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Consumption with Durability

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Consumption with Durability[*]

Aylin Seckin[†]

Résumé / Abstract

En utilisant la programmation dynamique, dans un modèle intertemporel de consommation et d'épargne avec incertitude de revenu et durabilité, nous avons trouvé une solution fermée pour la consommation et dérivé l'épargne précautionnelle. Avec l'hypothèse de durabilité, la consommation a une réaction plus intense au revenu permanent par rapport au modèle standard et aussi réagit aux niveaux passés du revenu permanent avec une alternation de signe. En plus, l'effet de l'incertitude de revenu sur la consommation est plus élevé par rapport au cas avec les préférences temps non séparables. C'est-à-dire, quand la force de durabilité augmente, le niveau de l'épargne précautionnelle contre l'incertitude de revenu doit être plus élevé.

Using a dynamic programming framework, in an intertemporal consumption-saving model with income uncertainty and durability, we have found a closed form solution for consumption. Also, the precautionary savings term arising from the riskiness of income is formally derived. With durability assumption, consumption has a higher response to permanent income than the one in the standard model, and also responds to a distributed lag of past permanent incomes with alternating signs. Furthermore, the effect of income uncertainty is greater on consumption relative to the case with time-separable preferences. That is, as the strength of durability increases, the level of precautionary savings against income uncertainty will be higher.

Mots Clés : Durabilité, épargne précautionnelle

Keywords: Durability, precautionary savings

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Consumption with Durability

1. INTRODUCTION

In a seminal paper, Hall (1978) proposes an alternative way of testing the permanent income hypothesis (PIH), that is by using directly the Euler equation instead of estimating the final consumption function. The implication of this form is that lagged real disposable income will have little predictive power for consumption and since the innovation in income is unpredictable at time t and the best estimate of the future consumption is the current consumption. All other past values of consumption should add no predictive power to consumption.

More recent work on consumptin indicates that consumers do not smooth out consumption as much as predicted by the Life-Cycle Permanent Income Hypothesis (LIH-PIH). In fact, current consumption seems to be excessively sensitive to current and lagged income[1].

The aim of this study is to discuss the implications of durability in a consumption-saving model with uncertainty using a dynamic programming framework. We assume that consumption is the purchase of durable goods. Current consumption depends on past purchases of durable goods. By allowing non-separability in preferences in terms of durability in consumption, we find not only a closed form solution for the consumption function with income uncertainty and durability, but also explain the excess sensitivity puzzle of consumption. We show that changes in consumption are highly sensitive to past changes in income, while consumption growth is fluctuating more with durability which is another finding contrasting the predictions of LIH-PIH.

With durability assumption, consumption has a higher response to permanent income than in the standard model, but also responds to a distributed lag of past permanent incomes with alternating signs. Thus, consumption in this model exhibits a fluctuating growth rate and its variance is greater than the one of income Also, the precautionary savings, against the riskiness of income are higher with durability behavior in consumption.

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[1] See Flavin (1981), Hall and Mishkin (1982), Hayashi (1985), Dunn and Singleton (1986), Eichenbaum and Hansen (1990), Ferson and Constantinides (1991), Alessie, Devereux and Weber (1997).

The paper is organized as follows. In section 2, the model with income uncertainty, incorporating time non-separable CARA preferences with durability of consumption expenditures, has been developed and solved. Using dynamic programming and a conjecture for consumption, a closed form solution for consumption with durability has been obtained.The conclusion is given in section 3.

2. THE MODEL

The consumer's maximization problem is:

$$Max \quad E_t \sum_{t=0}^{\infty} \beta^t - \frac{1}{\Theta} e^{-\Theta[c_t + \alpha c_{t-1}]} \tag{1}$$

$$\{c_t\}, \{A_{t+1}\} \tag{2}$$

subject to :

$$A_{t+1} = (1+r)[A_t + y_t - c_t] \tag{3}$$

with lim $A_t(1+r)^{-t} = 0$

t→ ∞

A_0, c_0 given together with an exogenous expected time pattern of income y_t. We assume that y_t follows an autoregressive process with a normal distribution. That is, $y_t = \mu y_{t-1} + \omega_t$, $0 < \mu < 1$ where $\omega_t \sim i.i.d.$ with mean zero and variance σ^2_ω.

β is the discount factor, $\beta = \frac{1}{1+\rho}$ $0 < \beta < 1$, where ρ is the rate of time preference and α is the durability parameter, $0 < \alpha < 1$.

