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Pay-for-Luck in CEO Compensation: Matching and Efficient Contracting

Pierre Chaigneau
Nicolas Sahuguet

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Chaigneau : Finance Department, HEC Montréal and CIRPÉE
pierre.chaigneau@hec.ca
Sahuguet : Institute of Applied Economics, HEC Montréal and CIRPÉE
nicolas.sahuguet@hec.ca

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Abstract:
We develop a stylized model of efficient contracting with matching between firms and managers with state-contingent reservation utility. We show that the optimal contract is designed to retain and insure the manager. The retention motive explains pay-for-luck in executive compensation, while the insurance feature explains asymmetric pay-for-luck. This contract can be implemented with call options based on a single performance measure which generally does not filter out luck. When costs of involuntary managerial turnover differ across firms, and the abilities of different managers are more or less precisely estimated ex-ante, the model can also explain the observed association between pay-for-luck and bad corporate governance.

Keywords: CEO pay, corporate governance, pay-for-luck, stock-options

JEL Classification: D86, G34, J33

In recent years, CEO pay has attracted considerable attention, both in the popular press and in academic journals. This renewed interest was in part triggered by some puzzling observations such as pay-for-luck, i.e., the evidence that exogenous and contractible shocks to performance do nevertheless have an effect on CEO pay (Bertrand and Mullainathan (2001)). In response to this and other empirical findings which are inconsistent with a standard version of the principal-agent model, the managerial power or "skimming" hypothesis has been proposed as an alternative paradigm, most notably by Bertrand and Mullainathan (2001) and Bebchuk and Fried (2004). This strand of literature has documented a number of "anomalies", which are suggestive of corporate governance failures. Since then, many papers have shown that some of these apparent "anomalies" can actually be explained in an efficient contracting framework (see the literature review of Edmans and Gabaix (2009)), although some puzzles remain (Frydman and Jenter (2010)).

Bertrand and Mullainathan (2001) acknowledge that the principal-agent model may explain pay-for-luck, as in Oyer (2004). However, they argue that the facts that pay-for-luck is asymmetric, in the sense that CEOs tend to be more rewarded for good luck than they are penalized for bad luck (see also Garvey and Milbourn (2006)), and that pay-for-luck is stronger in firms with worse corporate governance provide supporting evidence for a skimming model of executive pay. In this paper, we explain these phenomena as the equilibrium outcome of a simple principal-agent model of efficient contracting.

As in Oyer (2004), we focus on the effect of a CEO's outside options on his compensation. Our baseline model builds on Harris and Holmstrom (1982). We consider a two-period principal-agent relationship between risk-neutral shareholders and a risk averse CEO with state-contingent reservation utility. Crucially, as in Harris and Holmstrom (1982), we assume that the firm can commit to employ the CEO next period, but the CEO cannot commit to work for the firm next period. In this setting, we show that a risk averse CEO accepts a lower pay in the first period in exchange for insurance against a low reservation utility in the second period. This is possible because the firm can commit to pay an endogenously generated "minimum wage" next period.

However, in case the reservation utility of the CEO is sufficiently high next period, the firm must adjust his pay upward to retain him. This is because the CEO cannot commit to work for the firm next period. Therefore, with an optimal contract derived without any restriction on the contracting space, remuneration will be downward rigid *and* sensitive to factors that affect the reservation wage of the manager (including luck) on the upside, so that pay-for-luck will be asymmetric.

Moreover, we show that stock-options provide both the required downward protection and the required upside participation, so that an optimal contract may be implemented with stock-options. This result is all the more interesting that the standard principal-agent model of effort choice does not explain the widespread use of stock-options in managerial compensation (Hall and Murphy (2002), Dittmann and Maug (2007)).[1] The model also predicts an increase in stock-options based compensation as general managerial skills become relatively more important than firm-specific skills. To the extent that this has been the case in the 1980s and the 1990s (as argued in Murphy and Zabojnik (2004) and Frydman (2007)), the model can explain the rise in options-based executive compensation in this period.

Our second main result is that pay-for-luck will be stronger (relative to pay-for-performance) in firms with a high cost of involuntary CEO turnover. Indeed, the pay of the CEO in the second period must be state-contingent for two reasons. First, firm performance in the first period provides some information on the CEO's ability, which affects his reservation wage in the second period. Second, changes in economic conditions ("luck") also affect the reservation wage of the CEO. But firm performance is all the more informative about managerial ability that this ability is not precisely estimated ex-ante – think about young CEOs, or CEOs with a short tenure, for example. It follows that state-contingent pay will put a higher weight on firm performance (rather than luck) for CEOs whose ability is more uncertain. In addition, if we assume that dismissing a CEO and hiring a new one is costly (Taylor (2010)), and that this cost differs across firms, then we may expect firms with a high firing cost to hire CEOs

[1] This being said, postulating loss aversion (Dittmann, Maug, and Spalt (2010)) or a larger action set for the manager (Dittmann and Yu (2010)) improves its explanatory power.

whose ability is more precisely estimated ex-ante. Our simple matching model of CEOs and firms indeed yields this intuitive result. To summarize, firms with high dismissing costs will hire CEOs whose ability is more precisely estimated ex-ante, and the contingent pay of these CEOs will consequently involve more pay-for-luck (rather than pay for performance) than the contingent pay of other CEOs. Thus, the model predicts that the degree of pay-for-luck relative to pay-for-performance will tend to be higher in firms with high dismissing costs. Given that it is efficient to dismiss CEOs with low ability in our model, a high cost of dismissal can be interpreted as an indicator of bad corporate governance.[2]

Lastly, as already noted for example in Murphy and Zabojnik (2004), the skimming hypothesis is hard to reconcile with the facts that, in the past decades, CEO pay increased *and* corporate governance improved. Our model can contribute to explain this phenomenon – although many other factors are probably behind the rise in CEO pay (see, e.g., Gabaix and Landier (2008)). Indeed, we show that an improvement in corporate governance, whether across the board or confined to the subset of badly governed firms, has a spillover effect that increases CEO pay in all firms. The intuition is that corporate governance heterogeneity attenuates competition for CEOs and enables well-governed firms to earn rents. Improvements in governance improve the bargaining position of CEOs and reduce these rents.

The hypothesis at the core of our model is that retention is an important determinant of CEO compensation. There is empirical support for this notion: Rajgopal, Shevlin and Zamora (2006) present evidence that CEO pay is structured to match the state-contingent outside employment opportunities of managers, while Oyer and Schaefer (2005) emphasize the limitations of the incentives-based explanation for the adoption of broad-based stock-options plans.

