

2003s-67

Auction versus Dealership Markets

Moez Bennouri

Série Scientifique
Scientific Series

Montréal
Décembre 2003

© 2003 Moez Bennouri. Tous droits réservés. *All rights reserved.* Reproduction partielle permise avec citation du document source, incluant la notice ©.

Short sections may be quoted without explicit permission, if full credit, including © notice, is given to the source.

CIRANO

Le CIRANO est un organisme sans but lucratif constitué en vertu de la Loi des compagnies du Québec. Le financement de son infrastructure et de ses activités de recherche provient des cotisations de ses organisations-membres, d'une subvention d'infrastructure du ministère de la Recherche, de la Science et de la Technologie, de même que des subventions et mandats obtenus par ses équipes de recherche.

CIRANO is a private non-profit organization incorporated under the Québec Companies Act. Its infrastructure and research activities are funded through fees paid by member organizations, an infrastructure grant from the Ministère de la Recherche, de la Science et de la Technologie, and grants and research mandates obtained by its research teams.

Les organisations-partenaires / The Partner Organizations

PARTENAIRE MAJEUR

. Ministère du développement économique et régional [MDER]

PARTENAIRES

. Alcan inc.
. Axa Canada
. Banque du Canada
. Banque Laurentienne du Canada
. Banque Nationale du Canada
. Banque Royale du Canada
. Bell Canada
. BMO Groupe Financier
. Bombardier
. Bourse de Montréal
. Développement des ressources humaines Canada [DRHC]
. Fédération des caisses Desjardins du Québec
. Gaz Métropolitain
. Hydro-Québec
. Industrie Canada
. Ministère des Finances [MF]
. Pratt & Whitney Canada Inc.
. Raymond Chabot Grant Thornton
. Ville de Montréal

. École Polytechnique de Montréal
. HEC Montréal
. Université Concordia
. Université de Montréal
. Université du Québec à Montréal
. Université Laval
. Université McGill

ASSOCIE A :

. Institut de Finance Mathématique de Montréal (IFM²)
. Laboratoires universitaires Bell Canada
. Réseau de calcul et de modélisation mathématique [RCM²]
. Réseau de centres d'excellence MITACS (Les mathématiques des technologies de l'information et des systèmes complexes)

Les cahiers de la série scientifique (CS) visent à rendre accessibles des résultats de recherche effectuée au CIRANO afin de susciter échanges et commentaires. Ces cahiers sont écrits dans le style des publications scientifiques. Les idées et les opinions émises sont sous l'unique responsabilité des auteurs et ne représentent pas nécessairement les positions du CIRANO ou de ses partenaires.

This paper presents research carried out at CIRANO and aims at encouraging discussion and comment. The observations and viewpoints expressed are the sole responsibility of the authors. They do not necessarily represent positions of CIRANO or its partners.

Auction versus Dealership Markets *

Moez Bennouri[†]

Résumé / Abstract

Cette étude propose une comparaison entre deux structures d'échanges dans les marchés financiers: les marchés aux enchères et les marchés de contreparties. Les marchés aux enchères sont concentrés et régis par les ordres alors que les marchés de contreparties sont fragmentés et régis par les prix. Par rapport à la littérature, cette comparaison se base sur les deux dimensions qui distinguent les deux structures, à savoir le timing de soumettre des ordres (marchés dirigés par les ordres et marchés dirigés par les prix) et le niveau de concentration dans les deux marchés (centralisation et fragmentation). De plus, la comparaison utilise différentes mesures de performances des marchés: robustesse aux problèmes d'asymétrie d'information, efficacité informationnelle, variance des prix, agressivité des ordres des informés et la liquidité du marché. On montre que l'utilisation des deux dimensions qui distinguent les deux structures aboutit à des résultats parfois complètement contraires à ceux préconisés dans d'autres études utilisant une seule des deux dimensions. En effet, on montre que les marchés aux enchères sont moins sensibles aux problèmes d'asymétrie d'information et sont plus efficaces. Pour la variance des prix, l'agressivité des stratégies des informés et la profondeur du marché, la comparaison dépend du nombre d'agents dans les marchés.

Mots clés : marchés aux enchères, marchés de contrepartie, performances des marchés, concentration.

This paper compares two market structures, namely auction and dealership markets defined respectively as centralized order-driven and fragmented quote-driven markets. Our approach departs from previous works comparing these market mechanisms by considering both the timing of order submission (quote versus order-driven) and trading concentration (centralized versus fragmented) as dimensions differentiating these trading structures. We compare markets using measures of market viability, informational efficiency, price variance, informed trading aggressiveness and market liquidity. We find that this approach changes dramatically the results of previous works comparing these trading structures. Indeed, we prove that auction markets are less sensitive to asymmetric information problem and they exhibit higher level of informational efficiency than dealership markets. Moreover, we find that the relative magnitude of price variance, informed trading aggressiveness and market depth in both structures depend on the market thickness.

Keywords: Auction Markets, dealership markets, market performances, concentration.

Codes JEL : D43, D44, D82

* I am grateful for their helpful comments to participants at the NFA 2002 congress (Ban., 27-28 September 2002) where earlier version of this paper was presented. I also gratefully acknowledge useful comments and suggestions from Bruno Biais, Catherine Casamatta, Sonia Falconieri, Pascal François, Jean-Charles Rochet and Hyun Song Shin. Financial support by the Initiative on the New Economy (INE) program of SSHRC (Canada) is gratefully acknowledged. All errors are mine.

[†] Assistant Professor, HEC Montréal, 3000 Chemin de la Côte Sainte-Catherine, Dept. of Finance, Montréal (Québec), H3T 2A7 Canada, e-mail: moez.bennouri@hec.ca

1 Introduction

Since the mid-eighties, financial markets were subject to spectacular structural, technological and regulatory changes. All these changes gave rise to a large number of trading structures resulting in each market establishing its own functioning rules. The rules governing the functioning of any market can be of two kinds: structural or organizational. Structural rules regulate the trading method and market participants. Organizational rules instead discipline some practical aspects such as the minimum price tick size, capacity of trading for brokers, delays of disclosure of information about trades, etc...

In spite of this diversity, each trading structure may be described as an hybrid version of two basic market structures: auction and dealership markets. Understanding the relative merits of each of these pure trading mechanisms is an important issue from a normative standpoint because it would allow to better design the specific hybrid version. Comparing these structures would also help to deal with the optimal structure issue.

Auction and dealership markets differ along many dimensions and in rather subtle ways. Two dimensions may however be considered as the main structural properties distinguishing them. These dimensions are the degree of concentration of trading (centralized versus fragmented) and the timing of order submission for liquidity providers (quote and order-driven markets). Auction markets are concentrated order-driven markets while dealership markets are fragmented quote-driven.

In this paper, we compare these trading structures by looking at both dimensions distinguishing them. In an asymmetric information framework, these dimensions lead to different levels of information available to market participants and in particular to liquidity providers. In order-driven markets, traders act simultaneously without knowing the prices and liquidity providers observe traders' order flow before they choose their strategies. Conversely, in quote-driven markets, dealers begin the trading process by posting prices. A main consequence of the different timing of order submission is that liquidity providers compete differently. In fact, in the order-driven markets they compete on quantities while in the quote-driven markets they are engaged in price competition. Furthermore, dealers in fragmented markets have no information about trading costs or trading sizes of their competitors because trades are executed in bilateral meetings. They can only try to elicit some private information from individual orders. In contrast, in concentrated markets, market makers learn more information about the final asset value because they trade with more than only one counterpart.

In the literature, the comparison between auction and dealership markets is generally based either on the timing of order submission (see for instance, Pithyachariyakul (1986), Madhavan (1992), Shin (1996), Bernhardt and Hughson (1996) and Viswanathan and Wang (2002)), or on the concentration of trading (see e.g. Mendelson (1987), Pagano and Röell (1996) and Biais (1993)). The introduction of both dimensions to distinguish auction and dealership markets in our work is motivated by two main considerations.

First, we believe that introducing concentration in auction markets is a more realistic representation of financial markets having a continuous auction structure. By assuming that auction markets are centralized, we argue that the limit-order book may contain more than only one order submitted by traders even when trading occurs continuously. This argument suggests that, at each moment, each submitted order could be executed either against outstanding orders on the limit order book or against orders coming from liquidity providers; hence, each trader should consider the fact that her order is in competition with orders submitted by other traders. In the previous literature, this kind of model describes periodic auction markets (or batch auctions), so that the difference between batch markets and continuous markets is merely based on centralization in the sense that auction markets are the ones where there is only one order submitted by traders.¹ This approach ignores that orders in periodic markets are submitted sequentially between execution rounds. So, in periodic markets, each trader chooses his optimal strategy given the information he can infer from the existing order flow.²

Second, intuition suggests that there is a trade off between concentration of trading and timing of order submission with respect to their impact on the market performances. This is also supported by some existing results in the literature. For instance, Madhavan (1992) proves that fragmented quote-driven markets are less sensitive to asymmetric information than fragmented order-driven markets. However, when we introduce the concentration of trading to characterize auction markets, it is clear that because of the higher transparency, concentrated order-driven markets may become more viable than fragmented quote-driven markets.

Another important contribution of our paper is to compare auction and dealership market with respect to different measures of market quality like market viability, price variability, trading aggressiveness of informed traders and market liquidity. In addition to this, we evaluate each trading structure from the point of view of a) policy makers, who need to choose the

¹See Madhavan (1992) and Pagano and Röell (1996).

²Under this distinction between continuous and periodic auction markets, the modeling of trade in batch markets must be carried out by using a dynamic model so as to consider the adjustments of each trader's strategy to those already displayed on the screen.

optimal trading structure; b) investors, who need to decide in which market to invest, and c) firms, which have to choose where to be listed.

We show that concentration decreases the informational disadvantage of liquidity providers leading to lower sensitivity of auction markets to asymmetric information. In the same way, we prove that concentration allows a higher informational transmission among market participants generating higher informational efficiency in auction markets. Empirically, these results suggest that auction structure should be more suitable for markets with higher level of asymmetric information. By studying IPOs, Falconieri, Murphy and Weaver (2003) document that the level of underpricing is higher in the NASDAQ (a dealership like structure) than in the NYSE (an auction like structure). This is exactly what our theoretical result predicts.

For informed agents aggressiveness, price variance and market liquidity, we find that comparison between markets depend on their thickness. For the first measure, it is commonly argued that informed traders will follow more aggressive strategies in quote-driven markets because of their opacity. If we consider concentration, relative opacity is higher in fragmented quote-driven markets which would accentuate the use of aggressive strategies by informed agents in these markets. However, by introducing concentration, an opposite effect arises. Indeed, concentration decreases the informational advantage of informed agents because of the higher information transmission among market participants. In this case, informed agents may trade more aggressively in auction markets in order to counterbalance the loss of informational advantage. We prove that this latter effect dominates the first one in thick markets. Hence, in thick markets, informed agents strategies are more aggressive in auction markets than in dealership markets, and *vice versa*.

A similar reasoning may be applied to price variance. Price distortions in order-driven markets increase price variance in these markets. Even if concentration amplifies this first effect, an opposite effect arises. Indeed, less asymmetric information decreases the variance of prices. This latter effect will dominate in thick markets, leading to lower price variance in auction markets.

Finally, concentration and timing of order submission have opposite effects on market depth. Concentration leads indeed to lower marginal effect of individual orders, making concentrated markets deeper. On the other hand, quote-driven markets seem to be deeper because competition between liquidity providers in these markets is based on prices leading to lower price sensitivity to traders' orders. We prove that, the concentration effect dominates when the number of liquidity providers is sufficiently large, and *vice versa*.

The remainder of this paper is organized as follows: the next section spells out the model which is based on Glosten (1989) and Madhavan (1992). Then, in section 3, auction and dealership markets are characterized. Thereafter, in sections 4 and 5, equilibria in dealership and auction markets are derived and some of their properties are described. The comparison is exposed in section 6, and finally we conclude by some remarks and possible extensions. All proofs are in the Appendix.

2 The model

We consider a simple one-period model in which agents liquidate their positions after trading occurrence.³ There are two assets in the market: risk-free asset (cash), and a risky asset with a stochastic liquidation value denoted by \tilde{v} .

Two types of agents participate in the market: Traders and liquidity providers (or market makers). Each of the N risk averse traders (indexed by $i = 1, \dots, N$) chooses a trading strategy that maximizes his expected utility given his information set H_i . This set contains his private information that represents his trading motivations, public information and the information related to the trading structure. On the other hand, M risk neutral⁴ market makers (indexed by $m = 1, \dots, M$) provide liquidity to traders. They behave strategically and maximize their expected profits conditional on their information sets ϕ_m . These sets contain public information and information related to the trading mechanism.