The CARA utility function is assumed with $\Theta > 0$, the coefficient of absolute risk aversion. We will assume that the rate of interest is certain and equal to the rate of time preference, $\frac{1}{1+r} \equiv \frac{1}{1+\rho} = \beta$. The individual makes decision about current period consumption, c_t, and the next period asset holdings, A_{t+1}, subject to his budget constraint. Given the expectations of the future (assuming that all present and past variables are observable), one can solve the individual's maximization problem using dynamic programming.

The first order condition is:

$$\begin{aligned} & e^{-\Theta[c_t+\alpha c_{t-1}]} + \alpha\beta E_t e^{-\Theta[c_{t+1}+\alpha c_t]} \\ = \; & \beta(1+r)E_t[e^{-\Theta[c_{t+1}+\alpha c_t]} + \alpha\beta e^{-\Theta[c_{t+2}+\alpha c_{t+1}]}] \qquad (4) \end{aligned}$$

Now, let $\widehat{c}_t = c_t + \alpha c_{t-1}$. For future use, it will be convenient to express the budget constraint in terms of $\widehat{c}_t$. By arranging terms, we can write

$$\sum_{i=0}^{\infty} \beta^i c_{t+i} = (\frac{1-\alpha\beta}{1-\alpha^2\beta^2}) \sum_{i=0}^{\infty} \beta^i \widehat{c}_{t+i} - (\frac{\alpha(1-\alpha\beta)}{1-\alpha^2\beta^2}) c_{t-1}. \tag{5}$$

Given the transversality condition, the lifetime budget constraint in this problem is:

$$\sum_{i=0}^{\infty} \beta^i c_{t+i} = A_t + \sum_{i=0}^{\infty} \beta^i y_{t+i} \tag{6}$$

Substituting $\sum_{i=0}^{\infty} \beta^i c_{t+i}$ from (5) into (6),

$$\sum_{i=0}^{\infty} \beta^i \widehat{c}_{t+i} = \frac{(1-\alpha^2\beta^2)}{(1-\alpha\beta)} \{A_t + \sum_{i=0}^{\infty} E_t \beta^i y_{t+i}\} + \alpha c_{t-1}. \tag{7}$$

Then, the first order condition for the dynamic programming problem can be stated as:

$$e^{-\theta \hat{c}_t} = E_t[(1-\alpha\beta) e^{-\theta \hat{c}_{t+1}} + \alpha\beta e^{-\theta \hat{c}_{t+2}}] \tag{8}$$

where

$$\widehat{c}_{t+i} = c_{t+i} + \alpha c_{t+i-1} \qquad i = 0, 1, 2, ...$$

Following Caballero (1990), the conjectured solution for $\hat{c}_{t+i}$ is the following:

$$\hat{c}_{t+i} = \Gamma_{t+i-1} + \Phi_{t+i-1}\hat{c}_{t+i-1} + v_{t+i} \tag{9}$$

where Γ, Φ and v are terms to be determined. After substituting for the conjectured solution, the first order condition becomes:

$$e^{\Theta[\Gamma_t + E_t v_{t+1} - \frac{\Theta}{2}\sigma_v^2]} = [(1-\alpha\beta) + \alpha\beta e^{-\Theta[\Gamma_{t+1} + E_t v_{t+2} - \frac{\Theta}{2}\sigma_v^2]}] \tag{10}$$

The above equality will hold when

$$\Gamma_t = -E_t v_{t+1} + \frac{\Theta}{2}\sigma_v^2 \text{ and } \Gamma_{t+1} = -E_t v_{t+2} + \frac{\Theta}{2}\sigma_v^2. \tag{11}$$

Note that if v_t is *i.i.d.* with $E_t v_{t+1} = E_t v_{t+2} = 0$, $\Gamma = \frac{\Theta}{2}\sigma_v^2$, a constant. On the other hand, if v_t is serially correlated, then Γ_t will be time-varying.

We will now proceed to substitute our conjectured solution for $\hat{c}_t$ into the budget constraint (7).