Pay-for-luck is not predicted by the standard moral hazard model of efficient contracting with a risk averse agent: if a worker's pay must be variable for incentive purposes, then the informativeness of the performance measure should be maximized (Holmstrom (1979)), so that

[2]The variables used in Bertrand and Mullainathan (2001) – the existence of a large shareholder, CEO tenure, the board size, and the fraction of insider directors – can also be interpreted as proxies for the cost or difficulty of CEO dismissal.

it is optimal to filter out exogenous shocks to performance. The model of Oyer (2004) explains pay-for-luck as the outcome of efficient contracting by invoking the retention motive and renegotiation costs (see also Himmelberg and Hubbard (2000), and Edmans and Gabaix (2009) for additional references), but it does not explain the facts that pay-for-luck is asymmetric and is associated with poor corporate governance. Gopalan, Milbourn and Song (2009) and Feriozzi (2010) propose some alternative hypotheses to explain asymmetric pay-for-luck, related respectively to strategy choice and the implicit incentives emanating from the threat of bankruptcy.

Section 1 presents the model. Section 2 derives the optimal compensation contract, for any given firm-manager match. Section 3 describes the relevant performance measure for managerial compensation, and introduces stock-options. Section 4 describes the matching equilibrium. Section 5 discusses the results. Section 6 concludes. All proofs are in the Appendix.

1 The model

We consider a two-period economy in which firms compete for CEOs.

A firm can be run either by a "CEO" or by a "manager". CEOs and managers are risk averse with utility function $u(\cdot)$ ($u' > 0$, $u'' < 0$). The ability a of a CEO is normally distributed with mean $\bar{a} > 0$ and variance σ_a^2. We assume a limited supply of CEOs: there are m firms and n CEOs, with $m > n$. There is also an infinite supply of managers, whose expected ability is normalized to zero.[3] The reservation utility of managers is normalized at zero, and the reservation utility of CEOs will be endogenously determined.

Firms are risk neutral and maximize their expected profits. In every period, the gross profits (before compensation of the manager) of a firm depend on three factors: its CEO's ability a, business conditions $\tilde{L}$, and an unobservable idiosyncratic shock $\tilde{\epsilon}_t$. We assume that $\tilde{\epsilon}_t$ is normally distributed with mean zero and variance σ_ϵ^2, and independent from other random variables for any $t \in \{1, 2\}$.[4] We assume that $\tilde{L}$ is a random variable with positive support, which is also

[3] The ability of managers could also be a random variable, but this would not generate additional insights.

[4] Assuming that both $\tilde{\epsilon}_t$ and a are normally distributed makes the Bayesian updating of beliefs on a tractable.

independent from other random variables – we do not need to specify the distribution of $\tilde{L}$. We will use the notation $\tilde{L}$ to denote the random variable $\tilde{L}$, and the notation L to denote its realization at the end of period 1, which is observable and contractible. We will refer to L as "luck", since it represents a shock which is not under the control of the manager or CEO but which nevertheless has an effect on firm value and on the CEO's productivity (see below), such as the price of crude oil for an oil company for instance.[5] We assume that the gross profits of the firm in period t, for $t \in \{1, 2\}$, are realized at the end of the period and write as:

$$\pi_t = (\alpha + s_t a + \tilde{\epsilon}_t)\tilde{L} \tag{1}$$

where $s_t > 0$. On the one hand, the superior average ability of CEOs relative to managers increases expected profits. On the other hand, realized profits are informative about CEO ability. Note that luck interacts with CEO ability. This is important: without this interaction – with an additive structure, say – the reservation wage or "value" of a CEO would not depend on luck. Thus, the complementarity between a and L matters, while the multiplicative structure used in (1) is for simplicity. In practice, there are reasons to believe that the importance of CEO ability for firm value is not independent of business conditions.

The variable s_t represents the accumulated experience and firm-specific skills of the CEO. Following Murphy and Zabojnik (2004), we let $s_t = 1$ if the CEO worked for the firm in period $t - 1$, and $s_t = \gamma \in (0, 1)$ otherwise. This makes CEO turnover intrinsically costly. This cost of CEO turnover is related to the relative importance of general managerial skills as opposed to firm-specific skills in managerial jobs. For example, if general skills predominate, then s approaches one, in which case CEOs are more easily replaceable.

For any firm, the net profits (henceforth "profits") in any period are the gross profits net of compensation costs. Both gross profits and net profits are observable and contractible. For

[5] Gabaix and Landier (2008) provide strong evidence that the dollar effect of CEO "talent" on firm value is increasing in firm value. Therefore, an exogenous shock to firm value (L) also affects the value of CEO talent or ability.

simplicity, we assume a zero interest rate and no time discounting. Firms pay out their net profits realized over period 1 and 2 to shareholders at the end of the second period. We also assume that there is a market for each firm's shares at the beginning of the second period, following the realization of the luck shock $\tilde{L}$, where stock prices are established by competitive and risk neutral investors. We denote by V the (endogenously determined) market value of a given firm at the end of the first period.[6]

Crucially, we assume that a firm can commit to a long-term contract, but a CEO cannot. While firms can and do propose enforceable long-term contracts to their employees, constraints on involuntary servitude prevent employees from forgoing (ex-ante) the option to quit a job. This one-sided-commitment assumption is natural and was introduced in Harris and Holmstrom (1982), and Holmstrom and Ricart i Costa (1986). We also assume that a manager can neither save nor borrow, so that he does not transfer income from one period to another.[7]

Two types of contracts are feasible. Any firm can propose a spot contract to a manager or CEO at the beginning of the first period, and a spot contract to a manager or CEO at the beginning of the second period. In this case, the employment of the first period manager/CEO terminates at the end of the first period. Alternatively, the firm can propose a long-term contract to a CEO at the beginning of the first period.[8] This contract specifies the wage that the firm commits itself to pay the CEO in periods 1 and 2. It should be stressed that even a CEO who is dismissed is entitled to this second period wage, as specified in the contract. In this case, the wage can be interpreted as a severance payment.[9] This is without loss of generality, since we will show that both the compensation of the CEO in period 2 and the dismissal rule are contingent on the expected ability of the CEO at the beginning of the second period and on the

[6]These two assumptions imply that there exists a performance measure (firm value V) which captures both the past performance of the manager and market conditions. In practice, there are reasons to believe that firm value will indeed reflect these two factors.

[7]As in Harris and Holmstrom (1982) and Holmstrom and Ricart i Costa (1986), the optimal contract is such that the saving restriction is inconsequential.

[8]We show in the next section that we can ignore long-term contracts for managers, as they can be replicated by a sequence of spot contracts.

[9]We show later that the expected ability of any dismissed CEO is less than zero, so that his reservation wage is also negative. This notably implies that no firm would be willing to employ the CEO for a nonnegative wage, and that no profitable renegotiation is possible for dismissed CEOs.

luck shock.

