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**The Value of Multivariate Model Sophistication:  
An Application to pricing Dow Jones Industrial  
Average Options**

*Jeroen V.K. Rombouts, Lars Stentoft, Francesco Violante*

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# The Value of Multivariate Model Sophistication: An Application to pricing Dow Jones Industrial Average Options<sup>\*</sup>

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## Résumé / Abstract

We assess the predictive accuracy of a large number of multivariate volatility models in terms of pricing options on the Dow Jones Industrial Average. We measure the value of model sophistication in terms of dollar losses by considering a set of 248 multivariate models that differ in their specification of the conditional variance, conditional correlation, and innovation distribution. All models belong to the dynamic conditional correlation class which is particularly suited because it allows to consistently estimate the risk neutral dynamics with a manageable computational effort in relatively large scale problems. It turns out that the most important gain in pricing accuracy comes from increasing the sophistication in the marginal variance processes (i.e. nonlinearity, asymmetry and component structure). Enriching the model with more complex correlation models, and relaxing a Gaussian innovation for a Laplace innovation assumption improves the pricing in a smaller way. Apart from investigating directly the value of model sophistication in terms of dollar losses, we also use the model confidence set approach to statistically infer the set of models that delivers the best pricing performance.

**Mots clés/Keys words** : Option pricing, economic loss, forecasting, multivariate GARCH, model confidence set.

**Codes JEL** : C10, C32, C51, C52, C53, G10

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# 1 Introduction

Multivariate volatility models have been used extensively to model financial data such as stock returns, interest rates, and exchange rates. The resulting dynamics take into account the interaction and correlation between assets and are therefore more realistic. Forecasts from these models are typically needed as inputs in empirical asset pricing. For example, there is quite a large body of work on evaluating a small number of low dimensional models in terms of portfolio management, see e.g. Fleming, Kirby, and Ostdiek (2001), Fleming, Kirby, and Ostdiek (2003), Engle and Colacito (2006), Voev (2009), and Chiriac and Voev (2011). Much less attention has been devoted to the evaluation of multivariate volatility models in the context of derivative pricing where neglecting the correlation may lead to severe mispricings. The main reason for this likely is the lack of a flexible general framework. For example, the existing applications of discrete time models in e.g. van den Goorbergh, Genest, and Werker (2005) and Zhang and Guégan (2008) are limited to low dimensional, i.e. bivariate, models and the multivariate stochastic volatility models of e.g. Gouriéroux and Sufana (2010) and Da Fonseca, Grasselli, and Tebaldi (2007) are inherently very complex and rely heavily on having option data available for calibration. In a recent paper, Rombouts and Stentoft (2011) fill this gap by directly modeling the dynamics of the underlying stocks using multivariate models for asset returns. They demonstrate the existence of an equivalent martingale measure, characterize the risk neutral dynamics, and provide a feasible way for pricing options.

This paper investigates the value of model sophistication by considering a large number of high dimensional GARCH models and measuring performance in terms of Dow Jones Industrial Average (DJIA) option pricing accuracy. This approach is appealing because we get immediately a sense of the short, medium, and long run forecasting behavior of the models, rather than focussing on specific horizons. Furthermore, since the option price does not only depend on the conditional variance matrix but also on other conditional moments, we are actually able to evaluate the full forecasting model, i.e. level, variance, and innovation distribution. The setup we work with specifies the multivariate risk neutral return distribution and therefore allows to price any option written on the vector of underlying assets. We choose to price the index option for which true prices are available, so that we can measure the models' pricing accuracy by contrasting predicted option prices with the observed ones. The accuracy of the pricing is evaluated by means of several loss functions like in Hansen and Lunde (2005)

for example. Apart from investigating directly the value of model sophistication by ordering the models according to their pricing performances, we also use the model confidence set (MCS) approach of Hansen, Lunde, and Nason (2011), to statistically infer the set of best models.

In the empirical option pricing literature, there is strong evidence that models beyond standard Gaussian dynamic volatility models can substantially improve pricing performance when using univariate models. Examples in the continuous time stochastic volatility context are among others Bakshi, Cao, and Chen (1997), Bates (2000), Bates (1991), Eraker (2004), and Pan (2002). More advanced discrete time models have been used in Christoffersen and Jacobs (2004), Christoffersen, Heston, and Jacobs (2006), Christoffersen, Jacobs, Ornathanalai, and Wang (2008), Heston and Nandi (2000), Hsieh and Ritchken (2005), Stentoft (2005), and Stentoft (2008). Our aim is to identify the degree of multivariate model sophistication required to obtain the most accurate pricing. In principle, the higher the model complexity the higher the precision of the pricing, thus its economic value measured in dollars. However, as argued by Giacomini and White (2006), in real data applications more sophisticated models may overfit the data, and thus be outperformed by (possibly) misspecified but more parsimonious models characterized by a lower estimation uncertainty. In this paper, we estimate 248 multivariate GARCH models differing in three dimensions: their specification of the conditional variance, conditional correlation, and innovation distribution. We focus on dynamic conditional correlation (DCC) models which is not coincidental. In fact, being characterized by idiosyncratic risk premia, the risk neutral dynamics for the vector of underlying asset returns provided in Rombouts and Stentoft (2011) allow for the usual factorization of the likelihood of these models, see Engle (2002). Thus, the models can be consistently estimated in two steps (first the variances and then the correlation matrix) rendering the estimation feasible in the thirty dimensional setting of the DJIA constituents.

With respect to the model for the conditional variances of the marginal processes, we consider the entire universe of suitable univariate GARCH models available in the literature, see Xekalaki and Degiannakis (2010) and Bollerslev (2010) for extensive surveys. By doing this, we are able to quantify the economic value of different strategies in modeling the conditional variance, i.e., i) as a linear versus non-linear function of the squared innovations, ii) modeling asymmetry versus the symmetric response of the variance to the sign of the shocks, iii) di-

rect specification of the conditional variance versus a functional transformation, iv) constant versus time varying parameters, and v) single components versus multi-components models. In terms of correlation dynamics, we first aim to assess whether allowing for simple common dynamics in correlation outperforms the hypothesis of constant correlation. Then, we further increase the complexity of the models by allowing for idiosyncratic dynamics (i.e., diagonal DCC) and asymmetry in the correlation (i.e., asymmetric DCC) respectively. Finally, since the variable of interest, i.e. the expected index price at some future point in time, is an integral under the future risk neutral price density, and therefore not only a function of the first two conditional moments, we evaluate the multivariate models not only under the assumption of a Gaussian innovation distribution but also under the multivariate Laplace distribution which has fatter tails.

We evaluate the model performance using a direct evaluation of economic quantities (option prices). This is in contrast to Laurent, Rombouts, and Violante (2011) and Caporin and McAleer (2010) for example, who perform large scale comparisons of multivariate GARCH models using statistical criteria to evaluate statistical quantities (variance matrices). These papers rank volatility models on the basis of their ability to replicate volatility paths, and being the latter unobservable, need a proxy (typically computed by using high frequency data) of the conditional variance matrix. This requires that the loss function must have a specific functional form in order to avoid distortions in the ranking (Hansen and Lunde (2006) and Patton (2009), Laurent, Rombouts, and Violante (2009)). The advantage of our evaluation is that by forecasting option prices for which we observe the true prices, we do not need to rely on a proxy. Thus, the problem of inconsistency of the ranking does not arise and virtually any loss function can be used.

Our results suggest that in general more complicated models provide better option pricing forecasts. It turns out that the most important improvement in pricing comes from increasing the sophistication in the marginal variance processes. Enriching the model with more complex correlation models, and relaxing a Gaussian innovation for a Laplace innovation assumption provides smaller economic gains when considering the entire sample of option prices. Overall, increasing model sophistication can reduce the dollar loss up to 60%. The model that performs best, according to the MCS test, is the two component threshold GARCH in combination with an asymmetric DCC structure and Laplace innovations. When we look at the pricing

performance for different levels of moneyness, we find that the out-of-the-money contracts, which have a low average price, are more difficult to price but increasing the complexity of the model still pays off generally. Also, regarding the maturity of the options, it turns out that the best performance is for short maturity options, with very close performance between the different models.

It should be noted that, as the DJIA options are based on an equally weighted portfolio, the model comparison is based on average dynamics. However, the evaluation could equally well be conducted in terms of other types of contracts, for instance options on the maximum or minimum of the assets, which depend on other aspects of the forecasted multivariate distribution. Unfortunately, data on such options is not readily available and we therefore limit our attention to the exchange traded average options.

The rest of the paper is organized as follows: Section 2 introduces the Dow Jones Industrial Average data. In Section 3, we discuss the theoretical framework used for pricing multivariate options, including the various multivariate models to be considered. Section 4 contains the empirical results. Finally, Section 5 contains concluding remarks.

## **2 Data**

In this section, we introduce the Dow Jones Industrial Average data. We start by defining the index. We then provide descriptive statistics for the returns of the 30 constituents as well as for the aggregate index. Finally we provide an overview of the index option data.

### **2.1 DJIA return data**

The Dow Jones Industrial Average (DJIA) is a price-weighted index, which is composed of 30 of the largest and most liquid stocks listed on the NYSE and NASDAQ exchanges. The index is calculated by dividing the sum of the prices of the component stocks by a number called the DJIA divisor or Dow divisor. The index divisor is updated periodically and adjusted to offset the effect of stock splits and any other change in the component stocks. This ensures consistency of the index value through time.

As of November 21, 2005, the companies in Table 1 constituted the DJIA index. At that

**Table 1: Description of Dow Jones constituent companies**

	Ticker	Company	Industry	Date Added
1	AA	Alcoa	Aluminum	1959-06-01
2	AIG	American International Group	Property & Casualty Insurance	2004-04-08
3	AXP	American Express	Consumer finance	1982-08-30
4	BA	Boeing	Aerospace & defense	1987-03-12
5	CAT	Caterpillar	Construction & mining equipment	1991-05-06
6	C	Citigroup	Banking	1997-03-17
7	DD	EI DuPont de Nemours	Commodity chemicals	1935-11-20
8	DIS	Walt Disney	Broadcasting & entertainment	1991-05-06
9	GE	General Electric	Conglomerate	1907-11-07
10	GM	General Motors	Automobiles	1925-08-31
11	HD	Home Depot	Home improvement retailers	1999-11-01
12	HON	Honeywell International	Aerospace & defense	1925-12-07
13	HPQ	Hewlett-Packard	Diversified computer systems	1997-03-17
14	IBM	International Business Machines	Computer services	1979-06-29
15	INTC	Intel	Semiconductors	1999-11-01
16	JNJ	Johnson & Johnson	Pharmaceuticals	1997-03-17
17	JPM	JPMorgan Chase	Banking	1991-05-06
18	KO	Coca-Cola	Beverages	1987-03-12
19	MCD	McDonald's	Restaurants & bars	1985-10-30
20	MMM	3M	Diversified industrials	1976-08-09
21	MO	Altria Group	Cigarettes	1985-10-30
22	MRK	Merck	Pharmaceuticals	1979-06-29
23	MSFT	Microsoft	Software	1999-11-01
24	PFE	Pfizer	Pharmaceuticals	2004-04-08
25	PG	Procter & Gamble	Non-durable household products	1932-05-26
26	T	AT&T	Telecommunication	1999-11-01
27	UTX	United Technologies Corporation	Aerospace, heating/cooling	1939-03-14
28	VZ	Verizon Communications	Telecommunication	2004-04-08
29	WMT	Walmart	Broadline retailers	1997-03-17
30	XOM	Exxon Mobil	Integrated oil & gas	1928-10-01

Note: This table provides the ticker, company name, industry, and the date when added to the index for the 30 constituents of the DJIA as of November 21, 2005.

time the value of the Dow divisor was 0.1249, and thus the index was given by

$$DJIA_t = \frac{1}{0.1249} \sum_{i=1}^N P_{i,t} \quad (1)$$

where  $P_{i,t}$  is the price of the  $i$ 'th company at time  $t$ . As the Dow divisor was less than one, the index value was actually larger than the sum of the prices of the components, a fact stemming from the numerous stock splits which have occurred with index constituents throughout the existence of the DJIA.