Hence,

$$\sum_{i=0}^{\infty} \beta^i \hat{c}_{t+i} = \hat{c}_t \sum_{i=0}^{\infty} \beta^i + \sum_{i=1}^{\infty} \beta^i \sum_{j=1}^{i} \Gamma_{t+j-1} + \sum_{i=1}^{\infty} \beta^i \sum_{j=1}^{i} v_{t+j} \tag{12}$$

Substituting (12) into (7) yields:

$$\begin{aligned}
&\frac{1}{1-\beta}\hat{c}_t + \sum_{i=1}^{\infty}\beta^i\sum_{j=1}^{i}\Gamma_{t+j-1} + \sum_{i=1}^{\infty}\beta^i\sum_{j=1}^{i}v_{t+j} \\
&-(\frac{1-\alpha^2\beta^2}{1-\alpha\beta})\{A_t + E_t\sum_{i=0}^{\infty}\beta^i\{y_{t+i}-E_t y_{t+i}\} + \sum_{i=0}^{\infty}\beta^i E_t y_{t+i}\} \\
&-\alpha c_{t-1} = 0 \qquad (13)
\end{aligned}$$

where we have added and subtracted the terms $E_t y_{t+i}$. Taking expectations on both sides of the budget constraint to obtain:

$$\begin{aligned}
\hat{c}_t &= (1-\beta)(\frac{1-\alpha^2\beta^2}{1-\alpha\beta})\{A_t + \sum_{i=0}^{\infty}\beta^i E_t y_{t+i}\} \qquad (14) \\
&\quad -(1-\beta)\sum_{i=1}^{\infty}\beta^i\sum_{j=1}^{i}\Gamma_{t+j-1} + (1-\beta)\alpha c_{t-1}
\end{aligned}$$

where we have used the fact that since ω_{t+i} is an *i.i.d.* random variable with zero mean , the innovation to consumption v_{t+i} will also have a mean of zero.

Substituting for $\hat{c}_t$ back into the budget constraint (7). Since the resulting expression must be equal to zero, we have:

$$\sum_{i=1}^{\infty}\beta^i\sum_{j=1}^{i}[v_{t+j} - (\frac{1-\alpha^2\beta^2}{1-\alpha\beta})\mu^{i-j}\omega_{t+j}] = 0 \qquad (15)$$

For this sum to be equal to zero, each of the terms must be zero.

Therefore,

$$v_{t+1} = [\frac{(1-\beta)(1-\alpha^2\beta^2)}{(1-\beta\mu)(1-\alpha\beta)}]\omega_{t+1} \tag{16}$$

We have solved for the explicit relationship between v_{t+1} and ω_{t+1}.

By using the definition of permanent income which is:

$y_t^p \equiv (1-\beta)(A_t + \sum_{i=0}^{\infty} \beta^i E_t y_{t+i})$ and (14), c_t can be written as:

$$c_t = (\frac{1-\alpha^2\beta^2}{1-\alpha\beta})y_t^p - \alpha\beta c_{t-1} - (1-\beta)\sum_{i=1}^{\infty}\beta^i\sum_{j=1}^{i}\Gamma_{t+j-1} \tag{17}$$

where

$$\sum_{i=1}^{\infty}\beta^i\sum_{j=1}^{i}\Gamma_{t+j-1} = \frac{\beta}{(1-\beta)^2}\Gamma \tag{18}$$

Given our assumption that ω_{t+i}'s are *i.i.d.* with zero mean, so that the v_{t+i}'s are *i.i.d.* with zero mean$\Rightarrow \Gamma_t = \Gamma = \frac{\Theta}{2}\sigma_v^2 \quad \forall t$. That is from (16), we obtain,

$$\Gamma = \frac{\Theta}{2}[\frac{(1-\beta)(1-\alpha^2\beta^2)}{(1-\beta\mu)(1-\alpha\beta)}]^2\sigma_\omega^2 \tag{19}$$

Then (17) becomes:

$$c_t = (\frac{1-\alpha^2\beta^2}{1-\alpha\beta})y_t^p - \alpha\beta c_{t-1} - \frac{\Theta\beta(1-\beta)(1-\alpha^2\beta^2)^2}{2(1-\beta\mu)^2(1-\alpha\beta)^2}\sigma_\omega^2 \tag{20}$$

or solving the difference equation for c_t,

$$\begin{aligned} c_t &= \underbrace{(\frac{1-\alpha^2\beta^2}{1-\alpha\beta})y_t^p}_{1} - \underbrace{(\frac{1-\alpha^2\beta^2}{1-\alpha\beta})\alpha\beta\sum_{i=0}^{\infty}(-\alpha\beta)^i y_{t-i-1}^p}_{2} \\ &\quad - \underbrace{\frac{\Theta\beta(1-\beta)(1-\alpha^2\beta^2)}{2(1-\beta\mu)(1-\alpha\beta)}\sigma_\omega^2}_{3} \end{aligned} \tag{21}$$

The first term shows a higher response to what is perceived to be permanent income. The second term stands for the response of consumption to lagged income through lagged permanent income. Due to durabiliy, there is an alternating response of consumption to past changes in income: following a higher income in period t-1, there will be a decrease in consumption in period t, arising from the fact that the consumer, with higher income in period t-1 buys the durables that he or she wants and in period t, he or she does not need anymore those products or services already bought them previously. The third term indicates that with durability there is a precautionary premium. With durability, the effect of income uncertainty is greater on consumption. The higher is the strength of durability, the higher will the precautionary savings against income uncertainty.

