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Imposing Curvature Conditions on Flexible Functional Forms for GNP Functions

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Abstract:
This paper empirically investigates the implications of the imposition of convexity in output prices and concavity in factor endowments on flexible functional forms for the GNP function. Using macroeconomic data for Switzerland, we estimate the Translog and the Symmetric Normalized Quadratic forms to investigate the manner with which curvature restrictions are imposed, the extend of curvature violations and the robustness of estimated elasticities. We also compare the predictive accuracy of the aforementioned flexible functional forms. Our result show that concavity in factor endowments is violated much more often than convexity in output prices. For the Translog, the date at which local restrictions are imposed matters a great deal in terms of remaining curvature violations in the sample, but far less for estimated elasticities. In contrast, we found that the size and sign of elasticities vary across functional forms. In-sample forecasting analysis demonstrates that the Translog model significantly dominates the Symmetric Normalized Quadratic.

Keywords: GNP function, flexible functional forms, curvature violations, elasticities

Classification JEL: D24, C30

Résumé:
Ce papier analyse empiriquement l'impact de l'imposition de la convexité dans le prix des biens produits et de la concavité dans les quantités de facteurs de production sur l'estimation des formes fonctionnelles flexibles dans le cadre des fonctions de Produit National Brut (PNB). En utilisant les données macroéconomiques de la Suisse, nous estimons les formes fonctionnelles Translog et Quadratique Symétrique Normalisée avec et sans imposition des conditions de courbures, et enquêtons sur l'étendue de la violation des restrictions et la robustesse des élasticités estimées. Nous évaluons également la capacité prédictive intra-échantillon des deux formes fonctionnelles. Nos résultats montrent que la concavité dans la dotation en facteurs est violée plus souvent que la convexité dans le prix des biens produits. Pour la Translog, la date à laquelle les restrictions locales sont imposées a un impact important sur le nombre de violations des conditions de courbure et cela sur l'ensemble de l'échantillon, mais a peu d'influence sur les élasticités estimées. En revanche, nous avons constaté que la taille et le signe des élasticités varient selon les deux formes fonctionnelles. L'analyse du pouvoir prédictif intra-échantillon montre que la forme Translog domine nettement la Quadratique Symétrique Normalisée.

Mots clés: Fonction de PNB, formes fonctionnelles flexibles, conditions de courbure, élasticités

1 Introduction

The imposition of theoretical curvature restrictions on functional forms is an important problem in several areas of applied economics, such as demand analysis, production economics and international trade.[1] This problem stems from the popularity of specific flexible functional forms like the Almost Ideal Demand System of Deaton and Muellbauer (1980), the Generalized Leontief of Diewert (1974) and the Transcendental Logarithmic or Translog developed by Christensen et al. (1973) for which curvature properties are often violated in practice. Given that theoretical consistency must guide the selection of a functional form (e.g., see Lau (1986, p. 1520)), several studies have examined the imposition of theoretical regularity conditions without sacrificing flexibility to maintain the appeal of the aforementioned functional forms.

In practice, theoretical properties can be imposed globally or locally depending on the functional form.[2] In some cases, global restrictions destroy the flexibility of the functional form and local restrictions provide a more appealing tradeoff. The first attempt to impose curvature conditions on flexible functional forms (hereafter FFF) was made Lau (1978). The imposition of local curvature conditions was also studied by Gallant and Golub (1984) using a nonlinear optimization procedure to estimate parameters for constrained FFF. Morey (1986) provides an excellent overview of the early literature. More recently, Moschini (1999) and Ryan and Wales (2000) showed that local restrictions can be imposed on the associated Slutsky matrix as proposed by Ryan and Wales (1998).

Global curvature restrictions were first derived by Diewert and Wales (1987). They compared different flexible functional forms and discussed the stability of estimated parameters and elasticities. Curvature restrictions can also be dealt with through the imposition of inequality restrictions in a Bayesian estimation framework. Terrell (1996) compared three flexible functional forms while imposing monotonicity and curvature conditions under different prior distributions. Elasticities are computed by drawing many sets of parameters and keeping only the ones for which theoretical conditions are valid. As such, the curvature property is not

[1] The expenditure, cost, and input distance functions are concave in product prices, input prices and inputs, respectively. The profit function is convex in product and input prices, the indirect utility function is quasi-convex in product prices and the GNP/revenue function is convex in product prices and concave in factor endowments.

[2] When the property is imposed globally the restrictions do not depend on the model's exogenous variables unlike

imposed ex-ante. Griffiths et al. (2000) and Lariviere et al. (2000) also used a Bayesian estimation to investigate the impact of regularity (monotonicity + curvature) conditions on the signs and magnitudes of elasticities.[3] Even though Kohli (1991) is a well-known contribution, few papers discuss curvature conditions in the context of the GNP function (see Kohli (1992, 1993) and Tombazos (2003)). These papers mentioned the issue of imposing curvature restrictions, but do not thoroughly assess its impact in terms of regularity violations, robustness of estimated parameters or elasticities and forecasting performance.

This paper empirically investigates the implications of the imposition of curvature conditions on flexible GNP functions. In practice, the properties can be imposed directly on the functional form before the estimation is implemented. Alternatively, one can do the estimation without curvature restrictions and then test if the curvature properties are maintained. If curvature conditions are violated, one can then decide to impose them locally or globally depending on the functional form. Our empirical application focuses on Switzerland.[4]

Contributions pertaining to demand systems and cost functions make up the bulk of the literature on curvature restrictions. Far less research has examined the imposition of curvature restrictions on GNP functions. We wish to fill this void by analyzing the implications of local curvature restrictions on two popular flexible functional forms : the Translog (hereafter TL) and the Symmetric Normalized Quadratic (hereafter SNQ). Unlike expenditure and cost functions, two sets of restrictions are needed to deal with the curvature of GNP functions : convexity in output prices and concavity in factor endowments. Our investigation provides insights about the flexible functional forms' performance in terms of curvature coverage, the robustness of elasticities and predictive accuracy comparisons.

We found much variation in the number of violations of curvature restrictions depending

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when it is imposed locally.

[3]In contrast, Diewert and Wales (1987), Fisher et al. (2001), Feng and Serletis (2007) and Serletis and Shahmoradi (2007) have relied on a classical estimation framework. Many papers have focused on consumer theory applications (e.g., Fisher et al. (2001), Barnett and Serletis (2008)) while technological/cost functions applications have been done by Gallant and Golub (1984), Terrell (1996), Griffiths et al. (2000), Ryan and Wales (2000) and Feng and Serletis (2007). More recently, Chapda Nana et al. (2012) proposed an application of GNP function to analyze impact of regional trade agreement for Canada and the United States.

[4]The choice of Switzerland is primarily motivated by the fact that it is a small and stable open economy, unlike Canada which embarked on a long transition when it began negotiating a free trade agreement with the United States. Kohli (1992) also used Swiss data and constitutes a pertinent comparison in spite of differences in sample periods.

on the date at which the restrictions are imposed. Thus, searching for a date might be worth the effort if one intends to report elasticities at various points in time. Concavity in factor endowments is violated more often than convexity in output prices. Estimated elasticities had the size and sign suggested by the literature for the two models. Demand for factors are inelastic and more volatile for the Translog form. Using the Diebold and Mariano test, the predictive accuracy analysis demonstrates that the TL model significantly outperforms the SNQ for in-sample forecasting.

The reminder of the paper is organized as follows. Section 2 briefly describes the theoretical foundations and properties of the GNP function. In Section 3, we present the Translog and Symmetric Normalized Quadratic flexible functional forms along with the procedures to impose curvature restrictions on each of them. The data and econometric issues are discussed in Section 4, the estimations and the assessment of the impact of curvature imposition on flexible functional forms and estimated elasticities are presented in Section 5 and in-sample forecasting analysis is provided in Section 6. The final section concludes the paper.

2 The Gross National Product Function

The GNP function is used in empirical international trade to analyse resource allocation and production for an economy. For an economy, let us assume that the GNP function $G(p,x,t)$, is conditioned by I outputs and J factor endowments. It is defined as the maximum that can be produced by an economy through optimal resource allocation given its technology, factor endowments $x=(x_1,...,x_J)$ and output prices $p=(p_1,...,p_I)$. The technology can be represented by :

$$G(p,x,t) \equiv \max_{y_i \geq 0} \sum p_i y_i \qquad s.t \qquad \sum x_i \leq x \tag{1}$$

where $y_i = f(x_i,t)$ denotes the production function of good i at time t. The GNP function results from equilibrium conditions pertaining to full employment on the market for production factors, perfect competition and restrictions on the technology such that a production function is positive, increasing, concave and homogenous of degree one. Thus, changes in output prices induce changes in input allocation that maintain production on the production possibility frontier. Accordingly, the GNP function is increasing and homogenous of degree one in prices, increasing and homogenous of degree one in endowments, twice differentiable and convex in prices and twice differentiable and concave in endowments (Wong (1995)).