In Table 2 descriptive statistics are provided for the 30 constituents as well as for the DJIA itself. We use daily raw return from December 30, 1997 to December 27, 2006, for a total of 2246 observations. The assets listed exhibit typical features common to many other financial data. The table shows that for 19 out of the 30 stocks returns are negatively skewed and for all 30 stocks returns are leptokurtic. For some the kurtosis is orders of magnitude

**Table 2: Descriptive statistics for Dow Jones companies**

Ticker	min	mean	max	stddev	skewn	kurt
AA	-11.66	0.03135	13.152	2.3054	0.24098	5.5447
AIG	-11.02	0.029612	10.46	1.9135	0.077733	6.2767
AXP	-14.614	0.043158	12.018	2.1839	-0.086318	6.3712
BA	-11.532	0.044143	8.4307	1.956	-0.13626	6.3866
CAT	-17.115	0.050373	16.838	2.1603	0.062693	9.2984
C	-15.686	0.055666	10.296	2.1275	-0.20395	6.686
DD	-11.695	0.0038985	9.4125	1.892	0.062353	6.2373
DIS	-20.289	0.0068601	14.203	2.2575	-0.24388	10.4
GE	-11.287	0.032381	11.743	1.8493	0.053849	6.9385
GM	-15.045	-0.0017424	16.647	2.298	0.10738	6.9799
HD	-33.877	0.035111	12.128	2.3967	-1.3012	23.364
HON	-19.079	0.014932	24.852	2.3863	-0.25409	15.002
HPQ	-20.701	0.024279	18.991	2.8403	-0.062388	8.5292
IBM	-16.892	0.02721	12.366	2.0634	-0.12532	10.602
INTC	-24.888	0.0064339	18.335	2.9955	-0.45827	8.8805
JNJ	-17.252	0.037201	7.8925	1.5193	-0.61117	13.253
JPM	-19.977	0.030022	14.873	2.3612	0.10734	8.67
KO	-11.072	-0.0057238	9.2162	1.6461	-0.17626	7.8956
MCD	-13.716	0.033771	10.31	1.8828	-0.056628	7.396
MMM	-9.3837	0.037487	10.5	1.6286	0.19183	6.3402
MO	-14.853	0.045488	14.882	2.0627	-0.19121	11.009
MRK	-31.171	0.0021853	12.251	1.9419	-1.9682	35.278
MSFT	-16.958	0.036521	17.869	2.2697	-0.19865	9.3492
PFE	-11.817	0.012271	9.271	1.9887	-0.30771	6.328
PG	-37.658	0.029849	9.0972	1.7905	-4.3843	92.943
T	-13.538	0.01585	8.8338	2.0324	-0.082488	5.9627
UTX	-33.195	0.066822	9.3762	2.0068	-2.0997	38.005
VZ	-12.609	0.013821	11.565	1.918	0.065695	6.9257
WMT	-10.26	0.043204	9.0151	1.9876	0.14496	5.6743
XOM	-8.8397	0.048885	10.485	1.564	0.054154	5.5619
DJIA	-7.396	0.021	6.1547	1.11	-0.1239	6.6772

This table provides descriptive statistics for the 30 constituents of the DJIA. The sample period is from December 30, 1997, to December 27, 2006, for a total of 2246 observations. The bottom line provides the equivalent statistics for the index itself. Results are for raw returns in percentage terms.

larger than that of the Gaussian distribution. This is particularly striking for PG, UTX, and MRK which are also the most negatively skewed. The returns on the index are also highly negatively skewed and leptokurtic. As expected the standard deviation of the index is lower than the individual stock standard deviations showing the diversification effect. Finally, as can be seen in Figure 1 all the unconditional correlations are positive. For some of the return pairs the correlation can reach up to 74% percent, i.e., between JP Morgan and Citigroup. Some stocks are only slightly correlated with the others, e.g., Altria Group. In general, all the other correlations span from about 20% to 50%. The average correlation is close to 30%.

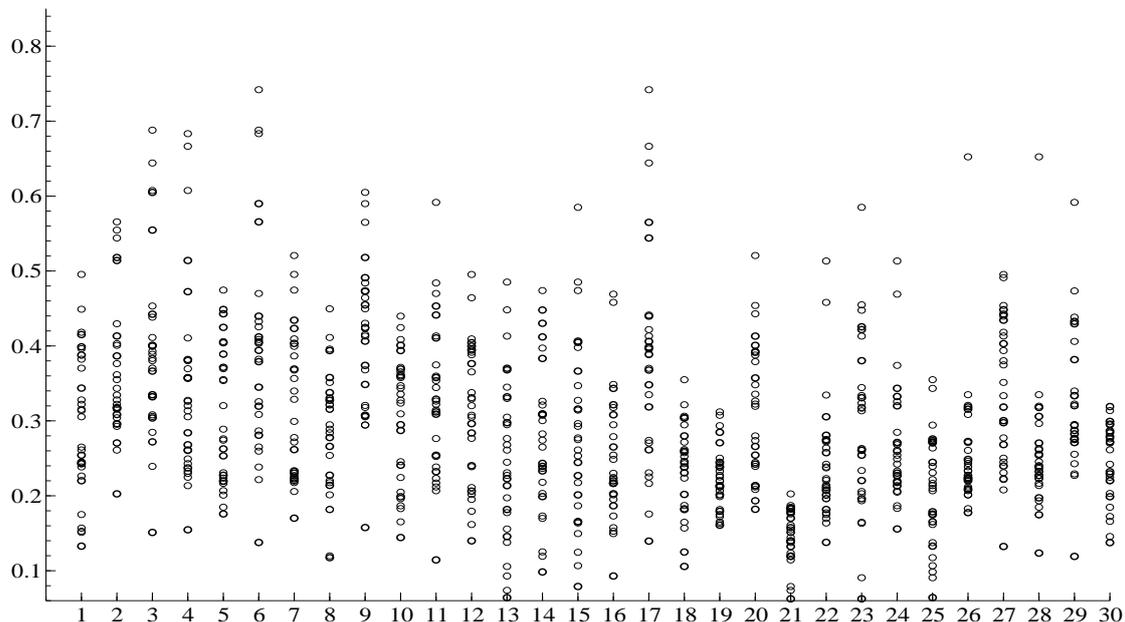


Figure 1: Correlations for each stock with the other DJIA stocks (See Table 1 for the names of the stocks)

## 2.2 DJIA option data

Options on the DJIA were introduced at the Chicago Board of Options Exchange, or CBOE, in 1997. Since this time the options have grown to become some of the most popular index options worldwide. The main reason for this it that investors are able to trade a broad market by making one DJIA trading decision rather than making the many decisions involved with investing in the numerous individual stocks. The DJIA options are European style options and are based on 1/100th of the index level. The options are cash settled and the underlying dollar value is equal to the index level multiplied by \$100.

In this paper, we consider DJIA call options for 2006. However, since the total number of options traded for 2006 is 54,939 this is clearly too time consuming to deal with and for this reason we apply a number of filters to the sample that are standard in the option pricing literature. First, we only consider options on a weekly basis. This is done to balance the tradeoff between having a long enough time period to be of interest and to have a number of options which is computationally reasonable. Specifically, we choose options traded on

**Table 3: Properties of the DJIA 30 index options data set**

Moneyiness				
	OTM	ITM	DITM	ALL
Price	1.30	2.25	7.71	4.13
number	406	334	497	1237
Time to maturity in days				
	ST	MT	LT	ALL
Price	4.24	3.93	4.17	4.13
number	428	335	474	1237

Average price in USD and number of call contracts in the cells of this table. The moneyiness and maturity categories are defined in the text.

Wednesdays as these are less likely to be affected by the so-called weekend effects. Second, we only consider options which have less than 252 trading days to maturity and, third, options for which the traded volume on a given Wednesday exceeded 5 contracts. Applying these filters yields a sample of 1,237 call options.

In Table 3, we provide descriptive statistics for the options in terms of the number of contracts and their average prices. We tabulate data for various categories of maturity measured in trading days,  $Mat$ , and moneyiness measured as  $Mon = S / (K \exp(-r_t Mat))$ , where  $S$  is the value of the underlying,  $K$  is the strike price, and  $r_t$  is the risk free interest rate. As the risk free rate we use the LIBOR rate which starts at 4.4% and ends at 5.2% in 2006. The moneyiness categories are divided into out of the money (OTM), with  $Mon < 1$ , in the money (ITM), with  $1 \leq Mon < 1.02$ , deep in the money (DITM), with  $Mon \geq 1.02$ . The maturity categories are divided into short term (ST), with  $Mat < 24$ , medium term (MT), with  $24 \leq Mat < 45$ , and long term (LT), with  $Mat \geq 45$ . The moneyiness and maturity categories are defined such that we have roughly the same number of contracts in each category while keeping the definitions meaningful.

### 3 Pricing options on the DJIA

Letting  $\mathcal{F}_t$  denote the information set up to time  $t$ , we assume that the underlying return process  $R_{j,t} = \ln(P_{j,t}/P_{j,t-1})$  for  $j = 1, \dots, N$  can be characterized by

$$R_{j,t} = \mu_{j,t} - \Psi_t(-e_j) + \varepsilon_{j,t}, \quad (2)$$

where  $P_{j,t}$  is the price level of asset  $j$  on day  $t$ . The term  $\Psi_t(\cdot)$  denotes the conditional cumulant generating function of  $\varepsilon_{j,t}$ , and  $e_j$  is an  $N$  dimensional vector of zeros except for position  $j$  where it is 1. Thus, we have that  $\ln(E_{t-1}[\exp(\varepsilon_{j,t})]) = \Psi_t(-e_j)$ , and it follows that

$$\begin{aligned} E_{t-1}[P_{j,t}/P_{j,t-1}] &= E_{t-1}[\exp(\mu_{j,t} - \Psi_t(-e_j) + \varepsilon_{j,t})] \\ &= \exp(\mu_{j,t}). \end{aligned} \quad (3)$$

The specification in (2) therefore implies that  $\mu_{j,t}$  is measurable and can be interpreted as the expected gross rate of return for asset  $j$ . The error term  $\varepsilon_t = (\varepsilon_{1,t}, \dots, \varepsilon_{N,t})'$  can be decomposed as

$$\varepsilon_t = \Sigma_t^{1/2} \eta_t. \quad (4)$$

The innovation term  $\eta_t$  in (4) is identically and independently distributed with a  $N$ -variate absolutely continuous distribution function  $F$  (i.e. the physical measure) with first two moments respectively equal to  $E[\eta_t] = 0$  and  $E[\eta_t \eta_t'] = I_N$ . The latter assumption makes  $\Sigma_t$  the conditional variance matrix of  $\varepsilon_t$ . Various specifications for this volatility matrix will be provided below in Section 3.3.

In the next section, we make two choices about the innovation distribution and explain how risk neutralization is performed. We then discuss how, by using a simple specification for the rate of return, estimation can be made easier. Finally, we explain how, based on the derived dynamics, options can be priced using Monte Carlo simulation.

#### 3.1 Risk neutralization

To perform option pricing, we need the risk neutral dynamics, i.e. the dynamics under  $Q$ . Rombouts and Stentoft (2011) show that, building on Christoffersen, Elkamhi, Feunou, and

Jacobs (2010), under the risk neutral measure the conditional cumulant generating function of  $\varepsilon_t$  is given by

$$\Psi_t^Q(u) = \Psi_t(\nu_t + u) - \Psi_t(\nu_t), \quad (5)$$

where  $\nu_t$  is a  $N$ -dimensional vector sequence. In fact, by using a multivariate generalization of the exponential affine Radon-Nikodym derivative, this provides a unique equivalent martingale measure if and only if

$$0 = \Psi_t(\nu_t - e_j) - \Psi_t(\nu_t) - \Psi_t(-e_j) + \mu_{j,t} - r_t, \quad (6)$$

for all  $j = 1, \dots, N$ .