Then the change in consumption is:

$$\begin{aligned} c_{t+1} - c_t &= \frac{\Theta}{2(1+\alpha)}[\frac{(1-\beta)(1-\alpha^2\beta^2)}{(1-\beta\mu)(1-\alpha\beta)}]^2\sigma_\omega^2 + [\frac{(1-\beta)(1-\alpha^2\beta^2)}{(1-\beta\mu)(1-\alpha\beta)}]\omega_{t+1} \quad (22) \\ &\quad -[\frac{(1-\beta)(1-\alpha^2\beta^2)}{(1-\beta\mu)(1-\alpha\beta)}]\alpha\sum_{i=0}^{\infty}(-\alpha)^i\omega_{t-i}. \end{aligned}$$

where the first term represents the precautionary premium, the second term is the "unanticipated changes in income" and the third term is the "anticipated changes in income"or averages of previous income innovations.

In order to examine the excess sensitivity and excess smoothness puzzles, we examine the marginal propensity to consume. The marginal propensity to consume out of an unanticipated income change ($\frac{\Delta c}{\Delta y}$) with durability is defined as the change in current consumption from an innovation in income.

Hence, from (22),

$$MPC_D = \frac{(1-\beta)(1-\alpha^2\beta^2)}{(1-\beta\mu)(1-\alpha\beta)} > MPC_{NoD} = \frac{(1-\beta)}{(1-\beta\mu)}$$

That is, the response of consumption to current changes in income is higher than would be expected without having durability, i.e., $\alpha = 0$. (Since $\frac{(1-\alpha^2\beta^2)}{(1-\alpha\beta)} > 1$.

Second, notice also that consumption responds to all past (anticipated) income innovations .With $\alpha = 0.8$ for example, current consumption responds relatively strongly to past anticipated innovations in income. This implies that with a positive coefficient on ω_{t-i} , consumption will have a higher rate of growth and with a negative coefficient it will have a lower growth rate. (With $\alpha = 0$, this term disappears). Both results are in conformity with the "excess sensitivity" puzzle and with the empirical findings of Campbell and Deaton (1989).

Overall, with durability, consumption has a higher response to y_t^p than in the usual model, but also responds to a distributed lag of past permanent incomes, i.e., to past perceptions of the income stream, thus exhibiting a fluctuating rate of growth of consumption.

Now, let us look at variances rather than levels of change of consumption. Note that with durability in consumption the variance of consumption is higher than the case with no durability.

$$Var\Delta c_t^D = [\frac{(1-\beta)(1-\alpha^2\beta^2)}{(1-\beta\mu)(1-\alpha\beta)}]^2\sigma_\omega^2$$

since in the case without durability we have,$Var\Delta c_t = [\frac{(1-\beta)}{(1-\beta\mu)}]^2\sigma_\omega^2$ for a AR(1) process. As the income process approaches a random walk, $\mu \to 1$, $Var\Delta c_t = Var\Delta y_t = \sigma_\omega^2$. With durability, $Var\Delta c_t = \left(\frac{1-\alpha^2\beta^2}{1-\alpha\beta}\right)^2 \sigma_\omega^2$, i.e. even when $\mu = 1$, $Var\Delta c_t > Var\Delta y_t$.

3. CONCLUSION

In this intertemporal consumption-saving model with income uncertainty, using a dynamic programming framework, we have found a closed form solution for the consumption function with income uncertainty and durability. Also, the precautionary savings term arising from the riskiness of income is formally derived. With durability assumption, consumption (considered as current purchases of durable goods) has a higher response to permanent income than in the standard model, but also responds to a distributed lag of past permanent incomes with alternating signs. Thus, response of consumption to current changes in income is higher than the case with no durability and the variance of consumption is greater than the one of income.

With durability, the effect of income uncertainty is greater on consumption relative to the case with time-separable preferences. That is, as the strength of durability increases, the level of precautionary savings against income uncertainty will be higher. The individual has to choose a higher level of precautionary savings than the one with no durability in consumption.

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