Following Hotelling's lemma, the supply of output can be obtained by differentiation of the GNP function with respect to output prices :

$$y_i = \partial G\left(p,x,t\right)/\partial p_i, i=1,...,I$$

Young's theorem implies that :

$$\partial y_i / \partial p_j = \partial^2 G\left(p,x,t\right) / \partial p_i \partial p_j = \partial y_j / \partial p_i, i=1,...,I; j=1,...,J$$

The convexity (in prices) property ensures that the matrix of second derivatives is positive semi-definite which implies that diagonal elements are non-negative : $\partial y_i/\partial p_j \geq 0$. Under the aforementioned assumptions (perfect competition and restrictions on technology), GNP can be expressed in terms of the value of endowments or of outputs since $\sum w_k x_k = \sum p_i y_i$, with w_k representing the price of factor k. Appealing to the envelope theorem, the relationship between factor prices and factor endowments can be obtained by :

$$\partial G\left(p,x,t\right)/\partial x_k = p_i \partial y_i/\partial x_k, i=1,...,I; k=1,...,J$$

Combining the previous results yields:

$$\partial w_k/\partial p_i = \partial^2 G\left(p,x,t\right)/\partial x_k \partial p_i = \partial y_i/\partial x_k$$

The technology allows us to analyse substitution possibilities for the economy. For this purpose, we define the matrix of price, quantity, and time elasticities and semielasticities of output supply, input inverse demand and technological change by :

$$E = \begin{bmatrix} E_{pp} & E_{px} \\ E_{xp} & E_{xx} \end{bmatrix}$$

where

$$E_{pp} = \left[\partial \ln y_i/\partial \ln p_j\right], \quad E_{px} = \left[\partial \ln y_i/\partial \ln x_k\right],$$
$$E_{xp} = \left[\partial \ln w_k/\partial \ln p_i\right], \quad E_{xx} = \left[\partial \ln w_k/\partial \ln x_l\right],$$

According to the adopted functional form, E can be calculated in detail as follows :

$$E_{pp} = \left(diag\left(\nabla_p G\right)\right)^{-1} \times \nabla_{pp} G \times diag\left(p\right), \quad E_{px} = \left(diag\left(\nabla_p G\right)\right)^{-1} \times \nabla_{px} G \times diag\left(x\right)$$
$$E_{xp} = \left(diag\left(\nabla_x G\right)\right)^{-1} \times \nabla_{xp} G \times diag\left(p\right), \quad E_{xx} = \left(diag\left(\nabla_x G\right)\right)^{-1} \times \nabla_{xx} G \times diag\left(x\right)$$

Where $\nabla_{pp}G, \nabla_{px}G, \nabla_{xp}G, \nabla_{xx}G$ are the sub-Hessians of G with respect to the elements of p and x; for example, $\nabla_{px}G = \partial^2 G/\partial p \partial x$. The convexity in price of G implies that $\nabla_{pp}G$ is positive semi-definite and the concavity in input, $\nabla_{xx}G$ is negative semi-definite.[5]

We focus on four types of elasticities commonly reported in the empirical trade literature. The block E_{pp} embodies output supply and input demand price elasticities. The elements of E_{px} represent the impact of factor endowments on output quantities for a given output price and are often referred to as Rybczinsky (R) elasticities. The element of E_{xp} corresponds to the impact of changes in output prices on input prices or Stolper-Samuelson (SS) elasticities. Finally, the block E_{xx} includes inverse input demand elasticities. The magnitude of the SS effects is of great interest because the gains and losses of factors can be assessed by looking at how much their prices decrease or increase.

To derive supply and input demand or share equations for empirical purposes, a functional form must be specified. There are many functional forms to chose from. Ideally, a functional form should satisfy all regularity conditions imposed by economic theory while remaining flexible to approximate the technology. The imposition of curvature conditions can destroy the flexibility of the functional form. In what follows, we compare and test different approaches to impose curvature conditions in the context of the GNP function with two flexible functional forms on data for Switzerland.

3 The flexible functional forms

This section describes how to impose convexity and concavity restrictions on two popular flexible functional forms of the GNP function : the Translog and the Symmetric Normalized Quadratic.

[5]Wiley et al. (1973) showed that a sufficient condition for matrix A to be negative semi-definite is $A = -TT'$, where T is a lower triangular matrix. Similarly, Diewert and Wales (1987) show that A is positive semi-definite if it can be written as $A = TT'$.

3.1 The Translog

The TL introduced by Christensen et al. (1973) is the most popular functional form. In the context of GNP function with factor endowments $x=(x_1,...,x_J)$ and output prices $p=(p_1,...,p_I)$, it has the following structure :

$$\ln G(p,x,t)=\alpha_0+\beta_t t+\frac{1}{2}\phi_{tt}t^2+\sum_{i=1}^{I}\alpha_i\ln p_i+\sum_{k=1}^{J}\beta_k\ln x_k+\frac{1}{2}\sum_{i=1}^{I}\sum_{j=1}^{I}\gamma_{ij}\ln p_i\ln p_j \tag{2}$$
$$+\frac{1}{2}\sum_{k=1}^{J}\sum_{l=1}^{J}\phi_{kl}\ln x_k\ln x_l+\sum_{i=1}^{I}\sum_{k=1}^{J}\delta_{ik}\ln p_i\ln x_k+\sum_{i=1}^{I}\delta_{it}(\ln p_i)t+\sum_{h=1}^{J}\phi_{kt}(\ln x_j)t$$

where the following restrictions must be imposed for the adding-up, homogeneity and symmetry properties to be verified.

$$\sum_{i=1}^{I}\alpha_i=\sum_{k=1}^{J}\beta_k=1,\sum_{j=1}^{I}\gamma_{ij}=\sum_{l=1}^{J}\phi_{kl}=\sum_{k=1}^{J}\delta_{ik}=\sum_{i=1}^{I}\delta_{it}=\sum_{k=1}^{J}\phi_{kt}=0$$

$$\gamma_{ij}=\gamma_{ji},\phi_{kl}=\phi_{lk}$$

These parametric restrictions greatly reduce the number of parameters to be estimated at the empirical stage.

Differentiating the TL GNP function (2) with respect to $\ln p_i$ yields the output share of good i:

$$s_i\equiv\partial\ln G/\partial\ln p_i=\alpha_i+\sum_{j=1}^{I}\gamma_{ij}\ln p_j+\sum_{k=1}^{J}\delta_{ik}\ln x_k+\delta_{it}t,\qquad i,j=1,...,I \tag{3}$$

Similarly, differentiating the Translog GNP function (2) with respect to $\ln x_k$ yields the input share of production factor j:

$$s_k\equiv\partial\ln G/\partial\ln x_k=\beta_k+\sum_{l=1}^{J}\phi_{kl}\ln x_l+\sum_{i=1}^{I}\delta_{ik}\ln p_i+\phi_{kt}t\qquad k,l=1,...,J \tag{4}$$

If we define $G_i\equiv\partial G(p,x,t)/\partial p_i$ and $G_{ij}\equiv\partial^2 G(p,x,t)/\partial p_i\partial p_j$, the following identity is verified :

$$\frac{\partial^2\ln G}{\partial\ln p_i\partial\ln p_j}=\theta_{ij}\frac{p_iG_i}{G}-\frac{p_ip_jG_iG_j}{G^2}+\frac{p_ip_jG_{ii}}{G} \tag{5}$$

where $\theta_{ij}=1$ if $i=j$ and is 0 otherwise, and $G\equiv G(p,x,t)$. More compactly, (5) can also be written as :

$$\frac{\hat{p}.\nabla_{pp}G.\hat{p}}{G} = \Delta - \hat{s} + \hat{s}.\hat{s}' \quad (6)$$

where $\hat{p}$ and $\hat{s}$ are respectively diagonal matrices whose elements are price vectors and vector of shares $\left(s_i = \frac{p_i G_i}{G}, i = 1,...,I \right)$ and $\Delta = \left[\delta_{ij} \right]$ a symmetric matrix of parameters. Similarly, we have :

$$\frac{\partial^2 \ln G}{\partial \ln x_k \partial \ln x_l} = \omega_{kl} \frac{x_k G_k}{G} - \frac{x_k x_l G_k G_l}{G^2} + \frac{x_k x_l G_{kl}}{G} \quad (7)$$

with $\omega_{kl} = 1$ if $k = l$ and is 0 otherwise, and $G_k = \partial G(p,x,t)/\partial x_k$ and $G_{kl} = \partial^2 G(p,x,t)/\partial x_k \partial x_l$. Then (7) is equivalent to

$$\frac{\hat{x}.\nabla_{xx}G.\hat{x}}{G} = \Phi - \hat{s} + \hat{s}.\hat{s}' \quad (8)$$

with $\Phi = [\phi_{kl}]$, $s_k = \frac{x_k G_k}{G}, k = 1,..., J$.