More concretely, in this paper we consider two distributional assumptions, the multivariate Gaussian distribution and the fatter tailed multivariate Laplace distribution. When  $F$  is the multivariate Gaussian distribution, the conditional Laplace transform of  $\varepsilon_t$  in (4) is given by

$$\mathbb{E}_{t-1} \exp(-u' \varepsilon_t) = \exp\left(\frac{u' \Sigma_t u}{2}\right), \quad (7)$$

and therefore given (5) we obtain

$$\begin{aligned} \Psi_t^Q(u) &= \Psi_t(\nu_t + u) - \Psi_t(\nu_t) \\ &= \frac{1}{2} (\nu_t + u)' \Sigma_t (\nu_t + u) - \frac{1}{2} \nu_t' \Sigma_t \nu_t \\ &= u' \Sigma_t \nu_t + \frac{1}{2} u' \Sigma_t u. \end{aligned} \quad (8)$$

Thus, it follows that the risk neutral dynamics remain Gaussian although with a shifted mean. This shift in the mean is required to compensate investors for the risk associated with investing in the underlying risky assets.

Under the Gaussian assumption we may further solve (6) directly for any specification of  $\mu_{j,t}$ . This can be done by substituting (7) into (6) upon which we obtain the following

$$\begin{aligned} 0 &= \frac{1}{2} [(\nu_t - e_j)' \Sigma_t (\nu_t - e_j) - \nu_t' \Sigma_t \nu_t - e_j' \Sigma_t e_j] + \mu_{j,t} - r_t \\ &= -e_j' \Sigma_t \nu_t + \mu_{j,t} - r_t, \end{aligned}$$

for all  $j = 1, \dots, N$ . Solving for  $\nu_t$  we obtain

$$\nu_t = \Sigma_t^{-1}(\mu_t - r_t), \quad (9)$$

where  $\mu_t$  and  $r_t$  are the corresponding vectors of the gross rate of return and the risk-free interest rate respectively.

The risk neutralization under the multivariate Laplace distribution can be approximated by the one under the Gaussian distribution. To see this, note that the conditional cumulant generating function of the (zero mean) multivariate Laplace distribution, see Eltoft (2006), is given by

$$\Psi_t(u) = -\log\left(1 - \frac{u'\Sigma_t u}{2}\right). \quad (10)$$

If we consider the first order Taylor expansion of  $\log\left(1 - \frac{u'\Sigma_t u}{2}\right)$  around 1, then we obtain

$$\Psi_t(u) \approx \frac{u'\Sigma_t u}{2}, \quad (11)$$

which is indeed consistent with the (zero mean) multivariate Gaussian cumulant generating function. Therefore, while we can apply the same risk neutralization under both distributional assumptions, the multivariate Laplace distribution will generate fatter tails in the predicted future price distribution.

### 3.2 Rate of return specification

To simplify matters further, we make the assumption that the conditional gross rate of return is given as

$$\mu_t = r_t + \Sigma_t \odot I_N \lambda.$$

This means that the risk premium for asset  $j$  only depends on the variance of the asset and as a function of this it remains constant through time, i.e.  $\lambda_j$  is constant. Compared to a more general specification this has the effect of making estimation easier as this can be done for each asset individually.

The particular choice of conditional mean also implies that the risk neutral dynamics are simple. To see this, substitute (9) and the mean specification into (8) to obtain

$$\Psi_t^Q(u) = u'\Sigma_t \odot I_N \lambda + \frac{1}{2}u'\Sigma_t u. \quad (12)$$

Thus, for this particular choice the risk neutral mean equals  $-\Sigma_t \odot I_N \lambda$ , which is readily interpreted as the compensation an investor requires for holding risky assets. Moreover, from the specification it makes sense to refer to  $\lambda$  as the price of risk. Finally, it should be noted that we assume for simplicity that interest rates are constant.

**Table 4: Models for the conditional correlations**

$\Sigma_t = (\Sigma_t \odot I_N)^{1/2} R_t (\Sigma_t \odot I_N)^{1/2}$ $Cor_t = (Q_t \odot I_N)^{-1/2} Q_t (Q_t \odot I_N)^{-1/2}$		
CCC	$Q_t = \bar{Q}$	Bollerslev (1990)
DCC	$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha(u_{t-1}u'_{t-1}) + \beta Q_{t-1}$	Engle (2002)
DDCC	$Q_t = (1 - \beta)\bar{Q} - A\bar{Q}A' + Au_{t-1}u'_{t-1}A' + \beta Q_{t-1}$	Cappiello et al. (2006)
ADCC	$Q_t = (1 - \alpha - \beta)\bar{Q} - \Gamma\bar{U}\Gamma' + \alpha(u_{t-1}u'_{t-1}) + \beta Q_{t-1} + \Gamma(u_{t-1}u'_{t-1} \odot 1_{u_{t-1} < 0} 1'_{u_{t-1} < 0})\Gamma'$	Cappiello et al. (2006)
$\bar{Q} = E[u_{t-1}u'_{t-1}]$ $\bar{U} = E[u_{t-1}u'_{t-1} \odot 1_{u_{t-1} < 0} 1'_{u_{t-1} < 0}]$ $u_{i,t} = \epsilon_{i,t}/h_{i,t}$		

### 3.3 Volatility models

We now define the various multivariate GARCH models that will be applied to the  $N = 30$  stocks in the DJIA. If we define  $R_t = (R_{1,t}, \dots, R_{N,t})'$  then given the above discussion on risk neutralization the econometric model can be written as

$$\begin{aligned} R_t &= r_t + (\Sigma_{m,t} \odot I_N)\lambda - \frac{1}{2}(\Sigma_{m,t} \odot I_N) + \epsilon_t \\ \epsilon_t &= \Sigma_{m,t}^{1/2}\eta_t, \end{aligned} \quad (13)$$

where  $\Sigma_{m,t}$  defines the conditional variance matrix for model specification  $m$ . To make estimation of this 30 dimensional model feasible, the number of parameters being easily higher than 100 in our setting, we consider only models for which the conditional variance matrix can be decomposed as follows

$$\Sigma_{m,t} = (\Sigma_{m,t} \odot I_N)^{1/2} Cor_{m,t} (\Sigma_{m,t} \odot I_N)^{1/2}, \quad (14)$$

where the first part,  $(\Sigma_{m,t} \odot I_N)^{1/2}$ , contains the  $N$  conditional standard deviations on the diagonal and where  $Cor_{m,t}$  is the conditional correlation matrix.

We consider four specifications for the conditional correlation matrix, see Table 4, ranging from constant correlations to dynamic correlations that take into account an asymmetry effect. With respect to the models for the marginal variances  $h_t$ , we consider most of the models in the (extensive) glossary of Bollerslev (2010). Since the number of models amounts to 34, see Table 5, we decide to group them by common characteristics. This approach allows us to

appreciate the value of model sophistication in a concise manner. We distinguish models in five ways: i) linearity in  $\epsilon_t^2$ , ii) symmetry related to impact of shocks, iii) direct specification of conditional variance, iv) constant parameters, and v) component type of specifications. More details are given in Table 6.

With respect to the estimation of the model parameters, the specification given in (13) is particularly appealing because it does not include spill over terms in the conditional mean and thus it allows for the factorization of the likelihood as suggested in Engle (2002). In fact, under Gaussianity of the innovations in (13) the total log-likelihood can be decomposed in the sum of the log-likelihoods of the marginal densities and the (multivariate) log-likelihood of the joint density of the centered devolatilized returns. Thus, the estimation can be carried out in two steps: first, the first two conditional moments of the marginal processes and second, the correlation dynamics of the joint process. Further, in order to ease the computational burden of the likelihood of the correlation driving process, we replace  $\bar{Q}$  and  $\bar{U}$  by the unconditional second moment of  $u_t$  and  $u_t \odot 1_{u_t < 1}$  respectively, see Engle and Mezrich (1995), which saves 465 parameters. Note also that, imposing conditional Gaussianity to all models allows for a quasi maximum likelihood interpretation of the two step estimator independently of the assumed distribution (Gaussian or Laplace).

The models are estimated using data from December 31, 1997 to December 30, 2005 which amounts to 2000 observations, results are too numerous to report but are available on request. To keep computation time reasonable, the parameter estimates are kept fixed over the 52 weeks for which we price option contracts in 2006. In fact, this is also the reason why we consider one year of options. It takes roughly one day on one processor to estimate and price options for each of the 272 model specifications. Due to lack of accuracy in the numerical optimization during the parameter estimation, 24 specifications involving combinations of Alt-garch, sqrt-garch and vgarch have been excluded from the analysis.

### 3.4 Option pricing

The theoretical value at time  $t$  of the European call option with a strike price equal to  $K$  and maturity in  $T$  days is