At the beginning of the second period, any CEO can resign, in which case he forgoes his contractual second period payment, but earns his reservation wage W_2, which is derived in the next section. A firm can dismiss a CEO who has not resigned at an extra cost k, with $k \geq 0$, and hire a new CEO (or a manager) on the spot market. The parameter k represents the cost of involuntary CEO turnover. It notably reflects some unmodeled but important factors such as the cost of coordination problems which impede internal governance, the cost of the severance package that an entrenched CEO may obtain, or the disruptive effect of a disorderly or lengthy dismissal procedure.[10]

2 The optimal long term contract

2.1 Equilibrium on the spot market for CEOs

We first derive the updated beliefs of firms about the ability of a given CEO at the end of the first period, after profits π_1 at his firm and the luck shock L have been observed. The updated expected ability $\hat{a}$ of a CEO is calculated using Bayes' rule:

$$\hat{a} = \frac{1}{\gamma} \frac{\gamma^2 \sigma_a^2 (\pi_1/L - (\alpha - \gamma \bar{a})) + \sigma_\epsilon^2 \gamma \bar{a}}{\gamma^2 \sigma_a^2 + \sigma_\epsilon^2} \tag{2}$$

In the second period, a firm can choose to hire a manager with zero ability on a spot contract, thereby making a (deterministic) profit of αL. A firm can also compete for CEOs. Consider a CEO with expected ability $\hat{a}$. The firm that employed him in the first period is willing to pay up to $\hat{a} L$ to employ him in the second period. All other firms with vacant positions are

[10] Taylor (2010) finds that the cost of involuntary CEO turnover consists of two components: the direct cost to the firm, and the personal cost to the board, which includes directors' ties to the CEO, their effort, and a lower probability of nomination at other boards. In our model, we only consider the direct cost in terms of firm profits for simplicity, but our results would be robust to a more general interpretation of k that includes the personal cost to the board. Indeed, the value of the firm derived in section 3 only matters when the CEO is not dismissed (so that k is irrelevant), and the dismissal rule derived in (6) would remain the same (the only difference is that (5) would not be interpreted narrowly as firm profits, but as the objective function of the board, which would then include the personal cost of dismissal).

only willing to pay up to $\gamma \hat{a} L$ to hire this CEO.[11] Competition between firms drives the second period reservation wage of a CEO with expected ability $\hat{a}$ to:

$$W_2(\hat{a}, L) = \gamma \hat{a} L. \tag{3}$$

Any given CEO can earn this wage in the second period, whether he is employed under a spot contract or a long-term contract. It follows that a firm which employed a CEO in the first period only needs to match the compensation in (3) to retain him in the second period. For a given L, the set of reservation wages described in (3) clears the market for CEOs at the beginning of the second period.

In the first period, any CEO can earn a wage of $W_1 = \gamma \bar{a} E[\tilde{L}]$ with a spot contract.

Managers are in unlimited supply, they have zero ability and a reservation wage of zero. Their first and second period compensation under either a spot or a long-term contract is therefore zero.

2.2 The dismissal rule

A firm will optimally dismiss a CEO at the beginning of the second period if and only if it increases its expected profits in the second period. On the one hand, the expected second period profits of a firm which does not dismiss its CEO at the beginning of the second period are

$$(\alpha + \hat{a})L - w_2 \tag{4}$$

where w_2 denotes the (as yet undetermined) compensation of the CEO in the second period under his contract. On the other hand, the expected second period profits of a firm which dismisses its CEO at the beginning of the second period and hires either a new CEO or a

[11] This corresponds to the additional profits generated by a given CEO relative to a zero-ability manager.

manager on the spot market are[12]

$$\alpha L - k - w_2 \tag{5}$$

where w_2 is again the second period compensation of the dismissed CEO. Comparing the expressions in (4) and (5) yields the optimal firing rule: a firm will dismiss its CEO if and only if

$$\hat{a} < -\frac{k}{L} \tag{6}$$

Dismissing a CEO under a long-term contract is optimal if the expected ability of the CEO in place is lower than a threshold. This threshold is decreasing in the cost k of changing the CEO, which is expected, and increasing in the luck shock L. The intuition is that the ability of the CEO matters all the more that L is high, since firms profits are multiplicative in a and L.

2.3 The optimal long-term contract

The optimal long-term contract is defined by a first period wage $w_1^\star$ and a second period wage $w_2^\star(\hat{a}, L)$, where $w_1^\star$ and $w_2^\star(\hat{a}, L)$ solve the following optimization problem:

$$\min_{\langle w_1, w_2(\hat{a}, L)\rangle} w_1 + E[w_2(\hat{a}, L)] \tag{7}$$

$$w_2(\hat{a}, L) \geq W_2(\hat{a}, L) \qquad \text{for all } \hat{a}, L \tag{8}$$

$$u(w_1) + E[u(w_2(\hat{a}, L))] \geq \bar{U} \tag{9}$$

where $\bar{U}$ denotes the (endogenously determined) reservation level of utility of the CEO over two periods at the beginning of the first period.[13] Note that $\bar{U}$ will not affect the shape of the optimal long-term contract.

The firm sets the pay of a given CEO in the first and second period to minimize the ex-

[12]We use the fact that the second period wage of a newly hired CEO is as in (3), whereas it is zero for a manager.

[13]In a competitive equilibrium, this outside option corresponds to the highest utility offered by other firms proposing optimal contracts.

pected cost of compensation subject to two types of participation constraints. First, the state-contingent participation constraints in (8) guarantee that the firm matches the reservation wage of the CEO in the second period state-by-state (this constraint is not binding when the firm dismisses its manager).[14] Second, the constraint in (9) guarantees that the CEO accepts the long-term contract at the beginning of the first period – which is the case if and only if the expected utility associated with the long-term contract exceeds the reservation utility $\bar{U}$ of the CEO, (i.e., the expected utility achieved with a sequence of spot contracts, or with the long-term contracts offered by other firms.).

The first-order conditions with respect to w_1 and $w_2(\hat{a}, L)$ are respectively:

$$\mu u'(w_1) = 1 \tag{10}$$

$$\mu u'(w_2(\hat{a}, L)) = 1 - \lambda(\hat{a}, L) \qquad \text{for all } \hat{a}, L \tag{11}$$

where $\frac{\lambda(\hat{a},L)}{Pr(\hat{a},L)}$ and μ are respectively the (nonnegative) Lagrange multipliers associated with the constraints (8) and (9), where $\lambda(\hat{a}, L) \geq 0$ satisfy the complementary slackness condition:

$$\lambda(\hat{a}, L)(W_2(\hat{a}, L) - w_2(\hat{a}, L)) = 0 \qquad \text{for all } \hat{a}, L \tag{12}$$

where $W_2(\hat{a}, L)$ is as in (3). Since the second-order conditions for minimization are satisfied, this immediately yields the form of the optimal long-term contract:

Proposition 1: Optimal contract

The optimal long-term contract is characterized by a first period wage of w_1^* and a second period wage of

$$w_2^{\star}(\hat{a}, L) = \max\{w_1^*, \gamma \hat{a} L\} \tag{13}$$

[14] On the one hand, a firm dismisses its CEO if and only if $\hat{a} < -\frac{k}{L}$. On the other hand, (8) is binding if and only if $\hat{a} \geq \frac{w_1^*}{\gamma L}$, which is positive. It follows that, in cases when the firm dismisses its CEO, $\hat{a}$ is negative so that (8) cannot be binding.