Assuming that $G(p,x,t) > 0, \hat{p} > 0$ and $\hat{x} > 0$, $\nabla_{pp}G$ is a positive semi-definite matrix (respectively $\nabla_{xx}G$ is negative semi-definite) if and only if $Hp = \Delta - \hat{s} + \hat{s}.\hat{s}'$ is positive semi-definite (respectively $Hx = \Phi - \hat{s} + \hat{s}.\hat{s}'$ is negative semi-definite). Hence, applying the procedure of Wiley et al. (1973), Hp and Hx can be written as :

$$Hp = TT' \text{ and } Hx = -KK', \quad (9)$$

where T and K are lower triangular matrices.

Conditions (6) and (8) depend on the exogenous variables of the model and at different sample points. Thus, imposing global regularity conditions would impair the flexibility of the model and local imposition is more suitable. Knowing that we have two curvature conditions, their local imposition can be made at the same reference date or at two different dates as proposed by Kohli (1991).

3.1.1 Imposing curvature condition at the same observation/date

At the reference date t^*, $p_i^* = 1, x_k^* = 1, s_i = \alpha_i$ and $s_k = \beta_k$. According to (6) and (9), G is convex in output prices if at the reference point,

$$Hp_{ij} = \delta_{ij} - \theta_{ij}\alpha_i + \alpha_i \alpha_j = (TT')_{ij}, i, j = 1,..., I \quad (10)$$

By the same token, but from (8) and (9), G is concave in input quantities if at reference point:

$$Hv_{kl} = \phi_{kl} - \omega_{kl}\beta_k + \beta_k\beta_l = -KK'_{kl}, k, l = 1, ..., J \tag{11}$$

with $I = 4$ and $J = 2$, $T = \left[\tau_{ij}\right]$ and $K = \kappa_{kl}$ defined as follows :

$$T = \begin{bmatrix} \tau_{11} & 0 & 0 & 0 \\ \tau_{12} & \tau_{22} & 0 & 0 \\ \tau_{13} & \tau_{23} & \tau_{33} & 0 \\ \tau_{14} & \tau_{24} & \tau_{34} & \tau_{44} \end{bmatrix} \text{and} K = \begin{bmatrix} \kappa_{11} & 0 \\ \kappa_{12} & \kappa_{22} \end{bmatrix}$$

By replacing elements of T in (10), we get the following reparametrization of δ_{ij} :

$$\begin{aligned}
\delta_{11} &= \tau_{11}^2 + \alpha_1 - \alpha_1^2; \\
\delta_{12} &= \tau_{12}\tau_{11} - \alpha_1\alpha_2; \\
\delta_{13} &= \tau_{13}\tau_{11} - \alpha_1\alpha_3; \\
\delta_{14} &= \tau_{14}\tau_{11} - \alpha_1\alpha_4; \\
\delta_{22} &= \tau_{12}^2 + \tau_{22}^2 + \alpha_2 - \alpha_2^2; \\
\delta_{23} &= \tau_{12}\tau_{13} + \tau_{22}\tau_{23} - \alpha_2\alpha_3; \\
\delta_{24} &= \tau_{12}\tau_{14} + \tau_{22}\tau_{24} - \alpha_2\alpha_4; \\
\delta_{33} &= \tau_{13}^2 + \tau_{23}^2 + \tau_{33}^2 + \alpha_3 - \alpha_3^2; \\
\delta_{34} &= \tau_{13}\tau_{14} + \tau_{23}\tau_{24} + \tau_{33}\tau_{34} - \alpha_3\alpha_4; \\
\delta_{44} &= \tau_{14}^2 + \tau_{24}^2 + \tau_{34}^2 + \tau_{44}^2 + \alpha_4 - \alpha_4^2.
\end{aligned} \tag{12}$$

Similarly, from K and (11), we obtain ϕ_{kl} :

$$\begin{aligned}
\phi_{11} &= -\kappa_{11}^2 - \beta_1^2 + \beta_1; \\
\phi_{12} &= -\kappa_{11}\kappa_{12} - \beta_1\beta_2; \\
\phi_{22} &= -\kappa_{22}^2 - \beta_2^2 + \beta_2.
\end{aligned} \tag{13}$$

The reparametrizations represented by equations (12) and (13) guarantee that the TL GNP function is convex in outputs prices and concave in factor endowments at least at the reference point. It is also clear that the flexibility of the Translog form is not destroyed because the $I(I+1)/2$ elements of Δ are replaced by the $I(I+1)/2$ of T and the $J(J+1)/2$ elements of Φ are replaced by the $J(J+1)/2$ of K.

3.1.2 Imposing curvature condition at different observation dates

As proposed by Kohli (1991) but not yet tested, the alternative procedure to impose curvature restrictions locally in the case of the TL functional forms for GNP function is to impose restrictions at different points. As in the former case, the objective is to ensure that Hp (respectively Hx) is positive semi-definite (respectively negative semi-definite) at the observation date t_1 (respectively t_2), with $t_1 \neq t_2$.

The input and output share equations (3) and (4) can be written as :

$$s_{it} = \hat{\alpha}_i + \sum_{j=1}^{I} \gamma_{ij} \;\; \ln p_{jt} - \ln p_{jt_1} \;\; + \sum_{k=1}^{J} \delta_{ik} \;\; \ln x_{kt} - \ln x_{kt_1} \;\; + \delta_{it} \;\; t - t_1 \qquad (14)$$

and

$$s_{kt} = \hat{\beta}_k + \sum_{l=1}^{J} \phi_{kl} \;\; \ln x_{lt} - \ln x_{lt_2} \;\; + \sum_{i=1}^{I} \delta_{ik} \;\; \ln p_{it} - \ln p_{it_2} \;\; + \phi_{kt} \;\; t - t_2 \qquad (15)$$

where

$$\hat{\alpha}_i = \alpha_i + \sum_{j=1}^{I} \gamma_{ij} \ln p_{jt_1} + \sum_{k=1}^{J} \delta_{ik} \ln x_{kt_1} + \delta_{it} t_1$$

and

$$\hat{\beta}_k = \beta_k + \sum_{l=1}^{J} \phi_{kl} \ln x_{lt_2} + \sum_{i=1}^{I} \delta_{ik} \ln p_{it_2} + \phi_{kt} t_2$$

It can be shown from (14) and (15) that $\hat{\alpha}_i = s_{it_1}$ and $\hat{\beta}_k = s_{kt_2}$.

Following the same procedure as for the imposition of curvature restrictions at the same date, we can write, using (9) and (14), the convexity restrictions as:

$$Hp_{ij} = \delta_{ij} - \theta_{ij}\hat{\alpha}_i + \hat{\alpha}_i\hat{\alpha}_j = \;\; TT' \;\; _{ij}, i, j = 1, ..., I \qquad (16)$$

where $\theta_{ij} = 1$ if $i = j$ and is 0 otherwise. For concavity, we rely on (9) and (15) to show that:

$$Hx_{kl} = \phi_{kl} - \omega_{kl}\hat{\beta}_k + \hat{\beta}_k\hat{\beta}_l = \;\; -KK' \;\; _{ij}, k, l = 1, ..., J \qquad (17)$$

where $\omega_{kl} = 1$ if $k = l$ and is 0 otherwise.

Hence, we can impose convexity and concavity at different dates by applying the same reparametrization as in equation (12) and (13) and replacing in each equation α_i by $\hat{\alpha}_i$ or β_k by $\hat{\beta}_k$. Finally, the new system to be estimated is made up of equations (14) and (15) with restrictions given by equations (16) and (17). The difference here is that the output share equations and input share equation are normalized at different dates. The estimation is implemented in the same manner as when only one date is used.

3.2 The Symmetric Normalized Quadratic

The SNQ functional form was developped by Diewert and Wales (1987), but Kohli (1993) was the first to use it to approximate a GNP function. The SNQ with technological change can be written as :

$$G(p,x,t)=\frac{1}{2}(\mathbf{b}'\mathbf{x})\left(\mathbf{p}'\mathbf{A}\mathbf{p}/\mathbf{a}'\mathbf{p}\right)+\frac{1}{2}(\mathbf{a}'\mathbf{p})\left(\mathbf{x}'\mathbf{B}\mathbf{x}/\mathbf{b}'\mathbf{x}\right)$$
$$+\mathbf{p}'\mathbf{C}\mathbf{x}+\mathbf{p}'\mathbf{D}\mathbf{x}t+\frac{1}{2}d_{tt}(\mathbf{a}'\mathbf{p})(\mathbf{b}'\mathbf{x})t^2 \tag{18}$$

where $A=\left[a_{ij}\right]$ and $B=\left[b_{kl}\right]$ are unknown symmetric parameters matrices of dimensions $I\times I$ and $J\times J$ respectively. C and D represent other parameter matrices of dimensions $I\times J$; $\mathbf{a}$ and $\mathbf{b}$ are two predetermined vectors of dimension I and J, respectively; d_{tt} is a scalar to be estimated. As in Kohli (1993), we also require that $\sum a_{ij}=0,\sum b_{kl}=0,\sum\alpha_i=1,\sum\beta_k=1$. For empirical reasons, we set $\alpha_i=1/I\ (i=1,...,I)$ and $\beta_k=1/J\ (k=1,...,J)$.