$$C_t(T, K) = e^{-rt(T-t)} E_t^Q [\max(K - S_T, 0)], \quad (15)$$

**Table 5: Models for the conditional variance**

Garch	$h_t = \beta_0 + \beta_1 \epsilon_{t-1}^2 + \beta_2 h_{t-1}$	Bollerslev, 1986
GJR	$h_t = \beta_0 + \beta_1 \epsilon_{t-1}^2 + \beta_2 \epsilon_{t-1}^2 1_{\epsilon_{t-1} < 0} + \beta_3 h_{t-1}$	Glosten et al, 1993
Alt-Garch	$h_t = \beta_0 + \beta_1 (\epsilon_{t-1}^2 / h_{t-1}) + \beta_2 h_{t-1}$	Knight and Satchell, 2002
$\beta$ -Garch	$h_t = \beta_0 + (\beta_1 1_{\epsilon_{t-1} > 0} + \beta_2 1_{\epsilon_{t-1} < 0}) \epsilon_{t-1}^{2\beta_4} + \beta_3 h_{t-1}$	Guegan and Diebolt, 1994
Agarch(a)	$h_t = \beta_0 + \beta_1 \epsilon_{t-1}^2 + \beta_2 \epsilon_{t-1} + \beta_3 h_{t-1}$	Engle, 1990
Agarch(b)	$h_t = \beta_0 + \beta_1  \epsilon_{t-1} ^{\beta_4} + \beta_2 \epsilon_{t-1} + \beta_3 h_{t-1}$	Engle, 1990
Nagarch	$h_t = \beta_0 + \beta_1 (\epsilon_{t-1} + \beta_2 \sqrt{h_{t-1}})^2 + \beta_3 h_{t-1}$	Engle and Ng, 1993
Vgarch	$h_t = \beta_0 + \beta_1 (\epsilon_{t-1} / \sqrt{h_{t-1}} + \beta_2)^2 + \beta_3 h_{t-1}$	Engle and Ng, 1993
$\sqrt{\cdot}$ -Garch	$h_t = \beta_0 + \beta_1 (\epsilon_{t-1} / \sqrt{h_{t-1}} + \beta_2 \sqrt{h_{t-1}})^2 + \beta_3 h_{t-1}$	Heston and Nandi, 2000
Vsarch(a)	$h_t = \beta_0 + \beta_1 \epsilon_{t-1}^2 + \beta_2 (\epsilon_{t-1}^2 / h_{t-1}) 1_{\epsilon_{t-1} > 0}$ $- \beta_2 (\epsilon_{t-1}^2 / h_{t-1}) 1_{\epsilon_{t-1} < 0} + \beta_3 h_{t-1}$	Fornari and Mele, 1995
Vsarch(b)	$h_t = \beta_0 + \beta_1 \epsilon_{t-1}^2 + \beta_2 (\epsilon_{t-1}^2 / h_{t-1}) 1_{\epsilon_{t-1} < 0} + \beta_3 h_{t-1}$	Fornari and Mele, 1995
Vsarch(c)	$h_t = \beta_0 + \beta_1 \epsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 \epsilon_{t-1}^2 1_{\epsilon_{t-1} > 0} - \beta_3 \epsilon_{t-1}^2 1_{\epsilon_{t-1} < 0}$ $+ \beta_4 (\epsilon_{t-1}^2 / h_{t-1} - \beta_5) 1_{\epsilon_{t-1} > 0} - \beta_4 (\epsilon_{t-1}^2 / h_{t-1} - \beta_5) 1_{\epsilon_{t-1} < 0}$	Fornari and Mele, 1996
Narch(a)	$h_t = \beta_0 + \beta_1  \epsilon_{t-1} ^{\beta_3} + \beta_2 h_{t-1}$	Engle and Bollerslev, 1986
Narch(b)	$h_t^{\beta_4/2} = \beta_0 + \beta_1  \epsilon_{t-1} ^{\beta_4} + h_{t-1}^{\beta_4/2}$	Higgins and Bera, 1992
Aparch	$h_t^{\beta_5/2} = \beta_0 + \beta_1 ( \epsilon_{t-1}  - \beta_3 \epsilon_{t-1})^{\beta_5} + \beta_4 h_{t-1}^{\beta_5/2}$	Ding et al, 1993
Hgarch	$h_t^{\beta_6/2} = \beta_0 + \beta_1 \beta_6 h_{t-1}^{\beta_6/2} ( \epsilon_{t-1} / (\sqrt{h_{t-1}}) - \beta_2 $ $- \beta_3 (\epsilon_{t-1} / (\sqrt{h_{t-1}} - \beta_2))^{\beta_5} + \beta_4 h_{t-1}^{\beta_6/2}$	Hentschel, 1995
Atgarch	$\sqrt{h_t} = \beta_0 + \beta_1  \epsilon_{t-1}  1_{\epsilon_{t-1} < 0} + \beta_2  \epsilon_{t-1}  1_{\epsilon_{t-1} > 0} + \beta_3 \sqrt{h_{t-1}}$	Crouhy and Rokinger, 1997
Tarch	$\sqrt{h_t} = \beta_0 + \beta_1  \epsilon_{t-1}  + \beta_2  \epsilon_{t-1}  1_{\epsilon_{t-1} < 0} + \beta_3 \sqrt{h_{t-1}}$	Zakoian, 1990
TS-Garch	$\sqrt{h_t} = \beta_0 + \beta_1  \epsilon_{t-1}  + \beta_2 \sqrt{h_{t-1}}$	Taylor and Schwert, 1989
log-Garch	$\log(h_t) = \beta_0 + \beta_1 \log(\epsilon_{t-1}^2) + \beta_2 \log(h_{t-1})$	Geweke, 1986, Pantula, 1986
Egarch	$\log(h_t) = \beta_0 + \beta_1 (\epsilon_{t-1} / h_{t-1})$ $+ \beta_2 ( \epsilon_{t-1} / h_{t-1}  - \sqrt{2/\pi}) + \beta_3 \log(h_{t-1})$	Nelson, 1990
March	$h_t = \beta_0 + \beta_1 F(\epsilon_{t-1}) + \beta_2 h_{t-1}$ $F(\epsilon_t) = \sin(\beta_3 \epsilon_t^2) 1_{\beta_3 \epsilon_t^2 < \pi/2} + 1_{\beta_3 \epsilon_t^2 \geq \pi/2}$	Friedman et al, 1989
ST-Garch	$h_t = \beta_0 + (\beta_1 + \beta_2 F(\epsilon_{t-1})) \epsilon_{t-1}^2 + \beta_3 h_{t-1}$	
LST	$F(\epsilon_t) = (1 + \exp(-\beta_4 \epsilon_t))^{-1} - 0.5$	Gonzalez-Rivera, 1998
EST	$F(\epsilon_t) = (1 - \exp(-\beta_4 \epsilon_t^2))$	Hagerud, 1996
GLST	$F(\epsilon_t) = (1 - \exp(-\beta_4 \epsilon_t^2)) / (1 + \exp(-\beta_4 (\epsilon_t^2 - \beta_5)))$	Lubrano, 1998
GEST	$F(\epsilon_t) = (1 - \exp(-\beta_4 (\epsilon_t - \beta_5)^2))$	Lubrano, 1998
ANST-Garch	$h_t = \beta_0 + \beta_1 \epsilon_{t-1}^2 + \beta_2 h_{t-1} + (\beta_3 + \beta_4 \epsilon_{t-1}^2 + \beta_5 h_{t-1}) F(\epsilon_{t-1})$ $F(\epsilon_t) = (1 + \exp(-\beta_6 \epsilon_t))^{-1}$	Nam et al, 2002
Cgarch	$h_t = q_t + \beta_3 (\epsilon_{t-1}^2 - q_{t-1}) + \beta_4 (h_{t-1} - q_{t-1})$ $q_t = \beta_0 + \beta_1 (\epsilon_{t-1}^2 - h_{t-1}) + \beta_2 q_{t-1}$	Engle and Lee, 1993
Acgarch	$h_t = q_t + \beta_4 (\epsilon_{t-1}^2 - q_{t-1})$ $+ \beta_5 (\epsilon_{t-1}^2 1_{\epsilon_{t-1} < 0} - 0.5 q_{t-1}) + \beta_6 (h_{t-1} - q_{t-1})$ $q_t = \beta_0 + \beta_1 (\epsilon_{t-1}^2 - h_{t-1})$ $+ \beta_2 (\epsilon_{t-1}^2 1_{\epsilon_{t-1} < 0} - 0.5 h_{t-1}) + \beta_3 q_{t-1}$	Engle and Lee, 1993
2C(.)Garch*	$h_t = \beta_4 s_t + (1 - \beta_4) l_t$ $s_t = \beta_1 + \beta_2 \epsilon_{t-1}^2 + \beta_3 s_{t-1}$ $l_t = \beta_0 \epsilon_{t-1}^2 + (1 - \beta_0) l_{t-1}$	Ding and Granger, 1996

\*In the 2C(.)Garch the dynamics of the short term component  $s_t$  can be replaced by more sophisticated specifications. We consider other than the simple Garch also Tarch and Nagarch. The three specifications are denoted 2C(I)Garch, 2C(I)Tarch and 2C(I)Nagarch, respectively, where the I denotes the integrated Garch (IGarch) specification used for the long term component  $l_t$ . We also consider a different parameterization where the long term component  $l_t$  is a standard stationary Garch model. The corresponding three specifications are denoted 2C(G)Garch, 2C(G)Tarch and 2C(G)Nagarch, respectively. See Bollerslev (2010) for the exact bibliographic references.

**Table 6: Characterizations of conditional variance models**

lin	Garch, Alt-Garch, GJR, Vsarch (a,b,c), 2C(G)-Garch, 2C(I)-Garch, Cgarch, Acgarch
non lin	TSgarch, Narch(a,b), March, Egarch, $\beta$ -Garch, Tarch, Atgarch, Agarch (a,b), Nagarch Vgarch, $\sqrt{\cdot}$ -Garch, Aparch, Hgarch, LST-Garch, EST-Garch, GLST-Garch, GEST-Garch ANST-Garch, 2C(I)-Tarch, 2C(G)-Tarch, 2C(I)-Nagarch, 2C(G)-Nagarch
sym	Garch, Alt-Garch, TSgarch, Narch(a,b), March, EST-Garch, GLST-Garch 2C(G)-Garch, 2C(I)-Garch, Cgarch
asym	Egarch, GJR, $\beta$ -Garch, Atgarch, Tarch, Agarch(a,b), Nagarch, Vgarch, $\sqrt{\cdot}$ -garch Aparch, Hgarch, Vsarch(a,b,c), LST-Garch, GEST-Garch, ANST-Garch, 2C(I)-Tarch 2C(G)-Tarch, 2C(I)-Nagarch, 2C(G)-Nagarch, Acgarch
$h_t$	Garch, Alt-Garch, Narch(b), March, GJR, $\beta$ -Garch, Agarch(a,b), Nagarch, Vgarch $\sqrt{\cdot}$ -Garch, Vsarch(a,b,c), LST-Garch, EST-Garch, GLST-Garch, GEST-Garch, ANST-Garch 2C(G)-Nagarch, 2C(I)-Nagarch, 2C(G)-Garch, 2C(I)-Garch, Cgarch, Acgarch
$f(h_t)$	TSgarch, Narch(a), Egarch, Tarch, Atgarch, Aparch, Hgarch, 2C(G)-Tarch, 2C(I)-Tarch
cst par	Garch, Alt-Garch, TSgarch, Narch(a,b), March, Egarch, Gjr, $\beta$ -Garch, Tarch, Atgarch Agarch(a,b), Nagarch, Vgarch, $\sqrt{\cdot}$ -Garch, Aparch, Hgarch, Vsarch(a,b,c), 2C(G)-Garch 2C(I)-Garch, 2C(G)-Tarch, 2C(I)-Tarch, 2C(G)-Nagarch, 2C(I)-Nagarch, Cgarch, Acgarch
st	EST-Garch, GLST-Garch, LST-Garch, GEST-Garch, ANST-Garch
std	Garch, Alt-garch, TSgarch, Narch(a,b), March, Egarch, GJR, $\beta$ -Garch, Tarch, Atgarch Agarch(a,b), Nagarch, Vgarch, $\sqrt{\cdot}$ -Garch, Aparch, Hgarch, Vsarch(a,b,c), LST-garch EST-Garch, GLST-Garch, GEST-Garch, ANST-Garch
comp	2C(I)-Garch, 2C(G)-Garch, 2C(I)-Tarch, 2C(G)-Tarch, 2C(I)-Nagarch, 2C(G)-Nagarch Cgarch, Acgarch

Notes: lin (non lin): models linear (non linear) in  $\epsilon_t^2$ , sym (asym): models with symmetric (asymmetric) impact of shocks wrt the sign of the shock,  $h_t$  ( $f(h_t)$ ): models where the variance (a function of the variance) is explicitly modelled, cst (st): models with constant parameters (smooth transition), std (comp): single component (2-component). The total number of specifications is 34. Categories in each pair are mutually exclusive. See Table 5 for the precise functional forms.

where  $E_t^Q$  means that the expectation is taken under the risk neutral measure  $Q$  and where  $S_T$  is the value at time  $T$  of the underlying index. While closed form solutions to (15) do exist in a few cases, i.e. the constant volatility Gaussian case, this is not so for more general specifications of the underlying dynamics. However, even in this case it remains possible to price the options using numerical methods.

We use a Monte Carlo approach which is easy to implement in the current setting as the models are simple to simulate from under the risk neutral distribution. The use of Monte Carlo methods for option pricing goes back at least to Boyle (1977). For the European option example in (15), given initial values of  $P_t$ ,  $r_t$ ,  $\Sigma_t$ , and the parameter estimates of a model, we simulate  $B$  paths of length  $T - t$  for the return process under the risk neutral measure. More precisely

$$\begin{aligned} R_{t+i}^{(b)} &= \mu_{t+i} - \frac{1}{2} \Sigma_{t+i}^{(b)} \odot I_N + \varepsilon_{t+i}^{(b)} \\ \varepsilon_{t+i}^{(b)} &= -\Sigma_{t+i}^{(b)} \odot I_N \hat{\lambda} + \Sigma_{t+i}^{1/2(b)} \eta_{t+i}^{(b)} \\ \mu_{t+i} &= r_{t+i|t} + \Sigma_{t+i}^{(b)} \odot I_N \hat{\lambda}, \end{aligned} \tag{16}$$

where  $\eta_{t+i}^{(b)} \sim F(0, I_N)$  with  $F$  either the Gaussian or Laplace distribution,  $b = 1, \dots, B$  denotes the number of paths chosen to be sufficiently large (fixed to 20,000 in the application), and  $r_{t+i|t}$  is the conditional expectation of the risk free rate which is assumed fixed and equal to  $r_t$ . Given each return path, we compute the predicted price of each stock at maturity as

$$P_T^{(b)} = P_t \exp \left( \sum_{i=1}^{T-t} R_{t+i}^{(b)} \right), \tag{17}$$

and the index  $S_T^{(b)}$  using (1). Then, an estimate of the price of a call option is given by

$$\hat{C}_t(T, K) = e^{-r_t(T-t)} \frac{1}{B} \sum_{b=1}^B \max \left( K - S_T^{(b)}, 0 \right), \tag{18}$$

where  $S_T^{(b)}$  is the terminal index value simulated under the risk neutral dynamics for the  $b$ th path. More generally, the advantages of generating the multivariate price paths is that it can be used in any dimension and consequently any type of derivative contract on the underlying assets can be priced. In addition, with a Monte Carlo approach American options could also be priced using e.g. the Least-Squares Monte Carlo method of Longstaff and Schwartz (2001), for which the mathematical foundation was provided in Stentoft (2004).