The level of the first period wage w_1^* is determined using the first period participation constraint in (9). It notably depends on $\bar{U}$, and we assume that parameter values are such that w_1^* is positive. A long term contract is fully determined by the first-period wage w_1^*. The second period wage is equal to w_1^*, unless the participation constraint in (8) binds given $\hat{a}$ and L, in which case the second period wage is equal to the reservation wage $W_2(\hat{a}, L)$, which is larger than w_1^*. At this stage, we do not specify how this state-contingent payment is implemented.

2.4 Interpretation

The optimal long-term contract displays three interesting features.

First, as in Harris and Holmstrom (1982), the risk averse CEO is insured by the firm in the second period: should his reservation wage fall below w_1^* in the second period, the firm will nevertheless pay w_1^*. This will be either the second period wage of the CEO, or his severance payment in case he is dismissed at the beginning of the second period. The cost of this insurance is a first period wage w_1^* lower than what the CEO could obtain on the spot market. That is, the CEO pays an insurance premium in the first period to be insured against adverse realizations of his reservation wage in the second period.[15] Otherwise, if the reservation wage of the CEO in the second period is above w_1^*, the firm matches it. The CEO cannot be fully insured by the firm (with a constant wage in the second period) because the CEO cannot commit in the first period to work for the firm in the second period. To summarize, the optimal contract features downside protection, with a downward rigid second period wage for insurance purposes, and upside participation for retention purposes. The assumption of one-sided commitment is crucial to obtain this result.

Second, the compensation of the CEO in the second period depends on business conditions,

[15] They stem from two sources: updating of beliefs regarding the CEO's ability, and the luck shock. In Harris and Holmstrom (1982), the only source of uncertainty relates to the updating of beliefs.

or "luck" (L). The reason is the same as in Oyer (2004): compensation adjusts to the level required to retain the CEO, and this level in turn depends on business conditions.

Third, pay for luck is asymmetric. Indeed, the pay of the CEO is sensitive to luck (at the margin) if and only if the luck shock is sufficiently positive:

Proposition 2: Asymmetric pay for luck

For any given value of $a + \tilde{\epsilon}$,

$$\frac{dw_2^\star(\hat{a}, L)}{dL} > 0 \text{ if } L > \underline{L} \qquad \text{and} \qquad \frac{dw_2^\star(\hat{a}, L)}{dL} = 0 \text{ if } L < \underline{L} \tag{14}$$

$$\text{where} \qquad \underline{L} \equiv \frac{1}{b}\Big(w_1^* - \kappa - \frac{\sigma_a^2}{\sigma_\epsilon^2 + \sigma_a^2}(a + \tilde{\epsilon})\Big)$$

This result highlights that, with an optimal contract, CEOs are "rewarded for good luck", but they are not symmetrically "penalized for bad luck": pay-for-luck is asymmetric. This is due to the insurance against adverse states of the world (including bad luck) provided by the optimal contract to risk averse CEOs.

As long as the expected termination costs are not too large, a long-term contract dominates a sequence of spot contracts. This is because a long-term contract allows to partly insure the CEO in the second period. In the remainder of the paper, we assume that this is the case.[16]

3 Implementation with stock-options

In equilibrium, the second period compensation of a CEO depends on his expected ability $\hat{a}$ and on the luck shock L. In this section, we show that the optimal long-term contract can be implemented by giving stock-options to the CEO. We also discuss how the optimal compensation

[16]This assumption can be microfounded by assuming that σ_a^2 and k are sufficiently low.

contract depends on the model parameters.

We first show that the optimal second-period compensation of the CEO can be expressed as a function of firm value at the beginning of the second period and the luck shock. To this end, we first derive firm value, V. In a competitive market for firm shares with risk neutral shareholders, firm value at the beginning of the second period is:

$$V = \pi_1 - w_1 + E[\pi_2|\pi_1, L] - w_2(\hat{a}, L) = \pi_1 - w_1^* + (\alpha + \hat{a})L - \max\{w_1^*, \gamma\hat{a}L\} \tag{15}$$

if $\hat{a} > -\frac{k}{L}$. Otherwise, the incumbent CEO is dismissed at the beginning of the second period, with a compensating payment of w_1^*, while the new CEO receives a fixed wage, so that firm value does not matter for compensation purposes. Recall that

$$\hat{a} = \frac{1}{\gamma}\frac{\gamma^2\sigma_a^2(\pi_1/L - (\alpha - \gamma\bar{a})) + \sigma_\epsilon^2\gamma\bar{a}}{\gamma^2\sigma_a^2 + \sigma_\epsilon^2}.$$

Substituting $\pi_1 = V + w_1^* - (\alpha + \hat{a})L + \max\{w_1^*, \gamma\hat{a}L\}$, and isolating $\hat{a}$, we get that:

$$\begin{aligned} w_2^\star(\hat{a}, L) &= w_1^* \text{ if } \gamma\hat{a}L \leq w_1^* \\ w_2^\star(\hat{a}, L) &= \gamma\hat{a}L = \left(\frac{\gamma^2\sigma_a^2 + \sigma_\epsilon^2}{\gamma^2\sigma_a^2} + \frac{1-\gamma}{\gamma}\right)^{-1}\left(V + w_1^* - 2\alpha L + \gamma\bar{a}L\left(\frac{\sigma_\epsilon^2}{\gamma^2\sigma_a^2} - 1\right)\right) \text{ otherwise} \end{aligned}$$

We thus get:

$$w_2^\star(\hat{a}, L) = \max\{w_1^*, \psi(w_1^* + V + \eta L)\} \tag{16}$$

$$\text{where} \qquad \psi \equiv \left(\frac{\gamma^2\sigma_a^2 + \sigma_\epsilon^2}{\gamma^2\sigma_a^2} + \frac{1-\gamma}{\gamma}\right)^{-1} \qquad \text{and} \qquad \eta \equiv \gamma\bar{a}\left(\frac{\sigma_\epsilon^2}{\gamma^2\sigma_a^2} - 1\right) - 2\alpha \tag{17}$$

Notice that $\psi < 1$.

We now specify how the optimal contract described in (13) and (16) can be implemented with stock-options on a measure which is constructed to incorporate both changes in firm value and luck. Consider the measure $P(V, L)$, constructed as

$$P(V, L) = V + \eta L \tag{18}$$

Then the state-contingent payment $w_2^\star(\hat{a}, L)$ in (13) can be implemented by making payments to the agent contingent on the measure P:

$$w_2^\star(\hat{a}, L) = \max\{w_1^*, \psi w_1^* + \psi P(V, L)\} \tag{19}$$

This immediately leads us to this important result:

Proposition 3: Optimal contract and stock options

The state-contingent optimal payment in (19) may be implemented by giving the CEO a fixed wage w_1^* and ψ stock-options on P with exercise price $\kappa = \frac{w_1^*(1-\psi)}{\psi}$, which vest and expire at the beginning of the second period, after the firm has decided whether to retain or to dismiss its CEO.