Using (1) and (18), the output supply and inverse input demand functions are :

$$\mathbf{y}=(\mathbf{b}'\mathbf{x})\left(\mathbf{A}\mathbf{p}/\mathbf{a}'\mathbf{p}\right)-\frac{1}{2}(\mathbf{b}'\mathbf{x})\left(\mathbf{p}'A\mathbf{p}/\mathbf{a}'\mathbf{p}\right)^2$$
$$+\frac{1}{2}\mathbf{a}\left(\mathbf{x}'\mathbf{B}\mathbf{x}/\mathbf{b}'\mathbf{x}\right)+\mathbf{C}\mathbf{x}+\mathbf{D}\mathbf{x}t+\frac{1}{2}d_{tt}\mathbf{a}(\mathbf{b}'\mathbf{x})t^2 \tag{19}$$

and

$$\mathbf{w}=\frac{1}{2}\mathbf{b}\left(\mathbf{p}'\mathbf{A}\mathbf{p}/\mathbf{a}'\mathbf{p}\right)+(\mathbf{a}'\mathbf{p})\left(\mathbf{B}\mathbf{x}/\mathbf{b}'\mathbf{x}\right)$$
$$-\frac{1}{2}\mathbf{b}(\mathbf{a}'\mathbf{p})\left(\mathbf{x}'\mathbf{B}\mathbf{x}/\mathbf{b}'\mathbf{x}\right)^2+\mathbf{C}'\mathbf{p}+\mathbf{D}'\mathbf{p}t+\frac{1}{2}d_{tt}(\mathbf{a}'\mathbf{p})\mathbf{b}'t^2 \tag{20}$$

Referring to Diewert and Wales (1987) and Kohli (1993), the SNQ functional form for the GNP function is globally convex in output prices, if and only if A is positive semi-definite, and globally concave in input quantities, if and only if B is negative semi-definite. We can then impose concavity and convexity globally by implementing the technique of Wiley et al. (1973).

As for the TL, A is positive semidefinite if it can be written as : $A = \Omega\Omega'$, where $\Omega = \left[\omega_{ij}\right]$ is a lower triangular matrix. Hence, if $I = 4$ and $\sum a_{ij} = 0$, we have[6] :

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} = \begin{bmatrix} \omega_{11}^2 & \omega_{11}\omega_{12} & \omega_{11}\omega_{13} \\ \omega_{11}\omega_{12} & \omega_{21}^2 + \omega_{22}^2 & \omega_{12}\omega_{13} + \omega_{22}\omega_{23} \\ \omega_{11}\omega_{13} & \omega_{12}\omega_{13} + \omega_{22}\omega_{23} & \omega_{13}^2 + \omega_{23}^2 + \omega_{33}^2 \end{bmatrix}$$

Similarly, B is negative semidefinite if it can be written as $B = -\Psi\Psi'$. With $\Psi = \psi_{kl}$ $J = 2$ and $\sum b_{kl} = 0$, we get the following reparametrization $b_{11} = -\psi_{11}^2$.

3.3 Curvature checking process

Morey (1986) provides a complete presentation of necessary and sufficient algebraic conditions for different curvature properties used in applied economics. These conditions revolve around the properties of matrices. A matrix is positive (respectively negative) semi-definite if and only if all of its principal minors are nonnegative (respectively alternate in sign); or equivalently all eigenvalues are nonnegative (respectively nonpositive). Thus, to evaluate the coverage of local restrictions or the extent of curvature violations, we can proceed as follows :

- for global imposition as in the case of SNQ functional form, we check if matrix A has only nonnegative eigenvalues and if B has only nonpositive eigenvalues;

- for local imposition, we check whether the estimated hessian matrix (equations (16) and (17)) have the desired properties at each point in the sample. As Ryan and Wales (2000) maintain, the imposition of local curvature restrictions does not guarantee that regularity conditions will hold at all of the points in the sample.

4 Data and estimation method

The time series used in our analysis come from OECD statistics and the Main Economic

[6]With the condition $\sum a_{ij} = 0$, the last element in each row of matrix A is a linear combination of the remaining elements in that row. Owing to the symmetry of A, we can obtain all the elements of A simply by estimating a ($I - 1 \times I - 1$) matrix where we had deleted the last row and the last column. Finally, with linear algebra, we know that if A is semi-positive definite its leading principal sub-matrix is also semi-positive definite. The same reasoning is applied to B.

Indicators (MEI) database. The sample covers the period 1970-2007. Following Kohli (1991), we rely on two factors of production, capital (K) and labour (L), and four variable output quantities imports (M), exports (X), investment goods (I) and consumption goods (C). Imports are considered as intermediate products, requiring various domestic services such as unloading, transportation, storage, repackaging, marketing and retailing, before being consumed. Indeed, Kee et al. (2008) contends that a large part of the growth in world trade can be attributed to vertical specialization. In this context, the treatment of imports as intermediate products is an appealing feature of the GNP function framework. Because exports must be tailored to suit the specificities of importing countries, they are differentiated from domestic goods. The data considered are price and quantity series for the six variables. Details about the construction of the dataset are presented in the appendix. Basic descriptive statistics for the data are presented in Table 1.[7] All the price series were normalized to one in 2002.

The equations to be estimated are in the system formed by (3) and (4) for the TL and (19) and (20) for the SNQ. The first thing to assess before the estimation is the stochastic specification of each system. This issue is very important for our study particularly when estimating share equations summing to 1 as in the TL case. We estimate our model using the iterative method for seemingly unrelated equations proposed by Zellner (1962).[8] Because Switzerland can be viewed as a small open economy with no capacity to influence its terms of trade, the endogeneity issue can be ignored. Otherwise, three-stage least squares would have to be considered, provided one could find appropriate instruments. We also assume absence of structural change and hence stable parameters.

We assume that the dependant variable in equation i of any system for the TL or the SNQ functional form is specified with additive disturbance u_i. Specially, we assume that $\mathbf{u} : N\ 0, \Lambda \otimes I_T$, where 0 is a null vector, Λ is the $n \times n$ symmetric positive matrix (n is the number of equations in the system), and I_T is the identity matrix. With this stochastic specification, the corresponding system can be written in matrix form as :

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[7]We can notice that the negative sign associated with the import share is explained by the treatment of imports as intermediate inputs.
[8]See for example Kohli (1978, 1992), Kohli and Werner (1998), or Sharma (2002).

$$\mathbf{F}_t = \mathbf{f}\left(h_t, \varpi\right) + \mathbf{u}_t, \tag{21}$$

where F_t is the vector of explained variables, ϖ is the parameter vector to be estimated, h_t is the vector of exogenous variables and $\mathbf{f}\left(h_t, \varpi\right)$ is the right-hand side of the system equation formed by (3) and (4) for the TL and (19) and (20) for the SNQ. This corresponds to a non-linear system of equations.

The stochastic specification proposed for equation (21) allows for contemporaneous correlation but not for autocorrelation. Autocorrelation of order one of the error term in (21) can be accommodated by writing :

$$\mathbf{u}_t = R\mathbf{u}_{t-1} + \mathbf{e}_t \tag{22}$$

where $R = \left[R_{ij}\right]$ is a matrix of unknown parameters and $\mathbf{e}_t$ is a non-auto-correlated vector of disturbance with constant variance matrix. As discussed in Holt (1998), the matrix R can take many forms. In the case of a system of share equations, as for the TL, Berndt and Savin (1975) showed that the matrix R must be diagonal.

Finally, any system of the form of equation (21) can be represented by the following non-constrained model :

$$\mathbf{F}_t = \mathbf{f}\left(h_t, \varpi\right) + R\mathbf{F}_{t-1} - R\mathbf{f}\left(h_{t-1}, \varpi\right) + \mathbf{e}_t \tag{23}$$

Evidence of autocorrelation can be tested by conducting a likelihood ratio test for the constrained model ($R = 0$) against the auto-correlated alternative with a degree of freedom equal to the number of coefficients in the matrix R.

We estimate the different systems using an iterative version of the method proposed by [32] for seemingly unrelated regression equations, which is equivalent to maximum likelihood estimation. All estimations were performed with the nonlinear command in TSP/Givewin for the Translog functional form and Shazam for the SNQ specification.[9] The starting values for each optimization were chosen randomly.

[9] In Shazam, the form of the matrix *R* can be specified by changing the option of the command nl for "auto" for a diagonal matrix with the same coefficient or for "dhro" for a diagonal matrix with different terms or for "across" for a full rank matrix.

5 Empirical results

5.1 Parameter estimates

Tables 2 and 3 present estimated parameters, goodness of fit statistics, autocorrelation test results and statistics about curvature violations for the Translog and SNQ models.[10] The results compare the two estimated models with and without curvature restrictions. For the two models the coefficient of determination is high (over 0.9) and many parameters are significant. The null of no autocorrelation is rejected regardless of wether regularity properties are imposed or not. Autocorrelation tests when global restrictions are imposed require a diagonal matrix R (degree of freedom $=6$) for the first sub-model and a full matrix for the second (degree of freedom $=36$) in the case of the SNQ model. For the TL, the autocorrelation matrix R is diagonal with two different parameters (degree of freedom $=2$).