Each trader i is assumed to have a negative exponential utility function $U(W_i) = -e^{-\rho W_i}$, where ρ is the coefficient of risk aversion and W_i is his final wealth.

Trader i 's private information is described by a vector (s_i, ω_i) ; s_i is his private signal about the final value of the risky asset and ω_i is his initial endowments⁵. We assume that, for the rest of the market, endowments are normally distributed with mean 0 and precision π_ω . The private signal of trader i is modeled as a noisy observation of the final value:

$$\tilde{s}_i = v + \tilde{\epsilon}_i.$$

We assume that, for all i , $\tilde{\epsilon}_i$ are independently normally distributed with mean 0 and precision π_ϵ . It is publicly known that the final value of the risky asset is normally distributed with

³As in Madhavan (1992), we can easily extend this analysis to a multi-period framework where private information lasts only one period. To simplify notation we omit this extension without loss of generality.

⁴As suggested by Pagano and Röell (1993) and Gould and Verrechia (1985), market makers should be sufficiently less risk averse than traders so as to keep a certain level of market viability. Moreover, risk neutrality in this model avoids the inventory costs problems that risk averse market makers would face.

⁵We could also see ω_i as a liquidity shock for the traders' portfolios.

mean μ and precision π_v ; therefore, given his signal s_i , the trader i considers that \tilde{v} is normally distributed with mean s_i and precision π_ϵ .

This structure of private information with two sources of uncertainty allows the existence of different trading motivations and the introduction of adverse selection problems in this model of asymmetric information. Indeed, when a liquidity provider observes a large purchase [respectively sell] order, he cannot know whether it comes from an information-based trader, *i.e.*, a trader having a good signal (s_i is high) [respectively, s_i is low], or from a liquidity-based trader who trades for hedging reasons because of his initial endowments ($-\omega_i$ is large) [ω_i is large].⁶

For each agent i , when q_i is the quantity of risky assets demanded (or offered) and p is the related unit price, his final wealth is $\tilde{W}_i = (q_i + \omega_i)\tilde{v} - pq_i$. If $q_i > 0$, the trader i is a buyer and if $q_i < 0$ he is a seller. Since his final wealth is normally distributed and he has an exponential utility function, the objective function of trader i is:

$$E[\tilde{W}_i/H_i] - \frac{\rho}{2}var[\tilde{W}_i/H_i]$$

where $E[./H_i]$ and $var[./H_i]$ are expectation and variance operators conditional on H_i .

Public information contains market exogenous parameters (e.g., number of agents for each type, absolute risk aversion), the distributions of the asset's liquidity value, the noise about private observations and trader's initial endowments of risky assets. The last component of all agents' information sets are those related to market structures; they will be presented in the following description of market organizations.

2.1 Auction markets (or centralized order-driven markets)

In auction markets, traders begin the trading process by choosing their strategies given their information sets, and then, simultaneously, submit their orders to be displayed on the screen. After observing the traders' order flow, market makers determine their trading strategies and submit their orders. All orders are accumulated and executed at a single clearing price.

These markets are order-driven because trading is started up by traders' orders. All strategies are chosen before the price is fixed. Rational traders will then submit quantity-price

⁶As in Biais, Martimort and Rochet (2000), this setting is a two dimensional adverse selection problem which is technically extremely complex to solve. CARA utility functions and normal distributions allow to reduce this problem to a one dimensional adverse selection problem using a variable which is a linear combination of private signals and liquidity shocks.

schedules.⁷ So, for each possible price, the trader will adjust his beliefs about the asset's final value given the information transmitted in this price. He considers the effect of his strategy on the market clearing price.

In the model of Madhavan (1992), it is considered that at each trading round in continuous order-driven markets there exists only one trader in the market. On the contrary, we assume in this work that auction markets are also centralized. Under this more general modeling (in some sense, without restriction on market liquidity), we assume that, at each trading round, there are N traders submitting orders for the risky asset. Consequently, each trader should not consider only his own effect on the price but also the effects of his competitors' orders.

The role of market makers in these markets is to provide liquidity for traders. They are institutional investors or intermediaries who respond to the order flow displayed on the screen. Because these markets are characterized by a higher degree of transparency, market makers are assumed to be symmetrically informed about market parameters and order flow. Each market maker submits a quantity-price schedule that maximizes his expected profits given his competitors' trading strategies. Therefore, competition between them is based on quantities and their effect on the equilibrium price is drawn through the effect of their orders on the market clearing condition.

An important aspect distinguishing trading structures is the information sets available for each market participant before choosing his trading strategy. Each agent i 's information set H_i contains no information about markets since traders will begin the trading process. For market makers, ϕ_m contains the aggregate order flow Q submitted by traders.⁸ Therefore:

$$H_i = \{PI, (s_i, \omega_i)\} \quad \text{for all } i = 1, \dots, N$$

and

$$\phi_m = \{PI, Q\} \quad \text{for all } m = 1, \dots, M$$

(where PI = public information).

We denote the vector of traders' demand functions by $\vec{Q} = \{q_1(p), \dots, q_N(p)\}$ and the vector of dealers' demand functions by $\vec{d} = \{d_1(p), \dots, d_M(p)\}$. For all i , let \vec{Q}_{-i} be the \mathbb{R}^{N-1} vector of

⁷See Brown and Zhang (1997) where traders are compelled to submit market orders in auction markets (even though they term this structure "dealer markets").

⁸This represents a low level of transparency for auction markets. However, we can imagine a more transparent auction market in which market makers observe orders separately. Within the present model (normal distributions, CARA utility function and rational expectation framework), this higher transparency has no effect on equilibrium outcomes.

all the other traders, *i.e.*, $\vec{Q}_{-i} = \{q_1(p), \dots, q_{i-1}(p), q_{i+1}(p), \dots, q_N(p)\}$. We define \vec{d}_{-m} for dealers in the same way. We define an equilibrium in auction markets as follows:

Definition 1 : *The Bayesian Nash equilibrium in auction markets is defined by the set (p^*, \vec{Q}, \vec{d}) such that:*

(i) *the equilibrium price p^* satisfies the market clearing condition:*

$$\sum_{m=1}^M d_m(p^*) + \sum_{i=1}^N q_i(p^*) = 0 \quad (1)$$

(ii) *for all $i \in \{1, \dots, N\}$, $q_i^*(p)$ is trader i 's strategy satisfying his optimality condition, given the trading strategies of other market participants and given his information set:*

$$q_i^*(p) \in \operatorname{argmax}_{q(p)} \{E[\widetilde{W}/H_i, p, \vec{d}, \vec{Q}_{-i}] - \frac{\rho}{2} \operatorname{var}[\widetilde{W}/H_i, p, \vec{d}, \vec{Q}_{-i}]\} \quad (2)$$

(iii) *for all $m \in \{1, \dots, M\}$, $d_m^*(p)$ is the market maker's trading strategy satisfying the optimality of his expected profits conditional on the trading strategies of other market participants and his information set:*

$$d_m^*(p) \in \operatorname{argmax}_{d_m(p)} \{E[(\tilde{v} - p)d_m(p)/\phi_m, p, \vec{Q}, \vec{d}_{-m}]\} \quad (3)$$

subject to non negativity.

2.2 Dealership markets (or fragmented quote-driven markets)

Each trading period in dealership markets may be divided in two sub-periods. In the first sub-period, dealers begin by setting their bid-ask prices. Unlike market makers in auction markets, dealers compete in prices because of their status of price setters. Then, each trader chooses the best price for his unique order among dealers' quotations.⁹

Dealership markets are quote-driven since trading is guided by quotes set by different dealers. Because he cannot observe traders' orders, each dealer sets a price-quantity schedule so that, for each possible order size, he will extract all available information from this order.

These markets are also fragmented because trading occurs after pairwise meetings between dealers and traders. Hence, during the trading process, neither the dealer nor the trader is

⁹Note that it is supposed that traders cannot split their orders among dealers. This may occur, for instance, because of higher fixed transaction fees. See Dennert (1993) and Biais, Martimort and Rochet (2000) for models of dealer competition where traders are allowed to split their orders among dealers. Biais, Foucault and Salanié (1998) prove the importance of allocation rules when traders can split their orders among dealers.

informed about other simultaneous trades on the market, if there is any. Because of this opacity, the price is affected by only one order.

Since they move first in the dealership trading process by setting their price-quantity schedules, dealers have no private information either about order flow or about the asset's value. This suggests that dealers, like market makers in auction markets, are symmetrically informed before they set their prices. This occurs either because of the Mandatory Last Trade Displaying rule or by assuming that each dealer can observe his competitors' pricing function and can perfectly learn all their private information. Obviously, we can consider another modeling of dealership markets with a higher degree of opacity when last trade publication is not mandatory¹⁰ and where dealers cannot extract all private information from the pricing functions of informed dealers, or they cannot observe them.¹¹

Since dealers compete on prices, using the standard argument for "Bertrand" games in these markets where risk neutral dealers are symmetrically informed, the unique Nash equilibrium for each of them is to set prices at the break-even level, *i.e.*, prices equal the asset's expected value conditional on the order size and dealers' information set. In fact, if dealers set prices to make positive expected profits, it is always profitable for one dealer to undercut them.¹²

The information sets are $\phi_m = \{PI\}$ for dealers and $H_i = \{p(\cdot), PI, (s_i, \omega_i)\}$ for traders. In dealership markets, the equilibrium is defined by the dealer's common pricing function $p(\cdot)$ and traders' orders $q(s_i, \omega_i)$ as follows:

Definition 2 : *The equilibrium in dealership markets is a differentiable price function and a corresponding demand $q(s_i, \omega_i)$ such that:*

(i) *prices satisfy the zero-expected profit condition for dealers:*

$$p(q) = E[\tilde{v}/\phi_m, q] = E[\tilde{v}/PI, q]. \quad (4)$$

¹⁰See Madhavan (1995) for a theoretical analysis of the mandatory trade publication in fragmented and centralized markets. Gemmil (1996) provides an empirical evidence on the irrelevance of delayed publication on market liquidity by comparing the SEAQ's liquidity under three regimes of publication. He argues that competition between dealers prevents them from exploiting the potential advantage that a delay provides.

¹¹See Biais, Martimort and Rochet (2000) for a model of competition between market makers in a mechanism design framework. In another context, Bernhardt and Hughson (1996) analyse the case of a competitive dealers environment and consider the effect of the price tick size on the strategic behavior of dealers. See also Dennert (1993) for a game theoretical analysis of competition between dealers.

¹²More precisely, price function is equal to the break-even level because of market opacity and because of the fact that traders cannot split their order among dealers. Under the same conditions of risk neutrality, symmetric information and price competition, when traders are allowed to split their orders and dealers are allowed to observe all trading strategies of traders with their competitors, Biais, Martimort and Rochet (2000) prove that, with a finite number of dealers, equilibrium prices are different from the break-even level.

(ii) each trader i maximizes his expected utility given the pricing function and his information set:

$$q(s_i, \omega_i) \in \operatorname{argmax}_q \{E[U(W_i(q)/H_i, p)]\}. \quad (5)$$

Once equilibria in both markets are defined and trading structures are presented, we derive these equilibria and establish their principal features.

3 Equilibrium in auction markets

We will focus on symmetric linear equilibria to make comparison between auction and dealership mechanisms more tractable. As suggested in Madhavan (1992), one may argue that these linear equilibria are “the most natural equilibria given the computational burden facing agents in the economy”. As proposed in definition 1, the equilibrium is Bayesian-Nash. In this equilibrium, each trader determines his optimal trading strategy in order to maximize his expected profits given his conjectures about the trading strategies of the other informed traders. The conjecture of each identical informed trader must be correct conditional on each trader’s information.

Proposition 1 :

Let ψ and ψ_m be defined as follows: $\psi = \pi_v + \pi_\epsilon + (N - 1)\pi$ and $\psi_m = \pi_v + N\pi$; where $\pi = \pi_\epsilon / (1 + \frac{\rho^2}{\pi_\epsilon \pi_\omega})$. When

$$\left\{ \frac{\pi_\epsilon [(M + N - 1)(2\psi - \psi_m) + \pi_v]}{[(M + N - 1)\psi_m - \pi_v]} < \frac{\rho^2}{\pi_\omega} \right\}$$

there exists an equilibrium for auction markets in which

(i) The strategy function of each market maker m is:

$$d_m(p) = \zeta(\mu - p), \quad (6)$$

(ii) The demand function of each trader i is:

$$q_i(s_i, \omega_i, p) = \alpha\mu + \beta s_i - \gamma\omega_i - \theta p, \quad (7)$$

(iii) the equilibrium price is:

$$p = \mu + \frac{1}{M\zeta}Q; \quad (8)$$

with $Q = \sum_{i=1}^N q_i$ and $\alpha, \beta, \gamma, \theta$ and ζ are positive constants defined in the Appendix. Otherwise, there is no linear equilibrium and market breaks down.