## 4 Evaluating option pricing models

We start by explaining which loss functions we use to contrast predicted option prices to their realizations. Next, we investigate the pricing accuracy of the models (distinguished by similar characteristics for the marginal variances, type of correlations, and innovation distribution) for all options, and then we repeat the analysis for the different categories of moneyness and maturity. Finally, we provide inference, using the model confidence set approach, on predictive accuracy of each individual model in order to identify the set of superior models.

### 4.1 Evaluation criteria

We will analyze the pricing performance of the models measured in dollar losses. The predicted option prices are contrasted with the observed option prices using the mean absolute deviation, or MAD, loss and mean absolute relative deviation, or MARD, loss which measures the deviation in percentage points. The two loss functions are respectively defined as  $W^{-1} \sum_n^W |C_n - \hat{C}_n|$  and  $W^{-1} \sum_{n=1}^W \left| \frac{C_n}{\hat{C}_n} - 1 \right|$  where  $W$  is the number of options considered. Figure 2 visualizes the difference between loss functions.

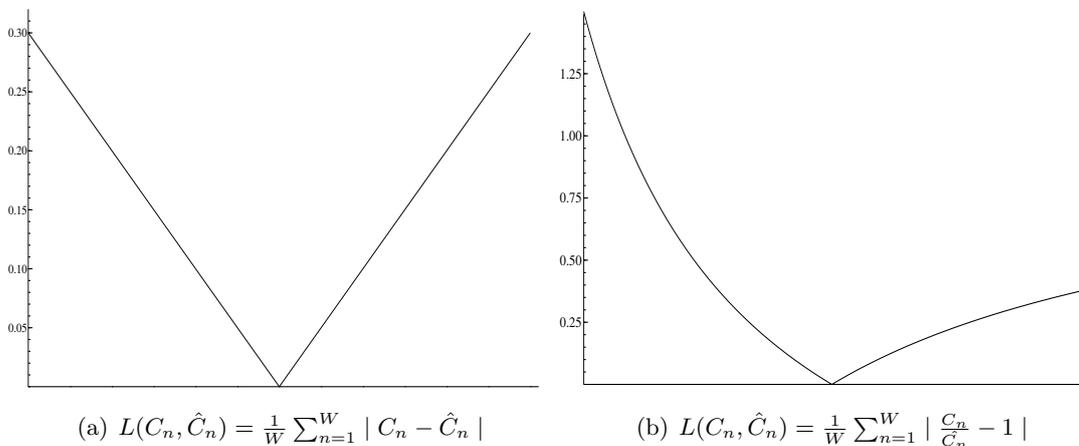


Figure 2: MAD (left) - MARD (right)

The choice of these loss functions is not coincidental. In fact, the MAD loss function interprets the pricing accuracy in average dollar losses and is symmetric in the sense that over and under predictions are penalized equally. The MARD loss function measures deviations in percentage terms in a unit free way, and as can be seen from Figure 2 is slightly asymmetric,

assigning heavier weights to underpredictions.

## 4.2 Overall results

Table 7 summarizes the results for the variance model groups identified in Table 6 for each correlation specification and choice of innovation distribution. The evaluation is based on the MAD loss for each of these categories. For space considerations, the results for the MARD loss function are found in Appendix 1 and will be commented only briefly.

With respect to the contribution of the marginal variance specification to the pricing accuracy, we find that modelling nonlinearity, asymmetry and functional transformations of the conditional variance yields systematic economic gains. Christoffersen and Jacobs (2004) and Stentoft (2005) also find empirically that nonlinearities and asymmetries are important in univariate option pricing models. Also in line with recent literature, see e.g. Christoffersen, Jacobs, Ornathanalai, and Wang (2008), we find that the largest gains are obtained when moving from a standard single component to a two component approach where the conditional variance is expressed as a linear combination of a long term (typically slow moving) component and a short term one. When comparing these two modelling approaches, for a given correlation specification and innovation distribution we obtain gains of up to 45%. However, increasing model sophistication does not always pay off. This is the case for smooth transition models which are systematically and largely outperformed by simpler specifications.

If we focus on the contribution of increasing the model sophistication of the correlation structure, we find that allowing for dynamic correlations produces better price forecasts, though the gains are of a much smaller order of magnitude compared to the marginal variance specifications. For example, moving from constant correlations to dynamic correlations provides gains only up to 3%. Increasing further the complexity of the correlation model provides only marginal, if not negligible, gains. In fact, in the cases of the symmetric marginal variance specification with Gaussian innovations and the component specifications with Laplace innovations the asymmetric correlation specifications perform the worst.

Finally, when comparing the performance for the two choices of innovation distributions, Table 7 shows that pricing with Laplace innovations in general brings considerable gains. Similar findings are reported in Christoffersen, Heston, and Jacobs (2006) and Stentoft (2008) for univariate models. In the multivariate framework presented here, allowing for non-Gaussian

**Table 7: Mean absolute deviation losses in dollars within each class of models**

Panel A: Gaussian innovations										
Characteristics of the marginal variances										
	lin vs. non lin		sym vs. asym		$h_t$ vs. $f(h_t)$		cst par vs. st		std vs. comp	
CCC	0.533	0.529	0.588	0.501	0.539	0.508	0.518	0.598	0.570	0.398
DCC	0.527	0.521	0.581	0.494	0.532	0.499	0.511	0.591	0.562	0.409
DDCC	0.523	0.519	0.577	0.492	0.530	0.499	0.507	0.588	0.560	0.389
ADCC	0.523	0.517	0.591	0.488	0.529	0.495	0.505	0.587	0.559	0.387

Panel B: Laplace innovations										
Characteristics of the marginal variances										
	lin vs. non lin		sym vs. asym		$h_t$ vs. $f(h_t)$		cst par vs. st		std vs. comp	
CCC	0.510	0.441	0.504	0.441	0.498	0.369	0.448	0.516	0.485	0.375
DCC	0.502	0.432	0.489	0.432	0.490	0.361	0.440	0.508	0.476	0.367
DDCC	0.499	0.432	0.489	0.433	0.489	0.359	0.440	0.508	0.476	0.379
ADCC	0.497	0.428	0.486	0.430	0.486	0.355	0.437	0.505	0.472	0.379

Note: lin (non lin): models linear (non linear) in  $\epsilon_t^2$ , sym (asym): models with symmetric (asymmetric) impact of shocks wrt the sign of the shock,  $h_t$  ( $f(h_t)$ ): models where the variance (a function of the variance) is explicitly modelled, cst (st): models with constant parameters (smooth transition), std (comp): standard GARCH (component GARCH). See Table 6 for the specific models belonging to each group. With respect to the marginal variance specifications, this table is read by pairwise columns, i.e. each pair identifies two mutually exclusive groups of models aggregated by common characteristics which allows to compare simple specifications (left column) to more complex ones (right column).

innovations often improves the performance above and beyond that of adding flexibility to the correlation or variance specifications. For example, changing the distribution for linear models with a CCC specification decreases the pricing errors by 0.023\$, from 0.533\$ to 0.510\$, whereas allowing for non linearities in the variance or dynamics and asymmetries in the correlation specification only reduces the pricing errors by 0.006\$ and 0.010\$, respectively. However, note that once a component structure is used the benefit of allowing for non-Gaussian innovations is marginal. This is in line with Christoffersen, Dorion, Jacobs, and Wang (2010) who find that the generalized error distribution does not improve much on a Gaussian innovation assumption for the component model when valuing options with univariate models.

As suggested above, the specification for the marginal variances plays a dominant role in improving pricing performances, followed by the choice for the distribution and, only marginally by the specification of the correlation dynamics. To this regard, the bubble plot in Figure 3 provides some more specific insights. Each bubble identifies a cluster of correlation models (CCC, DCC, DDCC, ADCC) characterized by the same specifications for the variances of the marginal processes. The bubbles are centered around the within cluster average loss, their diameter is the range computed on the MAD, which is used as a measure of the within cluster dispersion. The horizontal axis represents the MAD and the vertical axis the MARD. Black and grey bubbles identify the models using the Gaussian and Laplace densities respectively.

With respect to the different choice of the marginal variances (Figure 3, top), we see that aggregates of models cover a relatively wide range, from 0.24\$ to 0.71\$, with a high degree of between models dispersion. Also, at this lower level of aggregation, the potential marginal gains that can be obtained from deviating from the standard Gaussian assumption, for given choice of the variance specification, appear clear. Zooming in on the top of the classification (Figure 3, bottom), we see that, consistently with Tables 7 and 11, the most accurate models all share similar characteristics, i.e., asymmetry, non-linearity, indirectly modelling the variance as a functional transformation, and most relevantly the two component structure. The latter class of models accounts for 7 out of 12 models, with the 2C(I)Tarch specifications appearing largely superior to the others. Figure 3 (bottom) also allows to measure the contribution of using a fatter tailed distribution. If we consider again the 2C(I)Tarch spec-

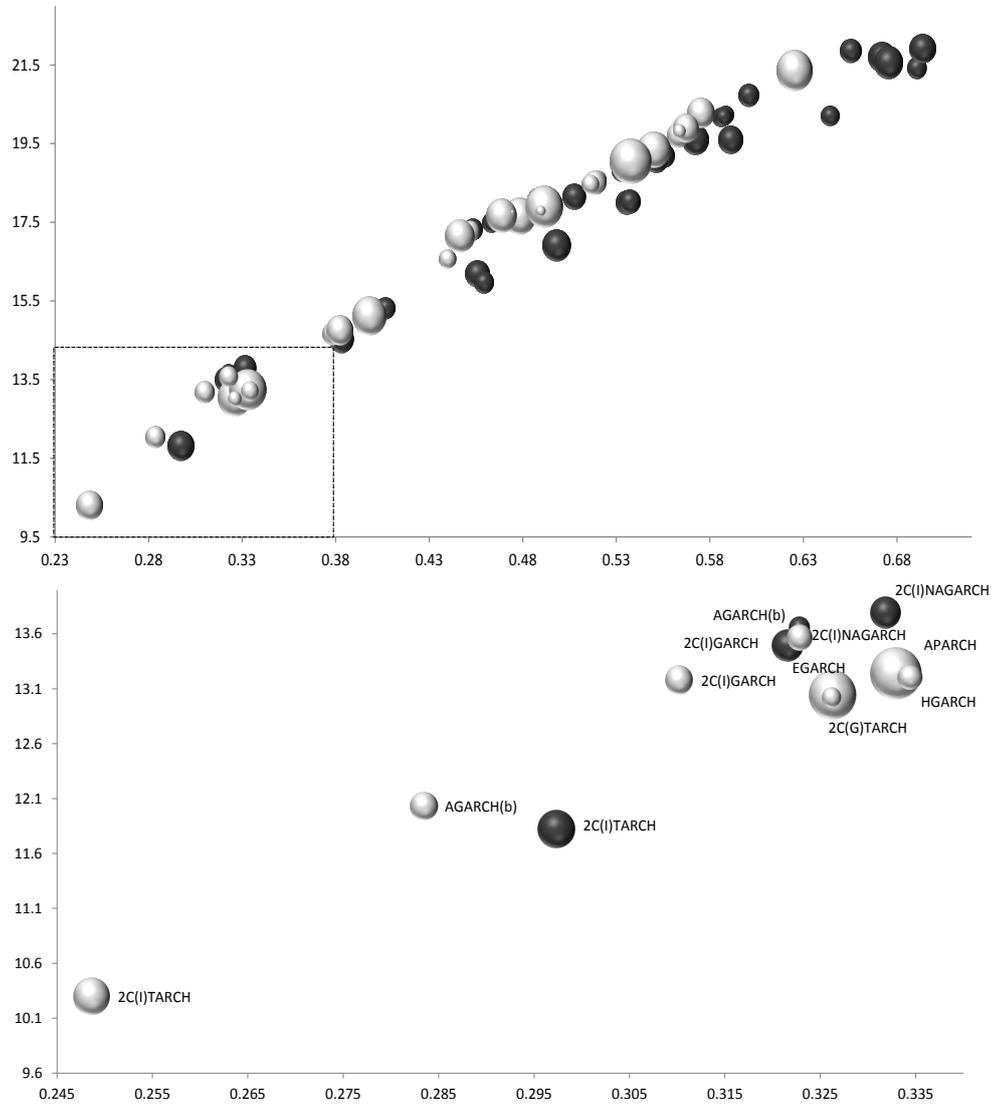


Figure 3: MAD and MARD losses for all options

Note: MAD (x-axis) and MARD (y-axis). Each bubble identifies a group of correlation models (CCC, DCC, DDCC, ADCC) characterized by the same specifications for the variance of the marginal processes. The bubbles are centered around the within group average loss, their diameter is the range computed on the MAD. Black and grey bubbles identify the models using the Gaussian and Laplace densities respectively. The second plot zooms in on the square in the first plot.

ifications, the models using Laplace conditional density perform about 20% better than the same specifications using the Gaussian density. The same holds to a similar extent for the other clusters of models.