This notably implies that the options of a dismissed CEO are out-of-the-money and therefore worthless. Three important comparative static results follow:

Corollary: Under the optimal contract, $\frac{d}{dL}P(V, L) \neq 0$, $\frac{d\eta}{d\gamma} < 0$, and $\frac{d\psi}{d\gamma} > 0$.

First, the stock-options are not completely indexed: the luck shock L is not fully filtered out of the measure P. Intuitively, this is because the reservation wage of the CEO in the second period depends on L.[17] Furthermore, the degree of pay-for-luck relative to pay-for-performance (or pay-for-ability), which is measured by η, is increasing in σ_ϵ^2, and decreasing in σ_a^2: there

[17] More precisely, firm value net of luck, which may be viewed as a measure of "pure performance", matters for the reservation wage insofar as it leads to updating on the manager's ability. Likewise, luck matters for the reservation wage insofar as the latter is sensitive to "economic conditions".

will be more pay-for-luck relative to pay-for-performance when firm value V is a noisy measure of CEO ability a, and when the initial uncertainty on the ability of the executive is low. This suggests that pay-for-luck will be relatively strong for non-CEOs (whose σ_ϵ^2 is high), as well as for old CEOs or CEOs with a long tenure (whose σ_a^2 is low). On the contrary, young CEOs or CEOs with a short tenure should be less paid for luck, all else equal. In the limit, as the ratio $\frac{\sigma_\epsilon^2}{\sigma_a^2}$ tends to infinity, η also tends to infinity, and state-contingent remuneration only depends on luck. Note that η can be positive, in which case the measure P contains more luck than firm value V.

Second, the degree of pay-for-luck relative to pay-for-performance is decreasing in γ, which measures the relative importance of general managerial skills as opposed to firm-specific skills. As γ approaches zero, there is only pay-for-luck and no pay-for-performance. Indeed, if only firm-specific skills matter, then the CEO's ability is irrelevant for other firms, so that his reservation wage does not depend on his ability, but only on luck. On the contrary, our model predicts that a rise in the importance of general managerial skills (as proxied by γ) should be accompanied by a reduction in pay-for-luck relative to pay-for-performance.

Third, the quantity ψ of stock-options given to the CEO is an increasing function of γ. To the extent that general skills did indeed become progressively more important in the 1980s and the 1990s, as argued by Murphy and Zabojnik (2004) and Frydman (2007), then our model can explain why CEOs received increasing amounts of stock-options over this period (Frydman and Jenter (2010), figure 2).

It is noteworthy that the CEO is not more exposed to risk with this long-term contract than he would be with a spot contract – he is even less exposed to risk with the long-term contract because of the embedded insurance. Indeed, the CEO is only exposed to variations in his reservation wage – on the upside. Firm value V consists in two components: some exogenous shocks which can be filtered out (here represented by L), and the "pure measure of performance", i.e., firm profits once exogenous influences (such as L) have been removed. Since the beliefs on the CEO's ability are updated based on this "pure measure of performance", any residual noise in this measure will *also* affect the updated beliefs on the CEO's ability, and therefore his reservation wage.[18]

Finally, it should be noted that the firm could simply commit to paying the manager the fixed wage w_1^* in periods 1 and 2, and adjust his pay at the beginning of the second period depending on $\hat{a}$ and L. While this is certainly possible, this would not be optimal if we were to assume that there exists a renegotiation or transaction cost, no matter how small. Even with an arbitrarily small renegotiation cost, the long-term contract described in this section would strictly dominate the contract described above in this paragraph. By contrast, in Oyer (2004), with an arbitrarily small renegotiation cost, it will almost always[19] be optimal to use spot contracts instead of long-term contracts.[20]

4 Matching equilibrium

The literature on pay-for-luck has documented a negative correlation between pay-for-luck and measures of corporate governance. This empirical fact is at the root of the skimming theory, which states that CEOs "set their own pay" in badly governed firms (Bertrand and Mullainathan (2001)). To incorporate this important dimension in our model, we develop a model of matching between CEOs and firms.

[18]Here our results differ from Oyer's (2004). In Oyer, the tradeoff is between exposing the CEO to risk (by indexing his pay on some variable which is imperfectly correlated with his reservation wage) and incurring renegotiation or transactions costs with interim re-contracting. We also differ from standard models of moral hazard, where the optimal contract is the outcome of a tradeoff between exposing the agent to a noisy measure of his effort and providing incentives for effort.

[19]As long as the degree of risk aversion and the variance of the performance measure are strictly positive.

[20]This is not the only potential reason why it is preferable to use an explicit contract rather than an ex-post adjustment to relate the compensation of the CEO pay in the second period to firm value at the end of the first period. For example, suppose that the CEO can invest (at a cost) in firm-specific skills at the beginning of the first period. There will then typically be a time-inconsistency problem: ex-ante, the firm would like to commit to pay the CEO more for increases in firm value, so that the CEO invests efficiently in firm-specific skills. However, ex-post, since investment in firm-specific skills does not increase the reservation wage of the CEO, it is in the interests of the firm to renege on this promise. In this perspective, an explicit contract may be used as a commitment device.

4.1 A matching model of CEOs and firms

On the one hand, we assume that firms differ along one dimension: the cost k of CEO dismissal. This cost can be viewed as a measure of (bad) governance. In general, corporate governance measures the extent to which the firm is managed in the interests of the providers of funds. In our model, the only relevant differences across firms are the estimated ability of the CEO at the beginning of the second period and the cost of CEO dismissal. In addition, all CEOs have the same expected ability ex-ante. Therefore, in our model, corporate governance measures the extent to which a firm can dismiss a CEO with low estimated ability at the beginning of the second period and hire a more able CEO or manager instead. This is captured by the cost k. Accordingly, we will henceforth refer interchangeably to the cost of dismissal or to governance, bearing in mind that we consider corporate governance only insofar as it matters for involuntary CEO turnover.[21]

On the other hand, we assume that different managers have different σ_a^2, so that managers can be ranked according to the variance of their ability. That is, managers' ability is more or less precisely estimated ex-ante.[22] There are no information asymmetries: for each manager, the value of σ_a^2 is common knowledge, but neither the firms nor the manager observe a.