In proposition 1, $\left\{ \frac{\pi_\epsilon[(M+N-1)(2\psi-\psi_m)+\pi_v]}{[(M+N-1)\psi_m-\pi_v]} < \frac{\rho^2}{\pi_\omega} \right\}$ is a necessary and sufficient condition for equilibrium existence in auction markets. The left hand side may be interpreted as a measure of asymmetric information between traders and market makers. It indeed depends on ψ and ψ_m which are the adjusted precisions of the asset's value distribution for respectively informed traders and market makers. The information conveyed by each conjectured trader's strategy allows an additional precision about the asset's value of π ; then, for each trader, given his conjectures about the $(N-1)$ other traders, the precision of the asset's value is the sum of his precision given his information set $(\pi_v + \pi_\epsilon)$ and the $(N-1)$ additional precision (so $(N-1)\pi$). On the other hand, for each market maker, given his conjectures about the traders' strategies, the precision of the distribution of the asset's value is the additional precision inferred by theses strategies $(N\pi)$ and the *ex ante* precision π_v . Note that this measure of asymmetric information increases when π_ϵ is large relative to π_v , *i.e.*, when traders' private information is precise compared to public information about the final asset's value. Conversely, when π_v is high and π_ϵ is low, *i.e.*, when private signal does not present a substantial improvement of information about the final asset's value, then $\frac{\pi_\epsilon[(M+N-1)(2\psi-\psi_m)+\pi_v]}{[(M+N-1)\psi_m-\pi_v]}$ is low, which may be interpreted as lower asymmetric information.

The right hand side of the condition for equilibrium existence $\left(\frac{\rho^2}{\pi_\omega}\right)$, may be seen as a measure of liquidity-motivated trading which depends on trader's risk aversion and the precision of liquidity shocks. When traders are more risk averse or when the variance of their initial endowments is high (π_ω is low), traders are more likely to be liquidity motivated. Hence, equilibrium in auction market exists when asymmetric information is less important than non-information trading motivation. Otherwise, market makers' informational disadvantage relative to traders is so severe that they cannot avoid negative expected profits and market breaks down.

Obviously, higher private information precision or lower precision of prior distribution of the asset's value for uninformed market makers lead to higher asymmetric information relatively to liquidity measure and then to lower market viability. On the other hand, in order to study the effects of ρ and π_ω on market viability, we need to study the variations of both measures with these parameters. As mentioned above, non-information motivation measure is increasing in ρ and decreasing in π_ω . For the asymmetric information measure, substitution of the values of π , ψ and ψ_m , in the left hand side of the equilibrium existence condition gives

$$\frac{\pi_\epsilon[(M+N)\pi_v + (M+N-1)\pi_\epsilon(2 + \frac{(N-2)\pi_\epsilon\pi_\omega}{\pi_\epsilon\pi_\omega + \rho^2})]}{(M+N-2)\pi_v + N(M+N-1)\frac{\pi_\epsilon^2\pi_\omega}{\pi_\epsilon\pi_\omega + \rho^2}},$$

or equivalently

$$\frac{2\pi_\epsilon[N(M+N-1)\pi_\epsilon + (M+2N-2)\pi_v]}{N[(M+N-2)\pi_v + N(M+N-1)\frac{\pi_\epsilon^2}{\pi_\epsilon + \frac{\rho^2}{\pi_\omega}}]} + (N-2)\frac{\pi_\epsilon}{N}.$$

Then the asymmetric information measure is increasing in ρ and decreasing in π_ω . In lemma 1 (see the appendix), we study the effect of increasing liquidity motivation on auction market viability. We find that the marginal effect of an increase in the liquidity trading measure on the asymmetric information measure is lower than 1. Then, when we increase risk aversion or initial endowments variance, liquidity measure increases more than the asymmetric information measure. Thus, the viability of auction markets increases with risk aversion and the variance of initial endowments.

Further, in the proof of proposition 1, we show that ζ is positive. Hence, market makers buy when prices are low and sell when prices are high. This means that liquidity providers have a stabilizing behavior, even in a model where they behave as profit-maximizers and not as social welfare maximizers.

Finally, we can easily see that the measure of asymmetric information is a decreasing function of N . Intuitively, when the number of traders on the market increases, market makers gather more information from observed variables (the precision of their learned information is $[\pi_v + N\pi]$) which decreases their informational disadvantage and then, induces them to take the opposite side of the market, leading to higher viability for auction markets.

As for traders' number, the measure of asymmetric information decreases with the number of market makers. Indeed, a larger number of market makers decreases the marginal amount of asymmetric information supported by each one of them, what reduces their reluctance to take the opposite side of the market and then enhances market viability. The equilibrium in the limit case, *i.e.*, when the number of market makers is extremely large, is characterized in Proposition 2.

Proposition 2 : *With market maker's free entry, i.e., when $M \rightarrow +\infty$, equilibrium in auction markets always exists and:*

- (i) $\beta \rightarrow b_0$
- (ii) $\alpha \rightarrow 0$
- (iii) $\zeta \rightarrow 0$
- (iv) $\gamma \rightarrow \frac{\rho b_0}{\pi_\epsilon}$
- (v) $\theta \rightarrow b_0$.

with b_0 is the largest possible value of β given by

$$b_0 = \frac{\pi_\varepsilon - \pi}{\rho} - \frac{\pi\psi}{\rho(\pi_v + (N-1)\pi)}$$

Proposition 2 shows that free entry of market makers in auction markets raises traders' aggressiveness (β takes the maximal value that satisfies the dealers' second order condition). This occurs because of the centralization feature of auction markets. Indeed, since market makers are numerous, the marginal effect of one trader's order on the equilibrium price is sufficiently small to allow him to trade more aggressively according to his private information and to carry less about market makers' adjustments about the final value of the asset. Furthermore, as predicted, the quantity demanded by traders (which is finite) is shared between all market makers, and their individual order size tends towards zero.

In the following proposition, we derive some features of the equilibrium in auction markets, when it exists, in order to use them subsequently when we begin comparison between structures.

Proposition 3 : *When symmetric linear equilibrium exists for auction markets, then:*

(i) *equilibrium price is not semi-strong form efficient. This efficiency level is reached with market makers' free entry,*

(ii) *the variance of the equilibrium price tends towards zero when $N \rightarrow +\infty$,*

(iii) *with free entry, the quoted bid-ask spread tends to $\frac{2\pi}{b_0\pi_v}$.*

In part (i) of proposition 3 we prove that equilibrium price in auction markets is not semi-strong form efficient because of market makers' quantity-based competition which induces a difference between equilibrium price and the expected value of the asset conditional on the extracted information from the price function and public information. This difference disappears with free entry because competition between market makers, even if it is quantity-based, becomes a perfect competition leading price to the semi-strong form efficiency level. In part (ii), it is shown that, when the number of traders becomes sufficiently large, market makers' precision of extracted information increases and price tends towards the asset's value v . Finally, in part (iii), we show that the quoted bid-ask spread in auction markets (which is equal to $p(1) - p(-1)$) tends to its minimal value with free entry because, in this case, numerous market makers share the risk between them and the "marginal" informational risk borne by each of them is minimized.

4 Equilibrium in dealership markets

Equilibrium in these markets is derived as in Glosten (1989) and Madhavan (1992).¹³ By using our notations we get the following proposition.

Proposition 4 : *In dealership markets, equilibrium is defined by $(p(q), q_i(s_i, \omega_i))$ such that: for all q*

$$p(q) = \mu + \frac{\rho\pi_\epsilon\pi_\omega}{\pi_v\rho^2 - \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)}q \quad (9)$$

and

$$q_i(s_i, \omega_i) = \frac{\pi_v\rho^2 - \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)}{\rho[\pi_v\rho^2 + \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)]}[-\pi_\epsilon\mu + \pi_\epsilon s_i - \rho\omega_i]. \quad (10)$$

This equilibrium exists only if $\pi_v\rho^2 > \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)$; Otherwise, there is no equilibrium price schedule and market breaks down.

In proposition 4, a necessary and sufficient condition for equilibrium existence is

$$\frac{\pi_\epsilon(\pi_\epsilon + \pi_v)}{\pi_v} < \frac{\rho^2}{\pi_\omega}.$$

If traders' signal precision is sufficiently high, or dealers' prior precision about the final security's value is sufficiently low, this leads to market failure. Intuitively, when information asymmetry is large, then dealers' informational disadvantage relative to traders is so severe that they cannot make non negative expected profits. They will refuse to make transactions. Nevertheless, if traders' non-information related motivations for trade are important, dealers are urged to take the opposite side of the market even with information asymmetry. Liquidity related motives for trade depend on traders' risk aversion and the precision of their initial endowments.

Contrary to auction markets, note that the equilibrium outcomes in dealership markets depend neither on the number of traders nor on that of dealers. This is directly linked to the structure of the trading in these markets. Symmetric behavior of dealers and the fact that trading is involved in dealership markets after pairwise meetings between traders and dealers entail this independence between equilibrium and the number of each type of market participants in the market. However, the extreme fragmentation considered in these model where each dealer cannot receive more than one order, induces that this equilibrium outcomes are realized for $M > N$.

¹³See proposition 1 in Glosten (1989) and proposition 1 in Madhavan (1992).

As in auction markets, we derive in the following proposition some useful features of the equilibrium in dealership markets.

Proposition 5 : *In dealership market, when equilibrium exists, we have:*

- (i) *prices are semi-strong form efficient,*
- (ii) *price variance is equal to $\frac{\pi}{(\pi_v + \pi)^2}$,*
- (iii) *the quoted bid-ask spread is an increasing function of π_ϵ and π_ω and a decreasing function of π_v and ρ .*

The explicit bid-ask spread in these markets is an increasing function of the trader's order size, reflecting the response of dealers to the asymmetric information problem they face. Furthermore, the quoted bid-ask spread is an increasing function of π_ϵ and π_ω , and a decreasing function of ρ and π_v . Thus, when trader's private information about the asset's value is more precise or when their initial endowments are less variable, dealers' beliefs that trading is information-motivated increases leading to an increase in the adverse selection problem, and consequently to an increase in the bid-ask spread. On the contrary, when traders are more risk averse or when dealers have enough information about the risky asset (π_v is higher), asymmetric information is no longer severe and traders are more likely to be liquidity-motivated which induces a decrease of bid-ask spreads.

5 Market performances and trading structures

In this section, we compare trading equilibria in both structures. Different indicators are used to measure market performances. For each measure, we will discuss the implications of our results on policy makers and investors decisions.

5.1 Market viability

Market viability reflects the ability of the trading structure to be less sensitive to asymmetric information between different participants. In particular, it measures the willingness of liquidity providers (market makers in auction markets and dealers in dealership markets) to take the opposite side of the market even when they are informationally disadvantaged relative to traders. From Proposition 1 and Proposition 4, market viability in both markets is measured by the conditions of existence of equilibria. We say that one market is more viable when

its equilibrium existence condition is satisfied each time the equilibrium exists on the other market.¹⁴

Proposition 6 : *Auction markets are always more viable than dealership markets; indeed, equilibrium existence condition in auction markets is satisfied each time trading in dealership markets exists. Moreover, when*

$$\pi_\epsilon < \left(1 - \frac{2}{M + N}\right) \frac{\rho^2}{\pi_\omega},$$

then for

$$\frac{M\pi_\epsilon^2\pi_\omega}{(M + N - 2)\rho^2 - (M + N)\pi_\epsilon\pi_\omega} - (N - 1) \frac{\pi_\epsilon^2\pi_\omega}{\rho^2 + \pi_\epsilon\pi_\omega} < \pi_v < \frac{\pi_\epsilon^2\pi_\omega}{\rho^2 - \pi_\epsilon\pi_\omega},$$

equilibrium exists only in auction markets.

Madhavan (1992) shows that fragmented quote-driven markets are more robust to problems of asymmetric information than fragmented order-driven markets. Proposition 6 states that introducing concentration in order-driven markets contradicts this results since in this case auction markets are more viable than fragmented quote-driven markets. This is directly attributable to the higher level of information conveyed in concentrated markets; indeed, this higher level of information decreases the market makers' informational disadvantage relative to dealers operating in more opaque markets which incites them to take the opposite side of the market in auction structure. For some values of private information precision, it is possible that the precision of public information is such that auction markets exist but dealership markets do not. Again, this is directly attributable to the concentration that enhances market makers' informativeness and incites them to provide liquidity on auction markets while dealers will not since their informational disadvantage is so high that they cannot trade without bearing losses.¹⁵

¹⁴In this work, we consider only linear equilibria for auction markets. This restriction has no effect on the result related to the market viability. We prove indeed that auction market are more viable than dealership markets even under this restriction. This result holds when we consider other equilibria in auction markets.