### 4.3 Results across moneyness and maturity

Table 8 reports MAD losses along the different categories of moneyness as defined in Table 3. The OTM contracts, which have a low average price, are more difficult to price (relative errors spanning from 30% to 40% as can be seen in Table 12 of the Appendix) and increasing the complexity of the model still pays off generally. First, with respect to the marginal variances we find again that the largest improvement is for the two component models (up to 55%) and that the other pairwise categories give less pronounced differences than for all the contracts together. Figure 4 (top) illustrates that indeed the individual models are characterized by substantially different performances. Second, regarding the correlation models it turns out that idiosyncratic dynamics outperforms asymmetry under a Gaussian innovation assumption only. Third, relaxing the Gaussian assumption to the Laplace distribution delivers important gains of larger size than for all the contracts above.

The overall picture of the ITM and DITM options is rather similar to the entire aggregate of options. As can be seen from the bubble plot in Figure 4, the absolute dollar losses are similar to the OTM contracts for good models. It is also clear that increasing the model sophistication of the marginal variances still provides gains, though to a lower extent than what is observed for OTM contracts. The distributional assumption and the choice of the correlation model plays a marginal, if not negligible, role. However, looking solely at the performances of models based on the MAD provides only a partial understanding of the results. In fact, it has to be mentioned that, unlike OTM options, ITM and DITM contracts are characterized by a much higher value (see Table 3). Thus, when the pricing accuracy is measured in terms of relative errors, losses drop into the 11%-16% range for ITM options and between 4.5% and 5.6% for DITM options, which shows a dramatic increase in the degree of accuracy with respect to the profitability of the category of options. The two component models again show overall the best performances. The performance of less accurate models also improves showing a large reduction in across model variability, hence the value of model sophistication declines rapidly as the moneyness increases. Evidence of this is the very tight

interval in which all models cluster, in particular, when used to price DITM contracts.

Table 9 reports the MAD losses along the different categories of maturity as defined in Table 3. From this it is clear that short maturity options are much more accurately priced, with errors within the range of 7.4% and 9.5%, than medium and long term options, with errors in the 8.4%-13% and 20.3%-29.8% range, respectively. With respect to the short maturity options, model sophistication pays off to a lesser extent, and as before more complex marginal variances specifications take the largest share of the potential gains. While there is some gain in allowing for dynamics in the correlation, the simplest common dynamics specification (DCC) seem to be sufficient. Finally, regarding the innovation distribution, deviating from Gaussianity does not yield systematic gains.

As the maturity increases, the choice of an adequate model and thus the value of model sophistication becomes striking. As appears from Figure 4 (bottom), not only performances of all models deteriorate (both in absolute and relative terms) but also the forecasts become less informative (i.e. they converge to their unconditional levels) and model performances become more dispersed.

#### 4.4 Model confidence sets

The MCS approach of Hansen, Lunde, and Nason (2011) allows to identify, from an initial set of  $M$  models, a subset of forecasts that contains the best forecast at a confidence level  $\alpha$ . Let us denote the initial set of option price forecasts  $\mathcal{M}^0 : \{\hat{C}_{m,n} \in \mathcal{M}^0 \forall m = 1, \dots, M\}$ , where  $n = 1, \dots, W$  denotes the sequence of option contracts with true price  $C_n$  and which are considered as a cross-section. The starting hypothesis is that all forecasts in  $\mathcal{M}^0$  have equal forecasting performance, measured by a loss function  $L_{m,n} = L(C_n, \hat{C}_{m,n})$ . Here,  $L_{m,n}$  is either the MAD or the MARD loss function as defined in Section 4.1. Let  $d_{ij,n} = L_{i,n} - L_{j,n} \forall i, j \in \mathcal{M}^0$  define the relative performance of forecast  $i$  and  $j$  for contract  $n$ . The null hypothesis takes the form  $H_{0,\mathcal{M}^0} : E(d_{ij,n}) = 0, \forall i, j = 1, \dots, M$ . We use the ‘deviation’ statistic defined as  $T_D = M^{-1} \sum_{i \in \mathcal{M}^0} t_i^2$ , where  $t_i = \sqrt{W} \bar{d}_i / \sqrt{\text{Var}(\sqrt{W} \bar{d}_i)}$  represents the standardized relative performance of forecast  $i$  with respect to the average across forecasts,  $\bar{d}_i = M^{-1} \sum_{j \in \mathcal{M}^0} \bar{d}_{ij}$  and  $\bar{d}_{ij} = W^{-1} \sum_{n=1}^W d_{ij,n}$  is the sample loss difference between forecast  $i$  and  $j$ . An *i.i.d.* bootstrap scheme, producing 10,000 resamples, is used to obtain the distribution under the null. If the null is rejected, an elimination rule removes the forecast with

**Table 8: Absolute deviation losses in dollars by moneyness**

Panel A: Gaussian innovations										
Characteristics of the marginal variances										
	lin vs. non lin		sym vs. asym		$h_t$ vs. $f(h_t)$		cst par vs. st		std vs. comp	
OTM (406 contracts)										
CCC	0.779	0.777	0.893	0.720	0.794	0.740	0.753	0.900	0.846	0.555
DCC	0.770	0.767	0.884	0.712	0.784	0.729	0.744	0.891	0.836	0.572
DDCC	0.763	0.761	0.873	0.705	0.778	0.723	0.737	0.882	0.829	0.538
ADCC	0.766	0.763	0.879	0.706	0.781	0.724	0.739	0.885	0.832	0.540
ITM (334 contracts)										
CCC	0.460	0.435	0.487	0.419	0.459	0.401	0.431	0.494	0.471	0.344
DCC	0.451	0.424	0.477	0.410	0.449	0.390	0.422	0.484	0.461	0.349
DDCC	0.452	0.428	0.479	0.412	0.452	0.393	0.424	0.486	0.464	0.335
ADCC	0.445	0.420	0.473	0.404	0.445	0.385	0.416	0.479	0.457	0.329
DITM (497 contracts)										
CCC	0.382	0.388	0.408	0.376	0.385	0.390	0.379	0.422	0.410	0.307
DCC	0.381	0.384	0.404	0.373	0.382	0.385	0.376	0.418	0.406	0.316
DDCC	0.375	0.383	0.400	0.372	0.379	0.386	0.374	0.416	0.405	0.303
ADCC	0.376	0.382	0.402	0.370	0.379	0.383	0.373	0.416	0.405	0.301
Panel B: Laplace innovations										
Characteristics of the marginal variances										
	lin vs. non lin		sym vs. asym		$h_t$ vs. $f(h_t)$		cst par vs. st		std vs. comp	
OTM (406 contracts)										
CCC	0.736	0.620	0.735	0.609	0.717	0.498	0.616	0.756	0.694	0.511
DCC	0.727	0.607	0.723	0.597	0.705	0.484	0.618	0.743	0.681	0.500
DDCC	0.717	0.605	0.719	0.598	0.701	0.481	0.617	0.742	0.679	0.516
ADCC	0.718	0.601	0.718	0.595	0.699	0.477	0.615	0.739	0.676	0.518
ITM (334 contracts)										
CCC	0.446	0.377	0.426	0.380	0.432	0.310	0.387	0.436	0.416	0.329
DCC	0.435	0.367	0.415	0.370	0.422	0.299	0.377	0.427	0.405	0.319
DDCC	0.434	0.367	0.415	0.372	0.422	0.297	0.378	0.428	0.406	0.330
ADCC	0.431	0.362	0.411	0.368	0.418	0.293	0.374	0.423	0.400	0.329
DITM (497 contracts)										
CCC	0.367	0.338	0.354	0.342	0.364	0.303	0.340	0.374	0.361	0.295
DCC	0.363	0.334	0.349	0.338	0.359	0.301	0.336	0.370	0.357	0.292
DDCC	0.364	0.334	0.351	0.339	0.360	0.300	0.338	0.371	0.358	0.299
ADCC	0.362	0.331	0.348	0.336	0.357	0.297	0.334	0.368	0.354	0.299

Note: lin (non lin): models linear (non linear) in  $\epsilon_t^2$ , sym (asym): models with symmetric (asymmetric) impact of shocks wrt the sign of the shock,  $h_t$  ( $f(h_t)$ ): models where the variance (a function of the variance) is explicitly modelled, cst (st): models with constant parameters (smooth transition), std (comp): standard GARCH (component GARCH). See Table 6 for the specific models belonging to each group.

**Table 9: Absolute deviation losses in dollars by maturity**

Panel A: Gaussian innovations										
Characteristics of the marginal variances										
	lin vs. non lin		sym vs. asym		$h_t$ vs. $f(h_t)$		cst par vs. st		std vs. comp	
ST (428 contracts)										
CCC	0.189	0.173	0.186	0.173	0.186	0.157	0.176	0.185	0.182	0.162
DCC	0.186	0.168	0.182	0.168	0.180	0.156	0.171	0.181	0.177	0.159
DDCC	0.194	0.179	0.192	0.178	0.192	0.162	0.181	0.191	0.188	0.167
ADCC	0.183	0.167	0.180	0.166	0.180	0.151	0.169	0.179	0.175	0.156
MT (335 contracts)										
CCC	0.412	0.380	0.423	0.371	0.409	0.341	0.381	0.425	0.408	0.325
DCC	0.403	0.370	0.413	0.363	0.399	0.330	0.372	0.416	0.398	0.326
DDCC	0.412	0.377	0.420	0.370	0.407	0.337	0.379	0.422	0.406	0.322
ADCC	0.399	0.365	0.410	0.357	0.395	0.326	0.367	0.411	0.394	0.311
LT (474 contracts)										
CCC	0.929	0.955	1.068	0.888	0.951	0.942	0.919	1.094	1.035	0.664
DCC	0.924	0.945	1.061	0.881	0.937	0.944	0.911	1.085	1.025	0.690
DDCC	0.898	0.927	1.035	0.862	0.963	0.916	0.891	1.064	1.005	0.637
ADCC	0.918	0.941	1.055	0.874	0.938	0.926	0.906	1.079	1.021	0.650

Panel B: Laplace innovations										
Characteristics of the marginal variances										
	lin vs. non lin		sym vs. asym		$h_t$ vs. $f(h_t)$		cst par vs. st		std vs. comp	
ST (428 contracts)										
CCC	0.197	0.177	0.190	0.179	0.193	0.158	0.181	0.190	0.187	0.170
DCC	0.193	0.174	0.187	0.175	0.189	0.155	0.177	0.188	0.183	0.166
DDCC	0.193	0.173	0.185	0.175	0.189	0.154	0.177	0.186	0.182	0.169
ADCC	0.191	0.171	0.185	0.174	0.187	0.152	0.176	0.185	0.180	0.168
MT (335 contracts)										
CCC	0.411	0.352	0.392	0.355	0.398	0.296	0.362	0.395	0.381	0.321
DCC	0.400	0.342	0.381	0.346	0.388	0.286	0.352	0.387	0.371	0.312
DDCC	0.400	0.342	0.382	0.348	0.389	0.284	0.353	0.388	0.372	0.320
ADCC	0.398	0.338	0.378	0.342	0.385	0.281	0.348	0.384	0.368	0.309
LT (474 contracts)										
CCC	0.862	0.743	0.852	0.735	0.845	0.610	0.750	0.896	0.828	0.598
DCC	0.854	0.730	0.839	0.724	0.833	0.599	0.739	0.882	0.816	0.588
DDCC	0.844	0.729	0.839	0.726	0.830	0.597	0.739	0.883	0.815	0.610
ADCC	0.844	0.723	0.835	0.721	0.827	0.591	0.735	0.878	0.809	0.612

Note: lin (non lin): models linear (non linear) in  $\epsilon_t^2$ , sym (asym): models with symmetric (asymmetric) impact of shocks wrt the sign of the shock,  $h_t$  ( $f(h_t)$ ): models where the variance (a function of the variance) is explicitly modelled, cst (st): models with constant parameters (smooth transition), std (comp): standard GARCH (component GARCH). See Table 6 for the specific models belonging to each group.