5.2 Curvature coverage and selection of date of imposition

Results related to curvature violations are reported in the last two rows of Tables 2 and 3, in Tables 4 and 5, and finally in Figure 1. In the case of SNQ, we check if regularity properties are respected globally i.e. if the estimated A and B matrices are respectively positive and negative semi-definite after imposition. Table 4 confirms that the restrictions on A are global. When the curvature conditions are not imposed, some eigenvalues of the matrix A are negative, thus violating the definition of a positive semi-definite matrix. All eigenvalues become positive or null when the model is restricted. The imposition of curvature conditions does not change anything for matrix B, which has negative eigenvalues in both situations (restricted and unrestricted).

For the TL, the local theoretical restrictions can be imposed at the same date or at different dates. First, we consider imposing restrictions at the same date. Summary results about local violations comparing the restricted model and the unrestricted are reported in Table 2.[11] The performances of the TL and SNQ restricted and unrestricted GNP functions are similar because the imposition of curvature restrictions significantly reduce or completely eliminate

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[10]The system R^2 is computed as in McElroy (1977).

[11]The model was estimated 38 times, corresponding to the local imposition of properties at each date of our sample. Results reported in Table 2 are those obtained for one of the best date of imposition, i.e. 2000.

curvature violations. When the TL model is not restricted, there are 38 convexity and 0 concavity violations. The TL performs much better on concavity relative to the unrestricted SNQ. The imposition of curvature conditions can improve the performance on convexity violations while increasing the number of concavity violations.

Concavity and convexity restrictions can be imposed at different dates. The results of this comparison are reported in Figure 1. The performance of the TL in terms of curvature violations varies a lot depending on the date of imposition. Clearly, one property can be violated more or less frequently than the other. Nonetheless, concavity in factor endowments fails more often than convexity in prices. In addition, the coverage performance is better at the end of the sample. There is much variation in the number of violations between 1970 and 1980.

Table 5 presents the number of violations in curvature conditions associated with the restricted Translog model when convexity and concavity are imposed at different points of observation.[12] In this table, for each pair of years, the first row presents the number of convexity violations, and the second row the concavity violations. The diagonal corresponds to cases when the local imposition of both sets of curvature restrictions is done at the same observation/date. For example, when convexity is imposed in 1974 and concavity in 1976, the number of convexity and concavity violations are respectively 2 and 12. When the two restrictions are imposed in 1974, the number of violations are 1 and 3 for convexity and concavity respectively, compared with 0 and 12 when the restrictions are imposed in 1976. In all, we have 4 violations when the convexity and concavity restrictions are imposed in 1974, which is far less than the 12 violations when both sets of restrictions are imposed in 1976 and the 14 violations when the dates of imposition of convexity and concavity are allowed to differ.

Based on the evidence produced from our dataset, it seems that practitioners have potentially much to gain by searching for the best date at which to impose both sets of restrictions if we consider only the number of remaining violations once local restrictions are imposed. However, the gains from searching for the optimal combination of date may not be worth the effort given the small difference in total violations between the "best" date along the diagonal and the "best" date off the diagonal.

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[12] We chose to present the results for the period 1970 to 1980 because the results for 1981 to 2007 provide similar

5.3 Curvature coverage and elasticities

To better understand the consequences of imposing theoretical properties at the same points in the case of the Translog model, we now compare estimated elasticities when concavity and convexity restrictions are imposed at two different dates, 1978 and 2000.[13,14] We saw previously that imposing local curvature restrictions at 1978 led to a maximum overall number of violations of 29, with 27 for concavity and 2 for convexity while imposing concavity and convexity at year 2000 led to zero violation at other years in the sample. Table 6 presents elasticities derived from the Translog model when curvature restrictions are imposed at 1978 and 2000. All elasticities are evaluated at each year and we report the mean and standard error of each elasticity type for the entire sample (1970- 2007).

The first part of Table 6 shows price elasticities of import demand and output supply. The own-price elasticities of the different goods have the expected sign and are surprisingly similar regardless of the date of imposition. Even if the cross-price elasticities have the same sign and their difference across date of curvature imposition is smaller, as illustrated by the cross-price elasticities involving imports and domestic consumption, they are all significantly different.[15]

The quantity elasticities of inverse factor demands are low and robust in sign to the date of imposition of the theoretical properties. However, in absolute value the elasticities are clearly lower when the restrictions are imposed at year 1978. The differences are not surprising considering that concavity in factor endowments violations remain high when curvature restrictions are imposed at 1978. Table 3 shows that imposition of concavity affected some estimated parameters more than others, but it was impossible to infer how elasticities would be impacted.

The third part of Table 6 illustrates the effect of a change in factor endowment on

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insights. Naturally, they are available upon request.

[13]We do not present the analysis for the case where theoretical properties are imposed at different dates because the results are qualitatively the same.

[14]A more complete and supplementary discussion of elasticities is proposed in the next subsection when comparing elasticities for the TL and SNQ.

[15]We conducted a matched pairs test of the mean difference for each elasticity with curvature restrictions imposed locally at 1978 and 2000.

import demand and output supply. Regardless of the date of imposition of the theoretical properties (1978 or 2000), the sign and the size of the corresponding Rybczynski elasticities are similar, but the elasticities are estimated with sufficient precision to be statistically different. Globally, consumption goods are affected with the same magnitude by an increase of the quantity of capital or labour used while imports and investments are mostly affected by changes in capital and exports by changes in labour.

The fourth and last part of Table 6 displays the Stolper-Samuelson elasticities which differ considerably depending the year at which theoretical properties are imposed (1978 vs 2000). The cost of labour is sensitive to increase in each output price, particularly those fof exports. As anticipated, an increase in the import price reduces the price of capital, while increases in other output prices (principally those of investment goods) have the opposite effect.

What can be learned from these comparisons? As confirmed by matched pairs tests, there are significant differences between elasticities estimated with local curvature restrictions imposed at different dates. We repeated the comparisons with different dates and obtained similar results. The good news is that the differences between elasticities are often quite small, at least for the Swiss data. Thus, the search for an appropriate date for local curvature restrictions must be

5.4 Elasticities and Functional forms : Translog vs SNQ

We evaluate the robustness of elasticities to the choice of functional form which is tantamount to assessing the method of imposing curvature restrictions (global vs local). Table 7 reports the means and the standard errors for the four group of elasticities associated with the TL and SNQ GNP functions. The size and the sign of the elasticities are consistent with those reported by Kohli (1992), who also used Swiss data and Kohli (1993; 1991, p. 236).[16] In what follows, we define ε_{ij}^{f} as the elasticity of variable i with respect to the change in variable j, estimated with functional form f; $i, j = M, X, I, C, K, L$ and $f = SNQ, TL$.

[16] We also looked at the evolution of each elasticity over time and generally, the estimated elasticities are significant, have the expected sign and tend to be larger in absolute value near the end of the sample. The corresponding 31 figures are available upon request.

Own-price elasticities for import demand and output supply, $\varepsilon_{ii}^{f} \quad i = M, X, I, C$, have the expected sign (negative for imports and positive for others) and are significant. In general absolute values of SNQ own-price elasticities are inferior to their Translog counterparts, which means that outputs are more sensitive to their own-price movement according to the Translog. This outcome was also reported by Kohli(1991, p.236) when comparing different functional forms. Table 7 shows that own-price elasticities are positive for export, investments and consumption and negative for imports at all the sample dates for both models.

Cross-price elasticities are useful to analyse substitutability of outputs. The SNQ and TL cross-price elasticities between imports and exports are qualitatively different as the sign of ε_{MX} is negative in the case of the SNQ and positive in the case TL. For ε_{XM}, the signs are inverted, positive for the SNQ and negative for the TL. A positive (negative) sign means that an increase in the price of imports increases (decreases) the supply of exports. The signs of ε_{MX}^{SNQ} and ε_{XM}^{SNQ} are consistent with Kohli (1993; 1991, p.236) and the sign of ε_{MX}^{TL} and ε_{XM}^{TL} are consistent with Kohli (1992). The cross-price elasticity ε_{MI}^{SNQ} is positive and ε_{XI}^{SNQ} is negative, showing that an increase in the price of investment stimulates the demand for imports and reduces export supply, but ε_{IM}^{SNQ} is negative and ε_{IX}^{SNQ} is positive, implying that increases in the prices of imports and exports respectively decrease and increase the demand of investment. Kohli (1993; 1991, p.236) found similar results. For the TL, cross-price elasticities of imports with respect to consumption are very small and fluctuate around zero over the sample period. Our estimation also shows that investment is a substitute for imports ($\varepsilon_{MI} > 0$ and $\varepsilon_{IM} < 0$) and a complement for consumption ($\varepsilon_{IC} < 0$ and $\varepsilon_{CI} < 0$). Those relations are consistent with Kohli (1993; 1991, p.236) and economically logical. Imports are considered as intermediate inputs in our model and investment tends to be correlated to domestic demand. However, the relationship between consumption and investment goods is not robust across flexible functional forms. The signs of ε_{IC} and ε_{CI} are negative for the TL form and consistent with the literature while for the SNQ form, the signs of ε_{IC} and ε_{CI} are both positive, meaning that both goods are substitutes for each other.