¹⁵Glosten (1994) gets a similar result about the viability of auction markets. In his model, he proved, under more general conditions, that auction markets (which have the same structure as the limit order book analyzed in that work) does not invite competition from third market dealers, while other trading institutions do. However, the comparison in Glosten (1994) is focused on the outcome for an agent who is in competition with the existing limit order book (with an infinite number of market makers) by offering a liquidity providing services. It is shown that this agent will always earn negative expected profits. On the contrary, these expected profits may be positive if this agent would compete with another existing trading structure. This may be interpreted here by considering that another trading alternative enhances the market viability in dealership markets contrary to auction markets.

Several regulating incentives and decision rules for investors may be deduced from this result. First, consider a firm going public and having the possibility to choose between dealership and auction structures. The IPOs environment is characterized by a higher level of asymmetric information. Underpricing is also another peculiar feature of IPOs that is mainly explained by arguments related to the level of asymmetric information between agents. Proposition 6 suggests that firms are better off going public in an auction structure since this will increase the probability of success of their introductions. In terms of underpricing, this means that it should be higher in dealership markets because of the lower robustness of this structure to asymmetric information. This result is confirmed empirically in Falconieri, Murphy and Weaver (2003) where it is documented that the level of underpricing is higher in the NASDAQ (a dealership like structure) than in the NYSE (an auction like structure).

Second, market viability in practice is related to trading halts and circuit breakers. Proposition 6 suggests that a firm quoted in both structures will be less exposed to trading halts, caused by asymmetric information problem, than a firm quoted only in a dealership structure.¹⁶

Third, from a policy making point of view, dealership markets need a higher standards of information disclosure about firms than auction markets in order to alleviate the asymmetric information problem.

5.2 Traders' aggressiveness

Trading aggressiveness of informed traders is measured in both markets by the marginal effect of increasing one trader's private signal on his trading strategy. This reflects the importance of private information for this informed trader in his trading strategy. In auction markets, it is equal to β and in dealership markets to:

$$\beta_D = \frac{\pi_\epsilon[\pi_v\rho^2 - \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)]}{\rho[\pi_v\rho^2 + \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)]}$$

which is equal to:

$$\frac{\pi_v(\pi_\epsilon - 2\pi) - \pi_\epsilon\pi}{\rho(\pi_v + \pi)}.$$

It is commonly argued that, because of opacity, informed traders trade more aggressively in fragmented quote-driven markets than in fragmented order-driven markets. If we consider concentrated order-driven markets, the relative opacity of dealership markets is more important,

¹⁶Trading halts are activated for different reasons. Some of them are institutional like those related to regulatory rules of information disclosure and others may be strategically demanded by market makers (see Edelen and Gervais, 2003).

strengthening the argument of higher traders' aggressiveness in dealership markets. However, concentration has a positive effect on traders' aggressiveness in auction markets. Indeed, concentration leads to a lower informational advantage for informed traders who may therefore prefer to trade more aggressively in auction markets in order to compensate for their reduced informational advantage. In accordance with this intuition, we prove the following:

Proposition 7 : *For a finite M , when $(N - 1) > \frac{\pi_\epsilon(\pi_v + \pi)}{2\pi(\pi_\epsilon - \pi)}$, traders in auction markets trade more aggressively.*

Thus, when traders' number is sufficiently high, the second (positive) effect dominates since strategies convey higher information and the marginal effect of each trader is sufficiently low such that traders choose to be more aggressive in auction markets. With free entry, proposition 2 proves that β tends towards b_0 that is higher than β_D . Thus, trading is more aggressive in auction markets in that case.

5.3 Price variance

Before receiving their private information and when they face the choice between market organizations, traders will be concerned with the *ex ante* price variances in both markets. Alternatively, for regulators, price variance may be considered as a measure of price volatility and could be an important argument for choosing the market structure featuring lower price variance. From traders' point of view, expected prices are the same in both structures. Thus, because of their risk aversion the *ex ante* price variances may be considered as an important parameter of comparison between markets.

Comparing price variances in dealership and fragmented order-driven markets, Madhavan (1992) finds that price distortions caused by trading strategies of market makers lead to higher price variance in order-driven markets. If we consider concentration in auction market, this price distortions effect is amplified since we have in this case $(M + N)$ agents affecting the equilibrium price. Nevertheless, an opposite effect on price variance arises. Indeed, concentration reduces the asymmetric information between traders and market makers; then, all agents learn more information from their competitors' strategies inducing lower price variance. In line with this intuition, we prove the following.

Proposition 8 : *With a fixed M , if $N < (\frac{\pi_v}{\pi})^2$, price variance is higher in auction markets; when $(N - 1) > \frac{\pi_\epsilon^2(\pi_v + \pi)^2 - 4\pi^3(\pi_v + \pi_\epsilon)}{4\pi^4}$, price variance is lower.*

Hence, as suggested by the intuition, if N is sufficiently low, price distortion effect dominates the informational effect of concentration on price variance. However, when N is sufficiently high, information acquired by different market participants involves lower price variance in auction markets compared to dealership markets.

This result is supported by the empirical work in Jain (2002). In that work, it is documented that price volatility is higher in dealership markets than in auction markets. Market volatility, in each market, is measured by the price variance of the most active firms for which it is more likely to have a larger N .

In conclusion, regulators preferring markets with lower price variability will opt for an auction structure if they expect their market to be relatively deep (N high) and for a dealership structure if they consider that their market will be relatively thin (N is low).

5.4 Informational efficiency

It results from Proposition 3 and Proposition 5 that prices are semi-strong form efficient in dealership markets but not in auction markets. This result is directly linked to the difference of competition among liquidity providers. Price competition in dealership markets induces prices to reflect all publicly available information, whereas, in auction markets, since they are concentrated and market makers compete on quantities, each agent's strategy influences the equilibrium price leading to a non semi-strong form efficiency. With market making free entry, auctions markets attain this level of efficiency. In that case, quantity based competition converges, in terms of efficiency, to price based competition.

Note, however, that the semi-strong form efficiency of prices in dealership markets in this model is due to the specific assumptions leading to the expected zero-profits condition. When we introduce asymmetric information between dealers (no mandatory last trade publication) or the possibility of splitting trader's order among dealers,¹⁷ this level of efficiency should disappear.

An alternative measure of efficiency is the following:

$$e = 1 - \frac{\text{var}(\tilde{v}/p)}{\text{var}(\tilde{v})}. \quad (11)$$

This measure takes its values on $[0, 1]$, and the extreme values represent complete informational inefficiency or efficiency. It is zero (one) when p is completely uninformative (perfectly

¹⁷In this case, when dealers are symmetrically informed and with a certain level of transparency for markets, Biais, Martimort and Rochet (2000) prove that dealers' expected profits are strictly higher than zero.

informative) of the final value \tilde{v} .

Transparency of auction markets leads to higher information transmitted in the equilibrium price, and then to higher degree of efficiency. This is the intuition of the following proposition.

Proposition 9 : *If we use the informational efficiency measure defined in equation (11), auction markets are more efficient than dealership markets.*

5.5 Market liquidity

Since the model we use in this work is static, we use price related measures of market liquidity.¹⁸ Market depth or bid-ask spread are the appropriate measures. Because equilibria are linear, both measures lead to the same comparison. Let us then consider market depth as a measure of market liquidity. We say that one market is more liquid if its depth is higher. Market depths in auction and dealership markets are denoted respectively by Δ_A and Δ_D , with

$$\Delta_A = \frac{(2 - N)\pi_\epsilon + 2(N - 1)\pi + (N - 1)\rho\beta}{\pi_\epsilon(\pi_\epsilon - \pi - \rho\beta)}$$

and,

$$\Delta_D = \frac{\pi_v\rho^2 - \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)}{\rho\pi_\epsilon\pi_\omega} = \frac{\pi_v(\pi_\epsilon - \pi) - \pi(\pi_\epsilon + \pi_v)}{\rho\pi}.$$

Intuitively, because market depth is the effect of individual orders on prices, concentration leads to lower marginal effect of each individual trader on prices. Then, concentrated markets seem to be deeper. But, if we consider the second dimension distinguishing pure structures, *i.e.* the timing of action for market participants, an opposite argument arises. Indeed, quote-driven markets are deeper because price competition leads to a lower price sensitivity to trader's orders than in order-driven markets where competition between market makers is based on quantities.¹⁹

¹⁸In a dynamic framework, another dimension of liquidity is the one related to immediacy cost for trading over time (see for example Grossman and Miller, 1988).

¹⁹This argument may be proved using the equilibria derived in Madhavan (1992). In that work, if Δ_Q^* and Δ_O^* denote market depths in both markets (Q for quote-driven and O for order-driven), we have:

$$\Delta_Q^* = \Delta_D = \frac{\pi_v\rho^2 - \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)}{\rho\pi_\epsilon\pi_\omega}$$

and

$$\Delta_O^* = \frac{(M - 2)\pi_v\rho^2 - M\pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)}{(M - 1)\rho\pi_\epsilon\pi_\omega}.$$

So

Therefore, market participant’s timing of action and concentration have opposite effects on market depth, and once again, the liquidity based comparison between trading structures is not obvious.

Because we do not have an explicit formulation of β , a direct comparison between Δ_A and Δ_D is not possible. Nevertheless, one can obviously see that for the bounds of β (i.e., zero and b_0), the difference between Δ_A and Δ_D is negative for the lower bound and positive for the upper bound. Since, β is a continuous function of M , we conclude that if M is sufficiently large, concentrated order-driven markets are deeper than fragmented quote-driven markets. Intuitively, if quantity-based competition between market makers is greater, or sufficiently “efficient”, the positive effect of concentration on depth dominates the negative effect generated by the timing of action of market participants.

From regulators’ point of view, enhancing liquidity in auction markets should be done by encouraging market making, otherwise it is better for them to opt for a fragmented quote-driven structure where liquidity is greater. Similarly, investors and firms, looking “greedily” for liquidity when they face the choice of the market structure where they prefer to act or to be quoted, should study the level of competition between market makers in auction markets. If it is deemed sufficiently efficient, they will choose auction structure. Otherwise, it is optimal for them to opt for dealership markets.

6 Concluding Remarks

In this work, we establish a comparison between two financial market structures using the fact that they differ with respect to two dimensions: concentration and timing of action for different market participants. It turns out that some previous results on relative market performances are no longer true when we consider both concentration and timing of action. For instance, we proved that concentrated order-driven markets are less sensitive to asymmetric information than fragmented quote-driven markets, while Madhavan (1992) argued the contrary. In the same way, other results related to traders’ aggressiveness, price variability and market liquidity are derived, confirming the importance of using both dimensions distinguishing markets. Some

$$\Delta_Q^* - \Delta_O^* = \frac{\pi_v \rho^2 + \pi_\epsilon \pi_\omega (\pi_\epsilon + \pi_v)}{(M-1) \rho \pi_\epsilon \pi_\omega} > 0.$$

of these results are confirmed by several recent empirical works.

Nevertheless, this analysis lacks some features of financial market organizations that could influence investors or regulators' choices. Among others, we omitted the effects of inventory costs on liquidity providers' strategies, and execution risk faced by limit orders' submitters in auction markets (while in dealership markets, market makers provide them with an insurance against this risk).

For the latter, it is clear that introducing it as a parameter of choice for traders fosters dealership markets. So, depending on the importance that each investor attributes to this parameters relative to performances considered in our model, one can guess the choice of each trader. For investors and firms, there exists other regulating parameters which could be considered when they are choosing a trading structure; examples of such parameters are transaction fees and trading capacity of intermediaries.²⁰

For inventory costs, strategies of liquidity providers would be different in both markets and their risk aversion measure will affect different market performances. However, it is not clear whether this will be in favor of auction or dealership markets. An interesting field of research is to compare these market structures when we consider both the adverse selection and the inventory costs paradigms.²¹

Moreover, in this work auction and dealership markets are compared in a context where they are assumed to be separate entities. So, the interaction between them when both of them exist is ignored. Several works²² however prove that this point may have an important effect over the performances of these structures and then over the investment strategies of investors and the market structure choice of regulators.

Financial markets are rarely organized as pure fragmented quote-driven or centralized order-driven. In fact, each one may be seen as an hybrid version of these extreme organizations. For instance, at the Paris Bourse, which is organized as an electronic auction market, some trades may be executed on the off-exchange markets²³ so that trading is conducted without being

²⁰See Röell (1990) and Fishman and Longstaff (1992).