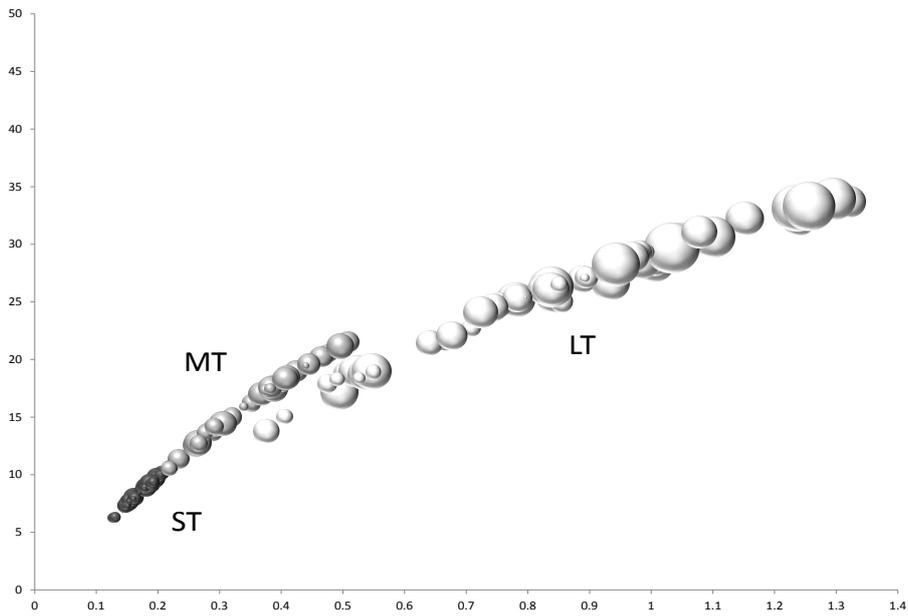
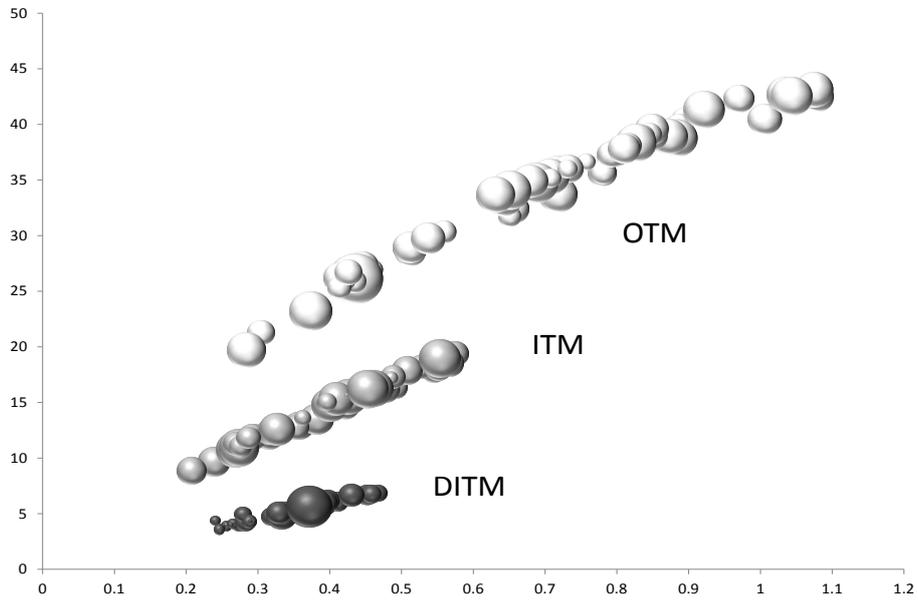


Figure 4: Models' ranking by Moneyness (top) and Maturity (bottom)

Note: MAD (x-axis) and MARD (y-axis). Each bubble identifies a group of correlation models (CCC, DCC, DDCC, ADCC) characterized by the same specifications for the variance of the marginal processes. The bubbles are centered around the within group average loss, their diameter is the range computed on the MAD.

the largest standardized relative performance. This process is repeated until non-rejection of the null occurs, thus allowing to construct a  $(1 - \alpha)$ -confidence set for the best forecast in  $\mathcal{M}^0$ .

The MCS approach has been applied for all options and for each category of moneyness and maturity. The results are summarized in Table 10 for the MAD loss. The MARD results are in line with this and presented in Table 14 of Appendix 1. When considering all option contracts, the MCS consists of a single model allowing for asymmetry in the variances and correlations, non-linearity and conditional leptokurtosis, i.e., the asymmetric DCC model with 2-component TARARCH marginal variances and Laplace conditional density. A more informative picture appears when we examine the contracts grouped by maturity/moneyness. As appears in Table 8, OTM and, to some extent ITM, contracts are the most difficult to price, showing the largest losses and loss differentials between models. The MCS is thus relatively small and reflects the large variability between model performances (within the range 0.27\$ - 1.08\$). The MCS does not reject the hypothesis of constant correlation (when coupled with a sufficiently sophisticated model for the marginal variances) but strongly supports the Laplace density against the Gaussian.

Differently, DITM contracts are relatively easier to price and hence characterized by relatively close differences between models and an overall good accuracy (within the range 0.24–0.47). In line with this evidence, the MCS suggests the absence of statistically relevant gains from the choice of the distribution or the model for the correlation (the hypothesis of constant correlation is not rejected). However, there is still evidence of significant gains from non-linearity, asymmetry, and 2-component modelling strategies.

A different picture emerges when we focus on the maturity structure of the set of contracts under consideration. Short maturity contracts show extremely small differences in performances between models, e.g., the losses lie in the band 0.12\$ - 0.21\$. However, the MCS delivers a singleton, which indicates that although models' sample performances are extremely close, they are also extremely stable thus allowing to efficiently discriminate between models. It is worth noting also that this is the only case where the Laplace assumption generates systematically larger losses than the pricing under Gaussianity. As the maturity increases, accurately pricing contracts becomes more difficult and we see large differences between model performances (within the range 0.37\$ - 1.30\$). For very long maturities, the

**Table 10: Model confidence sets (MAD loss,  $\alpha = 1\%$ )**

		Correlation	Variances	Distribution	Loss
All options		ADCC	2C(I)Tarch	Laplace	0.243
Moneyness	OTM	ADCC	2C(I)Tarch	Laplace	0.276
		DDCC	2C(I)Tarch	Laplace	0.278
		DCC	2C(I)Tarch	Laplace	0.286
		CCC	2C(I)Tarch	Laplace	0.298
		ADCC	Agarch(b)	Laplace	0.299
		DDCC	Agarch(b)	Laplace	0.301
		DCC	Agarch(b)	Laplace	0.303
		CCC	Agarch(b)	Laplace	0.315
	ITM	ADCC	2C(I)Tarch	Laplace	0.201
		ADCC	Agarch(b)	Gaussian	0.239
		DCC	Agarch(b)	Gaussian	0.239
		CCC	Agarch(b)	Gaussian	0.242
		ADCC	2C(I)Tarch	Laplace	0.244
		DDCC	Agarch(b)	Gaussian	0.244
		DDCC	2C(I)Tarch	Laplace	0.245
		DCC	2C(I)Tarch	Laplace	0.247
		CCC	2C(I)Tarch	Laplace	0.250
		ADCC	2C(I)Garch	Laplace	0.251
		DDCC	2C(I)Garch	Laplace	0.252
		CCC	2C(I)Garch	Laplace	0.253
		DCC	2C(I)Garch	Laplace	0.253
	DITM	ADCC	2C(I)Garch	Gaussian	0.254
		DDCC	2C(I)Garch	Gaussian	0.255
		DCC	2C(I)Garch	Gaussian	0.256
		CCC	2C(I)Garch	Gaussian	0.260
		ADCC	2C(I)Nagarch	Laplace	0.261
		DDCC	2C(I)Nagarch	Laplace	0.262
ADCC		2C(I)Nagarch	Gaussian	0.263	
CCC		2C(I)Nagarch	Laplace	0.264	
DCC		2C(I)Nagarch	Gaussian	0.264	
DCC		2C(I)Nagarch	Laplace	0.264	
DDCC		2C(I)Nagarch	Gaussian	0.266	
CCC		2C(I)Nagarch	Gaussian	0.269	
ADCC		2C(I)Tarch	Gaussian	0.270	
DCC		2C(I)Tarch	Gaussian	0.273	
DDCC		2C(I)Tarch	Gaussian	0.276	
ADCC		Agarch(b)	Laplace	0.277	
ADCC		Egarch	Laplace	0.277	
DDCC		Agarch(b)	Laplace	0.277	
DCC		Agarch(b)	Laplace	0.278	
DCC	Egarch	Laplace	0.280		
CCC	Agarch(b)	Laplace	0.286		
ST	ADCC	2C(I)Tarch	Gaussian	0.124	
	MT	ADCC	2C(I)Tarch	Laplace	0.212
Maturity		ADCC	2C(I)Tarch	Laplace	0.368
		DDCC	2C(I)Tarch	Laplace	0.370
		DCC	2C(I)Tarch	Laplace	0.378
		CCC	2C(I)Tarch	Laplace	0.388
	LT	DDCC	Agarch(b)	Laplace	0.401
		ADCC	Agarch(b)	Laplace	0.402
		DCC	Agarch(b)	Laplace	0.406
		CCC	Agarch(b)	Laplace	0.414

MCS delivers a relatively small MCS. Gaussianity is rejected as well as symmetry and linearity in the marginal variances. Also in this case there is no significant gain in modelling the dynamics of the conditional correlation. The rejection of the dynamic correlation specification is in some case somewhat surprising. In fact, it is likely that the functional forms of the correlations models considered in this paper are not sufficient in the thirty dimensional case of the DJIA (e.g. two DCC parameters for 435 correlations), and therefore yield biased correlation forecasts. Evidence of this has also been reported by Audrino (2011) and Laurent, Rombouts, and Violante (2011).

## 5 Conclusion

Multivariate volatility models have been used extensively to model financial data and forecasts from these models are typically used as inputs in empirical asset pricing. This is particularly important for derivative pricing where the underlying of some of the most liquidly traded options, the index options, are portfolios of equally weighted assets. Despite the large availability of multivariate models of asset returns, the problem of model selection based on derivative pricing accuracy has received little attention. This paper investigates the value of model sophistication by considering a large number of high dimensional GARCH models and measuring performance in terms of Dow Jones Industrial Average option pricing accuracy. Since the option price does not only depend on the conditional variance matrix but also on other conditional moments, we are actually able to evaluate the full forecasting model.