Table 7 also presents the effect of variations in factor endowment on input prices, for a given price of output. For either model, it is clear that an increase in a factor endowment decreases its own price and increases the price of the other factor. As discussed in the previous subsection, TL quantity elasticities for inverse factor demand are near zero and appear to be affected by the imposition of curvature restrictions.

The third part of Table 7 indicates average elasticities for the effect of a variation in factor endowment on the different outputs for both models. According to Rybczynski elasticities labour has a small impact on import demand and investments and a very small effect on consumption for the SNQ specification. For both TL and SNQ, outputs are much more sensitive to capital than to labor except for exports. In fact, the production of imports is not statistically affected by an increase in labor endowment. In the SNQ model, an increase in capital endowment has a negative effect on the production of exports. In addition, increases in labour endowment have an impact on exports particularly in the case of the SNQ form ($\varepsilon_{XL}^{SNQ} \geq \varepsilon_{XL}^{TL}$). The impact of an increase in capital endowment on exports is negative for the SNQ and positive for the TL. However, an increase in capital endowment induces increases in investment and consumption regardless of the functional form.

The fourth part of Table 7 contains the average elasticities sizing up the effects of output price increases on the factor prices for a given factor endowment. These elasticities are called Stolper-Samuelson elasticities (SS). By reciprocity, the signs of Stolper-Samuelson elasticities reflect those of Rybczynski elasticities (RR) and inverse factor demand elasticities. An increase in the price of imports decreases the price of capital and the price of labour in the SNQ case but it has no impact in the TL case. Moreover, in the TL case, labour and capital gain from an increase in the price of exports, investment and consumption. In the SNQ case, investment and consumption favour capital, but an increase in the price of exports is damaging for capital owners and favourable to labour. For both TL and SNQ, capital owners (labour) benefit most from increases in the price of investment (exports).

The estimates of ε_{LX} elasticities are all positive and thus have the same sign as in Kohli (1992; 1993; 1991, p.236). This means that an increase in the price of exports increases demand for labour relatively more than that of capital and hence increases the wage rate. However, $\varepsilon_{LX}^{SNQ} > \varepsilon_{LX}^{TL}$ means that the effect of an increase in price of exports is greater for the SNQ than for the TL. Regarding the incidence of increases in the prices of consumption and investment goods on the wage rate, the signs are all positive meaning that consumption and investment goods are "friends" of labour. These results are consistent with Kohli (1993; 1991, p.236). The price of labour is more sensitive to the consumption price under the TL specification ($\varepsilon_{LC}^{SNQ} \leq \varepsilon_{LC}^{TL}$). For capital, the impact of an increase in the price of imports is negative and quite similar for the SNQ and TL forms. This contrasts with the impact of an increase in the price of exports on the price of capital because the sign of TL and SNQ estimated elasticities differ. For the SNQ, the elasticities have negative sign and are significant, thus showing that an increase in the price exports reduces the price of capital. These results are consistent with Kohli (1993; 1991, p.236). The sign of the TL elasticity is positive as in Kohli (1992), hence indicating that rising export prices tend to inflate the rental rate. A similar outcome is observed for an increase in the price of investment goods, but the effect is stronger in the case of SNQ than for the TL ($\varepsilon_{KI}^{SNQ} \geq \varepsilon_{KI}^{TL}$). The sign differences in the TL and SNQ elasticities are most worrisome because they lead to opposite qualitative interpretations.

6 Forecasting evaluation

In the GNP function estimation literature, little attention has been paid to forecasting performances of different models. The objective of this section is to evaluate the predictive accuracy of the two flexible functional forms when curvature conditions are imposed. First we assess the predictive accuracy of each model and second, we test whether the difference in the predictive performance of the TL and the SNQ is significant. To evaluate the predictive accuracy, we rely on the two most popular measures, the Mean Square Error (MSE) and the Mean Absolute Error (MAE). Our statistical inference is based on the Diebold and Mariano (1995) test.[17] Due to the relative small size of our sample, we focus on in-sample comparisons.

Let s_t denote the series to predict and s_t^{TL} and s_t^{SNQ} the two competing predictions associated with the TL and SNQ forms. The forecast errors are : $e_t^{TL} = s_t - s_t^{TL}$ and $e_t^{SNQ} = s_t - s_t^{SNQ}$

[17] In demand system estimation for example, MSE and MAE criteria have been used by Kastens and Brester (1996) for forecasting performance comparison while Wang and Bessler (2003) used Diebold-Mariano test for predictive accuracy assessment.

with $t = 1,...,T$ and T is the sample size. The predictive accuracy of each series forecast is assessed by a loss function g which can be the MSE or MAE measure such as : $g(s_t, s_t^i) = g(e_t^i), i = TL, SNQ$. The model with the lowest loss value is the best in terms of in-sample forecasting performance.

The results about predictive accuracy based on the MSE and MAE measures are reported in Table 8. In the case of share equations, except for investment, the results indicate that the TL form has better in-sample predictive accuracy performance. This may be explained by the fact that for the TL form, we directly estimate the share equations whereas for the SNQ form, share are computed after estimation.

To validate our conclusions based on the predictive accuracy measures, we tested for significant differences in forecast accuracy with the Diebold and Mariano (1995) test statistic.[18] The null hypothesis of equal forecast accuracy for the two models is : $E(g(e_t^A)) = E(g(e_t^B))$ or $E(d_t) = 0$, where $d_t = g(e_t^A) - g(e_t^B)$ is the loss differential. The Diebold-Mariano test statistic is:

$$DM = [\hat{V}(\bar{d})]^{-1/2}\bar{d} \tag{24}$$

where $\bar{d}$ is the sample mean of the series d_t and $\hat{V}(\bar{d})$ the asymptotical variance of d_t. Over the null hypothesis of equal predictive accuracy, DM converges to a standard unit normal distribution. However, to avoid small-sample bias in the estimation of the variance of the mean loss differential, Harvey et al. (1997) proposed a modified Diebold-Mariano test statistic denoted DM*. For a general h-step ahead forecasting comparison, the modified Diebold-Mariano Test statistic is :

$$DM^* = [\frac{T + 1 - 2h + T^{-1}h(h-1)}{T}]^{-1/2} DM \tag{25}$$

with critical values readily available from the t-distribution with $T-1$ degrees of freedom.

We computed the modified Diebold-Mariano tests for the different share equations and the output supply and inverse demand equations. The results are presented in Tables 10 and 11

[18] The main advantage of this test is that it can be applied with many loss functions, and few hypotheses about the asymptotic distribution of forecast errors are needed.

with h=1 for a one-year horizon. For the share equations, the p-values for the consumption, capital and labour shares confirm that the TL outperforms the SNQ form regardless of the loss function chosen. Differences in forecasting performance for exports and investment are not significant. For the import share equation, when the MAE measure is used, the TL forecasting performance is superior to the SNQ, and the MSE criterion fails to reject equality in the predictive accuracy of the two models. Even if the predictive accuracy evaluation found that the TL is superior to the SNQ for 5 of 6 predicted shares, the Diebold-Mariano test showed that this domination is significant only for 3 shares: consumption, capital and labour.

For the output supply and inverse demand equations, the predictive accuracy evaluation shows that the SNQ is superior to the TL for import demand, export supply and inverse demand for capital. However, the TL is the best predictive model for consumption supply and the inverse demand for labour (see Table 9). The p-values show that the SNQ significantly outperforms the TL for the exports supply and inverse demand for capital, while for the consumption supply the SNQ is outperformed by the TL. For import demand and investment supply the differences are not significant. In the case of the inverse labour demand, the TL outperforms the SNQ for the MAE criteria and the equality of the predictive accuracy performances could not rejected for the MSE criteria. Nevertheless, we can conclude that the TL weakly dominates the SNQ in terms of predictive accuracy.

7 Conclusion

In this paper, we present an empirical investigation of the imposition of curvature conditions when estimating the GNP function with the Translog (TL) and the Symmetric Normalized Quadratic (SNQ) flexible functional forms over Switzerland aggregate economic data. The convexity in output prices and the concavity in factor endowments are curvature restrictions that can be imposed locally in the TL case and globally in the SNQ case.

We investigated the TL performance in terms of remaining curvature violations when the local restrictions are imposed at different dates. In the unrestricted TL, concavity in factor endowments was violated at all points, but there were no violations of the convexity in output prices property. The number of concavity violations remained high for several dates of local restrictions. This suggests that a search might be warranted, particularly if elasticities are to be reported at several points. Clearly, if the intent is to minimize the total number of curvature violations, finding an appropriate date is crucial. Our TL results show that the number of violations can decline from 29 for 1978 to 0 for 2000. Imposing local concavity restrictions and local convexity restrictions at different dates would be worthwhile if "too many" violations remained when forcing concavity and convexity to hold at the same date. However, we found that the gains in terms of curvature violations were small and that focussing on common dates was sufficient. Thus, the TL and SNQ can be equivalent in terms of curvature violations and other criteria must be used for comparison purposes.