²¹Brown and Zhang (1997) uses both paradigms to compare auction markets (termed limit order book markets) and another centralized order-driven market in which traders are compelled to submit market orders (termed dealer markets). Then, the difference between these markets is based on the higher information for dealers (observability of the traders' order flow) and the existence of the execution-price risk in their dealer markets. See also Hendershott and Mendelson (2000) for an analysis of the effects of introducing a competing trading structure on dealer markets, where both asymmetric information and inventory costs are introduced.

²²See for example Grossman (1990), Seppi (1990 and 1992) and Blume and Goldstein (1997) for a theoretical analysis of the interaction between trading structures.

²³Such tradings are called *opérations de contrepartie*.

displayed on the screen (at least before its execution). On the New York Stock Exchange, there is a monopolistic specialist for each stock which is in competition with market makers operating via a centralized system. So, two natural questions arise: Is it possible to derive an optimal trading structure as a combined version of the basic market organizations studied in this work? and, how this optimal mechanism could depend on market features and regulators' parameters of choice?

APPENDIX²⁴

Proof of proposition 1:

This proposition is an extension of Madhavan (1992)'s fragmented order-driven equilibrium construction. The proof constructs the linear Bayesian Nash equilibrium for auction markets by solving for an agent's best response to the conjectured strategies adopted by other agents and then shows that these conjectures are consistent.

* step 1: Traders' best replies

Suppose that a trader i with the information set $H_i = (s_i, \omega_i)$ will conjecture that:

1. trading strategy for all $j \in \{1, \dots, N\}$ and $j \neq i$ is :

$$q_j(p) = \alpha\mu + \beta s_j - \gamma\omega_j - \theta p. \quad (\text{A.1})$$

2. the trading strategy of market maker m , for all $m \in \{1, \dots, M\}$ is:

$$d_m(p) = \zeta(\mu - p) \quad (\text{A.2})$$

where β and γ are positive constants.

For this trader, if he chooses q_i , the market clearing condition is:

$$M\zeta(\mu - p) + q_i + \sum_{j \neq i} (\alpha\mu + \beta s_j - \gamma\omega_j - \theta p) = 0.$$

Then, the equilibrium price satisfies the following equality:

$$p = \left[\frac{M\zeta + (N-1)\alpha}{\lambda} \right] \mu + \frac{1}{\lambda} q_i + \frac{1}{\lambda} \sum_{j \neq i} (\beta s_j - \gamma\omega_j) \quad (\text{A.3})$$

where $\lambda = [M\zeta + (N-1)\theta]$. From equation (2) the trading strategy $q_i(p)$ satisfies:

$$q_i(p) \in \underset{q_i}{\operatorname{argmax}} \{ E[\tilde{v}/s_i, p, \vec{d}, \vec{Q}_{-i}](\omega_i + q_i) - q_i p - \frac{\rho}{2} \operatorname{var}[\tilde{v}/s_i, p, \vec{d}, \vec{Q}_{-i}](\omega_i + q_i)^2 \}.$$

Optimality conditions (the First and the Second Order Conditions) are

$$FOC : E[\tilde{v}/s_i, p, \vec{d}, \vec{Q}_{-i}] - p - \frac{\partial p}{\partial q_i} q_i - \rho \operatorname{var}[\tilde{v}/s_i, p, \vec{d}, \vec{Q}_{-i}](\omega_i + q_i) = 0 \quad (\text{A.4})$$

$$SOC : -2 \frac{\partial p}{\partial q_i} - q_i \frac{\partial^2 p}{\partial q_i^2} - \rho \operatorname{var}[\tilde{v}/s_i, p, \vec{d}, \vec{Q}_{-i}] < 0. \quad (\text{A.5})$$

²⁴Equations related to auction markets, dealership markets and comaprison analysis are respectively labeled A, D and C.

From (A.3), we have $\frac{\partial p}{\partial q_i} = \frac{1}{\lambda}$ and $\frac{\partial^2 p}{\partial q_i^2} = 0$, then the optimal trading order for i is:

$$q_i = \frac{E[\tilde{v}/s_i, p, \vec{d}, \vec{Q}_{-i}] - p - \rho var[\tilde{v}/s_i, p, \vec{d}, \vec{Q}_{-i}]\omega_i}{\rho var[\tilde{v}/s_i, p, \vec{d}, \vec{Q}_{-i}] + \frac{1}{\lambda}}. \quad (\text{A.6})$$

Let us turn to the computation of $E[\tilde{v}/s_i, p, \vec{d}, \vec{Q}_{-i}]$ and $var[\tilde{v}/s_i, p, \vec{d}, \vec{Q}_{-i}]$. Define z_i as follows:

$$z_i = \frac{1}{(N-1)\beta}[\lambda p - (M\zeta + (N-1)\alpha)\mu - q_i]. \quad (\text{A.7})$$

From equation (A.3), z_i may also be written as follows:

$$z_i = \frac{1}{(N-1)\beta}[\sum_{j \neq i} (\beta s_j - \gamma \omega_j)].$$

Thus, z_i is a realization of a random variable $\tilde{z}_i = \tilde{v} + \tilde{x}_i$ where $\tilde{x}_i \sim \mathcal{N}(0, [(N-1)\pi]^{-1})$ and

$$\pi = \frac{1}{\frac{1}{\pi_\epsilon} + \frac{\gamma^2}{\beta^2 \pi_\omega}} = \frac{\pi_\epsilon \pi_\omega}{\pi_\omega + \frac{\gamma^2 \pi_\epsilon}{\beta^2}}. \quad (\text{A.8})$$

Given the equilibrium price, trader i observes a realization of \tilde{z}_i ; this allows him to adjust his beliefs about the final value of the asset. Since all variables are normally distributed and stochastically independent, the trader's conditional expectation of \tilde{v} is:

$$E[\tilde{v}/s_i, p, \vec{d}, \vec{Q}_{-i}] = E[\tilde{v}/s_i, z_i] = \frac{\pi_v \mu + \pi_\epsilon s_i + (N-1)\pi z_i}{\psi} \quad (\text{A.9})$$

and the conditional variance is

$$var[\tilde{v}/s_i, p, \vec{d}, \vec{Q}_{-i}] = var[\tilde{v}/s_i, z_i] = \psi^{-1} \quad (\text{A.10})$$

where $\psi = \pi_v + \pi_\epsilon + (N-1)\pi$. Substituting (A.9) and (A.10) into (A.6) gives

$$q_i(p) = \frac{1}{r} \{ \mu [\beta \pi_v - \pi (M\zeta + (N-1)\alpha)] + \beta \pi_\epsilon s_i - \beta \rho \omega_i - p [\psi \beta - \lambda \pi] \} \quad (\text{A.11})$$

where $r = \beta \rho + \pi + \beta \frac{\psi}{\lambda}$.

Then, the trading strategy of i has the conjectured form with α , β , γ , and θ satisfying the following equations:

$$\alpha = \frac{\beta \pi_v - \pi (M\zeta + (N-1)\alpha)}{r} \quad (\text{A.12})$$

$$\beta = \frac{\beta \pi_\epsilon}{r} \quad (\text{A.13})$$

$$\gamma = \frac{\beta \rho}{r} \quad (\text{A.14})$$

$$\theta = \frac{\psi \beta - \lambda \pi}{r} \quad (\text{A.15})$$

*** step 2: dealers' best replies**

Suppose that the market maker m conjectures that traders' strategies are described as in (A.1) for $j \in \{1, \dots, N\}$ and that the trading strategies of his competitors are: $d_l(p) = \zeta(\mu - p)$ for all $l \in \{1, \dots, m-1, m+1, \dots, M\}$.

For this market maker, if his optimal order is $d_m(p)$, then the market clearing condition is:

$$d_m(p) + (M-1)\zeta(\mu - p) + \sum_{j=1}^N (\alpha\mu + \beta s_j - \gamma\omega_j - \theta p) = 0.$$

Therefore,

$$p = \mu \left[\frac{(M-1)\zeta + N\alpha}{\lambda_m} \right] + \frac{d_m(p)}{\lambda_m} + \frac{1}{\lambda_m} \sum_{j=1}^N (\beta s_j - \gamma\omega_j) \quad (\text{A.16})$$

with $\lambda_m = (M-1)\zeta + N\theta$. From equation (3), $d_m(p)$ satisfies:

$$d_m(p) \in \text{argmax}_{d_m} E[(\tilde{v} - p)d_m(p)/Q, p, \vec{Q}, \vec{d}_{-m}].$$

The optimality conditions are

$$\text{FOC} : E[\tilde{v}/Q, p, \vec{Q}, \vec{d}_{-m}] - p - \frac{\partial p}{\partial d_m} d_m(p) = 0 \quad (\text{A.17})$$

$$\text{SOC} : -2 \frac{\partial p}{\partial d_m} - d_m \frac{\partial^2 p}{\partial d_m^2} < 0. \quad (\text{A.18})$$

From (A.16), we have: $\frac{\partial p}{\partial d_m} = \frac{1}{\lambda_m}$ and $\frac{\partial^2 p}{\partial d_m^2} = 0$. Thus, from (A.17), the optimal trading order of m is:

$$d_m(p) = \lambda_m [E[\tilde{v}/Q, p, \vec{Q}, \vec{d}_{-m}] - p]. \quad (\text{A.19})$$

Given his conjectures, market maker m observes

$$Q = \sum_{j=1}^N q_j = N\alpha\mu - N\theta p + \sum_{j=1}^N (\beta s_j - \gamma\omega_j); \quad (\text{A.20})$$

then, given p and this observation, m should observe

$$z_m = \frac{Q - N\alpha\mu + N\theta p}{N\beta}. \quad (\text{A.21})$$

From (A.20), we have also that

$$z_m = \frac{1}{N} \sum_{j=1}^N (s_j - \frac{\gamma}{\beta} \omega_j).$$

Then, z_m is a realization of a random variable $\tilde{z}_m = \tilde{v} + \tilde{y}_m$, where \tilde{y}_m is a centered normal random variable with variance $[N\pi]^{-1}$ where π is defined in (A.8).

Consequently,

$$E[\tilde{v}/Q, p, \vec{Q}, d_{-m}^{\vec{}}] = E[\tilde{v}/z_m] = \frac{\pi_v \mu + N\pi z_m}{\psi_m}; \quad (\text{A.22})$$

with $\psi_m = \pi_v + N\pi$. Substituting (A.22) into (A.19) gives:

$$d_m(p) = \left\{ \frac{\pi_v \beta - \pi[(M-1)\zeta + N\alpha]}{\pi + \beta \frac{\psi_m}{\lambda_m}} \right\} \mu - \left\{ \frac{\psi_m \beta - \pi[(M-1)\zeta + N\theta]}{\pi + \beta \frac{\psi_m}{\lambda_m}} \right\} p. \quad (\text{A.23})$$

This solution takes the form of market makers' conjectured strategies when both terms are equal to ζ . So we have

$$\zeta = \frac{\pi_v \beta - \pi[(M-1)\zeta + N\alpha]}{\pi + \beta \frac{\psi_m}{\lambda_m}} \quad (\text{A.24})$$

and,

$$\zeta = \frac{\psi_m \beta - \pi[(M-1)\zeta + N\theta]}{\pi + \beta \frac{\psi_m}{\lambda_m}}. \quad (\text{A.25})$$

* step 3: existence of the equilibrium

The equilibrium exists when the equations system (A.12), (A.13), (A.14), (A.15), (A.24) and (A.25) for the five variables α , β , γ , θ and ζ has a solution.