This corresponds to a matching model with nontransferabilities, as studied in Legros and Newman (2007).[23] An equilibrium is defined by a matching function indicating which type of firm employs which type of manager in equilibrium, and equilibrium long-term contracts. The first condition for equilibrium is that the matching function be consistent, i.e., each manager is matched with only one firm. The second condition is that no firm can break its match and improve its expected profit by proposing a contract to an already matched manager that would prefer that contract. Legros and Newman (2007) derive sufficient conditions on the Pareto frontiers generated by a match that ensure positive or negative assortative matching. In our setup,

[21] The assumption that a better corporate governance results in a lower cost of CEO dismissal is also in Acharya, Gabarro, and Volpin (2011).

[22] We do not need to be more specific about the distribution of σ_a^2 across managers.

[23] Other matching models between managers and firms in the CEO compensation literature include Gabaix and Landier (2008), and Edmans, Gabaix, and Landier (2009).

the equilibrium is characterized by negative assortative matching:

Proposition 4: Negative assortative matching

First, the $m-n$ firms with the highest k are matched with managers. Second, given any two firms not matched with a manager and any two CEOs, the firm with the lowest k is matched with the CEO with the highest σ_a^2, while the firm with the highest k is matched with the CEO with the lowest σ_a^2.

A firm which can dismiss its CEO at a lower cost will be matched with a relatively more "risky" CEO. Intuitively, it is more likely that the estimated ability of a risky CEO (with a high σ_a^2) will fall below the firing threshold of any firm (which is necessarily negative). To minimize the costs of dismissals and the costs of inefficient continuation of CEOs with low ability, it is more efficient to match a risky CEO with a firm with a low k, i.e., a good corporate governance, according to our interpretation.[24]

Heterogeneity among firms and CEOs and the resulting matching equilibrium that we described has important implications, that we describe in the next two subsections.

4.2 Pay-for-luck and governance

First of all, our matching model can explain the Bertrand and Mullainathan (2001) finding that firms with bad corporate governance use contracts that display more pay for luck – note that the measures of governance used in Bertrand and Mullainathan (see footnote 2) can also be viewed as measures of the cost of CEO dismissal. Indeed, in a matching equilibrium, firms with different costs of CEO dismissal (or governance) are matched with CEOs whose ability is more precisely estimated, and conversely. This means that firms with different types of governance design optimal long-term contracts for different types of CEOs. Consequently, it is in principle

[24] The first part of Proposition 4 is not of great interest, since it is driven by the assumption that the variance of the ability of managers is zero.

possible that the observed differences in CEO pay across firms with different types of governance are explained by differences in CEO characteristics.

To take an extreme example, suppose that some CEOs have a known ability ($\sigma_a^2 = 0$). Since $\hat{a} = \bar{a}$ with probability one, only future business conditions, or "luck" (L), make the second period reservation wage of these CEOs stochastic. On the contrary, the reservation wage of CEOs with an unknown ability ($\sigma_a^2 > 0$) is stochastic for two reasons: future business conditions, and the updating of beliefs regarding their ability following their performance in the first period. We know from Proposition 4 that CEOs with a known ability will be matched to firms with high costs of dismissal (or bad governance), while CEOs with unknown ability will be matched to firms with low costs of dismissal (or better governance). In the former case, all the variability in the second period pay of the CEO will be attributable to luck. In the latter case, it will be attributable both to luck and to the updating of beliefs on the CEO's ability. That is, in firms with high costs of dismissal, CEOs will be exclusively paid for luck, but they will not be paid for performance (since firm performance is not informative about the CEO's ability, it is pure noise and will be filtered out of the contract). On the contrary, in firms with low costs of dismissal, CEOs will be paid for luck and for performance.

In a more general case where the ability of different managers is estimated with a different precision, our model relates the degree of pay-for-luck relative to pay-for-performance of a given CEO to the variance of his ability:

PROPOSITION 5:

$$\frac{\frac{dw_2^\star(\hat{a},L)}{dL}}{\frac{dw_2^\star(\hat{a},L)}{dV}} \quad \text{is (weakly) decreasing in } \sigma_a^2 \tag{20}$$

In turn, since CEOs with a high variance of ability are matched to firms with a high cost of dismissal, the model predicts that there is more pay-for-luck relative to pay-for-performance in firms with a higher cost of dismissal (or worse governance):

Corollary: Pay-for-luck and governance

$$\frac{\frac{dw_2^\star(\hat{a},L)}{dL}}{\frac{dw_2^\star(\hat{a},L)}{dV}} \quad \text{is (weakly) increasing in } k \tag{21}$$

The higher the cost of CEO dismissal, the more a firm pays its CEO for luck rather than for performance.

In summary, we have shown that firms with high costs of CEO dismissal will be matched with CEOs with a relatively low σ_a^2: these firms optimally select CEOs with a low variance of ability ("safe CEOs"). In addition, we know from the previous section that the degree of pay-for-luck relative to pay-for-performance is decreasing in σ_a^2. The model therefore predicts that pay-for-luck will be relatively stronger in firms with higher costs of CEO dismissal, i.e., a worse governance on this dimension.

4.3 Corporate governance spillovers

The matching model that we propose also generates corporate governance spillovers, whereby an improvement in governance in a subset of firms has spillover effects that increase CEO compensation in firms with a better governance. We denote the cost of CEO dismissal in a given firm i by k_i, with $k_1 < k_2 < \ldots < k_m < \ldots < k_n$.

Proposition 6: Corporate governance spillovers

If k_i diminishes for the set of firms $\{k_j, \ldots, k_J\}$, with $k_j \leq k_J$ and $k_j \leq k_m$ without changing the ranking of firms on this dimension, then CEO compensation increases in the set of firms with cost of dismissal $\{k_1, \ldots, \min\{k_{J-1}, k_m\}\}$.

In equilibrium, the expected profits of any given firm are constrained by the competition for CEOs. More precisely, the difference in expected profits between any given firm which employs a CEO and the next firm with a higher cost of CEO dismissal is increasing in the wedge in the cost of CEO dismissal between these two firms. Lower costs of CEO dismissal in a subset of firms reduce this wedge and therefore reduce expected profits in this subset of firms (except for the one with the highest cost of CEO dismissal, and except for firms which employ managers), and increase the compensation of their CEOs. It follows that these firms will be willing to bid higher to hire CEOs employed by firms with lower costs of CEO dismissals. In equilibrium, CEO pay must therefore increase in all firms with a better governance. In particular, an improvement in the quality of governance of badly governed firms (for example because of the widespread adoption of best practices) triggers an across the board increase in CEO pay.

Our model can explain the fact that CEO pay rose as corporate governance improved (Holmstrom and Kaplan (2003), Murphy and Zabojnik (2004)). This fact cannot be explained by the skimming hypothesis, which would predict the opposite.

5 Other predictions and empirical implications

In this section, we confront the predictions of our model to further empirical evidence.

