is like the usual GMM-based criteria, subject to the same shortcomings we will discuss.

[3] In an econometrics jargon this means that overidentifying restrictions tests may not have power against alternatives characterized by parameter variation. This is formally shown in Ghysels and Hall (1990a). They also provide several examples using the consumption-based CAPM.

diagnostics. Our general goal is to provide more rigor and structure to this issue. Moreover, it will be shown that some of the other diagnostics, like those examining pricing errors, are also prone to the shortcoming we will discuss. Finally, in several paper one finds explicit discussion of the desire to test the stability of parameters like the conditional betas (see Ferson (1990)), the covariance price of risk (see Harvey (1991)), the behavior of asset pricing model in energing market (see Harvey (1993)). Yet, the procedures hitherto adopted are not suited for testing this.

We will examine whether the conditional CAPM and nonlinear APT represent asset pricing models with time invariant parameters. This is important for practitioners who assess the market price of the various risk factors. Whenever an asset pricing structure is unstable it will naturally result in prediction errors and mispricing of risk. The analysis proposed here is also a natural extension of the model specification search since testing for structural change in parameters is one key element in assessing a model's reliability. For GMM estimation several tests have been developed in recent years. The one we will use are discussed in Andrews (1993)[4]. We apply the tests to the international conditional CAPM of Harvey (1991), the multifactor conditional CAPM of Ferson and Korajczyk (1995) and finally the nonlinear APT of Bansal, Hsieh and Visaranathan (1993). In section 2 we motivate the scope of our paper and discuss informally the test statistics, the technical details appearing in the Appendix. Section 3 briefly describes the two asset pricing models. Empirical results are documented in section 4. The paper concludes with section 5.

It should be noted that this paper is not simply an exercise in applying a set of diagnostic tests to a class of recently developed asset pricing models. The results of our investigation indicate that certain specifications appear more stable, and hence more satisfactory, than others. In particular, we find the nonlinear APT a more satisfactory specification. What appears most problematic about the conditional CAPM and APT is the specification of the projection equations for expected returns. Such findings help us directing our focus in the search for an adequate pricing function for assets.

2 Testing for Stable Asset Pricing Models

With the help of a simple example we will first discuss the scope and purpose of testing for structural stability in asset pricing models. Then we will move on to

[4]Tests for structural change for the GMM estimator can be divided in two categories, those assuming a known breakpoint and those without such assumption. The former are covered in Andrews and Fair (1988), Ghysels and Hall (1990a, b) and Dufour, Ghysels and Hall (1994). The second category is covered in Andrews (1993) and Ghysels, Guay and Hall (1994). An extensive Monte Carlo of tests belonging to the second category is documented in Ghysels and Guay (1994).

a discussion of the test statistics we will consider in our empirical work.

Let us concentrate on a very simplified version of the conditional CAPM to set the scene for discussion :

$$E\left[r_i\left(t,t+1\right)|Z_t\right]=\beta_t E\left[r_M\left(t,t+1\right)|Z_t\right] \tag{1}$$

where β_t is the time varying market beta to be specified more explicitly later and Z_t is a set of instruments. The excess return from t to $t+1$ on the market portfolio is measured by $r_M\left(t,t+1\right)$ while $r_i\left(t,t+1\right)$ is the excess return on any asset i. Equation (2.1) accommodates the fact, noted by Harvey (1989), Ferson and Harvey (1991, 1995) and Ferson and Korajczyk (1993), that market betas vary through time. Yet, once we admit that beta varies through time we must specify laws of motion for β_t. The conditional CAPM does that, namely with a single instrument it implies the following:

$$\beta_t=\frac{E\left[\left(r_M\left(t,t+1\right)-\delta_M Z_t\right)\left(r_i\left(t,t+1\right)-\delta_i Z_t\right)\mid Z_t\right]}{E\left[\left(r_M\left(t,t+1\right)-\delta_M Z_t\right)^2\mid Z_t\right]} \tag{2}$$

From the above equation we learn that two time invariant parameters, namely δ_M and δ_i, together with the projections on the instruments Z_t and the asset returns on the market portfolio and asset i determine the time variation in β_t. The two parameters are obtained via the projection equations:

$$E\left[r_i\left(t,t+1\right)|Z_t\right]=\delta_i Z_t \tag{3}$$

$$E\left[r_M\left(t,t+1\right)|Z_t\right]=\delta_M Z_t \tag{4}$$

The question we are interested in is whether this particular characterization of β_t is adequate and does not yield a systematic mispricing of risk factors. Combining equations (2.1) and (2.4) we can write the asset pricing equation as follows:

$$r_i\left(t,t+1\right)=\beta_t\delta_M Z_t+u_{it+1} \tag{5}$$

where $Eu_{it+1}Z_t=0$. If the restrictions of the conditional CAPM do not hold, we obtain as a generic alternative:

$$r_i\left(t,t+1\right)=\tilde{\beta}_t\tilde{\delta}_{Mt}Z_t+\tilde{u}_{it+1} \tag{6}$$

with $E\tilde{u}_{it+1}Z_t=0$ and $\tilde{\beta}_t$ is obtained from (2.2) replacing δ_M by $\tilde{\delta}_{Mt}$ and δ_i by $\tilde{\delta}_{it}$.[5] No specific laws for $\tilde{\delta}_{Mt}$ or $\tilde{\delta}_{it}$ and hence $\tilde{\beta}_t$ will not be explicitly used for

[5]This generic altenative emphasizes the fact that the specification of β_t is erroneous. Other sources of misspecification, such omitted factor risk are, at least for the moment, not considered here.

the moment. Testing whether (2.1) is an adequate model in the pricing of asset returns amounts to testing the hypothesis:

$$H_o : \begin{cases} \tilde{\delta}_{Mt} = \delta_M & \forall t = 1, \cdot\cdot, T \\ \tilde{\delta}_{it} = \delta_i & \forall t = 1, \cdot\cdot, T \end{cases} \tag{7}$$

so that *sole time variation in beta is that determined by the model.* It is worth noting that in (2.7) all parameters are tested jointly for stability. In several circumstances, however, the parameters involved play different roles and therefore depending on which ones are unstable, a different interpretation should be given. For instance, in the multifactor models which will be discussed later, one has a set of parameters that arise from purely ancillary statistical assumptions regarding projection equations besides parameters with an economic interpretation. To emphasize this distinction we will often conduct tests involving only a subset of the parameter vector. For the moment, however, we will proceed with discussing tests involving the entire vector

Continuing with this simple example it should be noted that testing the hypothesis in (2.7) is far more stringent than the usual overidentifying restrictions tests, often called J statistics, that have typically been used to diagnose the fit of an asset pricing model like the conditional CAPM. Since such models are estimated via GMM let us proceed by specifying the moment conditions of the model. Namely, equations (2.5) and (2.3) yield that:

$$E \begin{pmatrix} r_{it+1} - \tilde{\delta}_{it} Z_t \\ r_{Mt+1} - \tilde{\delta}_{Mt} Z_t \\ \tilde{\delta}_{it} Z_t \left[\left(r_{Mt+1} - \tilde{\delta}_{Mt} Z_t \right)^2 \right] - \left(r_{Mt+1} - \tilde{\delta}_{Mt} Z_t \right) \left(r_{it+1} - \tilde{\delta}_{it} Z_t \right) \tilde{\delta}_{Mt} Z_t \end{pmatrix} Z_t = 0 \tag{8}$$

where r_{jt+1}, is a short notation for $r_j\left(t, t+1\right) \;\; j = i, M$. The formulation in (2.8) represents the set of moment conditions involved in the GMM estimation procedure but does not impose the null hypothesis (2.7). The estimation of the conditional CAPM imposing fixed parameter δ_M and δ_i while the data are generated by (2.8) will yield GMM parameter estimates $\overline{\delta}_M$ and $\overline{\delta}_i$ which are some sort of sample average of the underlying $\tilde{\delta}_{Mt}$ and $\tilde{\delta}_{it}$ processes. Ghysels and Hall (1990b) show formally that overidentifying restrictions tests based on the moment conditions such as those in (2.8) but evaluated at fixed parameter estimates $\overline{\delta}_M$ and $\overline{\delta}_i$ may have a tendency *not* to reject the model. This problem is not just a theoretical curiosity. Indeed, we will provide numerous examples where this situation occurs in empirical asset pricing models. Hence, the usual diagnostic tests to judge the validity of a model are not adequate to detect systematic mispricing of asset returns because of parameter instability. It is worth noting parenthetically that besides J statistics other diagnostics are used to appraise the models we will present. Often these complementary diagnostic tests, particularly those based on pricing errors, have the same shortcomings as the J statistic. This will be discussed more elaborately at the end of the section. Our aim is to explicitly test the null hypothesis (2.7). Testing such hypothesis provides a more stringent evaluation of any asset pricing model as it addresses more explicitly the potential systematic mispricing of risk.

How do we go about testing for structural invariance of the model, i.e. verify whether (2.7) holds? As one can imagine, there are many ways to do this. Probably the simplest is to assume as an alternative that at some point in the sample there is a structural break, like for instance :

$$\tilde{\delta}_{jt} = \begin{cases} \delta_{j1} & t = 1, ..., \pi T \\ \delta_{j2} & t = \pi T + 1, ..., T \end{cases} \qquad j = M, i \tag{9}$$

where π determines the fraction of the sample before and after the assumed break point. If the break point πT were known our task would be relatively easy to perform. Something like estimating δ_{j1} and δ_{j2} and comparing both estimates to see whether they are significally different would be one way to proceed, which is often referred to as a Chow test. Unfortunately, in the present context we don't really want to assume π known. In recent years several procedures have been advanced to test the null hypothesis (2.7) against the alternative like (2.9) with unknown break point π. In the Appendix to the paper we provide a detailed description of the econometric procedures that were developed for GMM estimators by Andrews (1993). In the remainder of the section we will explain what these procedures amount to without actually providing any of the technical details. To facilitate our presentation let us denote parameter estimates for δ_{jh} , $h = 1, 2$, $j = i, M$ associated with a particular presumed break point πT as $\tilde{\delta}_{jh}$ (π). Suppose now we construct for each possible break point π between say $.2T$ and $.8T$ a test for structural change based on $\tilde{\delta}_{jh}$ (π), $h = 1, 2$.[6]Hence, for each break point πwe have a Wald-type statistic $W(\pi)$based on the two estimates before and after the break πT. The idea now is to combine the Wald statistics for all possible break points $\{W(\pi), \pi \in [.2, .8]\}$into a single test statistic. This can be done in a variety of ways. A first possibility is to take the maximum over $\pi$of all $W(\pi)$values, called SupW where Sup stands for supremum. Andrews (1993) suggested this type of test and tabulated its distribution under the null hypothesis appearing in (2.7).

The SupW test may be intuitively appealing as it picks the maximum evidence

[6]We have to leave a certain number of observations at each end of the sample in order to estimate $\tilde{\delta}_{j1}$ and $\tilde{\delta}_{j2}$. Therefore we have in this particular case 20% of the sample trimmed at each extreme. The trimming percentage determines for instance how many observations are used to compute the first estimate $\tilde{\delta}_{j1}(\pi)$ and last estimate $\tilde{\delta}_{j2}(\pi)$ with $\pi = .2T$ and $\pi = .8T$ respectively. The sample sizes T involved in our empirical applications made 20% a reasonable choice.

for a structural break. It is however not the only statistic one can think of. First, it should be noted that we prefer to use the SupLM test, that is to say the supremum Lagrangian Multiplier test rather than the SupW test simply because the former requires far less computations. Indeed, with the SupLM which is formally presented in equation (A.8) appearing in the Appendix, one does not compute all the parameter estimates $\widetilde{\delta}_{jh}(\pi)$ for each of the subsamples. Instead, the parameter estimates $\overline{\delta}_M$ and $\overline{\delta}_i$ obtained from the full sample are used. Since we will subject a great many asset pricing models to our test, computational efficiency has a strong appeal. Moreover, the statistical properties of the SupLM test are at least as good, if not better, than those of the SupW test (the Appendix provides the details again). Taking the supremum is not the only way to construct tests, and indeed we shall also consider an exponential LM test denoted ExpLM. It uses all the LM tests and combines them in a way that is in a certain sense optimal as explained in the Appendix.[7]

One may wonder by now why we focus exclusively on tests having a single break point as alternative. Surely, there are many other types of structural instabilities, like for instance cases where there are several breaks or where there are gradual movements in the δ_{ik} parameters. Constructing tests against all possible types of instabilities is simply impossible both statistically and practically. Fortunately, however, the situation is not that hopeless because the single unknown break point statistics have power against a large class of parameter instability patterns for beyond what appears explicitly as alternative in (2.9).