First, from (A.13) and (A.14) we have $\frac{\gamma}{\beta} = \frac{\rho}{\pi_\epsilon}$. Substitution into (A.8) allows us to conclude that π does not depend on equilibrium parameters:

$$\pi = \frac{\pi_\epsilon^2 \pi_\omega}{\pi_\epsilon \pi_\omega + \rho^2}.$$

Moreover, from (A.24) and (A.25) we have : $\theta = \beta + \alpha$. Therefore, if we substitute theses values of θ and ζ into different equations, the underlying system is simplified and we have to solve a new system with only three variables:

$$M\zeta + (N-1)\theta = \frac{(\pi_v + (N-1)\pi)\beta - \pi_\epsilon \alpha}{\pi} \quad (\text{A.26})$$

$$[(\pi_v + (N-1)\pi)\beta - \pi_\epsilon \alpha](\pi_\epsilon - \pi - \rho\beta) = \pi\beta\psi \quad (\text{A.27})$$

$$\zeta[\pi + \beta \frac{\psi_m}{\lambda_m}] = \pi_v \beta - \pi[(M-1)\zeta + N\alpha] \quad (\text{A.28})$$

In the following, we will endeavour to find conditions under which a solution for this system exists. From (A.27) we can write α as a function of β :

$$\alpha = \frac{\beta[(\pi_v + (N-1)\pi)(\pi_\epsilon - \pi - \rho\beta) - \pi\psi]}{\pi_\epsilon(\pi_\epsilon - \pi - \rho\beta)} \quad (\text{A.29})$$

and from (A.26) we have,

$$\zeta = \frac{\pi_v\beta - \alpha(\pi_\epsilon + (N-1)\pi)}{M\pi}. \quad (\text{A.30})$$

Then multiplying by $(M-1)$, adding $N\theta$ and considering the fact that $\theta = \alpha + \beta$ gives

$$[(M-1)\zeta + N\theta] = \frac{\beta[((M-1)\pi_v + MN\pi)\beta - \alpha((M-1)\pi_\epsilon - (M+N-1)\pi)]}{M\pi}.$$

Substituting the value of α in (A.29) gives:

$$[(M-1)\zeta + N\theta] = \frac{\beta\psi[(2M+N-2)\pi_\epsilon - 2(M+N-1)\pi - \rho(M+N-1)\beta]}{M\pi_\epsilon(\pi_\epsilon - \pi - \rho\beta)}; \quad (\text{A.31})$$

we denote

$$g(\beta) \stackrel{Def}{=} \frac{\beta\psi[(2M+N-2)\pi_\epsilon - 2(M+N-1)\pi - \rho(M+N-1)\beta]}{M\pi_\epsilon(\pi_\epsilon - \pi - \rho\beta)}.$$

We will then derive another relation between $[(M-1)\zeta + N\theta]$ and β using equation (A.28). From (A.26) we have:

$$[(M-1)\zeta + N\alpha] = \frac{\pi_v\beta - \alpha(\pi_\epsilon - \pi)}{\pi} - \zeta.$$

Substitution into the right hand side of (A.28) and simplification yield:

$$\frac{\zeta\beta\psi_m}{\lambda_m} = \alpha(\pi_\epsilon - \pi); \quad (\text{A.32})$$

substituting (A.30) and rearranging terms gives:

$$\lambda_m = \frac{\beta\psi_m[\pi_v\beta - \alpha(\pi_\epsilon + (N-1)\pi)]}{M\pi\alpha(\pi_\epsilon - \pi)}. \quad (\text{A.33})$$

Finally, substitution of (A.29) into (A.33) and simplification give the following second relation between λ_m and β :

$$\lambda_m = \frac{\beta\psi_m\psi[(2-N)\pi_\epsilon + 2(N-1)\pi + (N-1)\rho\beta]}{M(\pi_\epsilon - \pi)[(\pi_v + (N-1)\pi)(\pi_\epsilon - \pi - \rho\beta) - \pi\psi]} \stackrel{Def}{=} f(\beta). \quad (\text{A.34})$$

From the second order condition of market makers' maximization problem, λ_m has to be strictly positive. Thus, equilibrium exists when $f(\beta) > 0$ and $g(\beta) > 0$. This occurs when $\beta > \sup\{0, a_0\}$ and $\beta < b_0$ (with $a_0 \neq b_0$) with:

$$a_0 = \frac{(N-2)\pi_\epsilon}{(N-1)\rho} - \frac{2\pi}{\rho}$$

and

$$b_0 = \frac{\pi_\epsilon - \pi}{\rho} - \frac{\pi\psi}{\rho(\pi_v + (N-1)\pi)}$$

(note that b_0 is always greater than a_0 since $\pi_v > 0$).

If $b_0 \leq 0$, $f(\cdot)$ is not defined and the equilibrium does not exist; so b_0 has to be positive. Let us derive a necessary and sufficient condition for the equilibrium existence.

We can write the following

$$(f - g)(\beta) = K_1(\beta) \times h(\beta),$$

with

$$K_1(\beta) = \frac{\beta\psi}{M\pi_\epsilon\rho[\pi_\epsilon - \pi - \beta](\pi_\epsilon - \pi)(\psi_m - \pi)[b_0 - \beta]}$$

and

$$h(\beta) = X_1\rho^2\beta^2 + X_2\rho\beta + X_3,$$

where

$$\begin{aligned} X_1 &= -[(N-1)\pi_\epsilon\psi_m + (M+N-1)(\pi_\epsilon - \pi)(\psi_m - \pi)] \\ X_2 &= [(2N-3)\pi_\epsilon - 3(N-1)\pi]\pi_\epsilon\psi_m + (\pi_\epsilon - \pi)[(\psi_m - \pi) \\ &\quad [(3M+2N-3)\pi_\epsilon - 3(M+N-1)\pi] - (M+N-1)\pi\psi] \\ X_3 &= (\pi_\epsilon - \pi)\{2\pi\psi[(2M+N-2)\pi_\epsilon - 2(M+N-1)\pi] - \\ &\quad \pi_\epsilon\psi_m[(2M+2N-4)\pi_\epsilon - 2(M+2N-2)\pi]\}. \end{aligned} \tag{A.35}$$

Since, $K_1(\beta) > 0$ for all $\beta \in]0, b_0[$, we should study the sign of $h(\cdot)$ in order to state the equilibrium existence.

We can easily see that $X_1 < 0$. Then $h(\cdot)$ is a concave function over \mathbb{R} and it reaches its maximum at b' such that: $b' = \frac{-X_2}{2\rho X_1}$. Using the fact that $b_0 > 0$, we can easily prove that $b' > b_0$.²⁵

²⁵Indeed,

$$b' = \frac{(N-1)\pi_\epsilon\psi_m[\pi_\epsilon - \pi + \rho a_0]}{2\rho[(N-1)\pi_\epsilon\psi_m + (M+N-1)(\pi_\epsilon - \pi)(\psi_m - \pi)]} +$$

Therefore, $h(\cdot)$ is a strictly increasing function on $]0; b_0[$ and, proving the equilibrium existence is equivalent to show that $h(0) = X_3 < 0$; indeed, since $f(\beta) - g(\beta) \rightarrow +\infty$ when $\beta \rightarrow b_0$, then $h(b_0) > 0$, and if $h(0) < 0$, then from the intermediate value theorem we can conclude that $h(\cdot) = 0$ for some $\beta' \in]0; b_0[$, and therefore that $f(\beta') = g(\beta')$. Note also that by strict monotonicity of $h(\cdot)$, this equilibrium is the unique linear symmetric equilibrium.

Now, rearranging the terms of (A.35) and considering the fact that $(\pi_\epsilon - \pi)(\psi_m - \pi) = -\pi\psi + \pi_\epsilon\psi_m$ yield the following equality

$$h(0) = X_3 = 2(\pi_\epsilon - \pi)^2 \{[(2M + N - 2)\pi_\epsilon - 2(M + N - 1)\pi](\pi - \psi_m) + M\pi_\epsilon\psi_m\}.$$

Then, considering the fact that $[(2M + N - 2)\pi_\epsilon - 2(M + N - 1)\pi](\psi_m - \pi)$ is equal to

$$(M + N - 1)(\pi_\epsilon\psi_m - 2\pi\psi) + \pi_\epsilon((M - 1)\pi_v + MN\pi)$$

gives

$$h(0) = 2(\pi_\epsilon - \pi)^2 \{ \pi_\epsilon\pi_v - (M + N - 1)(\pi_\epsilon\psi_m - 2\pi\psi) \}.$$

Therefore, equilibrium in auction markets exists if and only if:

$$\pi_\epsilon\pi_v - (M + N - 1)(\pi_\epsilon\psi_m - 2\pi\psi) < 0.$$

Substituting the equation defining π as a function of π_ϵ , π_ω and ρ , this condition can be written as follows:

$$\frac{\pi_\epsilon[(M + N - 1)(2\psi - \psi_m) + \pi_v]}{[(M + N - 1)\psi_m - \pi_v]} < \frac{\rho^2}{\pi_\omega}.$$

Under this equilibrium existence condition, we can easily prove that trader's second order conditions are satisfied and that ζ , α , γ and θ are positive. ■

Lemma 1 : *The first derivative of the measure of asymmetric information relative to liquidity measure is lower than 1, i.e.,*

$$\frac{\partial \left[\frac{\pi_\epsilon[(M+N-1)(2\psi-\psi_m)+\pi_v]}{[(M+N-1)\psi_m-\pi_v]} \right]}{\partial \left(\frac{\rho^2}{\pi_\omega} \right)} < 1.$$

$$\frac{(\pi_\epsilon - \pi)(\psi_m - \pi)[(2M + N - 2)\pi_\epsilon - 2(M + N - 1)\pi + (M + N - 1)\rho b_0]}{2\rho[(N - 1)\pi_\epsilon\psi_m + (M + N - 1)(\pi_\epsilon - \pi)(\psi_m - \pi)]}$$

then,

$$b' - b_0 = \pi_\epsilon \left[\frac{\psi_m[(N - 1)\pi\psi - \pi_\epsilon\pi_v] + (\pi_\epsilon - \pi)(\psi_m - \pi)[(M - 1)\pi_v + MN\pi]}{2\rho(\psi_m - \pi)[(N - 1)\pi_\epsilon\psi_m + (M + N - 1)(\pi_\epsilon - \pi)(\psi_m - \pi)]} \right]$$

which is positive when b_0 is positive.

Proof of Lemma 1:

We have,

$$\partial\left[\frac{\pi_\epsilon[(M+N-1)(2\psi-\psi_m)+\pi_v]}{(M+N-1)\psi_m-\pi_v}\right] \bigg/ \partial\left(\frac{\rho^2}{\pi_\omega}\right) = 2(M+N-1)\pi_\epsilon^3[N(M+N-1)\pi_\epsilon + (M+2N-2)\pi_v] \{ \pi_\epsilon[N(M+N-1)\pi_\epsilon + (M+N-2)\pi_v] + (M+N-2)\pi_v \frac{\rho^2}{\pi_\omega} \}^{-2}$$

Moreover, we can easily show that

$$\begin{aligned} & \pi_\epsilon^2[N(M+N-1)\pi_\epsilon + (M+N-2)\pi_v]^2 - 2(M+N-1)\pi_\epsilon^3[N(M+N-1)\pi_\epsilon + (M+2N-2)\pi_v] \\ &= \\ & \pi_\epsilon^2[(M+N-2)^2\pi_v^2 + N(N-2)(M+N-1)^2\pi_\epsilon^2 + 2M(N-2)(M+N-1)\pi_\epsilon\pi_v], \end{aligned}$$

which is positive. Thus,

$$\begin{aligned} & 2(M+N-1)\pi_\epsilon^3[N(M+N-1)\pi_\epsilon + (M+2N-2)\pi_v] \\ & < \\ & \pi_\epsilon^2[N(M+N-1)\pi_\epsilon + (M+N-2)\pi_v]^2 \\ & < \\ & [\pi_\epsilon[N(M+N-1)\pi_\epsilon + (M+N-2)\pi_v] + (M+N-2)\pi_v \frac{\rho^2}{\pi_\omega}]^2. \end{aligned}$$

This ends the lemma's proof. ■

Proof of proposition 2:

Consider $\beta_* = \lim_{M \rightarrow +\infty} \beta$ where $\beta_* \in]0, b_0[$ and β_* must be different from 0 since we consider that, at the equilibrium, $\beta > 0$ for all $M > 1$ and $N > 1$.²⁶

We have, for the equilibrium solution β :

$$\lim_{M \rightarrow +\infty} f(\beta) = \lim_{M \rightarrow +\infty} g(\beta)$$

where,

$$\lim_{M \rightarrow +\infty} g(\beta) = \frac{\beta_* \psi (2\pi_\epsilon - 2\pi - \rho\beta_*)}{\pi_\epsilon (\pi_\epsilon - \pi - \rho\beta_*)} \quad (\text{A.36})$$

and

$$\lim_{M \rightarrow +\infty} f(\beta) = \frac{\beta_* \psi \psi_m (-(N-2)\pi_\epsilon + 2(N-1)\pi + \rho\beta_*)}{\lim_{M \rightarrow +\infty} M (\pi_\epsilon - \pi) [(\pi_\epsilon - \pi - \rho\beta_*) (\pi_v + (N-1)\pi) - \pi\psi]}.$$

From (A.36), $0 < \lim_{M \rightarrow +\infty} g(\beta) < +\infty$; it results that

$$\lim_{M \rightarrow +\infty} \{M (\pi_\epsilon - \pi) [(\pi_\epsilon - \pi - \rho\beta_*) (\pi_v + (N-1)\pi) - \pi\psi]\} \neq \infty.$$

²⁶note that $\beta = 0$ is out of the equilibrium path since, in this case, traders will not trade using their private signal.