Bertrand and Mullainathan (2001) find that "governance correlates very little with pay for performance, only with pay for luck."[25] This is also predicted by our model. Indeed, the sensitivity of pay for performance to σ_a^2 is zero, while the sensitivity of pay for luck to σ_a^2 is proportional to the sensitivity of η to σ_a^2, which is strictly negative. That is, the model does not predict any cross-sectional variation in pay-for-performance across firm-CEO matches. In addition, the model predicts that firms with better governance tend to be matched with CEOs

[25] As discussed in section 4, governance in our model is related to the cost of CEO dismissal, and the measures of governance used in Bertrand and Mullainathan (2001) can also be viewed as measures of the cost of CEO dismissal.

whose ability is more uncertain (i.e., with a high σ_a^2), and therefore tend to have relatively less pay-for-luck.

Even though we do not explicitly derive predictions on this dimension in our simple two-periods model, there are reasons to believe that σ_a^2 will decrease over the tenure of a CEO, as his ability becomes more accurately measured. An implicit prediction of our model is therefore that pay-for-luck should increase with CEO tenure. This is all the more interesting that Bertrand and Mullainathan derive a similar prediction with the skimming model: their hypothesis is that CEOs with a longer tenure are more entrenched, so that they can extract more monetary benefits in the form of asymmetric pay-for-luck. In addition, the variance of CEO ability is not related to pay-for-performance in our model, so that we expect no relationship between tenure and pay-for-performance. As is made clear in Bertrand and Mullainathan, the data suggest that there is indeed a positive relationship between tenure and pay-for-luck, but no relationship between tenure and pay-for-performance.[26] These two predictions, which are empirically validated, are common to the skimming hypothesis and the efficient contracting model.

This being said, Bertrand and Mullainathan also find that the positive relationship between tenure and pay-for-luck holds only for firms without a large shareholder on the board (with a large shareholder on the board, the relationship is not statistically significant). The skimming hypothesis can explain this, whereas our model of efficient contracting cannot.[27]

The model does not generate any prediction regarding the frequency of firm-CEO separations depending on a measure of corporate governance. On the one hand, firms with bad corporate governance will tend to hire CEOs with a more precisely estimated ability, which tends to reduce

[26]Garvey and Milbourn (2003) also find that relative performance evaluation, which consists in filtering out one type of exogenous shock, namely the market index, is stronger for younger CEOs. They interpret this finding as evidence that firms tend to let older (and more wealthy) CEOs hedge against market fluctuations themselves, since they are better able to do so than young CEOs.

[27]Obviously, this does not mean that no model of efficient contracting can explain this feature of the data. For example, the model of CEO discretion of Gromb, Burkart and Panunzi (1997) predicts that a CEO who is less monitored by shareholders (which will typically be the case with a large shareholder on the board) will take more initiatives. On the contrary, a CEO who is closely monitored by shareholders will tend to manage the firm more conservatively. All else equal, this suggests that the updating on CEO ability will tend to be much faster in the former case: the model would thus predict a stronger (positive) relationship between tenure and pay-for-luck for firms with a large shareholder on the board.

bad surprises and the associated forced turnover. On the other hand, CEOs whose ability are more precisely estimated may be older, and therefore closer to retirement, which tends to increase voluntary turnover. A priori, it is not clear which effect will dominate.[28] In addition, the model predicts that forced turnovers will follow a downward revision of shareholders' beliefs about CEO ability, but they will be unrelated to luck. In line with this latter prediction, Garvey and Milbourn (2006) do not find any statistically significant relationship between CEO turnover and luck.

6 Conclusion

This paper proposes a principal-agent model of efficient contracting which explains the main empirical regularities associated with the pay-for-luck phenomenon – notably asymmetric pay-for-luck and the association between pay-for-luck and bad corporate governance. Our analysis builds on Oyer (2004) and stresses the importance of the retention motive for CEO compensation. First, asymmetric pay-for-luck is due to the insurance against a deterioration of business conditions given by the firm to a risk-averse CEO. Second, when the cost of CEO dismissal varies across firms and the ability of different CEOs is more or less precisely estimated ex-ante, we show that the equilibrium matching between CEOs and firms is such that pay-for-luck is relatively more important in firms with a high cost of CEO dismissal, which can be interpreted as an indicator of bad governance. The model can also explain the use of stock-options in managerial compensation. Its predictions on a variety of dimensions are broadly consistent with the existing empirical evidence, but future work could specifically test some new hypotheses. In particular, we predict that an improvement in the governance of badly governed firms will have spillover effects that increase CEO pay in all firms. Our results contribute to a large recent literature which shows that the efficient contracting paradigm can actually explain a number of apparent anomalies (Edmans and Gabaix (2009)).

[28] Kaplan and Minton (2011) find no statistically significant relationship between CEO turnover and corporate governance.

We do not claim that our model integrates all the factors which are relevant for CEO compensation. In particular, we ignored the incentives aspect of the problem. This said, the ability of the model to explain many important stylized facts suggests that the often overlooked retention motive might be an important determinant of the structure (not just the level) of CEO compensation. Given that matching models can explain important features of CEO compensation, future work could build on this and other contributions (e.g., Gabaix and Landier (2008)) to develop a unified model of CEO pay which jointly explains the level and the structure of CEO compensation. This would enable us to better understand how competition and retention interplay to influence optimal compensation schemes.

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8 Appendix

Proof of Proposition 2:

According to (13), $\frac{dw_2}{dL} > 0$ if and only if $W_2(\hat{a}, L) > w_1^*$. According to (16), this is the case if and only if

$$\kappa + nV + (1-n)bL > w_1^*$$

Or

$$L > \frac{1}{b}\Big(w_1^* - \kappa - \frac{\sigma_a^2}{\sigma_\epsilon^2 + \sigma_a^2}(a + \tilde{\epsilon})\Big) \equiv \underline{L}$$

If this condition is not satisfied, then $w_2 = w_1^*$, and $\frac{dw_2}{dL} = 0$.

Proof of the Corollary of Proposition 3:

Comparing (19) and (13), we get, for $\gamma \hat{a} L \geq w_1^*$

$$\gamma \hat{a} L = n w_1^* + nP(V, L) \tag{22}$$

So that

$$\frac{d}{dL}\Big\{\gamma \hat{a} L\Big\} = n\frac{d}{dL}P(V, L) \tag{23}$$

Stock-options are fully indexed if and only if $\frac{d}{dL}P(V, L) = 0$. But this cannot be the case since the left-hand-side of (23) is different from zero, except on a set of measure zero (for $\hat{a} = 0$).

The other two comparative static results follow immediately from (17).

Proof of Proposition 4:

We will use the condition in proposition1-ii in Legros and Newman (2007) to prove that the matching will satisfy Negative Assortative Matching (NAM), that is firms with better corporate governance match with riskier CEOs.

For that consider two firms with firing costs $k > k^{'}$ and two CEOs indexed by risk $\sigma^2 < \sigma^{'2}$.