The J-statistic is not the only diagnostic, of course, used to judge the fit of asset pricing models. We would like to point out however, that our observations regarding the J statistic extend to other model diagnostics used to appraise the fit of conditional CAPM as well as the conditional and nonlinear APT models. We would like to conclude this section with a digression on this point providing some specific examples. Sometimes the so called pricing error of the model is examined. Continuing with the simple illustrative conditional CAPM model, this can be done in one of two ways which amount to a different augmentation of the moment conditions in (2.8).[8] Let us first define the pricing error to the asset pricing model as :

$$\widetilde{e}_{it} \equiv \widetilde{\delta}_{it} Z_t - \widetilde{\beta}_t \widetilde{\delta}_{Mt} Z_t \tag{10}$$

This error is defined under the true data generating process which does not necessarily impose the null hypothesis (2.9). Equation (2.10) represents the difference between the predicted excess return from the projection equation and that from the CAPM. Obviously, since the conditional CAPM is estimated with

[7] The optimality is only against a certain class of alternatives and only for the maximum likelihood environment . We will use both tests here side by side.

[8] The two moment conditions augmentations are for instance discussed in Harvey (1993).

fixed parameters, its pricing error will be evaluated using parameter estimates δ_i and δ_M. Therefore the pricing error actually being investigated is :

$$e_{it} \equiv \delta_i Z_t - \beta_t \delta_M Z_t \tag{11}$$

Straightforward algebra shows that the latter can be decomposed as :

$$e_{it} \equiv \widetilde{e}_{it} + (\delta_i - \delta_{it}) Z_t + \left(\beta_t \delta_M - \widetilde{\beta}_t \widetilde{\delta}_{Mt}\right) Z_t \tag{12}$$

Hence, the pricing error being examined is a mixture of model specification error and the pricing error defined in (2.10). The two moment conditions augmentations then considered are:

$$E\left(e_{it} - \mu_i\right) = 0 \tag{13}$$

with the hypothesis of interest being $\mu_i = 0$ and :

$$E e_{it} Z_{t-1} = 0 \tag{14}$$

The first augmentation of the conditional CAPM moment conditions obtained by combining (2.8) and (2.13), yields a test of the average pricing error very similar in spirit to the commonly used unconditional CAPM test of a zero intercept in a linear regression of the excess return of an asset on that of the market portfolio. The second augmentation, combining (2.8) and (2.14) tests whether the pricing error is conditionally predictable.

Since the diagnostic involving (2.14) is obviously more informative than that involving (2.13) we will discuss the former only[9]. From the discussion earlier in this section it is clear that a J statistic assessing the overidentifying restrictions of (2.8) *and* (2.14) combined is not really addressing the issue whether β_t equals $\widetilde{\beta}_t$. To say this differently, it is clear that evaluating the sample crossproduct of $e_{it} Z_{t-1}$ yields no or little information regarding the misspecification of the temporal dynamics of the market price for risk. Equation (2.12) can be viewed as another set of moment conditions involving time varying parameters like $(\delta_i - \delta_{it})$ which will remain undetected. A more rigorous test would amount to *jointly* test parameter stability, i.e. hypothesis (2.7), and the fact that the pricing error $\widetilde{e}_{it} \equiv e_{it}$ satisfies (2.14). To avoid overburdening the scope of our paper we will restrict our attention to simply testing the stability hypothesis (2.7) using the Andrews tests described earlier. Indeed, they already represent a significant amount of discriminatory power among the different models examined. In addition, the joint tests of (2.7) *and* (2.14) actually would require a class of tests different from the Andrews type tests involving a mixture of parameter stability and moment

[9] In a standard setup Z_t is a vector which includes a constant. This inplies that (2.14) encompasses (2.13).

condition tests. Such tests are formally discussed in Ghysels, Guay and Hall (1994). Applications in finance are discussed in Ghysels and Hall (1995).

3 A Review of the Conditional CAPM and Nonlinear APT

We turn our attention now to the two classes of asset pricing models considered in our empirical work, namely the conditional CAPM and the nonlinear APT. To describe both classes of models we follow Bansal and Viswanathan (1993) closely and start from the optimal portfolio allocation conditions of discrete time capital asset pricing models[10]. In an economy with N assets we obtain the following first order conditions:

$$E\left[\mathrm{MRS}\left(t,t+1\right)x_i\left(t,t+1\right)|\Omega_t\right] = \Pi\left(x_i\left(t,t+1\right)\right) for\ i=1,\ldots,N \tag{15}$$

where $x_i\left(t,t+1\right)$ is the one-period payoff of the ith asset at time $t+1$ that has time t price $\Pi\left(x_i\left(t,t+1\right)\right)$ with MRS$(t,t+1)$ the representative agent's marginal rate of substitution between t and $t+1$ consumption.

The expectation in (3.1) is conditional on the information set Ω_t. Equation (3.1) also holds when we replace MRS$(t,t+1)$ by its projection on the space of all one-period payoffs. Let us denote this projection as $P^{\star}_{t+1}$. Hansen and Jaganathan (1991) show this projection can be expressed as a linear combination of the N asset one-period payoffs represented by the vector $x\left(t,t+1\right) = \left[x_i\left(t,t+1\right)\right]_{i=1}^{N}$:

$$P^{\star}_{t+1} = \sum_{j=1}^{N} \alpha_{jt} x_j\left(t,t+1\right) \tag{16}$$

where the weights $\alpha_t = \left[\alpha_{jt}\right]_{j=1}^{N}$ satisfy:

$$\alpha_t = \left[E\left[x'_{t+1}x_{t+1}|\Omega_t\right]\right]^{-1}\Pi\left(x\left(t,t+1\right)\right) \tag{17}$$

While equations (3.2) and (3.3) represent a fundamental relationship in characterizing the pricing of assets it is not yet a "workable" model since it involves as many factors as there are assets, namely N factors. To make the model workable we need to reduce the set of factors, yet equations (3.2) and (3.3) tell us that this is unlikely to be attainable with a simple fixed linear relationship. This observation yielded the nonlinear APT of Bansal and Viswanathan (1993) and Bansal, Hsieh an Viswanathan (1993) and the conditional CAPM of Ferson (1985), Ferson and Harvey (1991, 1993) among others. We shall begin by briefly presenting

[10] See Lucas (1978), Breeden (1979), Stulz (1981), Huang (1987), Duffie and Zame (1989) among others.

the former and then continue with the second class of models. For the nonlinear APT we use equations (3.1) and replace the marginal rate of substitution by its projection onto Ω_{t+1}, yielding :

$$E\left[E\left[\mathrm{MRS}\left(t,t+1\right)|\Omega_{t+1}\right]x_i\left(t,t+1\right)|\Omega_t\right]=\Pi\left(x_i\left(t,t+1\right)\right) \tag{18}$$

Then, instead of using the projection onto the entire information we consider a vector P^b_{t+1} of well-diversified basis variables such that:

$$E\left[\mathrm{MRS}\left(t,t+1\right)|\Omega_{t+1}\right]=E\left[\mathrm{MRS}\left(t,t+1\right)\middle|P^b_{t+1}\right]=G\left(P^b_{t+1}\right) \tag{19}$$

with $G\left(\cdot\right)$ a well-behaved function chosen among a class of flexible functional forms.

Using the fact that $\Pi\left(x_i\left(t,t+1\right)\right)\in\Omega_t$ and normalizing the equation in (3.4) yields the following set of moment conditions :

$$E\left[\left(G\left(P^b_{t+1}\right)x_i\left(t,t+1\right)-1\right)Z_t\right]=0 \tag{20}$$

where Z_t is a set of instruments picked among the elements of Ω_t. Equation (3.6) forms the basis of a GMM estimation procedure for the parameters described the pricing kernel G $(\cdot)$. The set of Z_t instruments actually used in our empirical work will be described later since it coincides with those used in the conditional APT model. The elements entering P^b_{t+1} are the same as those used by Bansal, Hsieh and Viswanathan (1993) in their one-factor model namely :

$$P^b_{t+1}=\left(1+r_M\left(t,t+1\right),1+r_f\left(t,t+1\right)\right) \tag{21}$$

where $r_M\left(t,t+1\right)$ is the nominal return on the market and $r_f\left(t,t+1\right)$ the nominal yield to maturity on the Treasury bill next period.

What remains to be specified is a functional form for G $(\cdot)$. As the exact specification of the nonlinear pricing kernel is unknown, Bansal, Hsieh and Viswanathan (1993) suggest to approximate it with a polynomial series expansion, namely :[11]

$$\begin{aligned}G\left(P^b_{t+1}\right)=\beta_0+\beta_{1t}r_f\left(t,t+1\right)+\beta_{1M}r_M\left(t,t+1\right)+\beta_{2M}\left[r_M\left(t,t+1\right)\right]^2\\+\beta_{5M}\left[r_M\left(t,t+1\right)\right]^5\end{aligned} \tag{22}$$

As with regard to the asset x_i appearing in (3.6) we shall consider a set of size-sorted portfolios and industry-based classified portfolios which will also be used in the conditional APT . The details will be discussed in section 4.2. Finally it was noted in section 2 that the null hypothesis (2.7) was formulated for the entire

[11]As Bansal and Viswanathan (1993) explain, using the fith order rather than the third was partly motivated by the need to reduce collinearity between the various powers of the expansion.

parameter vector. For the nonlinear APT we will be interested in testing the five parameters in (3.8) jointly, of course, but also each parameter individually as well as for instance the parameters of the nonlinear part β_{2M} and β_{5M} separately.

We turn our attention next to two versions of the conditional CAPM, one considered by Harvey (1991) to study the pricing of international assets and another used by Ferson and Korajczyk (1995) to study predictable returns and risk in the U.S. Since the former of the two is the simplest and was already used as motivating example in section 2 we shall discuss it first. Again one can start from the observation that equations (3.2) and (3.3) do not directly yield a workable model, but instead of considering a nonlinear pricing kernel Harvey proposed to study expected returns for stock markets from a set of countries via their *conditional* beta with the return on a world market portfolio. Hence, modifying the traditional CAPM to its conditional version on obtains:

$$E\left[r_i\left(t,t+1\right) \mid \Omega_t\right] = \frac{Cov\left[r_i\left(t,t+1\right), r_M\left(t,t+1\right) \mid \Omega_t\right]}{Var\left[r_M\left(t,t+1\right) \mid \Omega_t\right]} E\left[r_M\left(t,t+1\right) \mid \Omega_t\right] \tag{23}$$

where $r_i\left(t,t+1\right)$ is the return on the market of country i. This formulation is of course different from the nonlinear APT but shares several features and objectives. In particular, it attempts to describe returns with a small set of factors, in this case one, and departs from the fixed linear representation of the traditional APT and CAPM models. To make the equation in (3.9) operational Harvey defined a set of projections, namely:

$$E\left[r_i\left(t,t+1\right)|\Omega_t\right] = Z_t\delta_i \tag{24}$$

$$E\left[r_M\left(t,t+1\right)|\Omega_t\right] = Z_t\delta_M \tag{25}$$

where Z_t is again a set of instruments (not necessarily the same as in the nonlinear APT) and the vectors δ_i and δ_M are (stable) parameter vectors defining the projections. One obtains a set of moment conditions suitable for GMM estimation of $\delta = \left[\delta_i\right]_{i=1}^N$ and δ_M via:

$$E\begin{pmatrix} \left(r_{t+1} - Z_t\delta\right)' \\ \left(r_{Mt+1} - Z_t\delta_M\right)' \\ \left(u_{Mt+1}^2 Z_t\delta - u_{Mt+1}u_{t+1}Z_t\delta_M\right)' \end{pmatrix}' \otimes Z_t = 0 \tag{26}$$

where $r_{t+1} = \left[r_i\left(t,t+1\right)\right]_{i=1}^N, u_t = r_t - Z_{t-1}\delta, u_{Mt} = r_{Mt} - Z_{t-1}\delta_M$ and $r_{Mt+1} = r_M\left(t,t+1\right)$. The moment conditions appearing in (3.12) generalize those in (2.8), Harvey included the following instruments in the estimation of the model (1) a constant, (2) a January Dummy, (3) lagged r_{Mt}, (4) the return on a 90-days

T-Bill minus that of a 30-days one, (5) the Moody Baa yield minus the Aaa one and (6) the dividend yield on the S&P500 minus the 30-days T-Bill return. The sample was similar to that in the first model, namely a set of monthly returns covering 16 OECD countries and Hong Kong from December 1969 to May 1989. The indices used were retrieved from the Morgan Stanley data base which also included the world equity index. The instruments used were from sources similar to these used by Bansal and Viswanathan. We refrain again from elaborating on the details as they are described the original work by Harvey (1991).