Therefore $\lim_{M \rightarrow +\infty} \{(\pi_\epsilon - \pi)[(\pi_\epsilon - \pi - \rho\beta_*)(\pi_v + (N-1)\pi) - \pi\psi]\} = 0$ and from this equality we conclude that: $\beta_* = b_0$. (ii), (iii), (iv) and (v) follow directly from equations (A.27), (A.26) and (A.13). ■

Proof of proposition 3:

(i) The equilibrium price is equal to $\mu + \frac{1}{M\zeta}Q$, then p and Q are equivalently informative for market makers, and we have:

$$E[\tilde{v}/p, Q] = E[\tilde{v}/p].$$

Yet, from (A.22),

$$E[\tilde{v}/p, Q] = E[\tilde{v}/z_m] = \frac{\pi_v\mu + N\pi z_m}{\psi_m}.$$

Substitution of z_m and Q respectively from (A.21) and the market clearing condition (equation (A.3)) yields the following equation of the conditional expectation of v :

$$E[\tilde{v}/p] = \frac{\pi_v\beta - \pi(M\zeta + N\alpha)}{\beta\psi_m}\mu + \frac{\pi(M\zeta + N\theta)}{\beta\psi_m}p.$$

Substitution of $(M\zeta + N\alpha)$ and $(M\zeta + N\theta)$ from (A.26) as functions of β gives

$$E[\tilde{v}/p] = \frac{\alpha(\pi_\epsilon - \pi)}{\beta\psi_m}\mu + \frac{\pi\psi(2\pi_\epsilon - 2\pi - \rho\beta)}{\pi_\epsilon\psi_m(\pi_\epsilon - \pi - \rho\beta)}p$$

which is equal to p only when $M \rightarrow +\infty$.

(ii) From the market clearing condition we can derive the following equation of p :

$$p = \left[\frac{M\zeta + N\alpha}{M\zeta + N\theta}\right]\mu + \frac{\beta}{M\zeta + N\theta} \left[\sum_{j=1}^N (s_j - \frac{\rho}{\pi_\epsilon}\omega_j)\right].$$

Then after substituting the value of $M\zeta + N\theta$ from (A.26), we have :

$$var(p) = \frac{N\pi_\epsilon^2(\pi_\epsilon - \pi - \rho\beta)^2}{\pi\psi^2(2\pi_\epsilon - 2\pi - \rho\beta)^2}; \tag{A.37}$$

which is equivalent to

$$var(p) = \frac{N\pi_\epsilon^2}{\pi\psi^2 \left[1 + \frac{\pi_\epsilon - \pi}{\pi_\epsilon - \pi - \rho\beta}\right]^2}.$$

Since β is in $]0, b_0[$ then $var(p)$ satisfies the following property:

$$\frac{N\pi}{\psi_m^2} < var(p) < \frac{N\pi_\epsilon^2}{4\pi\psi^2}. \tag{A.38}$$

Thus, we can conclude that $\lim_{N \rightarrow +\infty} var(p) = 0$.

(iii) The quoted bid-ask spread is equal to $p(1) - p(-1) = \frac{2}{M\zeta}$. From equation (A.30), if we compute the limit of $M\zeta$ as $M \rightarrow +\infty$ by considering the fact that $\beta \rightarrow b_0$ we find the announced result. ■

Proof of proposition 4:

This proof is similar to the proof of proposition 1 in Madhavan (1992). For the sake of completeness and in order to use some results derived in the proof we present it.

The equilibrium in dealership markets is defined by the couple $(p(q), q)$. Considering that dealers set a differentiable price function $p(\cdot)$, then a trader i , with information (s_i, ω_i) , chooses a trading strategy q_i satisfying his optimality condition

$$q_i \in \operatorname{argmax} E[U(\tilde{v}(\omega_i + q_i) - p(q_i)q_i)/p(\cdot), H_i] \quad (\text{D.1})$$

which is equivalent to

$$q_i \in \operatorname{argmax} \{E[\tilde{v}/s_i](\omega_i + q_i) - p(q_i)q_i - \frac{\rho}{2}(\omega_i + q_i)^2 \operatorname{var}[\tilde{v}/s_i]\}.$$

The first and second order conditions are respectively

$$-p'(q_i)q_i - p(q_i) + E[\tilde{v}/s_i] - \rho(\omega_i + q_i)\operatorname{var}[\tilde{v}/s_i] = 0 \quad (\text{D.2})$$

$$-p''(q_i)q_i - 2p'(q_i) - \frac{\rho}{\pi_v + \pi_\epsilon} < 0 \quad (\text{D.2}')$$

Since, $E[\tilde{v}/s_i] = \frac{\pi_v \mu + \pi_\epsilon s_i}{\pi_v + \pi_\epsilon}$, Substitution in (D.2) and rearrangement of terms give

$$\frac{\pi_v + \pi_\epsilon}{\pi_\epsilon} [p'(q_i)q_i + p(q_i)] - \frac{\pi_v}{\pi_\epsilon} \mu + \frac{\rho q_i}{\pi_\epsilon} = s_i - \frac{\rho}{\pi_\epsilon} \omega \quad (\text{D.3})$$

Then, given his pricing function $p(\cdot)$, the dealer should observe a noisy valuation of the final asset's value $s_i - \frac{\rho}{\pi_\epsilon} \omega_i$. Consider $\tilde{z} = \tilde{s}_i - \frac{\rho}{\pi_\epsilon} \tilde{\omega}_i$, then $\tilde{z} = \tilde{v} + \tilde{y}_0$ with

$$\tilde{y}_0 \sim \mathcal{N}(0, \pi),$$

where π , as in the proof of proposition 1, is equal to $\pi = \frac{\pi_\epsilon^2 \pi_\omega}{\pi_\epsilon \pi_\omega + \rho^2}$.

Thus, from the dealer's point of view, conditional expectation is:

$$E[\tilde{v}/z] = \frac{\pi_v \mu + \pi z}{\pi_v + \pi} \quad (\text{D.4})$$

and

$$\operatorname{var}[\tilde{v}/z] = \frac{1}{\pi_v + \pi}.$$

Substitution of (D.3) and the π value into (D.4) and introduction of the fact that $E[\tilde{v}/z] = E[\tilde{v}/q_i] = p(q_i)$ gives the following equation

$$p'(q_i)q_i\pi_\epsilon\pi_\omega(\pi_v + \pi_\epsilon) - p(q_i)\pi_v\rho^2 + \pi_v\rho^2\mu + \rho\pi_\epsilon\pi_\omega q_i = 0, \quad (\text{D.5})$$

or, equivalently:

$$p'(q_i)q_i = \frac{a}{b}(p(q_i) - \mu) - \frac{\rho\pi_\epsilon\pi_\omega}{b}q_i \quad (\text{D.6})$$

where $a = \pi_v\rho^2$ and $b = \pi_\epsilon\pi_\omega(\pi_v + \pi_\epsilon)$.

• *First case: $a = b$*

In this case, (D.6) is:

$$p'(q_i)q_i = p(q_i) - \mu - \frac{\rho\pi_\epsilon\pi_\omega}{b}q_i.$$

This is a first order differential equation in p with a second member. Its solution is

$$p(q_i) = \mu + Cq_i - \frac{\rho}{\pi_v + \pi_\epsilon}q_i \ln |q_i|, \quad (\text{D.7})$$

where C is the integration constant. If $C \leq 0$ then we have $p(-q_i) - p(q_i) = -2cq_i + 2\frac{\rho}{\pi_v + \pi_\epsilon}q_i \ln |q_i|$. Thus, for all $q_i > 1$ we have $p(-q_i) - p(q_i) > 0$. This represents an arbitrage opportunity which will be eliminated by inter-dealer trading. Then, this cannot be an equilibrium. If $C > 0$, substituting the value of $p(q_i)$ into the trader's second order condition gives the following:

$$-2C + 2\frac{\rho}{\pi_v + \pi_\epsilon}(\ln |q_i| + 1) < 0.$$

Then, the order size has to be lower than $\exp(\frac{C(\pi_v + \pi_\epsilon)}{\rho} - 1)$ to satisfy the dealer's second order condition, otherwise this condition will be violated. Thus, this cannot be an equilibrium.

• *Second case: $a \neq b$*

If we denote by

$$T(q_i) = p(q_i) - \mu + \frac{\rho\pi_\epsilon\pi_\omega}{b-a}q_i; \quad (\text{D.8})$$

and write (D.6) as a differential equation in T , we get

$$T'(q_i)q_i = \frac{a}{b}T(q_i). \quad (\text{D.9})$$

The solution to this differential equation is

$$T(q_i) = C_1 \text{sign}(q_i) |q_i|^{a/b}.$$

Then, from (D.8) we have:

$$p(q_i) = \mu - \frac{\rho\pi_\epsilon\pi_\omega}{b-a} q_i + C_1 \text{sign}(q_i) |q_i|^{a/b} \quad (\text{D.10})$$

substitution of the value of $p(q_i)$ in (D.10) into (D.2') gives the following relation

$$C_1 \frac{a}{b} \left[\frac{a-b}{b} - 2\pi_\epsilon\pi_\omega \right] |q_i|^{(a-b)/b} > \rho \frac{a+b}{(\pi_v + \pi_\epsilon)(b-a)}. \quad (\text{D.11})$$

If $a < b$ and $C_1 \leq 0$ then $p(-q_i) > p(q_i)$ for all $q_i > 0$. Indeed

$$p(-q_i) - p(q_i) = \frac{2\rho\pi_\epsilon\pi_\omega}{b-a} q_i - 2C_1 (q_i)^{a/(\pi_\epsilon + \pi_v)} > 0,$$

in this case we have an arbitrage opportunity and then this cannot be an equilibrium.

If $a < b$ and $C_1 > 0$ then from the trader's second order condition we can prove the existence of q_* and q^* such that this condition is violated for $q \notin [q_*, q^*]$. Then, this cannot be an equilibrium. Similarly, if $a > b$ and $C_1 < 0$ the trader's second order condition is violated for some values of q .

Finally, if $a > b$, and $C_1 \geq 0$, the second order condition is always satisfied, but these pricing functions are dominated by the linear pricing function where $C_1 = 0$. In fact, for all q_i we have:

$$\mu + \frac{\rho\pi_\epsilon\pi_\omega}{a-b} q_i < \mu + \frac{\rho\pi_\epsilon\pi_\omega}{a-b} q_i + C_1 |q_i|^{\frac{a}{b}},$$

and then the equilibrium price function is :

$$p(q_i) = \mu + \frac{\rho\pi_\epsilon\pi_\omega}{a-b} q_i. \quad (\text{D.12})$$

Given this pricing function we can easily derive the trader's strategy by substituting the value of $p(q_i)$ into (D.3) which gives:

$$q_i(s_i, \omega_i) = \frac{\pi_v \rho^2 - \pi_\epsilon \pi_\omega (\pi_\epsilon + \pi_v)}{\rho [\pi_v \rho^2 + \pi_\epsilon \pi_\omega (\pi_\epsilon + \pi_v)]} [-\pi_\epsilon \mu + \pi_\epsilon s_i - \rho \omega_i].$$

■

Proof of proposition 5:

(i) $E[\tilde{v}/p] = E[\tilde{v}/E(\tilde{v}/q)] = E[\tilde{v}/q] = p$, then prices in dealership market are semi-strong form efficient.