Consider the long term contracts offered by the firm with high costs of firing to both types of CEOs that lead to the same profit for the firm. To prove NAM, we just need to show that if the firm with low firing cost proposes long term optimal contracts to the CEOs that give them as much as the contracts proposed by the high cost firm, it would make larger profits when it proposes a contract to the risky CEO.

First, note that the expected utility of a CEO for a given long term contract depends only on w_1. This means that the low cost firm would propose the same long term contract as the high cost firm if it needs to ensure the same expected utility to the CEO.

The CEO is fired if the estimated ability of the current CEO is too low compared to the average ability of a new CEO. The dismissal condition is: $\hat{a} \leq -k/L = a^k(L)$.

The extra profit of a firm with low dismissal costs that proposes the same contract as a firm with bad corporate governance is thus for a given L : $\Phi_\sigma(-k/L)\left(k - k^{'}\right)$. When $\sigma^2 < \sigma^{'2}$, we have that $\Phi_\sigma\left(a^k(L)\right) > \Phi_{\sigma'}\left(a^k(L)\right)$ for $a < \bar{a}$. Given that $a^k(L) < \bar{a}$, we have that $\Phi_{\sigma_a}\left(a^k(L)\right)\left(k - k^{'}\right)$ increases as σ^2 increases. Integrating over L, we get the result of negative assortative matching .

Proof of Proposition 5:

Either $W_2 < w_1^*$, in which case $\frac{dw_2}{dL} = 0$ and $\frac{dw_2}{dV} = 0$, so that $\frac{\frac{dw_2}{dL}}{\frac{dw_2}{dV}}$ is independent of σ_a^2. Or $W_2 \geq w_1^*$, in which case $\frac{dw_2}{dL} = n(\frac{\sigma_\epsilon^2}{\sigma_a^2} + 1)$ and $\frac{dw_2}{dV} = n$, so that $\frac{\frac{dw_2}{dL}}{\frac{dw_2}{dV}} = \left(\frac{\sigma_\epsilon^2}{\sigma_a^2} + 1\right) - 1$, which is positive and decreasing in σ_a^2.

Proof of Proposition 6:

Suppose that the ranking of costs of dismissals at different firms is such that $k_1 < k_2 < \ldots < k_m < \ldots < k_n$. Below, we identify each firm by its cost of dismissal. Likewise, suppose that the ranking of CEOs is such that $\sigma_1 > \sigma_2 > \ldots > \sigma_m$, where the index a was dropped for readability. Below, we identify each CEO by the variance of his ability. Consider two firms with costs of dismissal k_i and k_{i+1}, for $i \in \{1, m\}$.

The structure of CEO compensation $\{w_1^\star\}_{i=1,\ldots,n}$ must be such that, for any $i \in \{1, m\}$:

(i) The CEO i, who is matched with firm k_i (because of negative assortative matching) with a wage $w_1^\star(k_i, \sigma_i)$, cannot get a strictly better offer at another firm: $w_1^\star(k_i, \sigma_i) \geq w_1(k_j, \sigma_i)$ for any $j \neq i$, where $w_1(k_j, \sigma_i)$ is the wage paid to CEO σ_i that would give firm k_j the same level of expected profits than under negative assortative matching and the structure of wages $\{w_1^\star\}_{i=1,\ldots,n}$.

(ii) Firm k_i cannot decrease $w_1^\star(k_i, \sigma_i)$ without violating (i).

First, we know from Proposition 4 that all firms k_i, for $i \geq m+1$, hire managers and make zero profits.

Second, the wage $w_1(k_i, \sigma_j)$ paid to a given CEO σ_j by firm k_i that leaves expected profits unchanged (see point (i) above) is strictly decreasing in k_i, for all firms $k_i \leq k_m$.

Third, we know from Proposition 4 that firm k_{m+1} hires a manager and therefore makes zero profits in each period, while firm k_m hires the CEO σ_m. By construction, $w_1(k_{m+1}, \sigma_m)$ is such that firm k_{m+1} is indifferent between hiring a manager or hiring the CEO σ_m at a wage $w_1(k_{m+1}, \sigma_m)$. We know from the second point above that, given all firms k_i, for $i \geq m+1$, $w_1(k_i, \sigma_m)$ is highest for firm $m+1$. Therefore, if we only consider the set of firms $\{k_m, \ldots, k_n\}$, condition (i) and (ii) are satisfied if and only if $w_1^\star(k_m, \sigma_m) = w_1(k_{m+1}, \sigma_m)$. In particular, since the firm k_{m+1} makes zero profits with CEO σ_m at wage $w_1(k_{m+1}, \sigma_m)$ (by construction of $w_1(k_{m+1}, \sigma_m)$), and because of the second point above, we know that firm m makes strictly positive profits.

Fourth, suppose that $w_1^\star(k_i, \sigma_i) = w_1(k_{i+1}, \sigma_i)$ is satisfied for all $i \in \{j, \ldots, n\}$, for some $j \in \{2, \ldots, m\}$, given conditions (i) and (ii). We will show that it is then also satisfied for $j-1$. As above, given the set of firms $\{k_{m+1}, \ldots, k_n\}$, $w_1(k_j, \sigma_{j-1})$ is highest for firm k_j. Therefore, as above, if we only consider the set of firms and CEOs $i \in \{j-1, \ldots, n\}$, condition (i) and (ii) are satisfied if and only if $w_1^\star(k_{j-1}, \sigma_{j-1}) = w_1(k_j, \sigma_{j-1})$. Repeating this algorithm, it can be shown by induction that, for any $i \in \{1, \ldots, n\}$:

$$w_1^\star(k_i, \sigma_i) = w_1(k_{i+1}, \sigma_i) \tag{24}$$

Now suppose that k_i diminishes for the set of firms $\{k_j, \ldots, k_J\}$, with $k_j \leq k_J$ and $k_j \leq k_m$, in such a way that the ranking of firms with respect to dismissal costs is unchanged. Since $w_1(k_{i+1}, \sigma_i)$ is strictly decreasing in k_{i+1}, it follows that $w_1(k_{i+1}, \sigma_i)$ strictly increases for all $i \in \{j-1, \ldots, \min\{J-1, m\}\}$, so that CEO pay in firm k_i, $w_1^\star(k_i, \sigma_i)$, strictly increases because of (24), for all $i \in \{j-1, \ldots, \min\{J-1, m\}\}$

But if $w_1^\star(k_{j-1}, \sigma_{j-1})$ strictly increases, this strictly decreases the expected profits at firm $j-1$, so that $w_1(k_{j-1}, \sigma_{j-2})$ strictly increases as well, by construction. Applying again (24) to $i = j-2$ shows that $w_1^\star(k_{j-2}, \sigma_{j-2})$ strictly increases. Repeating this algorithm for all $i \in \{1, \ldots, j-1\}$, it can be shown by induction that $w_1^\star(k_i, \sigma_i)$ strictly increases for all $i \in \{1, \ldots, \min\{J-1, m\}\}$.