In a recent paper Ferson and Korajczyk (1995) undertook a very thorough empirical investigation of risk and return for the U.S. using a multifactor conditional APT. The setup is very similar to that just described except that the moment conditions are a bit more elaborate because of the presents of a multitude of factors. For the multifactor conditional APT, Ferson and Korajczyk define the following set of moment conditions:

$$E\begin{bmatrix} r_i\,(t,t+1) - Z_t^{'}\delta_i \\ \left(F_{t+1}^{'} - Z_t^{'}\gamma_i\right)^{'} \\ \left(F_{t+1}^{'} - Z_t^{'}\gamma_i\right)\left(F_{t+1}^{'} - Z_t^{'}\gamma_i\right)\beta_i - F_{t+1}\left(r_i\,(t,t+1) - Z_t^{'}\delta_i\right) \end{bmatrix} Z_t^{'} = 0 \tag{27}$$

where F_t is a $K \times 1$ vector of factor-mimicking portfolios, β_i is a $K \times 1$ vector of the betas for asset i and Z_t is an $(L+1)$ vector of instruments. In contrast to the nonlinear APT and conditional CAPM, the model defined in (3.13), has parameters which play a different role which makes hypothesis testing also more interesting. Indeed, this more elaborate model has the advantage of separating projection equations and asset pricing moment conditions involving conditional betas. In (3.12) the third set of moment conditions does not involve any new parameters while in (3.13) the third set involves explicitly parameterized betas. The parameters δ_i and γ_i arise from purely anciallary statistical assumptions. Their instability means we have misspecified the projection equations. The instability of β_i, however, has a very different meaning and implication in (3.13). These are the most interesting parameters from asset pricing perspective.

Two alternative sets of risk factors were examined. The first consisted of economic variables similar to Chen, Roll and Ross (1986) and Ferson and Harvey (1991). Five representative economic variables were selected and mimicking portfolios were constructed using individual common stocks. The second approach was motivated by many previous studies of the APT and used the asymptotic principal components methods of Connor and Korajczyk (1986) to estimate the common factors. We followed step by step the specification of variables and instruments described by Ferson and Korajczyk and once again refrain from describing the specific details.

4 Empirical Results

We turn our attention now to the empirical evidence regarding the structural invariance of the three dynamic asset pricing models described in section 3. As there are three empirical models we will devote a subsection to each of them starting with the most simple of all, namely the conditional CAPM studied by Harvey (1991). The second subsection will cover the multifactor model of Ferson and Korajczyk (1995) while we conclude with the nonlinear APT.

4.1 Stable factors in the Conditional CAPM

In Table 4.1 we report empirical results of the international conditional CAPM described in equations (3.12) for the 17 countries covered by Harvey (1991) using exactly the same data and sample. The first column of Table 4.1 reports the overidentifying restrictions tests which are comparable with the tests reported by Harvey in column 6 of Table V of his paper. It should be noted that there are some slight differences between the results reported in the original paper and those appearing in Table 4.1 which are due to the difference in covariance estimator used in the GMM estimation. Our results are based on what should be asymptotically a more efficient estimator proposed by Andrews and Monahan (1992) which was not available at the time the original Harvey paper was written. A consequence of using this more efficient estimator is that there are some differences with the J-statistics reported by Harvey. Indeed, Harvey (1991) rejected the model (at for 5%) for Japan, Norway and Austria. We only reject the model for Austria, while the moment conditions for all other 16 countries seem to fit the data reasonably well when one uses the overidentifying restrictions as a guidance.

However, as we stressed before, the J statistic is a diagnostic test ill-equipped to scrutinize a model in terms of its structural invariance and by the same token the ability of a model to predict the market price for risk. Table 4.1 reports SupLM and ExpLM tests for each of the six instruments involved in Harvey's "common instrument" specification[12]. Each test statistic has two degrees of freedom as each instrument is associated with two parameters, one entering the projection on the country return and the other entering the world return equation. Let us consider an extreme case first. Take the results for France, for instance. According to the overidentifying restrictions test the model is not rejected at 10 %. Yet, for five of the six instruments we find evidence of instability (at 10 % significance level).

Hence, despite the favorable evidence according to the usual J-statistic it is clear that the return on France's market index cannot be satisfactorily priced

[12]To compute the statistics one has to specify the set of observations Π (see equations (A.5) and (A.7) in the Appendix). In all our computations we set $\Pi = [.2T, .8T]$. The choice of 20% trimming was motivated by the length of the sample as noted in section 2.

with the conditional CAPM. While the case of France is extreme in the sense that almost all instruments appear unstable we note that of the 17 countries eight have at least one unstable return-risk factor. These countries are Austria, Belgium, France, Hong Kong, The Netherlands, Sweden, Switzerland and the U.K. More interesting is to study whether any particular factor is more unstable than others. This is indeed the case, as appears from the results reported in Table 4.1. Which indicate that the interest rate spread series, i.e. the return for holding a 90-days US T-Bill for one month less the return on a 30-days T-Bill, is for seven countries an unstable risk factor (at least at 10 % significance). Next to the spread comes the lagged World return appearing as an unstable factor for five countries.

Table 4.1: Stable Factors in the Conditional CAPM - Analysis of 17 countries

Instruments		Constant		Lagged World Return		January Dummy		Interest Rate Spread		Junk Bonds		Dividends	
COUNTRY	J-TEST	SUP LM	EXP LM	SUP LM	EXP LM	SUP LM	EXP LM	SUP LM	EXP LM	SUP LM	EXP LM	SUP LM	EXP LM
Australia	6.5174	3.6311	0.7418	3.0362	0.3654	4.6616	0.8674	4.2902	0.6982	2.9770	0.5552	2.1704	0.3730
Austria	13.6066**	5.9826	1.2506	6.2740	1.1737	4.3686	1.0386	8.9242	2.7685*	6.7049	1.2651	6.8269	1.5649
Belgium	6.8188	5.2952	0.9927	9.1891	1.9403	6.4421	1.9249	11.0542*	1.7573	2.7133	0.6496	2.6847	0.5062
Canada	2.9671	4.6890	0.9997	4.9056	0.9114	3.9951	0.8899	3.2388	0.4490	3.4358	0.8351	5.2689	1.2920
Denmark	8.1489	4.3348	1.2592	6.3573	1.4528	3.0238	0.8263	4.7545	0.4073	4.0658	0.9185	2.7222	0.3955
France	10.2547	10.4621*	3.2905**	10.6398*	1.3758	9.7953	0.9751	11.2252*	3.1842*	11.2551*	2.8443*	11.2382*	2.6412*
Germany	3.4082	5.8450	1.5141	5.7787	1.1556	5.6670	0.9872	9.2200	1.5306	4.2970	1.1627	3.7410	0.8116
Hong-Kong	5.5435	9.3268	2.8960	5.9882	1.3431	2.3676	0.3319	11.1050*	2.6757	10.0280*	2.6980*	7.5126	2.3448
Italy	7.4231	7.0516	1.6570	5.4374	0.7784	2.4048	0.4844	12.9267**	2.6979*	6.5378	1.1373	6.7605	1.3404
Japan	9.6251	3.9931	0.5907	2.8982	0.5399	4.9859	1.0410	5.2808	0.5902	3.2186	0.4227	3.5220	0.5320
Netherlands	3.8889	8.9479	2.2940	6.6443	1.8772	7.4040	2.1625	9.6209	2.8014*	5.2552	1.2923	10.4580*	3.2942**
Norway	7.8014	4.6432	1.1367	9.7020	2.3309	3.5191	0.5844	6.5901	1.5024	6.1809	1.1902	5.5848	1.2164
Spain	8.6923	3.4196	0.6213	2.7424	0.4669	4.7209	0.7158	9.2328	1.6251	3.5783	0.4747	3.4557	0.5073
Sweden	7.0917	4.9210	0.6827	10.2503*	1.7013	5.3662	0.9731	5.0658	0.9265	4.8685	0.7173	6.8217	0.7296
Switzerland	9.3350	8.0102	2.3841	10.3749*	2.9198*	4.5855	0.9076	12.0330**	2.9852*	4.6948	1.1773	8.3915	2.2835
United Kingdom	1.0003	8.9989	2.2616	16.4854***	3.3758**	3.4458	0.8674	6.5070	1.1286	8.3893	1.8824	8.4515	1.5184
United-States	7.8994	2.4225	0.3708	3.2592	0.3690	6.0156	1.0762	9.1552	2.0155	2.7975	0.4562	3.3951	0.5439

Notes: The data are taken from Harvey (1991) where all details appear. The instruments are defined as follows: Lagged World Return is the excess return on the MSCI world index, the interest rate spread is the return for holding a 90-days US Tbill for 1 month less the return on a 30-days Bill, the Junk bond instrument is the yield on Moody's Baa rated bonds less the yield on Moody's Aaa rated bonds, the dividend instrument is the dividend yield on the S & P 500 stock index less the return on a 30-days Tbill.

This first of three empirical examples underscores several important points which motivates our study. We reported a set of models that would be found empirically acceptable, according to their overidentifying restrictions, for explaining the return on international stock markets with a pricing formula based on a set of common instruments. Using tests for structural stability, there is only for a small subset of countries, at least on the basis of the particular sample, supporting empirical evidence and a stable risk-return model. Moreover, quite often we also found the same unstable factors, in particular the interest rate spread and lagged world return. In practical terms this means that have to be careful using these instruments to yield a satisfactory dynamic conditional asset pricing model. In such circumstances either the model needs to be modified or else we need to search for a stable risk factor alternative.[13]

4.2 Stable factors in the Conditional Multifactor APT

Among the countries listed in Table 4.1 figures the U.S. The results show that we find the conditional CAPM a satisfactory specification since all loading coefficients for the six risk factors appear stable. Here we shall further explore this specification via a more detailed study of asset returns for the U.S. market. The data are those of Ferson and Korajzcyk (1995), that is to say a data set covering size-sorted returns for stocks appearing on the CRSP data set as well as those same asset returns classified by industry. To describe the empirical results let us return to equation (3.13) and recall the interpretation of each of the parameters. There are essentially three sets of moment conditions, the first two defining conditional expectations (linear projections) of asset returns while the third relates to the multifactor beta model. Indeed, we noted in section 3 that instability of the parameters δ_i and γ_i reflects a misspecification of the statistical models of predictable dynamics in returns or factor mimicking portfolios. In contrast, from an asset pricing perspective, fixed conditional betas is more a fundamental and a crucial assumption (see for instance Ferson (1990, table VIII) on this issue). We will first discuss the two sets of projection equations and then turn to the risk equation pricing.