(ii) From the market clearing condition we have:

$$p(q) = \mu + \left[\frac{\pi_\epsilon^2 \pi_\omega}{\pi_v \rho^2 + \pi_\epsilon \pi_\omega (\pi_\epsilon + \pi_v)} \right] \left\{ -\mu + s_i - \frac{\rho}{\pi_\epsilon} \omega_i \right\};$$

rearranging terms gives

$$p(q) = \left[\frac{\pi_v (\rho^2 + \pi_\epsilon \pi_\omega)}{\pi_v \rho^2 + \pi_\epsilon \pi_\omega (\pi_\epsilon + \pi_v)} \right] \mu + \left[\frac{\pi_\epsilon^2 \pi_\omega}{\pi_v \rho^2 + \pi_\epsilon \pi_\omega (\pi_\epsilon + \pi_v)} \right] \left\{ s_i - \frac{\rho}{\pi_\epsilon} \omega_i \right\}. \quad (\text{D.13})$$

Then, the price variance is :

$$\text{var}(p) = \left[\frac{\pi_\epsilon^2 \pi_\omega}{\pi_v \rho^2 + \pi_\epsilon \pi_\omega (\pi_\epsilon + \pi_v)} \right]^2 \frac{1}{\pi}. \quad (\text{D.14})$$

After simplification we can write:

$$\text{var}(p) = \frac{\pi}{(\pi_v + \pi)^2}. \quad (\text{D.15})$$

(iii) The quoted bid-ask spread is defined by $p(1) - p(-1)$. In dealership markets, it is equal to

$$\frac{2\rho\pi_\epsilon\pi_\omega}{\pi_v\rho^2 - \pi_\epsilon\pi_\omega(\pi_\epsilon + \pi_v)}. \quad (\text{D.16})$$

Given this relation we can easily prove that partial derivatives of the quoted bid-ask spread equation relative to π_ϵ , π_ω , ρ and π_v are positive for the first and second argument and negative for the latter. ■

Proof of proposition 6:

(i) Auction markets are more viable if their equilibrium exists each time the equilibrium in dealership markets exists. In other words, this is the case when:

$$\frac{\pi_\epsilon(\pi_\epsilon + \pi_v)}{\pi_v} < \frac{\rho^2}{\pi_\omega} \implies \frac{[(M + N - 1)\pi_\epsilon[2\psi - \psi_m] + \pi_v]}{(M + N - 1)\psi_m - \pi_v} < \frac{\rho^2}{\pi_\omega}.$$

From the first condition we have

$$\pi_\epsilon^2 \pi_\omega (\pi_\epsilon + \pi_v) < \pi_v \pi_\epsilon \rho^2$$

or,

$$\pi(\pi_\epsilon + \pi_v) < \pi_v(\pi_\epsilon - \pi).$$

Therefore, we get

$$\pi_\epsilon \pi_v > \pi(2\pi_v + \pi_\epsilon), \quad (\text{C.1})$$

and

$$(\pi_\epsilon - 2\pi) > 0. \quad (\text{C.2})$$

Now, we should prove that under these conditions, auction market's equilibrium existence condition is satisfied. We have:

$$(M + N - 1)(\pi_\epsilon \psi_m - 2\pi \psi) - \pi_\epsilon \pi_v = \\ (M + N - 2)\pi_\epsilon \pi_v + (M + N - 1)[(N - 2)\pi_\epsilon - \pi_v + 2(N - 1)\pi].$$

Using (C.1), we get:

$$(M + N - 1)(\pi_\epsilon \psi_m - 2\pi \psi) - \pi_\epsilon \pi_v > \\ \pi[(M + N - 1)(N - 1)(\pi_\epsilon - 2\pi) + (M + N - 3)\pi_v].$$

Since the right hand member of this equation is positive from (C.2), we have:

$$(M + N - 1)(\pi_\epsilon \psi_m - 2\pi \psi) - \pi_\epsilon \pi_v > 0.$$

(ii) Suppose that:

$$\pi_\epsilon < \left(1 - \frac{2}{M + N}\right) \frac{\rho^2}{\pi_\omega}, \quad (\text{C.3})$$

and

$$\frac{M\pi_\epsilon^2\pi_\omega}{(M + N - 2)\rho^2 - (M + N)\pi_\epsilon\pi_\omega} - (N - 1) \frac{\pi_\epsilon^2\pi_\omega}{\rho^2 + \pi_\epsilon\pi_\omega} < \pi_v < \frac{\pi_\epsilon^2\pi_\omega}{\rho^2 - \pi_\epsilon\pi_\omega}.$$

The second equation can be written as:

$$\frac{M\pi_\epsilon\pi}{(M + N - 2)\pi_\epsilon - 2(M + N - 1)\pi} - (N - 1)\pi < \pi_v < \frac{\pi_\epsilon\pi}{\pi_\epsilon - 2\pi}. \quad (\text{C.4})$$

By (C.3), we have $(M + N - 2)\pi_\epsilon - 2(M + N - 1)\pi > 0$, so $\pi_\epsilon - 2\pi > 0$.

Finally from these inequalities and (C.4), we have the following:

- $\pi_v[\pi_\epsilon - 2\pi] - \pi_\epsilon\pi < 0$, then dealership market breaks down.
- $(M + N - 1)[\pi_\epsilon \psi_m - 2\pi \psi] - \pi_\epsilon \pi_v > 0$, then the equilibrium in auction markets exists. ■

Proof of proposition 7:

First, we can easily verify that $0 < \beta_D < b_0$. Then, in order to compare trading aggressiveness in both markets, it is sufficient to compute $h(\beta_D)$ (see proof of proposition 1). In fact, since $h(\cdot)$ is an increasing function on $]0, b_0[$ and $h(\beta) = 0$, if $h(\beta_D) > 0$ then $\beta_D > \beta$ and conversely if $h(\beta_D) < 0$.

The function $h(\cdot)$ may also be written as follows:

$$h(\beta) = \pi_\epsilon \psi_m [\pi_\epsilon - \pi - \rho\beta] [(2 - N)\pi_\epsilon + 2(N - 1)\pi + (N - 1)\rho\beta] \\ - (\pi_\epsilon - \pi) [(\pi_v + (N - 1)\pi)(\pi_\epsilon - \pi - \rho\beta) - \pi\psi] \\ [(2M + N - 2)\pi_\epsilon - 2(M + N - 1)\pi - (M + N - 1)\rho\beta];$$

then,

$$h(\beta_D) = \pi \{ \pi_\epsilon \psi_m [2\pi_\epsilon + \pi_v - \pi] [\pi_\epsilon(\pi_v + \pi) - 2(N - 1)\pi(\pi_\epsilon - \pi)] \\ - (\pi_\epsilon - \pi) [2(\pi_\epsilon - \pi)(\pi_v + (N - 1)\pi) - \pi_\epsilon(\pi_v + \pi)] \\ [(M - 1)\pi_\epsilon(\pi_v + \pi) + 2(M + N - 1)\pi(\pi_\epsilon - \pi)] \}.$$

If we consider the fact that equilibrium conditions in both markets are satisfied, and that $\beta_D \in]0, b_0[$, we can argue that $h(\beta_D) < 0$ whenever²⁷,

$$\pi_\epsilon(\pi_v + \pi) - 2(N - 1)\pi(\pi_\epsilon - \pi) < 0$$

which is equivalent to:

$$(N - 1) > \frac{\pi_\epsilon(\pi_v + \pi)}{2\pi(\pi_\epsilon - \pi)}.$$

In this case $\beta_D < \beta$ and trading is more aggressive in auction markets. ■

Proof of proposition 8:

From (A.38) we have

$$\frac{N\pi}{\psi_m^2} < \text{var}(p^A) < \frac{N\pi_\epsilon^2}{4\pi\psi^2}$$

and from (D.18), $\text{var}(p^D) = \frac{\pi}{(\pi_v + \pi)^2}$.

(i) Consider that $N < (\frac{\pi_v}{\pi})^2$. Then, $(N - 1)[N\pi^2 - \pi_v^2] < 0$. This is equivalent to $(\pi_v + N\pi)^2 - N(\pi_v + \pi)^2 < 0$. Hence, $\text{var}(p^D) < \frac{N\pi}{(\pi_v + N\pi)^2} < \text{var}(p^A)$.

(ii) If $(N - 1) > \frac{\pi_\epsilon^2(\pi_v + \pi)^2 - 4\pi^3(\pi_v + \pi_\epsilon)}{4\pi^4}$, then we have:

$$4\pi^3\psi > \pi_\epsilon^2(\pi_v + \pi)^2.$$

Multiplying both sides by N and considering that $N\pi < \psi$, we can write $4\pi^2\psi^2 - N\pi_\epsilon^2(\pi_v + \pi)^2 > 0$ which is equivalent to:

²⁷Notice that this is just a sufficient condition for our result.

$$\frac{\pi}{(\pi_v + \pi)^2} > \frac{N\pi_\epsilon^2}{4\pi\psi^2}.$$

then, $\text{var}(p^A) < \frac{N\pi_\epsilon^2}{4\pi\psi^2} < \text{var}(p^D)$. ■

Proof of proposition 9:

The measure of efficiency for both markets is:

$$e_A = \frac{N\pi}{\pi_v + N\pi}$$

and

$$e_D = \frac{\pi}{\pi_v + \pi};$$

where the first equation is derived from the definition of e and the fact that²⁸ $E[\tilde{v}/p] = E[\tilde{v}/z_m] = \psi_m = \pi_v + N\pi$; and, the second equation is derived from the definition of e and (D.5).

A straightforward comparison between e_A and e_D gives the result. ■

References

- [1] Bernhardt, D. and E. Hughson, 1996, “Discrete Pricing and the Design of Dealership Markets”, *Journal of Economic Theory* (71), 148 - 182.
- [2] Biais, B., 1993, “Price Formation and Equilibrium Liquidity in Fragmented and Centralized Markets”, *Journal of Finance* (48), 105 - 124.
- [3] Biais, B., T. Foucault and F. Salanié, 1998, “Floors, Limit Order Markets and Dealer markets”, *Journal of Financial Markets* (1), 253-284.
- [4] Biais, B., D. Martimort and J-C. Rochet, 2000, “Competition Mechanisms in a Common Value Environment”, *Econometrica* (68), 799-837.
- [5] Blume, M.E. and M.A. Goldstein, “Quotes, Order Flow, and Price Discovery”, *Journal of Finance* (52), 221 - 244.

²⁸See the proof of proposition 3.

- [6] Brown, D.P. and Z.M. Zhang, 1997, "Market Orders and Market Efficiency", *Journal of Finance* (1997), 277-308.
- [7] Dennert, J., 1993, "Price Competition Between Market Makers", *Review of Economic Studies* (60), 735 - 751.
- [8] Edelen, R. and S. Gervais, 2003, "The Role of Trading Halts in Monitoring a Specialist Market", *Review of Financial Studies* (16), 263-300.
- [9] Falconieri, S., A. Murphy and D. Weaver, 2003, "From the IPO to the First Trade: Is Underpricing Related to the Trading Mechanism", mimeo, Tilburg University.
- [10] Fishman, M.J. and E. Longstaff, 1992, "Dual Capacity Trading in Future Markets", *Journal of Finance* (47), 643 - 671.
- [11] Gemmil, G., 1996, "Transparency and Liquidity: A Study of Block Trades on the London Stock Exchange Under Different Publication Rules", *Journal of Finance* (51), 1765 - 1790.
- [12] Glosten, L., 1989, "Insider Trading, Liquidity, and the Role of the Monopolist Specialist", *Journal of Business* (62), 211 - 236.
- [13] Glosten, L., 1994, "Is the Electronic Open Limit Order Book Inevitable?", *Journal of Finance* (49), 1127-1161.
- [14] Gould, J.P. and E. Verrechia, 1985, "The Information Content of Specialist Pricing", *Journal of Political Economy* (93), 66 - 83.
- [15] Grossman, S.J., 1990, "The Information Role of Upstairs and Downstairs trading", *Journal of Business* (65), 509 - 528.
- [16] Grossman, S.J. and M.H. Miller, 1988, "Liquidity and Market Structure", *Journal of finance*, (43), 617 - 633.
- [17] Jain, P., 2002, "Institutional Design and Liquidity at Stock Exchanges around the World", Working paper, Indiana University.
- [18] Kyle, A.S., 1989, "Informed Speculation with Imperfect Competition". *Review of Economic Studies* (56), 317 - 356.

- [19] Madhavan, A., 1992, "Trading Mechanisms in Securities Markets", *Journal of Finance* (47), 607 - 642.
- [20] Madhavan, A., 1995, "Consolidation, Fragmentation, and the Disclosure of Trading Information", *Review of Financial Studies* (8), 579 - 603.
- [21] Mendelson, H., 1987, "Consolidation, Fragmentation, and Market Performance", *Journal of Financial and Quantitative Analysis* (22), 189 - 207.
- [22] Pagano, M. and A. Röell, 1993, "Auction Markets, Dealership Markets and Execution Risk", in V. Conti and R. Hamanui eds: *Financial Markets Liberalization and the Role of Banks*; Cambridge University Press, Cambridge.
- [23] Pagano, M. and A. Röell, 1996, "Transparency and Liquidity: A Comparison of Auction and Dealership Markets With Informed Trading", *Journal of Finance* (51), 579 - 611.
- [24] Pithyachariyakul, P., 1986, "Exchange Markets: A Welfare Comparison of Market Maker and Walrasian Systems", *The Quarterly Journal of Economics* (101), 69 - 84.
- [25] Röell, A., 1990, "Dual Capacity Trading and The Quality of the Market", *Journal of Financial Intermediation* (1), 105 - 124.
- [26] Seppi, D., 1992, "Block Trading and Information Revelation Around Quarterly Earning Announcements", *Review of Financial Studies* (5), 281 - 306.
- [27] Shin, H.S., 1996, "Comparing The Robustness of Trading Systems to Higher Order Uncertainty", *Review of Economic Studies* (63), 39 - 59.
- [28] Viswanathan, S. and J. Wang, 2002, "Market Architecture: Limit-order Books versus dealership Markets", *Journal of Financial Markets* (5), 127-167.