Regarding the effect the date of imposition of curvature violations on TL elasticities, the latter turned up to be quite robust to changes in the year of imposition, even though the observed “small” differences were statistically significant. In contrast, the sign and magnitude of elasticities were not robust across functional forms. However, in terms of in-sample predictive accuracy based on the Mean Square Errors and Mean Absolute Errors loss functions, the TL performed better than the SNQ for 5 of 6 predicted shares. In addition, according to the test of equal forecast accuracy proposed by Diebold and Mariano (1995), the TL model significantly outperformed the SNQ form for 3 shares : consumption, capital and labour. In the case of the output supply and inverse demand equations, the SNQ significantly outperformed the TL for export supply and inverse demand for capital, while for consumption supply the TL was significantly better than the SNQ.

Our results suggest that a flexible form that allows for the global imposition of curvature restrictions may not be the best choice, contrary to the recommendation of Feng and Serletis (2007). When the number of curvature violations associated with a TL can be eliminated by choosing an appropriate date for the imposition of local curvature conditions, one must rely on the plausibility of elasticities and/or forecast accuracy to make a determination. In our application on Swiss data, the TL turned out to be a better choice than the SNQ.

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Appendix A : Data construction

As in most empirical studies using the GNP function, variables were constructed following some recommendations from Kohli (1991, chap 8). Working at an aggregate level, to evaluate domestic production or GNP, we used the income and expenditure approaches. We could thus obtain series not found in the OECD Main Economics Indicator database more easily. According to the income approach, the gross national product is equal to the sum of the value of exports and national expenditures minus imports. Imports and exports are evaluated net of foreign transfers. The value or volume of imports (M), exports (X), domestic consumption (C) and total investment I and the existing related prices (P_M, P_X, P_C and P_I) were extracted from the MEI database. To estimate the missing series, we use the fact that value is the product of volume and price.

In the second approach, we assume that the Gross National Product can be calculated as the sum of the factor remuneration (capital and labour). After approximating the remuneration of labour by the total salary paid per year in the economy, we obtained the amount of the remuneration of the capital by simple deduction with the first approach. We extracted series of confederation bond interest rates and used it as the price of capital (P_K) and we estimated the volume or quantity of capital (K). We used the unit labour cost of the economy as the price of labour (P_L) and by deflation, we obtained the volume of labour (L).

Finally, our data verified the following identity :

$$P_X X - P_M M + P_I C + P_I I = P_K K + P_L L.$$

Appendix B :Tables and Figure

Table 1: Descriptive statistics for GNP function variables (1970-2007)

Variable	Average	Min	Max
Prices :			
Import Price	92.2	69.1	105.3
Export Price	87.3	56.8	106.3
Consumption good Price	78.2	35.4	106.4
Investment Price	86.0	51.5	109.7
Quantities :			
Capital Quantity	106 220	33 095	252 120
Labour Quantity	214 380	162 190	290 900
Shares :			
Export Shares	−0.274	−0.327	−0.194
Import Share	0.271	0.235	0.317
Consumption good Share	0.732	0.641	0.791
Investment Share	0.272	0.220	0.361
Capital Share	0.382	0.317	0.487
Labour Share	0.618	0.513	0.683

Table 2: Translog Parameters estimates

Parameters'	Unrestricted	Curvature imposed locally
α_1	-0.2704 (0.0334)	-0.2841 (0.0118)
γ_{11}	-0.3277 (0.0973)	-0.1124 (0.0942)
γ_{12}	0.1457 (0.0933)	-0.0219 (0.0777)
γ_{13}	0.0580 (0.0719)	0.0203 (0.0499)
δ_{15}	-0.0067 (0.0271)	0.1070 (0.0238)
δ_{1t}	-0.0015 (0.0015)	-0.0038 (0.0012)
α_2	0.2760 (0.0222)	0.2533 (0.0133)
γ_{22}	-0.0070 (0.1359)	0.2472 (0.1275)
γ_{23}	0.0568 (0.0774)	-0.1605 (0.0507)
δ_{25}	0.0195 (0.0213)	0.0197 (0.0154)
δ_{2t}	-0.0003 (0.0011)	0.0014 (0.0007)
α_3	0.7547 (0.0283)	0.7678 (0.0076)
γ_{33}	0.0191 (0.0669)	0.3929 (0.0414)
δ_{35}	0.0035 (0.0280)	0.0192 (0.0196)
δ_{3t}	0.0012 (0.0011)	0.0006 (0.0007)
β_5	0.3388 (0.0168)	0.3071 (0.0106)
ϕ_{55}	0.0130 (0.0174)	0.2028 (0.0041)
ϕ_{5t}	-0.0032 (0.0005)	-0.0066 (0.0007)
R^2	0.9718	0.9522
Autocorrelation test ($R = 0$ vs $R \neq 0;$): $\chi^2_{.99}(2) = 9.21$		
Test statistics	22.92	15.61
Concavity violations	0	0
Convexity violations	38	0

Note : Standard errors in parentheses.

Table 3: Symmetric Normalized Quadratic Parameters estimates

Parameters'	Unrestricted	Curvature imposed globally
a_{11}	0.0303 (0.2329)	0.2411 (0.1051)
a_{12}	0.2735 (0.2608)	0.0862 (0.0612)
a_{13}	−0.4551 (0.1307)	−0.2656 (0.0879)
a_{22}	−0.5905 (0.4242)	0.0476 (0.0718)
a_{23}	0.3401 (0.2365)	−0.1101 (0.0903)
a_{33}	0.1536 (0.1597)	0.3061 (0.2296)
b_{55}	−0.0902 (0.0401)	−0.2371 (0.3146)
c_{15}	0.2506 (0.1016)	−0.1138 (0.2371)
c_{16}	−0.3597 (0.1195)	−0.4287 (0.1216)
d_{15}	0.0084 (0.0096)	−0.0011 (0.0082)
d_{16}	−0.0043 (0.0057)	−0.0054 (0.0048)
d_{tt}	−0.0000 (0.0005)	−0.0005 (0.0008)
c_{25}	0.1119 (0.1356)	0.6279 (0.3511)
c_{26}	0.0326 (0.1635)	−0.1094 (0.2485)
d_{25}	−0.0385 (0.0154)	−0.0384 (0.0173)
d_{26}	−0.0012 (0.0065)	−0.0004 (0.0119)
c_{35}	−0.0116 (0.0501)	0.0249 (0.0530)
c_{36}	1.1820 (0.0361)	1.167 2 (0, 0454)
d_{35}	0.0053 (0.0058)	0.0021 (0.0125)
d_{36}	−0.0075 (0.0032)	−0.0066 (0.0035)
c_{45}	0.1418 (0.0631)	0.0055 (0.2004)
c_{46}	0.2883 (0.0461)	0.4254 (0.2421)
d_{45}	−0.0134 (0.0048)	−0.0005 (0.0044)
d_{46}	0.0029 (0.0027)	−0.0007 (0.0072)
R^2	0.97	0.99
Autocorrelation test ($R = 0$ vs $R \neq 0$):	$\chi^2_{.99}(6) = 16.81; \chi^2_{.99}(36) = 54.78$	
Test statistics	43.2	133.8
Concavity violations	36	0
Convexity violations	38	0

Note : Standard errors in parentheses.

Table 4: Eigenvalues for matrix A and B

Matrix A		Matrix B	
Unrestricted	Restricted	Unrestricted	Restricted
0.5816	0.5939	0.0000	0.0000
0.0000	0.0169	−0.1804	−0.4742
−0.1466	0.0000	−	−
−0.9307	0.0000	−	−

Table 5: Numbers of curvature violations depending of the date of imposition (Curvature imposed at the different date)

		Date of imposition of concavity										
Date of imposition of convexity	Theoretical properties	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980
1970	Convexity	0	0	0	0	0	0	0	0	0	0	0
	Concavity	15	18	23	14	3	3	12	24	27	27	15
1971	Convexity	3	2	2	1	1	1	3	3	3	3	3
	Concavity	15	18	23	14	3	3	12	24	27	27	15
1972	Convexity	3	3	3	3	3	3	3	4	4	4	4
	Concavity	15	18	23	14	3	3	12	24	27	27	15
1973	Convexity	4	4	4	3	3	3	4	5	5	5	4
	Concavity	15	18	23	14	3	3	12	24	27	27	15
1974	Convexity	2	2	2	0	1	0	2	3	3	3	3
	Concavity	15	18	23	14	3	3	12	24	27	27	15
1975	Convexity	0	0	0	0	0	0	0	0	0	0	0
	Concavity	15	18	23	14	3	3	12	24	27	27	15
1976	Convexity	0	0	0	0	0	0	0	0	0	0	0
	Concavity	15	18	23	14	3	3	12	24	27	27	15
1977	Convexity	0	0	0	0	0	0	0	0	0	0	0
	Concavity	15	18	23	14	3	3	12	24	27	27	15
1978	Convexity	29	1	1	0	0	1	2	2	2	2	2
	Concavity	0	18	23	12	3	3	12	24	27	27	15
1979	Convexity	19	19	29	0	0	0	0	0	0	0	0
	Concavity	0	0	0	12	3	3	12	24	27	27	15
1980	Convexity	0	0	0	0	0	0	0	0	0	0	0
	Concavity	15	18	23	12	3	3	12	24	27	27	15