The first set of moment conditions are used to model predictability of portfolio returns over time and involve six instruments plus a constant. The instruments are fairly standard, namely (1) the level of the one-month T-Bill, (2) the dividend yield of the CRSP value-weighted NYSE stock index, (3) a detrained stock price level, (4) a measure of the slope of the term structure, (5) a quality-related yield spread in the corporate bind market, and (6) a January dummy. Obviously these instruments are quite similar to those appearing in the previous section,

[13] Our sample did not include, except for Hong Kong, many so called emerging markets which by their very nature of transition and globalization are prone to instabilities. Garcia and Ghysels (1994) focus more explicitly and exclusively on emerging market asset pricing.

though their precise definition differs slightly as can be verified by consulting the details of respectively Harvey (1991) and Ferson and Korajczyk (1995). Given that the instruments are quite similar let us first discuss the empirical results regarding the stability of the coefficients δ_i in (3.13) obtained from projecting the six instruments plus constant on size-sorted and industry-based portfolio returns. Obviously, one has to keep in mind that like in Harvey's model discussed in the previous section, these coefficients are estimated as part of a larger joint system involving moment conditions containing factor-mimicking portfolios. Those moment conditions will be discussed later.

Table 4.2: Stable Factors in the Conditional APT - Industry Classification with Principal Components Factors												
	Industry 1	Industry 2	Industry 3	Industry 4	Industry 5	Industry 6	Industry 7	Industry 8	Industry 9	Industry 10	Industry 11	Industry 12
J-TEST	39.4	45.2 **	21.6	51.6 ***	54.1 ***	31.6	33.2	39.2	58.0 ***	44.0 **	55.2 ***	43.1 *
	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM
δ_{ALL}	39.9 ***	52.8 ***	18.8	44.5 ***	38.5 ***	19.9 *	37.1 ***	14.3	21.6 *	25.1 **	88.2 ***	32.1 ***
δ_1	4.5	9.5 **	3.5	9.8 **	7.5 *	7.6 *	7.9 *	4.4	3.7	10.7 **	8.2 *	2.8
δ_2	22.2 ***	3.6	1.1	11.8 **	1.5	9.7 **	2.7	1.0	2.2	7.1	3.4	1.4
δ_3	4.8	13.0 ***	2.4	5.7	8.7 *	11.9 **	12.4 ***	4.7	0.9	8.3 *	20.6 ***	2.3
δ_4	3.7	15.8 ***	2.7	6.2	8.0 *	8.3 *	7.1	4.6	1.6	11.3 **	15.6 ***	2.3
δ_5	8.2 *	11.7 **	4.7	7.4 *	8.4 *	10.7 **	6.5	5.1	3.9	6.7	15.0 ***	6.0
δ_6	6.3	11.8 **	0.8	11.4 **	21.3 ***	9.4 **	8.3 *	2.0	3.7	12.7 ***	32.3 ***	12.7 ***
δ_7	1.4	3.7	3.7	6.6	1.6	4.7	2.5	3.8	8.3 *	6.9	10.0 **	2.3
γ_{1c}	71.1 ***	63.3 ***	9.8	17.5	48.7 ***	16.4	38.9 ***	20.1 *	102.4 ***	21.1 *	83.5 ***	83.6 ***
γ_{2c}	50.8 ***	79.2 ***	34.3 ***	45.2 ***	30.1 ***	22.6 **	31.2 ***	47.3 ***	80.4 ***	49.4 ***	85.4 ***	81.3 ***
γ_{3c}	65.8 ***	152.3 ***	16.5	45.4 ***	57.1 ***	35.2 ***	33.1 ***	33.5 ***	78.0 ***	48.8 ***	81.3 ***	93.4 ***
γ_{4c}	64.6 ***	70.7 ***	31.2 ***	62.7 ***	61.0 ***	37.9 ***	69.6 ***	78.8 ***	30.5 ***	56.7 ***	50.1 ***	53.1 ***
γ_{5c}	56.0 ***	65.6 ***	19.1	70.2 ***	30.0 ***	30.1 ***	55.8 ***	36.8 ***	52.6 ***	33.8 ***	65.9 ***	44.6 ***
β_{ALL}	82.0 ***	127.2 ***	9.8	67.0 ***	42.6 ***	52.5 ***	86.4 ***	74.9 ***	57.0 ***	106.6 ***	50.7 ***	132.7 ***
β_1	1.7	26.2 ***	4.5	25.9 ***	3.2	2.9	35.1 ***	9.9 **	0.7	15.4 ***	9.7 **	24.8 ***
β_2	24.0 ***	53.9 ***	5.7	35.5 ***	17.2 ***	12.4 ***	18.4 ***	23.4 ***	2.9	52.3 ***	7.2 *	21.4 ***
β_3	25.9 ***	66.1 ***	2.9	7.8 *	4.6	16.3 ***	31.1 ***	3.3	37.7 ***	52.7 ***	4.1	92.0 ***
β_4	5.4	21.6 ***	1.9	24.4 ***	2.5	3.2	23.8 ***	24.8 ***	4.3	18.4 ***	1.6	30.9 ***
β_5	8.9 **	7.9 *	2.6	28.7 ***	11.6 **	3.1	20.8 ***	6.8	8.4 *	12.5 ***	10.1 **	24.1 ***

Note: Tests for Ferson's model. δ_{ALL} test all coefficients of δ together (7 parameters), δ_i (i=1,2,...,7) test all coefficients of δ one by one (1 parameter each). γ_{ic} (i=1,2,...,5) test all coefficients of γ column by column (7 parameters by column). β_{ALL} test all coefficients of β together (5 parameters), β_i (i=1,2,...,5) test all coefficients of β one by one (1 parameters each).
*: 10% ; **: 5% ; ***: 1%

Table 4.3: Stable Factors in the Conditional APT - Size Classification with Principal Components Factors

	Size 1	Size 2	Size 3	Size 4	Size 5	Size 6	Size 7	Size 8	Size 9	Size 10
J-TEST	37.4	50.2 **	32.7	53.5 ***	39.2	26.1	52.8 ***	39.8	47.1 **	60.5 ***
	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM
δ_{ALL}	35.8 ***	19.1	37.2 ***	28.6 ***	21.8 *	29.8 ***	32.2 ***	78.8 ***	167.6 ***	189.4 ***
δ_1	2.2	2.4	3.7	4.9	5.4	2.8	3.6	30.8 ***	3.5	4.5
δ_2	2.4	4.9	4.6	15.6 ***	6.6	15.7 ***	5.7	7.7 *	2.6	25.6 ***
δ_3	4.0	2.3	4.4	5.3	5.6	2.6	4.5	34.2 ***	6.1	6.2
δ_4	3.1	2.7	4.7	5.5	5.4	2.1	4.9	35.7 ***	6.5	6.1
δ_5	2.3	1.1	3.4	17.1 ***	8.0 *	9.9 **	5.2	19.0 ***	22.4 ***	14.7 ***
δ_6	2.8	2.2	6.6	6.6	4.6	2.6	6.3	62.8 ***	3.4	12.8 ***
δ_7	5.7	6.9	9.6 **	3.6	6.5	7.7 *	3.2	10.9 **	2.2	9.8 **
γ_{1c}	46.7 ***	55.0 ***	34.1 ***	63.3 ***	51.4 ***	21.9 **	26.1 **	64.2 ***	203.1 ***	162.4 ***
γ_{2c}	68.30 ***	87.5 ***	80.6 ***	25.2 **	32.1 ***	28.4 ***	34.8 ***	38.4 ***	121.7 ***	92.2 ***
γ_{3c}	57.1 ***	86.4 ***	39.4 ***	53.5 ***	69.9 ***	27.0 ***	25.0 **	40.8 ***	309.1 ***	209.3 ***
γ_{4c}	78.9 ***	155.4 ***	99.0 ***	82.5 ***	97.0 ***	65.6 ***	42.1 ***	49.5 ***	119.1 ***	75.2 ***
γ_{5c}	47.5 ***	77.0 ***	73.4 ***	46.4 ***	41.9 ***	25.4 **	27.8 ***	55.9 ***	156.6 ***	110.4 ***
β_{ALL}	94.1 ***	50.9 ***	58.6 ***	24.7 ***	64.0 ***	29.9 ***	44.0 ***	81.9 ***	363.9 ***	319.7 ***
β_1	31.3 ***	23.1 ***	7.7 *	11.6 **	8.4 *	10.9 **	1.2	21.9 ***	14.5 ***	91.6 ***
β_2	9.6 **	5.4	32.1 ***	9.5 **	12.4 ***	23.0 ***	25.7 ***	58.0 ***	45.2 ***	60.8 ***
β_3	62.2 ***	24.6 ***	20.7 ***	10.4 **	30.0 ***	1.1	8.8 *	24.2 ***	306.9 ***	275.2 ***
β_4	27.5 ***	8.8 *	8.5 *	6.8	15.1 ***	9.6 **	4.9	7.4 *	135.2 ***	123.9 ***
β_5	38.7 ***	10.3 **	9.2 **	8.9 **	11.7 **	14.5 ***	11.9 **	22.9 ***	40.0 ***	90.2 ***

Note: Tests for Ferson's model. δ_{ALL} test all coefficients of δ together (7 parameters), δ_i (i=1,2,...,7) test all coefficients of δ one by one (1 parameter each). γ_{ic} (i=1,2,...,5) test all coefficients of γ column by column (7 parameters by column). β_{ALL} test all coefficients of β together (5 parameters), β_i (i=1,2,...,5) test all coefficients of β one by one (1 parameters each).
*: 10% ; **: 5% ; ***: 1%

Table 4.4: Stable Factors in the Conditional APT - Industry Classification with Economic Variables Factors

	Industry 1	Industry 2	Industry 3	Industry 4	Industry 5	Industry 6	Industry 7	Industry 8	Industry 9	Industry 10	Industry 11	Industry 12
J-TEST	36.7	35.2	27.8	36.5	31.4	37.7	27.5	48.8 **	34.6	26.2	49.8 **	33.5
	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM
δ_{ALL}	16.3	15.0	9.0	17.9	19.6	10.3	12.6	14.2	16.6	13.8	14.2	13.5
δ_1	1.7	0.6	5.1	1.5	10.0 **	2.3	1.9	1.8	4.3	7.7 *	2.1	1.8
δ_2	3.2	1.5	2.6	1.3	3.9	1.0	2.1	1.8	5.5	6.6	3.6	2.5
δ_3	3.9	1.2	4.9	1.5	12.0 **	1.4	1.7	1.3	3.7	5.7	2.6	2.6
δ_4	3.4	0.5	4.9	1.4	9.9 **	1.6	1.7	1.6	4.2	7.0	2.0	2.0
δ_5	5.8	2.5	3.2	2.9	7.3 *	2.8	2.1	3.6	4.3	5.5	1.5	3.4
δ_6	8.3 *	1.8	3.2	5.7	13.0 ***	3.0	3.3	1.4	1.8	7.7 *	2.7	3.2
δ_7	3.5	1.2	1.8	2.1	5.6	3.9	6.5	1.9	2.4	4.3	6.7	3.0
γ_{1c}	77.9 ***	76.4 ***	66.9 ***	62.6 ***	99.2 ***	69.5 ***	62.7 ***	62.5 ***	61.4 ***	81.6 ***	64.8 ***	71.7 ***
γ_{2c}	170.6 ***	112.0 ***	108.4 ***	95.7 ***	142.0 ***	95.8 ***	91.9 ***	92.5 ***	90.4 ***	131.3 ***	105.2 ***	97.9 ***
γ_{3c}	32.2 ***	27.8 ***	15.4	23.5 **	22.9 **	21.0 *	12.7	33.7 ***	22.7 **	17.6	25.4 **	16.2
γ_{4c}	31.0 ***	23.0 **	51.2 ***	19.3	17.9	20.0 *	15.7	37.3 ***	23.2 **	14.5	18.4	19.1
γ_{5c}	43.1 ***	16.1	9.0	16.0	20.5 *	12.6	9.8	27.0 ***	9.7	20.6 *	14.9	17.9
β_{ALL}	36.8 ***	17.6 *	7.1	8.7	39.4 ***	22.2 **	8.4	17.0 *	17.8 *	9.0	28.1 ***	19.8 **
β_1	10.3 **	7.8 *	2.1	5.5	1.0	2.9	3.1	3.9	4.1	2.4	1.6	9.4
β_2	10.7 **	9.1 **	1.0	2.5	5.5	3.6	3.0	5.0	2.0	4.8	3.1	16.2 ***
β_3	1.2	3.6	0.8	2.8	24.9 ***	3.9	3.3	2.0	5.8	2.5	19.9 ***	2.2
β_4	4.2	14.5 ***	1.1	4.2	4.2	0.8	0.8	2.8	8.5 *	1.7	5.8	0.7
β_5	18.2 ***	3.4	1.7	1.9	25.5 ***	11.0 **	5.0	14.3 ***	10.7 **	3.9	18.3 ***	4.4