In total we consider 248 multivariate GARCH models differing along three dimensions: the specification of the conditional variance, the conditional correlation, and the innovation distribution. The results of our application are first of all that in general more complicated models provide better option pricing forecasts. It turns out that the most important improvements in pricing come from increasing the sophistication in the marginal variance processes and from relaxing the Gaussian innovation for a Laplace innovation assumption. Enriching the model with more complex correlation models, and relaxing a Gaussian innovation for a Laplace innovation assumption improves the pricing in a smaller way. Overall, increasing model sophistication can reduce the dollar loss up to 60%. The model that performs best, according to the model confidence test, is the two component threshold GARCH in combination with an asymmetric DCC structure and Laplace innovations.

The results of this paper provide clear support for the use of models beyond standard multivariate volatility models for option pricing. However, the best models are often close to the boundary of sophistication. Since we consider only four correlation type specifications and two relatively close innovation densities, compared to the thirty-four univariate volatility models, it is expected that more flexible correlation models, e.g. Audrino and Trojani (2011), and especially innovation densities (which will require further developments for feasible option pricing) will further enhance the option pricing performance. An example is the dynamic copula approach of Fengler, Herwartz, and Werner (2010). Finally, given the evidence for example in Driessen, Maenhout, and Vilkov (2009), another challenging extension is to price correlation risk. This would require additional parameters and one step estimation in a thirty dimensional setup.

## Appendix 1: results for mean absolute deviation loss function

**Table 11: Mean absolute relative deviation losses in percentages within each class of models**

Panel A: Gaussian innovations										
Characteristics of the marginal variances										
	lin vs. non lin		sym vs. asym		$h_t$ vs. $f(h_t)$		cst par vs. st		std vs. comp	
CCC	18.86	18.17	19.72	17.67	18.84	17.22	18.05	19.88	19.25	15.43
DCC	18.48	17.75	19.45	17.55	18.55	16.39	17.75	19.56	18.96	15.28
DDCC	18.71	18.01	19.55	17.51	18.69	17.03	17.89	19.73	19.11	15.19
ADCC	18.55	17.70	19.19	17.30	18.41	16.81	17.60	19.54	18.81	15.03

Panel B: Laplace innovations										
Characteristics of the marginal variances										
	lin vs. non lin		sym vs. asym		$h_t$ vs. $f(h_t)$		cst par vs. st		std vs. comp	
CCC	18.47	16.43	18.17	16.38	18.10	14.34	16.68	18.45	17.62	14.84
DCC	18.24	16.15	17.90	16.11	17.86	14.04	16.41	18.22	17.36	14.57
DDCC	18.16	16.10	17.87	16.14	17.82	13.95	16.41	18.20	17.33	14.89
ADCC	18.12	16.00	17.81	16.05	17.75	13.84	16.33	18.11	17.22	14.88

Note: lin (non lin): models linear (non linear) in  $\epsilon_t^2$ , sym (asym): models with symmetric (asymmetric) impact of shocks wrt the sign of the shock,  $h_t$  ( $f(h_t)$ ): models where the variance (a function of the variance) is explicitly modelled, cst (st): models with constant parameters (smooth transition), std (comp): standard GARCH (component GARCH). See Table 6 for the specific models belonging to each group.

**Table 12: Absolute relative deviation losses in percentages by moneyness**

Panel A: Gaussian innovations												
Characteristics of the marginal variances												
	lin vs. non lin		sym vs. asym		$h_t$ vs. $f(h_t)$		cst par vs. st		std vs. comp			
					OTM (406 contracts)							
CCC	36.99	35.84	38.98	34.73	37.01	34.13	35.54	39.17	37.88	30.46		
DCC	36.69	35.46	38.70	34.45	36.69	33.69	35.23	38.87	37.54	30.88		
DDCC	36.67	35.46	38.61	34.37	36.68	33.70	35.17	38.84	37.56	29.95		
ADCC	36.80	35.28	38.54	34.16	36.54	33.47	35.01	38.67	37.38	29.84		
					ITM (334 contracts)							
CCC	16.34	15.28	16.75	14.97	16.19	14.09	15.31	16.81	16.27	13.23		
DCC	16.05	14.91	16.41	14.68	15.87	13.69	15.00	16.48	15.92	13.29		
DDCC	16.22	15.13	16.61	14.83	16.07	13.91	15.18	16.66	16.16	13.02		
ADCC	15.88	14.77	16.29	14.45	15.72	13.52	14.81	16.33	15.79	12.69		
					DITM (497 contracts)							
CCC	5.73	5.69	5.99	5.56	5.78	5.52	5.60	6.19	6.03	4.63		
DCC	5.69	5.61	5.92	5.50	5.72	5.44	5.54	6.12	5.95	4.73		
DDCC	5.71	5.68	5.96	5.55	6.23	5.51	5.59	6.18	6.02	4.60		
ADCC	5.63	5.57	5.89	5.44	5.67	5.39	5.49	6.08	5.92	4.51		

Panel B: Laplace innovations												
Characteristics of the marginal variances												
	lin vs. non lin		sym vs. asym		$h_t$ vs. $f(h_t)$		cst par vs. st		std vs. comp			
					OTM (406 contracts)							
CCC	36.14	32.19	35.97	31.88	35.39	28.25	32.61	36.41	34.51	29.10		
DCC	35.81	31.71	35.55	31.43	35.00	27.68	32.16	36.00	34.07	28.63		
DDCC	35.56	31.58	35.43	31.45	34.88	27.50	32.12	35.94	33.97	29.19		
ADCC	35.56	31.46	35.40	31.34	34.82	27.35	32.04	35.84	33.85	29.22		
					ITM (334 contracts)							
CCC	16.12	14.07	15.53	14.16	15.71	12.05	14.41	15.67	15.14	12.91		
DCC	15.79	15.18	15.18	13.85	15.40	11.70	14.07	15.40	14.82	12.57		
DDCC	15.78	13.72	15.18	13.91	15.42	11.63	14.11	15.40	14.81	12.89		
ADCC	15.68	13.57	15.06	13.76	15.28	11.48	13.97	15.26	14.65	12.84		
					DITM (497 contracts)							
CCC	5.61	5.14	5.40	5.20	5.58	4.52	5.19	5.64	5.50	4.50		
DCC	5.54	5.06	5.32	5.12	5.50	4.46	5.11	5.58	5.42	4.43		
DDCC	5.55	5.06	5.34	5.14	5.51	4.45	5.13	5.59	5.43	4.55		
ADCC	5.52	5.00	5.29	5.08	5.46	4.39	5.08	5.54	5.36	4.54		

Note: lin (non lin): models linear (non linear) in  $\epsilon_t^2$ , sym (asym): models with symmetric (asymmetric) impact of shocks wrt the sign of the shock,  $h_t$  ( $f(h_t)$ ): models where the variance (a function of the variance) is explicitly modelled, cst (st): models with constant parameters (smooth transition), std (comp): standard GARCH (component GARCH). See Table 6 for the specific models belonging to each group.

**Table 13: Absolute relative deviation losses in percentages by maturity**

Panel A: Gaussian innovations										
Characteristics of the marginal variances										
	lin vs. non lin		sym vs. asym		$h_t$ vs. $f(h_t)$		cst par vs. st		std vs. comp	
ST (428 contracts)										
CCC	9.37	9.31	9.26	8.59	9.23	7.85	8.74	9.17	9.03	8.12
DCC	9.18	8.39	9.04	8.42	9.02	7.62	8.55	8.98	8.81	8.08
DDCC	9.48	8.73	9.37	8.71	9.35	7.95	8.86	9.30	9.16	8.18
ADCC	9.09	8.30	8.99	8.27	8.94	7.52	8.44	8.88	8.73	7.81
MT (335 contracts)										
CCC	18.46	17.19	18.83	16.88	18.29	15.75	17.28	18.78	18.20	15.32
DCC	18.17	16.83	18.52	16.61	17.99	15.35	16.98	18.48	17.87	15.37
DDCC	18.42	17.04	18.72	16.76	18.22	15.54	17.16	18.66	18.10	15.14
ADCC	18.04	16.69	18.41	16.37	17.85	15.17	16.79	18.32	17.73	14.79
LT (474 contracts)										
CCC	27.70	27.51	29.81	26.43	27.91	26.73	27.00	30.34	29.21	22.12
DCC	27.57	27.28	29.65	26.28	27.73	26.48	26.83	30.15	29.02	22.62
DDCC	27.24	27.07	29.34	26.00	27.47	26.28	26.55	29.90	28.80	21.57
ADCC	27.44	27.18	29.54	26.10	27.64	26.34	26.69	30.03	28.93	21.73
Panel B: Laplace innovations										
Characteristics of the marginal variances										
	lin vs. non lin		sym vs. asym		$h_t$ vs. $f(h_t)$		cst par vs. st		std vs. comp	
ST (428 contracts)										
CCC	9.59	8.65	9.27	8.72	9.40	7.73	8.83	9.24	9.09	8.29
DCC	9.37	8.47	9.10	8.51	9.21	7.55	8.63	9.10	8.90	8.10
DDCC	9.38	8.41	9.06	8.52	9.19	7.48	8.63	9.04	8.86	8.21
ADCC	9.31	8.35	9.02	8.44	9.13	7.42	8.56	8.99	8.79	8.17
MT (335 contracts)										
CCC	18.35	16.13	17.79	16.18	17.89	14.00	16.50	17.81	17.22	15.09
DCC	18.03	15.78	17.45	15.84	17.57	13.59	16.14	17.54	16.88	14.73
DDCC	18.01	15.74	17.43	15.91	17.58	13.51	16.17	17.55	16.87	15.04
ADCC	17.93	15.61	17.34	15.78	17.47	13.38	16.06	17.43	16.73	15.00
LT (474 contracts)										
CCC	26.58	23.66	26.46	23.43	26.10	20.55	23.88	27.21	25.61	20.58
DCC	26.41	23.36	26.18	23.17	25.87	20.21	23.62	26.93	25.35	20.31
DDCC	26.20	23.30	26.14	23.19	25.79	20.11	23.61	26.92	25.30	20.82
ADCC	26.22	23.18	26.08	23.10	25.74	19.97	23.53	26.83	25.18	20.85

Note: lin (non lin): models linear (non linear) in  $\epsilon_t^2$ , sym (asym): models with symmetric (asymmetric) impact of shocks wrt the sign of the shock,  $h_t$  ( $f(h_t)$ ): models where the variance (a function of the variance) is explicitly modelled, cst (st): models with constant parameters (smooth transition), std (comp): standard GARCH (component GARCH). See Table 6 for the specific models belonging to each group.

**Table 14: Model confidence sets (MARD loss,  $\alpha = 1\%$ )**

		Correlation	Variances	Distribution	Loss
All options		ADCC	2C(I)TARCH	Laplace	10.0
Moneyness	OTM	ADCC	2C(I)TARCH	Laplace	19.3
		DDCC	2C(I)TARCH	Laplace	19.4
		EDCC	2C(I)TARCH	Laplace	19.8
		CCC	2C(I)TARCH	Laplace	20.4
	ADCC	AGARCH(b)	Laplace	21.0	
	DDCC	AGARCH(b)	Laplace	21.0	
	EDCC	AGARCH(b)	Laplace	21.2	
	CCC	AGARCH(b)	Laplace	21.8	
	DDCC	2C(I)TARCH	Gaussian	22.9	
	ITM	ADCC	2C(I)TARCH	Laplace	8.6
DITM	ADCC	2C(I)TARCH	Laplace	3.5	
Maturity	ST	ADCC	2C(I)TARCH	Gaussian	6.1
		ADCC	2C(I)TARCH	Laplace	6.1
		EDCC	2C(I)TARCH	Gaussian	6.2
		DDCC	2C(I)TARCH	Laplace	6.2
	MT	ADCC	2C(I)TARCH	Laplace	10.3
	LT	ADCC	2C(I)TARCH	Laplace	13.4
		DDCC	2C(I)TARCH	Laplace	13.5
EDCC		2C(I)TARCH	Laplace	13.8	
CCC		2C(I)TARCH	Laplace	14.2	
DDCC		AGARCH(b)	Laplace	14.9	
ADCC		AGARCH(b)	Laplace	14.9	
EDCC		AGARCH(b)	Laplace	15.1	
CCC	AGARCH(b)	Laplace	15.3		

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