Table 6: Average elasticities for Translog functional forms with curvature restrictions imposed at 1978 and 2000

	Restrictions imposed at 1978		Restrictions imposed at 2000	
Elasticities	Value	Standard Error	Value	Standard Error
i) Price elasticities of import demand and output supply				
ε_{MM}	−0.8961	0.0641	−0.8603	0.0678
ε_{MX}	0.2025	0.0220	0.1902	0.0229
ε_{MI}	0.7065	0.0365	0.6568	0.0370
ε_{MC}	−0.0129	0.0614	0.0133	0.0593
ε_{XM}	−0.2052	0.0280	−0.1929	0.0285
ε_{XX}	0.2133	0.0442	0.1875	0.0425
ε_{XI}	0.1319	0.0495	0.1364	0.0493
ε_{XC}	−0.1400	0.0502	−0.1310	0.0499
ε_{IM}	−0.2647	0.0256	−0.2462	0.0255
ε_{IX}	0.0492	0.0198	0.0509	0.0198
ε_{II}	0.2882	0.0075	0.2701	0.0084
ε_{IC}	−0.0727	0.0290	−0.0748	0.0289
ε_{CM}	0.0165	0.0581	−0.0102	0.0546
ε_{CX}	−0.1468	0.0612	−0.1377	0.0599
ε_{CI}	−0.2137	0.1063	−0.2193	0.1071
ε_{CC}	0.3440	0.1089	0.3672	0.1121
ii) Quantity elasticities of inverse factor demands				
ε_{LL}	−0.0502	0.0178	−0.0816	0.0148
ε_{LK}	0.0502	0.0178	0.0816	0.0148
ε_{KL}	0.0329	0.0160	0.0522	0.0173
ε_{KK}	−0.0329	0.0160	−0.0522	0.0173
iii) Rybczynski elasticities				
ε_{ML}	−0.0555	0.0677	−0.0123	0.0641
ε_{MK}	1.0555	0.0677	1.0123	0.0641
ε_{XL}	0.6316	0.0385	0.6342	0.0386
ε_{XK}	0.3684	0.0385	0.3658	0.0386
ε_{IL}	0.0811	0.0269	0.0986	0.0275
ε_{IK}	0.9189	0.0269	0.9014	0.0275
ε_{CL}	0.5060	0.0276	0.4994	0.0281
ε_{CK}	0.4940	0.0276	0.5006	0.0281
iv) Stolper-Samuelson elasticities				
ε_{LM}	0.0401	0.0460	0.0091	0.0434
ε_{LX}	0.4495	0.0215	0.4514	0.0216
ε_{LI}	0.1513	0.0325	0.1851	0.0298
ε_{LC}	0.3591	0.0360	0.3545	0.0364
ε_{KM}	−0.4668	0.0326	−0.4478	0.0317
ε_{KX}	0.1614	0.0175	0.1602	0.0175
ε_{KI}	1.0877	0.0191	1.0669	0.0197
ε_{KC}	0.2178	0.0396	0.2206	0.0397

Table 7: Average elasticities for Translog and SNQ functional forms

	Translog		SNQ	
Elasticities	Value	Standard Error	Value	Standard Error
i) Price elasticities of import demand and output supply				
ε_{MM}	−0.8603	0.0678	−0.4854	0.0517
ε_{MX}	0.1902	0.0229	−0.1703	0.0206
ε_{MI}	0.6568	0.0370	0.4198	0.1184
ε_{MC}	0.0133	0.0593	0.0972	0.0284
ε_{XM}	−0.1929	0.0285	0.1741	0.0271
ε_{XX}	0.1875	0.0425	0.0868	0.0149
ε_{XI}	0.1364	0.0493	−0.1385	0.0885
ε_{XC}	−0.1310	0.0499	−0.0205	0.0308
ε_{IM}	−0.2462	0.0255	−0.1539	0.0317
ε_{IX}	0.0509	0.0198	−0.0492	0.0305
ε_{II}	0.2701	0.0084	0.1864	0.0287
ε_{IC}	−0.0748	0.0289	0.0504	0.0058
ε_{CM}	−0.0102	0.0546	−0.1009	0.0347
ε_{CX}	−0.1377	0.0599	−0.0235	0.0309
ε_{CI}	−0.2193	0.1071	0.1369	0.0191
ε_{CC}	0.3672	0.1121	0.0526	0.0227
ii) Quantity elasticities of inverse factor demands				
ε_{LL}	−0.0816	0.0148	−0.2275	0.0930
ε_{LK}	0.0816	0.0148	0.2275	0.0930
ε_{KL}	0.0522	0.0173	0.1343	0.0398
ε_{KK}	−0.0522	0.0173	−0.1343	0.0398
iii) Rybczynski elasticities				
ε_{ML}	−0.0123	0.0641	0.0540	0.0898
ε_{MK}	1.0123	0.0641	1.0592	0.0798
ε_{XL}	0.6342	0.0386	1.1947	0.2639
ε_{XK}	0.3658	0.0386	−0.3789	0.0877
ε_{IL}	0.0986	0.0275	0.0116	0.0183
ε_{IK}	0.9014	0.0275	0.9893	0.0287
ε_{CL}	0.4994	0.0281	0.0534	0.0124
ε_{CK}	0.5006	0.0281	0.9947	0.0854
iv) Stolper-Samuelson elasticities				
ε_{LM}	0.0091	0.0434	−0.0410	0.0666
ε_{LX}	0.4514	0.0216	0.8361	0.1620
ε_{LI}	0.1851	0.0298	0.0266	0.0384
ε_{LC}	0.3545	0.0364	0.0380	0.0098
ε_{KM}	−0.4478	0.0317	−0.4704	0.0524
ε_{KX}	0.1602	0.0175	−0.1655	0.0420
ε_{KI}	1.0669	0.0197	1.1727	0.0631
ε_{KC}	0.2206	0.0397	0.4369	0.0588

Table 8: Predictive accuracy performances of the two Flexible Functional Forms : share equations

Share/Model	MSE ($\times 10^2$)	MAE
Imports		
TL	0.0536	0.0159
SNQ	0.0824	0.0225
Exports		
TL	0.0281	0.0129
SNQ	0.0368	0.0145
Consumption		
TL	0.0430	0.0159
SNQ	0.1311	0.0293
Investments		
TL	0.0483	0.0175
SNQ	0.0338	0.0126
Capital		
TL	0.0316	0.0130
SNQ	0.0974	0.0262
Labour		
TL	0.0317	0.0131
SNQ	0.0974	0.0262

Table 9: Predictive accuracy performances of the two Flexible Functional Forms : Output supply and inverse factor demand

Variables	MSE	MAE
Output supply		
Imports		
TL	0.0047	0.0491
SNQ	0.0077	0.0489
Exports		
TL	0.0080	0.0673
SNQ	0.0031	0.0400
Consumption		
TL	0.0007	0.0199
SNQ	0.0053	0.0537
Investments		
TL	0.0040	0.0496
SNQ	0.0035	0.0487
Inverse factor demand		
Capital		
TL	0.0258	0.1275
SNQ	0.0023	0.0366
Labour		
TL	0.0001	0.0088
SNQ	0.0005	0.0148

Table 10: Diebold and Mariano equality test results : share equations

Share	MSE	MAE
Imports	1.4160	2.0560
p-value	0.1651	0.0468
Exports	1.1369	0.7886
p-value	0.2629	0.4353
Consumption	3.0761	3.8692
p-value	0.0039	0.0004
Investments	−0.9776	−1.8200
p-value	0.3346	0.0768
Capital	3.3647	3.9581
p-value	0.0018	0.0003
Labour	3.3646	3.9580
p-value	0.0018	0.0003

Table 11: Diebold and Mariano equality test results : Output supply and inverse factor demand

Variables	MSE	MAE
Output supply		
Imports	0.0600	0.1074
p-value	0.9531	0.9150
Exports	−3.1655	−3.0797
p-value	0.0031	0.0039
Consumption	2.1831	3.8091
p-value	0.0354	0.0005
Investments	−0.4439	−0.1131
p-value	0.6597	0.9105
Inverse factor demand		
Capital	−3.9555	−5.5462
p-value	0.0003	0.0000
Labour	1.2773	2.0509
p-value	0.2094	0.0474

Figure 1: Number of curvature violations depending of the date of imposition(Curvature imposed at the same date)