Note: Tests for Ferson's model. δ_{ALL} test all coefficients of δ together (7 parameters), δ_i (i=1,2,...,7) test all coefficients of δ one by one (1 parameter each). γ_{ic} (i=1,2,...,5) test all coefficients of γ column by column (7 parameters by column). β_{ALL} test all coefficients of β together (5 parameters), β_i (i=1,2,...,5) test all coefficients of β one by one (1 parameters each).
*: 10% ; **: 5% ; ***: 1%

Table 4.5: Stable Factors in the Conditional APT - Size Classification with Economic Variables Factors										
	Size 1	Size 2	Size 3	Size 4	Size 5	Size 6	Size 7	Size 8	Size 9	Size 10
J-TEST	29.3	32.3	46.9 **	53.5 ***	50.7 **	46.5 **	39.5	37.4	30.1	54.9 ***
	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM
δ_{ALL}	13.2	11.1	14.0	10.9	16.4	19.1	31.7 ***	23.2 **	14.3	40.7 ***
δ_1	3.2	8.2 *	5.5	8.4 *	10.5 **	4.1	4.8	5.8	3.9	2.4
δ_2	1.7	2.4	6.5	4.3	9.6 **	1.7	10.1 **	7.9 *	11.2 **	3.8
δ_3	3.4	10.0 **	5.5	9.5 **	11.7 **	5.1	4.9	4.0	2.6	3.0
δ_4	2.5	8.4 *	4.8	8.2 *	10.2 **	4.0	3.2	4.5	4.0	3.0
δ_5	1.0	3.9	5.8	5.4	8.9 **	3.4	10.0 **	4.0	10.5 **	10.5 **
δ_6	3.8	7.7 *	2.9	7.4 *	6.6	8.9 *	5.8	2.0	2.6	2.1
δ_7	5.0	3.0	3.0	2.6	4.9	1.9	1.3	4.9	3.4	3.1
γ_{1c}	64.3 ***	59.7 ***	65.1 ***	61.7 ***	64.4 ***	75.4 ***	66.5 ***	57.2 ***	67.5 ***	75.2 ***
γ_{2c}	103.0 ***	105.0 ***	109.7 ***	115.0 ***	141.6 ***	126.4 ***	110.4 ***	98.8 ***	108.1 ***	108.9 ***
γ_{3c}	31.3 ***	26.0 **	13.9	19.9 *	30.0 ***	24.0 **	19.7 *	17.0	23.5 **	26.2 **
γ_{4c}	33.3 ***	28.8 ***	14.6	23.4 **	25.5 **	27.5 ***	24.1 **	18.8	24.4 **	22.0 **
γ_{5c}	12.2	8.2	9.7	10.8	13.8	17.2	29.1 ***	22.8 **	12.5	43.2 ***
β_{ALL}	8.6	9.5	7.3	16.2	13.1	18.5 **	18.6 **	17.5 *	18.1 *	39.2 ***
β_1	2.8	2.2	5.2	3.7	2.2	6.1	8.6 *	3.2	2.3	2.7
β_2	4.6	4.3	5.3	3.3	2.4	7.5 *	6.6	3.2	2.7	6.2
β_3	1.0	4.7	4.6	2.3	5.9	4.9	10.9 **	6.0	4.7	25.4 ***
β_4	2.5	1.7	1.3	6.3	4.7	8.5 *	3.7	10.5 **	12.9 ***	7.1
β_5	4.9	2.4	1.6	3.1	3.4	4.6	11.8 **	4.6	4.9	38.0 ***

Note: Tests for Ferson's model. δ_{ALL} test all coefficients of δ together (7 parameters), δ_i (i=1,2,...,7) test all coefficients of δ one by one (1 parameter each). γ_{ic} (i=1,2,...,5) test all coefficients of γ column by column (7 parameters by column). β_{ALL} test all coefficients of β together (5 parameters), β_i (i=1,2,...,5) test all coefficients of β one by one (1 parameters each).
*: 10% ; **: 5% ; ***: 1%

Tables 4.2 through 4.5 cover the empirical results for the conditional APT with a combination of industry-based and size-sorted portfolios using principal component factors and so-called economic factors. Both types of factors will be discussed shortly. For the moment let us concentrate on the J-statistics appearing in the top row of each table and the statistics listed in the rows labelled δ_{all} and $\delta_i, i = 1, \ldots, 7$.[14] The tests corresponding to δ_{all} will be joint tests for all seven instruments (the first being a constant), while the other measure each instrument individually. In Tables 4.2 through 4.5 we only report SupLM tests; a set of companion tables A.1 through A.4 appearing in the Appendix cover the ExpLM statistics. Let us focus on the results in Table 4.2. They cover the twelve industries selected by Ferson and Korajczyk.
According to the J-statistic we would reject the model for industries 2 (Finance/Real Estate), 4 (Basic Industries), 5 (Food/Tobacco), 9 (Utilities), 10 (Textile/Trade), 11 (Services) and to a lesser degree (i.e. only at 10 %) industry 12 (Leisure). Let us therefore concentrate on the remaining industries and examine the SupLM test associated with the δ_{all} row. Hence we focus on all the instruments together used to model the predictable part of returns and test whether such predictions can be done with a time invariant vector δ. For industry 1 (Petroleum), 7 (Capital Goods), and to a lesser extend industry 6 (Construction) there is evidence of instability. The only industries left, after using the J-statistics and SupLM statistic for δ_{all} as diagnostics, are industry 3 (Consumer Durables) and 8 (Transportation). For the other tables the results are not as dramatic. In particular, when looking at Table 4.4 where the industry classification is paired with an economic variables factor specification we would only reject the model for two industries, again solely on the basis of the two first diagnostic tests. Finally, it should also be noted that the individual tests for $\delta_i, i = 1, \ldots, 7$ listed in Tables 4.2 and 4.4 confirm mostly what is found on the basis of δ_{all}.

We turn our attention now to size-sorted portfolios, restricting again our attention first to the J and δ_{all} tests. The model appears to fit none of the size-sorted portfolios if we judge its performance not only on the basis of the overidentifying restrictions but also on the basis of the stability of the δ coefficients. If we were only to use the J tests the verdict would not have been as dramatic, with five out of ten portfolio specifications rejected. Unlike the results in Table 4.2 we should note that quite often the individual δ_i coefficients appear stable in Table 4.3 yet overall the parameter vector δ is time varying. When the model is estimated with portfolio-mimicking factors using economic factors instead of principal component we find again more favorable evidence as can be seen from the results in Table 4.5.

Let us examine now the second set of projection equations involving factor-

[14] The index J to δ_j is not to be confounded with index δ_i in (3.13). The latter referred to asst i and represents the entire vector (δ all tin the tables), while δ_j is an element of δ all.

mimicking portfolios denoted F_t in (3.13). It was noted at the end of section 3 that two types of factors were considered, a set of representative economic variables as in Chen, Roll and Ross (1986) or Ferson and Harvey (1991) and the commonly used principal component APT specification using methods discussed by Connor and Korajczyk (1986). To produce a Kx1 vector of factor-mimicking portfolios, with the factors either economic variables or principal components, Ferson and Korajczyk used a method proposed by Lehmann and Modest (1988) which they describe in detail in an Appendix to their paper. The second set of moment conditions in (3.13), like the first, involves projections on the set of instruments described before, to extract the predictable part of the Kx1 vector F_t . Since this is a multivariate process prediction with K=5 we focus on tests for each column which projects the entire set of instruments on the each of the 5 factor-mimicking portfolios. Hence, we use the notation $\gamma_{ic}, i = 1, \ldots, 5$ to denote the tests associated with each of the column vectors[15]. We turn our attention first to Tables 4.2 and 4.3 where the results are reported involving principle component factor-mimicking portfolios. The results are quite unambiguous. It is clearly impossible to predict with time invariant linear projections using the instruments in Z_t the five portfolios. With the economic factor specification appearing in Tables 4.4 and 4.5, there is clearly some improvement. Yet, there is never ever a specification of the model neither for industry-based portfolios nor for size-sorted ones, which yields results in all five elements of the F_t vector being adequately predictable via projections on Z_t . The best one can settle for is three out of the five projections being stable (see industries 7 and 12 in Table 4.4). Clearly this part of the conditional APT moment conditions needs to be improved upon, either by considering nonlinear projections and/or other instrumental variables.

In the remaining part of the model specification we turn our attention to the vector of conditional betas appearing in the third set of moment conditions (3.13). We have five parameters in β, as many as there are factor-mimicking portfolios. The tests reported in Tables 4.2 through 4.5 cover both joint tests, i.e. all elements of β, as well as individual tests for $\beta_j, j = 1, \ldots, 5$.[16] The results in the first set of two tables with the principal component specification is again revealing strong evidence of misspecification. Since from an asset pricing perspective, the instability of these parameters is more for reaching, this result is more significant then the previously reported instabilities. The economic variables specification on the other hand again yields more satisfactory results, particularly with the size-sorted portfolios. In fact, the $\beta's$ for small companies appear quite stable

[15]We could not perform an overall test for the entire matrix γ involving 35 coefficients as no critical values were available for that many coefficients. For reason of space we do not report individual tests nor tests associated with a particular instrument in this case.

[16]We use again the same convention for β_j as we did for γ_j.

in comparison to large companies as appears from the results in Table 4.5. This means, despite the problems with the specification of the projection equations discussed earlier, it appears that for some of the smaller firms in the sample there is a reasonable pricing equation which emerges.

A final comment is in order before moving to the nonlinear APT. The tables containing the ExpLM tests, which appear in the Appendix, largely confirm the results reported in Tables 4.2 through Tables 4.5. This means that our findings appear fairly robust regarding the presence of parameter instability

4.3 Stable factors in the Nonlinear APT

We turn now to the nonlinear APT proposed by Bansal, Hsieh and Viswana-than (1993). We did not attempt here to exactly replicate their data and estimates. Instead, for the purpose of comparison, we used the Ferson and Korajczyk data set of sized-sorted and industry-based portfolios to estimate the nonlinear APT specified in equation (3.8) using the same set of instruments as in the proceeding section. This means we have seven instruments, including a constant, to specify the moment conditions in (3.6). Since there are five parameters in equation (3.8) we have two overidentifying restrictions. The results are reported in two tables, one covering the asset returns for each of the ten sized-sorted portfolios and the other containing the industry-based portfolios. To streamline the presentation we have only reported the SupLM tests in the main body of the paper and deferred companion tables with ExpLM tests to the Appendix.

In Tables 4.6 and 4.7 we report, besides the J-test, tests for the stability of each of the five parameters in the nonlinear APT separately as well as two joint tests, one involving the parameters of the "nonlinear part", namely β_{2M} and β_{5M}, and one involving the joint set of five beta parameters. The results in Table 4.6 show that according to the J-test we reject the model for small firms only (size 1). However, if we look at the tests for parameter stability there are clearly problems with size categories 7, 9 and 10 and to a certain extend also size 2. All other size categories appear to be well fitted by a stable nonlinear APT model. In some sense this is far better than the conditional APT of the previous section, since for six portfolios the model seems acceptable. In particular, the results seem to indicate that the nonlinear APT fails to explain the return on very large firms, which are often used in speculative arbitrage strategies between broad market indices like the S&P100 and index futures and options. It also fails to explain returns on very small firms which probably are more affected by informed trading and idiosyncratic events. The nonlinear APT appears also quite successful if one looks at industry-based portfolios. In Table 4.7 we can see that for at least half of the twelve industries there is neither instability according to the SupLM tests nor rejection by the J-statistic. The industries where the model fails are : industry 2 (Petroleum), 3 (Consumer Durables), 5 (Food and Tobacco), 6 (Construction), 7 (Utilities) and 11 (Services). In each of these cases one must probably search for other risk factors to add to P^b_{t+1} such as the oil price (Petroleum), housing starts (construction), etc. or consider augmenting the polynomial expansion to accomplish a stable pricing kernel.

Table 4.6: Stable Factors in Nonlinear APT - Size Classification

	Size 1	Size 2	Size 3	Size 4	Size 5	Size 6	Size 7	Size 8	Size 9	Size 10
J-TEST	8.08 **	0.79	0.41	0.03	0.00	0.28	0.13	0.77	0.04	0.01
	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM
β_0	14.18 ***	7.97 *	2.68	2.92	1.10	1.18	12.17 **	3.70	10.00 **	14.78 ***
β_{1f}	16.20 ***	7.84 *	2.56	2.84	0.87	1.00	8.72 *	2.03	6.63	11.82 **
β_{1M}	14.18 ***	7.95 *	2.68	2.92	1.10	1.18	12.15 **	3.67	9.98 **	14.82 ***
β_{2M}	14.17 ***	7.94 *	2.68	2.93	1.09	1.18	12.14 **	3.65	9.95 **	14.85 ***
β_{5M}	14.14 ***	7.89 *	2.66	2.94	1.07	1.18	12.09 **	3.59	9.88 **	14.95 ***
β_{2M} & β_{5M}	14.27 **	8.91	2.94	3.75	2.28	2.18	12.51 **	6.04	10.90 *	16.70 ***
β_{ALL}	38.89 ***	12.30	7.65	6.05	4.95	3.65	19.77 **	10.70	15.14	19.43 **

Table 4.7: Stable Factors in Nonlinear APT - Industry Classification

	Industry 1	Industry 2	Industry 3	Industry 4	Industry 5	Industry 6	Industry 7	Industry 8	Industry 9	Industry 10	Industry 11	Industry 12
J-TEST	4.19	0.78	0.06	0.01	5.87	0.33	0.00	5.82	0.87	0.21	2.18	2.37
	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM	Sup LM
β_0	10.74 **	1.52	13.09 ***	5.45	7.18 *	9.21 **	4.92	2.46	19.85 ***	2.40	16.51 ***	5.77
β_{1f}	10.12 **	1.49	11.63 **	2.27	7.56 *	7.87 *	2.97	2.81	18.43 ***	1.34	18.85 ***	4.93
β_{1M}	10.72 **	1.54	13.03 ***	5.41	7.16	9.20 **	4.88	2.48	19.84 ***	2.34	16.57 ***	5.73
β_{2M}	10.69 **	1.55	12.97 ***	5.36	7.14	9.18 **	4.84	2.50	19.83 ***	2.28	16.63 ***	5.69
β_{5M}	10.61 **	1.59	12.77 ***	5.22	7.08	9.14 **	4.72	2.55	19.79 ***	2.11	16.79 ***	5.56
β_{2M} & β_{5M}	14.04 **	4.28	14.38 **	8.38	9.41	9.91	7.36	4.05	20.02 ***	12.02 **	21.40 ***	12.27 **
β_{ALL}	17.91 *	4.90	23.84 ***	21.34 **	11.41	15.45	12.03	5.57	31.00 ***	17.29 *	29.07 ***	13.61

Note: β_i test each parameter separately. β_{2M} & β_{5M} test these two parameters jointly. β_{ALL} test all parameters together.
*: 10% ; **: 5% ; ***: 1%

5 Conclusion

One should not trivialize the role of model specification and diagnostic in the formulation of empirical asset pricing models. We took several APT-type models of recent vintage which have as their key ingredient a time varying structure in factor-return tradeoffs. These models are at the same time sophisticated and fragile. They are sophisticated because they exploit dynamics in predictability and/or nonlinearities. But they are also fragile because they must deal with time varying betas and are therefore more prone to sources of misspecification and hence mispricing of assets. The role of this paper was to show (1) how serious the problem of parameter instability is and how relevant the quote from Fama (1991) appearing in the introduction is, (2) how the proposed diagnostics help us to identify where deficiencies exist and where progress is made. Finally, the paper also emphasized that the commonly used model diagnostics fall short of exposing the problems which exist. So far, the literature has focused on testing overidentifying restrictions for a set of moment conditions determined by the pricing kernel of the APT models. We argued, however, that such tests are highly inadequate in gauging the fit and (out-of- sample) use of the pricing formula determined by the model. The more stringent diagnostics we proposed helped us to identify the factors in the conditional CAPM and APT which appeared systematically unstable and therefore unreliable and helped us also to assess the relative merit of one model specification against another.

We found that for the U.S. market returns on size-sorted and industry-based portfolios are difficult to fit with principal component factors in a conditional APT . The pricing formula seem hardly usable as almost no parameters are stable. There is some improvement when a set of economic factors are used to price the assets, although the predictable part of the factor-mimicking portfolios seems still difficult to model. A far more stable formulation which emerged from our study is the nonlinear APT. It does fail for some size categories and some industries, but overall its performance seems far more satisfactory. With regard to both the conditional CAPM and APT, one could probably make a considerable improvement by trying more sophisticated projection formula instead of the simple linear projections on the instruments since most often these linear projections appear unstable. These are directions in which we should try to further explore the formulation and testing of the empirical models discussed here.

1 APPENDIX

In this Appendix we provide a more formal discussion of the tests for structural stability. To set the scene we first note that the models discussed in the previous section 3 can be expressed via a generic set of moment conditions:

$$E\left[f\left(x_{t+1},\Theta_o\right)\right]=E\left[e\left(y_{t+1},\Theta_o\right)\otimes Z_t\right]=0 \tag{A.1}$$

where Z_t is a set of instruments y_t a vector process containing all asset returns, factors, etc. entering the pricing kernels while Θ_o is the parameter vector governing the pricing function, the projection equations or conditional betas. Equations (3.6), (3.12) and (3.13) describe the specific examples considered in the empirical section 4. For the purpose of discussion we shall divide the parameter vector in two subvectors, namely $\Theta_o \equiv (\gamma_o, \delta_o)$. This division allows for cases where we are not always interested in testing the complete parameter vector Θ_o but only a subvector γ_o. We observed in section 2 that this is often done because the parameters involved in the moment conditons play very different roles. This leads to the following null hypothesis:

$$H_o:\gamma_t=\gamma_0 \qquad \forall t\geq 1 \text{ for some } \gamma_0\epsilon B\subset \mathbb{R}^p. \tag{A.2}$$

When no parameter δ_0 is present, one tests the entire parameter vector; a situation referred to as testing for pure structural change. Otherwise, one tests for partial structural change. The alternative hypothesis consists of a one-time change at some point $\pi\epsilon(0,1)$. Then, with sample size T, the change occurs at πT and can be formulated as:

$$H_{1T}(\pi):\gamma_t=\begin{cases}\gamma_1(\pi) & \text{for } t=1,\ldots,\pi T\\ \gamma_2(\pi) & \text{for } t=\pi T+1,\ldots,T\end{cases} \tag{A.3}$$

for some constants $\gamma_1(\pi),\gamma_2(\pi)\epsilon B\subset\mathbb{R}^p$. As π is assumed unknown or $\pi\epsilon\Pi\subset(0,1)$ a pre-specified subset Andrews (1993) proposed to compute Wald, LM and LR-like tests for all π in Π and consider statistics of the form $g\left(\{S_T(\pi),\pi\epsilon\Pi\}\right)$ where the statistic $S_T(\pi)$ equals $W_T(\pi), LM_T(\pi)$ or $LR_T(\pi)$ if Wald, LM or LR tests are computed. Andrews and Ploberger (1994) formulated a unifying framework for the choice of the function g depending upon the alternatives of interest. In particular, consider

$$g\left(\{S_T(\pi),\pi\epsilon\Pi\}\right)=(1+c)^{p/2}\int_{\Pi}\exp\left[\frac{1}{2}\frac{c}{1+c}TS_T(\pi)\right]dJ(\pi) \tag{A.4}$$

where $J(\pi)$ is a weight function over the values of $\pi \epsilon \Pi$ and c determines the direction for the power of the test. When $c \to \infty$, tests have power against distant alternatives giving greater weight to large structural changes. Such tests will be denoted ExpS_T as they are computed according to the following formulas corresponding to $c \to \infty$ in equation (A.4):

$$ExpS_T = \log \int_{\Pi} \exp\left[\frac{1}{2} T S_T(\pi)\right] dJ(\pi) \tag{A.5}$$

with $J(\pi)$ representing a uniform weighting scheme for all values over Π. The exponential statistics come in three forms, namely:

$$ExpW_T, ExpLM_T \text{ and } ExpLR_T. \tag{A.6}$$

An alternative design for the function g is of the "sup" form. It corresponds to a case where $c/(1+c)$ is equal to a constant and this constant goes to infinity. Andrews (1993) initially proposed such tests, namely:

$$\underset{\pi\epsilon\Pi}{Sup}\, W_T(\pi)\ \underset{\pi\epsilon\Pi}{Sup}\, LM_T(\pi) \text{ and } \underset{\pi\epsilon\Pi}{Sup}\, LR_T(\pi). \tag{A.7}$$

Of the six test statistics we shall only consider two, both of the LM variety. There are two reasons for confining our attention to the SupLM and ExpLM statistics. First, unlike their Wald and LR counterparts, they only require *one* estimation of the model over the entire sample. Second, based on Monte Carlo simulations Ghysels and Guay (1994) find that the LM statistics have, compared to the Wald and LR tests, very good power properties and show no notable size distortions.

To discuss the tests more formally, let $\hat{V}(\pi)\, i = 1, 2$ be the sample covariance matrices obtains from a standard GMM procedure with heteroskedasticity and autocorrelation consistent covariance matrix estimation [see, e.g., Hansen (1982), Gallant and White (1988), Hall (1993) or Ogaki (1993) for general discussion]. The LM statistic makes use of the full-sample GMM estimator $\left(\hat{\beta}, \hat{\delta}\right)$ and can be written as:

$$LM_T(\pi) = C_T(\pi)' \left(\hat{V}_1(\pi)/\pi + \hat{V}_2(\pi)/(1-\pi)\right)^{-1} C_T(\pi) \tag{A.8}$$

where $C_T(\pi)$ is computed as

$$C_T(\pi) = [I_P \ -\ I_P] \begin{bmatrix} \pi^{-1}\left(\hat{M}_1' \hat{S}_1^{-1} \hat{M}_1\right)^{-1} \hat{M}_1' \hat{S}_1^{-1} & 0 \\ 0 & (1-\pi)^{-1}\left(\hat{M}_2' \hat{S}_2^{-1} \hat{M}_2\right)^{-1} \hat{S}_2^{-1} \end{bmatrix} \sqrt{T}\,\overline{m}_T\left(\tilde{\beta}, \tilde{\delta}, \pi\right)$$

where $\overline{m}_T\left(\tilde{\gamma}, \tilde{\delta}, \pi\right)$ is the set of moment conditions $m(x_t, \gamma, \delta)$ stacked according to the sample split at π evaluated at the full sample estimates $\tilde{\gamma}$ and $\tilde{\delta}$:

$$\overline{m}_T(\beta, \delta, \pi) = \frac{1}{T} \sum_{t=1}^{\pi T} \begin{bmatrix} m(x_t, \gamma_1, \delta) \\ 0 \end{bmatrix} + \frac{1}{T} \sum_{t=T\pi+1}^{T} \begin{bmatrix} 0 \\ m(x_t, \gamma_2, \delta) \end{bmatrix}$$

while $\hat{M}_i = \hat{M}_i(\pi)$ is the score function of the sample moment conditions $m(x_t, \gamma_1, \delta)$ with respect to γ_i for $i = 1, 2$. Finally, $\hat{S}_i = \hat{S}_i(\pi)$ is the heteroskedasticity and autocorrelation consistent covariance estimator of the sample moment conditions for $i = 1, 2$. In our case we simplified the computations, as is typically done by using, the full sample estimates $\hat{M}_i(\pi) = \hat{M}$ and $\hat{S}_i(\pi) = \hat{S}$.

Table A.1: Stable Factors in the Conditional APT - Industry Classification with Principal Components Factors												
	Industry 1	Industry 2	Industry 3	Industry 4	Industry 5	Industry 6	Industry 7	Industry 8	Industry 9	Industry 10	Industry 11	Industry 12
J-TEST	39.4	45.2 **	21.6	51.6 ***	54.1 ***	31.6	33.2	39.2	58.0 ***	44.0 **	55.2 ***	43.1 *
	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM
δ_{ALL}	16.6 ***	21.1 ***	7.0 *	17.8 ***	14.4 ***	7.2 *	16.0 ***	5.2	8.4 **	8.0 **	38.4 ***	12.7 ***
δ_1	0.6	2.7 **	0.3	2.3 **	1.5	2.4 **	0.9	0.6	0.6	2.5 **	1.1	0.2
δ_2	7.1 ***	0.7	0.1	1.9 *	0.1	2.5 **	0.2	0.1	0.2	0.9	0.7	0.1
δ_3	0.6	4.2 ***	0.1	0.8	1.9 *	3.8 ***	2.3 ***	0.9	0.1	1.6 *	6.1 ***	0.2
δ_4	0.5	5.2 ***	0.2	1.3	1.7 *	2.6 **	1.0	0.7	0.2	2.7 **	3.6 ***	0.2
δ_5	1.2	2.3 **	1.1	0.8	2.9 **	3.4 **	1.6 *	1.4	1.1	1.3	4.3 ***	1.0
δ_6	0.4	2.7 **	0.0	2.2 **	6.9 ***	2.9 **	0.9	0.3	0.2	2.3 **	11.5 ***	2.5 **
δ_7	0.1	0.7	0.7	1.2	0.2	0.9	0.4	0.7	2.4 **	1.0	3.1 **	0.4
γ_{1c}	31.2 ***	26.9 ***	2.8	5.9	19.7 ***	6.1	15.6 ***	7.2 *	45.3 ***	7.6 *	36.3 ***	36.4 ***
γ_{2c}	21.5 ***	35.4 ***	13.2 ***	18.4 ***	10.8 ***	8.0 **	11.8 ***	20.5 ***	34.5 ***	20.1 ***	38.1 ***	35.7 ***
γ_{3c}	28.6 ***	70.9 ***	5.4	18.8 ***	24.2 ***	13.6 ***	12.5 ***	12.6 ***	34.4 ***	20.5 ***	35.6 ***	41.9 ***
γ_{4c}	30.3 ***	30.6 ***	12.8 ***	28.3 ***	25.8 ***	15.3 ***	30.6 ***	35.6 ***	11.7 ***	23.8 ***	21.1 ***	23.9 ***
γ_{5c}	24.3 ***	27.8 ***	7.2 *	30.9 ***	10.8 ***	11.0 ***	23.9 ***	15.0 ***	22.3 ***	14.1 ***	29.0 ***	20.0 ***
β_{ALL}	36.3 ***	58.7 ***	2.6	29.5 ***	15.2 ***	22.4 ***	38.7 ***	32.1 ***	23.7 ***	47.9 ***	20.9 ***	60.8 ***
β_1	0.2	9.3 ***	1.1	8.9 ***	0.1	0.2	13.7 ***	1.6 *	0.1	4.1 ***	1.4	9.9 ***
β_2	8.6 ***	20.8 ***	1.0	13.7 ***	4.7 ***	4.1 ***	5.7 ***	7.4 ***	0.3	21.7 ***	2.2 **	8.0 ***
β_3	9.9 ***	28.1 ***	0.7	1.1	0.4	4.2 ***	10.7 ***	0.2	14.0 ***	20.3 ***	0.4	40.6 ***
β_4	0.6	7.4 ***	0.3	9.3 ***	0.4	0.3	7.7 ***	8.3 ***	0.4	5.8 ***	0.1	12.0 ***
β_5	1.2	0.8	0.2	11.2 ***	2.5 **	0.8	6.6 ***	0.6	1.0	2.1 **	2.5 **	9.0 ***

Note: Tests for Ferson's model. δ_{ALL} test all coefficients of δ together (7 parameters), δ_i (i=1,2,...,7) test all coefficients of δ one by one (1 parameter each). γ_{ic} (i=1,2,...,5) test all coefficients of γ column by column (7 parameters by column). β_{ALL} test all coefficients of β together (5 parameters), β_i (i=1,2,...,5) test all coefficients of β one by one (1 parameters each).
*: 10% ; **: 5% ; ***: 1%

Table A.2: Stable Factors in the Conditional APT - Size Classification with Principal Components Factors

	Size 1	Size 2	Size 3	Size 4	Size 5	Size 6	Size 7	Size 8	Size 9	Size 10
J-TEST	37.4	50.2 **	32.7	53.5 ***	39.2	26.1	52.8 ***	39.8	47.1 **	60.5 ***
	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM
δ_{ALL}	12.8 ***	5.7	14.2 ***	10.7 ***	8.3 **	12.0 ***	11.0 ***	33.8 ***	77.5 ***	88.8 ***
δ_1	0.4	0.4	0.7	0.9	1.0	0.3	0.9	10.5 ***	0.5	0.9
δ_2	0.1	0.6	0.4	3.9 ***	1.0	4.9 ***	0.6	1.6 *	0.3	8.9 ***
δ_3	0.7	0.5	0.8	0.9	0.6	0.3	0.6	12.1 ***	1.2	1.1
δ_4	0.6	0.4	1.0	1.1	1.1	0.3	1.3	12.8 ***	1.4	1.2
δ_5	0.2	0.1	0.3	4.7 ***	1.9 *	3.2 **	0.5	6.5 ***	6.0 ***	5.6 ***
δ_6	0.7	0.4	0.8	1.2	0.4	0.2	0.5	26.0 ***	0.4	2.8 **
δ_7	1.6 *	1.7 *	2.2 **	0.5	0.6	1.4	0.6	2.2 **	0.3	1.7 *
γ_{1c}	20.3 ***	22.1 ***	12.4 ***	26.1 ***	21.0 ***	8.7 **	9.3 **	27.5 ***	96.2 ***	76.0 ***
γ_{2c}	29.8 ***	40.8 ***	35.4 ***	10.2 ***	12.9 ***	11.8 ***	14.2 ***	16.0 ***	54.8 ***	41.6 ***
γ_{3c}	23.6 ***	38.3 ***	16.4 ***	21.8 ***	31.3 ***	10.9 ***	7.8 **	15.9 ***	148.3 ***	98.4 ***
γ_{4c}	37.0 ***	73.0 ***	45.3 ***	38.8 ***	43.8 ***	29.7 ***	18.6 ***	21.1 ***	54.8 ***	32.5 ***
γ_{5c}	18.9 ***	33.8 ***	32.2 ***	19.7 ***	16.8 ***	10.6 ***	11.2 ***	22.7 ***	72.1 ***	49.8 ***
β_{ALL}	42.7 ***	21.1 ***	24.3 ***	8.9 ***	27.7 ***	12.0 ***	17.2 ***	36.6 ***	175.7 ***	154.0 ***
β_1	11.7 ***	7.9 ***	1.7 *	1.8 *	1.8 *	2.9 **	0.1	7.1 ***	2.9 **	41.3 ***
β_2	2.3 **	0.8	11.4 ***	1.5	2.5 **	8.7 ***	9.2 ***	24.7 ***	16.9 ***	24.7 ***
β_3	27.1 ***	8.0 ***	6.3 ***	1.9 *	11.0 ***	0.1	1.8 *	9.4 ***	147.2 ***	131.6 ***
β_4	10.4 ***	2.1 **	1.0	1.1	2.6 **	2.2 **	0.6	1.4	62.2 ***	56.6 ***
β_5	15.5 ***	2.0 *	2.5 **	1.8 *	3.0 **	4.8 ***	3.6 ***	6.7 ***	14.9 ***	39.7 ***

Note: Tests for Ferson's model. δ_{ALL} test all coefficients of δ together (7 parameters), δ_i (i=1,2,...,7) test all coefficients of δ one by one (1 parameter each). γ_{ic} (i=1,2,...,5) test all coefficients of γ column by column (7 parameters by column). β_{ALL} test all coefficients of β together (5 parameters), β_i (i=1,2,...,5) test all coefficients of β one by one (1 parameters each).
*: 10% ; **: 5% ; ***: 1%

Table A.3: Stable Factors in the Conditional APT - Industry Classification with Economic Variables Factors

	Industry 1	Industry 2	Industry 3	Industry 4	Industry 5	Industry 6	Industry 7	Industry 8	Industry 9	Industry 10	Industry 11	Industry 12
J-TEST	36.7	35.2	27.8	36.5	31.4	37.7	27.5	48.8 **	34.6	26.2	49.8 **	33.5
	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM
δ_{ALL}	4.8	4.6	3.0	5.5	6.6	3.3	3.7	3.6	5.0	4.3	4.1	4.4
δ_1	0.1	0.1	1.0	0.2	2.3 **	0.2	0.1	0.2	0.6	1.7 *	0.2	0.3
δ_2	0.1	0.2	0.6	0.1	0.9	0.1	0.2	0.4	0.6	1.8 *	0.2	0.2
δ_3	0.4	0.1	1.0	0.1	2.9 **	0.1	0.1	0.1	0.4	1.1	0.4	0.4
δ_4	0.3	0.1	1.0	0.2	2.3 **	0.1	0.1	0.2	0.7	1.5	0.2	0.2
δ_5	1.5	0.3	0.6	0.4	1.7 *	0.2	0.4	0.3	0.4	1.4	0.2	0.4
δ_6	1.0	0.1	0.6	0.8	3.2 **	0.2	0.2	0.1	0.2	1.7 *	0.4	0.4
δ_7	0.4	0.2	0.2	0.3	1.5	0.7	1.2	0.2	0.4	1.0	1.5	0.4
γ_{1c}	35.3 ***	34.2 ***	28.9 ***	27.3 ***	45.2 ***	30.8 ***	27.3 ***	27.5 ***	27.3 ***	36.5 ***	28.4 ***	31.8 ***
γ_{2c}	79.6 ***	52.4 ***	49.3 ***	43.0 ***	66.4 ***	44.7 ***	42.6 ***	42.3 ***	42.0 ***	60.1 ***	48.9 ***	44.9 ***
γ_{3c}	12.5 ***	10.7 ***	4.8	7.7 **	7.5 *	6.5	3.5	12.8 ***	7.7 **	5.7	8.5 **	5.3
γ_{4c}	11.9 ***	8.0 **	20.8 ***	7.3 *	6.0	6.6	5.5	14.5 ***	8.0 **	5.5	7.1 *	6.9 *
γ_{5c}	16.5 ***	5.1	2.8	5.3	7.0 *	3.6	2.7	9.5 ***	2.9	6.8 *	3.6	5.8
β_{ALL}	13.1 ***	5.0	2.1	2.0	16.1 ***	6.6 **	2.5	6.3 **	5.6 *	2.7	12.2 ***	6.9 **
β_1	1.6 *	1.5	0.6	0.7	0.1	0.6	0.5	0.8	0.4	0.5	0.1	2.3 **
β_2	1.8 *	2.3 **	0.2	0.4	1.2	0.8	0.5	0.9	0.4	1.2	0.3	4.2 ***
β_3	0.1	0.4	0.1	0.4	8.8 ***	0.6	0.2	0.3	1.3	0.3	7.6 ***	0.1
β_4	0.6	3.4 **	0.1	0.3	0.8	0.1	0.1	0.4	1.5	0.1	1.8 *	0.1
β_5	5.0 ***	0.5	0.2	0.2	9.8 ***	1.9 *	0.7	4.3 ***	3.2 **	0.8	7.6 ***	1.0

Note: Tests for Ferson's model. δ_{ALL} test all coefficients of δ together (7 parameters), δ_i (i=1,2,...,7) test all coefficients of δ one by one (1 parameter each). γ_{ic} (i=1,2,...,5) test all coefficients of γ column by column (7 parameters by column). β_{ALL} test all coefficients of β together (5 parameters), β_i (i=1,2,...,5) test all coefficients of β one by one (1 parameters each).
*: 10% ; **: 5% ; ***: 1%

Table A.4: Stable Factors in the Conditional APT - Size Classification with Economic Variables Factors

	Size 1	Size 2	Size 3	Size 4	Size 5	Size 6	Size 7	Size 8	Size 9	Size 10
J-TEST	29.3	32.3	46.9 **	53.5 ***	50.7 **	46.5 **	39.5	37.4	30.1	54.9 ***
	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM
δ_{ALL}	3.8	3.5	4.4	3.6	5.6	5.2	11.4 ***	7.5 *	4.3	16.2 ***
δ_1	0.5	1.9 *	0.9	2.0 *	2.5 **	0.6	0.6	1.3	0.4	0.1
δ_2	0.3	0.4	0.6	0.6	1.4	0.3	1.4	0.7	2.2 **	0.3
δ_3	0.6	2.4 **	0.8	2.3 **	2.8 **	0.9	0.7	0.6	0.3	0.5
δ_4	0.4	1.9 *	0.7	1.9 *	2.3 **	0.6	0.5	0.8	0.4	0.3
δ_5	0.1	0.5	1.1	0.8	2.0 *	0.4	2.5 **	0.5	2.5 **	2.2 **
δ_6	0.7	1.7 *	0.3	1.7 *	1.2	1.2	0.6	0.2	0.3	0.3
δ_7	1.5	0.6	0.3	0.3	0.5	0.3	0.1	0.6	0.5	0.5
γ_{1c}	27.3 ***	26.0 ***	28.2 ***	26.6 ***	28.0 ***	33.7 ***	29.8 ***	25.0 ***	30.0 ***	33.2 ***
γ_{2c}	46.3 ***	47.4 ***	50.7 ***	52.1 ***	65.1 ***	57.7 ***	51.5 ***	45.9 ***	50.6 ***	49.8 ***
γ_{3c}	10.5 ***	8.2 **	4.0	6.1	11.0 ***	7.9 **	7.0 *	6.5	6.9 *	10.8 ***
γ_{4c}	12.5 ***	10.0 ***	5.0	7.7 **	10.5 ***	9.6 ***	9.0 **	6.6	9.2 **	8.1 **
γ_{5c}	4.3	2.7	3.0	3.3	4.0	5.0	10.5 ***	7.5 *	3.3	17.4 ***
β_{ALL}	2.9	2.6	2.5	4.9	3.8	5.4 *	6.4 **	5.9 *	6.3 **	15.8 ***
β_1	0.6	0.4	1.1	0.5	0.4	1.5 *	2.7 **	0.6	0.5	0.2
β_2	1.1	0.8	1.3	0.6	0.4	2.1 **	1.9 *	0.6	0.5	0.8
β_3	0.1	0.7	0.6	0.4	1.4	0.6	2.3 **	1.5 *	0.9	8.0 ***
β_4	0.1	0.2	0.1	1.7 *	1.4	2.1 **	0.9	3.0 **	3.7 ***	1.4
β_5	0.5	0.3	0.2	0.4	0.5	0.5	2.5 **	0.7	0.8	15.0 ***

Note: Tests for Ferson's model. δ_{ALL} test all coefficients of δ together (7 parameters), δ_i (i=1,2,...,7) test all coefficients of δ one by one (1 parameter each). γ_{ic} (i=1,2,...,5) test all coefficients of γ column by column (7 parameters by column). β_{ALL} test all coefficients of β together (5 parameters), β_i (i=1,2,...,5) test all coefficients of β one by one (1 parameters each).
*: 10% ; **: 5% ; ***: 1%

Table A.5: Stable Factors in Nonlinear APT - Size Classification

	Size 1	Size 2	Size 3	Size 4	Size 5	Size 6	Size 7	Size 8	Size 9	Size 10
J-TEST	8.08 **	0.79	0.41	0.03	0.00	0.28	0.13	0.77	0.04	0.01
	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM
β_0	3.83 ***	1.46	0.28	0.13	0.05	0.08	3.79 ***	0.45	2.34 **	3.81 ***
β_{1f}	4.86 ***	1.41	0.28	0.14	0.04	0.06	2.28 **	0.22	1.25	2.72 **
β_{1M}	3.83 ***	1.45	0.28	0.13	0.05	0.08	3.78 ***	0.44	2.34 **	3.83 ***
β_{2M}	3.83 ***	1.45	0.28	0.13	0.05	0.08	3.78 ***	0.44	2.33 **	3.85 ***
β_{5M}	3.83 ***	1.44	0.28	0.13	0.05	0.08	3.76 ***	0.43	2.31 **	3.90 ***
β_{2M} & β_{5M}	4.00 **	1.77	0.37	0.72	0.16	0.28	3.87 **	1.10	2.65 *	5.44 ***
β_{ALL}	16.24 ***	3.23	1.92	1.27	0.64	0.66	7.03 **	2.98	4.69	6.57 **

Table A.6: Stable Factors in Nonlinear APT - Industry Classification

	Industry 1	Industry 2	Industry 3	Industry 4	Industry 5	Industry 6	Industry 7	Industry 8	Industry 9	Industry 10	Industry 11	Industry 12
J-TEST	4.19	0.78	0.06	0.01	5.87	0.33	0.00	5.82	0.87	0.21	2.18	2.37
	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM	Exp LM
β_0	3.27 **	0.15	3.74 ***	0.86	1.08	1.33	0.83	0.27	6.68 ***	0.39	4.62 ***	0.99
β_{1f}	3.02 **	0.15	3.20 **	0.28	1.19	1.04	0.43	0.31	6.10 ***	0.23	4.86 ***	0.75
β_{1M}	3.26 **	0.16	3.72 ***	0.86	1.08	1.33	0.83	0.27	6.67 ***	0.39	4.61 ***	0.98
β_{2M}	3.25 **	0.16	3.70 ***	0.85	1.09	1.33	0.82	0.27	6.66 ***	0.38	4.59 ***	0.97
β_{5M}	3.22 **	0.16	3.64 ***	0.83	1.09	1.32	0.80	0.27	6.64 ***	0.36	4.55 ***	0.94
β_{2M} & β_{5M}	4.66 **	0.40	4.19 **	1.37	1.78	1.45	1.22	0.55	6.98 ***	2.83 *	6.82 ***	2.91 *
β_{ALL}	6.06 *	0.96	8.43 ***	7.57 **	2.68	3.89	3.42	1.39	11.47 ***	5.68 *	9.98 ***	3.81

Note: β_i test each parameter separately. β_{2M} & β_{5M} test these two parameters jointly. β_{ALL} test all parameters together.
*: 10% ; **: 5% ; ***: 1%

References

[1] Andrews, D.W.K. (1993), "Tests for Parameter Instability and Structural Change with Unknown Change Point," *Econometrica* 61, 821-856.

[2] Andrews, D.W.K. and R. Fair (1988), "Inference in Econometric Models with Structural Change," *Review of Economic Studies* 55, 615-640.

[3] Andrews, D.W.K. and J.C. Monahan (1992), "An Improved Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimator," *Econometrica* 60, 953-966.

[4] Andrews, D.W.K. and W. Ploberger (1994), "Optimal Tests when a Nuisance Parameter is Present Only Under the Alternative", *Econometrica* 62, 1383-1414.

[5] Bansal, R., D. Hsieh, and S. Viswanathan, (1993), "A New Approach to International Arbitrage Pricing Theory," *Journal of Finance* 48, 1719-1747.

[6] Bansal, R. and S. Viswanathan (1993), "No Arbitrage and Arbitrage Pricing: A New Approach," *Journal of Finance* 48,1231-1262.

[7] Chamberlain, G., and M. Rothschild, (1983), "Arbitrage and Mean Variance Analysis on Large Asset Markets," *Econometrica* 51, 1281-1304.

[8] Chen, Nai-fu, R. Roll and S. Ross (1986) "Economic Forces and the Stock Market," *Journal of Business* 59, 383-403.

[9] Connor, G., (1984), "A Unified Beta Pricing Theory," *Journal of Economic Theory* 34, 13-31.

[10] Connor, G. and R.A. Korajczyk, (1989), "An Intertemporal Beta Pricing Model," *Review of Financial Studies* 2, 373-392.

[11] Dufour, J.M., E. Ghysels and A. Hall (1994) "Generalized Predictive Tests for Structural Change," *International Economic Review* 35, 199-229.

[12] Dumas, B., and B. Solnik, (1993), "The World Price of Exchange Rate Risk," Working Paper, Wharton School and HEC.

[13] Fama, E.F., (1991), "Efficient Capital Markets: II," *Journal of Finance* 46, 1575-1617.

[14] Ferson, W.E.,(1985), "Changes in Expected Risk Premiums and Security Risk Measures." *Proceedings of the European Finance Association.*

[15] Ferson, W.E. (1990), "Are the Latent Variables in Time-Varying Expected Returns Compensation for Consumption Risk?", *Journal of Finance* 45, 397-430.

[16] Ferson, W.E. and C.R. Harvey, (1993), "The Risk and Predictability of International Equity Returns," *Review of Financial Studies* 6, 527-566.

[17] Ferson, W.E. and C.R. Harvey, (1991), "The Variation of Economic Risk Premiums," *Journal of Political Economy* 99, 385-415.

[18] Ferson, W.E. and R. A. Korajczyk (1995), "Do Arbitrage Pricing Models Explain the Predictability of Stock Returns?", *Journal of Business* (forth-

coming).

[19] Gallant, A.R. and H. White (1988), *A Unified Theory of Estimation and Inference for Nonlinear Dynamic Models*, New York, Blackwell.

[20] Garcia, R. and E. Ghysels (1994), "Structural Change and Asset Pricing in Emerging Markets", Work in progress.

[21] Ghysels, E. and A. Guay (1994), "Structural Change Tests with Unknown Breakpoint: A Monte Carlo Investigation," Work in progress.

[22] Ghysels, E. and A. Hall (1990a), "A Test for Structural Stability of Euler Conditions Parameters Estimated via the Generalized Method of Moments Estimator," *International Economic Review* 31, 355-364.

[23] Ghysels, E. and A. Hall (1990b), "Are Consumption-Based Intertemporal Capital Asset Pricing Models Structural?," *Journal of Econometrics* 45, 121-139.

[24] Ghysels, E., A. Guay and A. Hall (1994), "Predictive Tests for Structural Change with Unknown Breakpoint," Paper presented at the Winter Meetings of the Econometric Society, Washington, D.C.

[25] Ghysels, E. and A. Hall (1995), "On the Predictive Stability of Asset Pricing Error", Work in progress.

[26] Hall, A. (1993), "Some Aspects of Generalized Method of Moments Estimation", *in* G.S. Maddala, C.R. Rao and H.D. Vinod (eds.), *Handbook of Statistics* 11, Amsterdam, North-Holland, Chapter 15.

[27] Hansen, L.P., (1982), "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica* 50, 1029-1054.

[28] Hansen, L.P. and R. Jaganathan (1991), "Implications of Security Market Data for Models of Dynamic Economies," *Journal of Political Economy* 99, 225-262.

[29] Harvey, C.R. (1991), "The World Price of Covariance Risk," *Journal of Finance* 46, 111-157.

[30] Harvey, C.R. (1993), "Predictable Risk and Returns in Emerging Markets", Discussion Paper Fuqua School of Business, Duke University.

[31] Huang, C. (1987), "An Intertemporal General Equilibrium Asset Pricing Model: The Case of Diffusion Information," *Econometrica* 55, 117-142.

[32] Huberman, G., (1982), "A Simple Approach to Arbitrage Pricing Theory," *Journal of Economic Theory* 28, 182-191.

[33] Ingersoll, J., (1984), "Some Results in the Theory of Arbitrage Pricing," *Journal of Finance* 39, 1021-1039.

[34] Lehmann, B.N. and D. Modest (1988) "The Empirical Foundations of the Arbitrage Pricing Theory," *Journal of Financial Economics* 21, 213-254.

[35] Lintner J., (1965), "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolio and Capital Budgets," *Review of Economics and Statistics* 47, 13-37.

[36] Lucas, R. (1978), "Asset Prices in an Exchange Economy," *Econometrica* 46, 1426-1446.

[37] Merton, R., (1973), "An Intertemporal Asset Pricing Model," *Econometrica* 41, 867-880.

[38] Ross, S.A., (1976), "The Arbitrage Theory of Capital Asset Pricing," *Journal of Economic Theory* 13, 341-360.

[39] Sharpe, W.F.(1964), "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *Journal of Finance* 19, 425-442.

[40] Stulz, R. (1981), "A Model of International Asset Pricing," *Journal of Financial Economics* 9, 383-406.