Transparency in the Financial System: Rollover Risk and Crises

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Abstract:
The paper presents a theory of optimal transparency in the financial system when financial institutions have short-term liabilities and are exposed to rollover risk. Our analysis indicates that transparency enhances the stability of the financial system during crises but may have a destabilizing effect during normal economic times. Thus, the optimal level of transparency is contingent on the state of the economy, with the regulator increasing disclosure in times of crises. Under this policy, however, an increase in disclosure signals a deterioration of the economy’s fundamentals, so the regulator has incentives to withhold information ex-post. In that case, the regulator may have to commit ex-ante to a degree of transparency which trades off the frequency and magnitude of financial crises. The analysis also considers the possibility that financial institutions, in an attempt to deal with rollover risk, either diversify their risks or increase the liquidity of their balance sheets.

Keywords: Banking regulation, bank run, disclosure, financial stability, global games

JEL Classification: G21, G24, G01
1 Introduction

Financial crises are often associated with demands for an increase in the transparency of the financial system. For instance, chapter 3 of the Squam Lake Report (2010) includes the following recommendation: “large financial institutions should report information about asset positions and risks to regulators each quarter” and “the systemic regulator should prepare an annual “risk of the financial system” report”. The fact remains, however, that the level of transparency is generally deemed to be optimal before crises, and that even during crises there is some reluctance to increasing transparency.\(^1\) This suggests that, while transparency certainly has its benefits, it may also come with costs. This paper studies the trade-offs faced by regulators when setting the level of transparency in the financial system.

We develop a stylized model of financial intermediation with rollover risk, in which financial institutions –banks– have exclusive access to a long-term investment technology that is illiquid.\(^2\) Banks are ex-ante identical but they differ ex-post in the quality of their investment technology, and hence, in the quality of their balance sheet. Investors may have information about the state of the economy, and hence about the average quality of banks in the financial system, but they do not know the relative quality of each individual bank. While banks cannot credibly communicate about their own quality, the regulator has access to information about their relative quality and can credibly disclose it to the public.\(^3\)

\(^1\)For example, the CEO of Deutsche Bank, Josef Ackermann, supported the principle of publishing the results of stress tests, but also cautioned that going public would be “very, very dangerous” if supporting mechanisms for European banks were not in place beforehand. \(\text{(The Economist, July 2010.)}\)

\(^2\)We use the term “bank” broadly, to refer to financial intermediaries that (partly) finance imperfectly liquid investments through short-term borrowings, and therefore, that are subject to rollover risk \(\text{(e.g., commercial and investment banks).}\)

\(^3\)Notice that the regulator internalizes the welfare of the entire financial system rather than the viability of a single bank, and also that his private information is about the relative strength of banks’ bal-
In this setting, we show that the optimal disclosure policy depends on the average quality of banks in the financial system. When the average quality is high enough that investors are willing to rollover their credit, it is optimal not to disclose information as transparency may expose lower-quality banks to a run. On the contrary, when the average quality is sufficiently low, the regulator will choose transparency, that is, it will disclose the quality of the balance sheet of each individual bank. Otherwise, if investors knew that the average quality was low but could not tell which banks are of higher relative quality, there would be a run on the whole banking system. This result relies on the threshold nature of the equilibrium which makes the probability of a run a non-linear function of a bank’s quality. That is, investors run on any given bank only if its expected quality is below some threshold. Thus, if economic conditions are such that many banks are well above this threshold, pooling these banks with a few lower-quality banks does not have a significant effect on their rollover risk, while it may avert runs on the lower-quality banks.

The result is consistent with the demands for an increase in transparency that typically accompany financial crises. For instance, following these demands, the European Union took the decision in July of 2010 to publish the results of stress tests on Europe’s 25 largest banks. The publication of these tests however was the subject of much debate, which can also be rationalized in the context of our model. On the one hand, Spanish authorities were in favor of releasing the stress test data arguing that investors were too gloomy about Spanish banks. On the other hand, Germany, where economic conditions were better than in Spain, was reluctant to release this information. (See The Economist, July 2010.)

Implementing the optimal disclosure policy, i.e., increasing transparency when the average quality of banks decreases, is not without difficulties. Consider a situation in
which the regulator has private information not only about the quality of each individual bank but also about the average quality of banks in the financial system. In such a case, we show that it is still optimal for the regulator to commit to disclosing information when average quality falls below a threshold. However, since investors perceive information disclosure as a sign of low average quality in the system, the regulator has incentives to renego on that commitment ex-post. This suggests that, in the initial stages of a crisis, regulators have a tendency to retain information from investors. Only if the crisis aggravates and is likely to become common knowledge among investors, does the regulator start disclosing more information to the public and transparency increases. To overcome this commitment problem, the regulator may have to rely on an independent entity and implement a disclosure policy that does not depend on the situation of the financial system.\footnote{Studying a non-contingent transparency policy is also interesting because switching from an opaque to a transparent regime may require a certain amount of time. For instance, US regulatory authorities released the results of stress tests performed on the nation’s major banks in May 2009, while the liquidity crisis had reached a peak around the collapse of Lehman Brothers, in September 2008.} In that case, the regulator faces a trade-off: a decrease in transparency reduces the frequency of bank runs, but runs affect more banks when they occur.

The optimal disclosure policy is also related to recent regulatory proposals that call for the need to distinguish risky assets according to their systemic component, e.g., Morris and Shin (2008). In the paper, we extend the basic model to make each bank’s idiosyncratic risk a choice variable and show that the optimal disclosure policy allows achieving some of the benefits of diversification while avoiding some of its costs. Specifically, when the average quality of banks in the financial system is high, the lack of public information about each individual bank insures banks against negative bank-specific shocks. Alternatively, when the average quality of banks is low, diversification can be costly since those banks that are liquidated hold, on average, higher-quality assets than in the absence of diversification.\footnote{When the Trouble Asset Relief Program –TARP– was created (October 2008) the US Treasury...} This result highlights the shortcomings of taking each bank’s asset volatility as a measure of aggregate risk for the financial...
system. Intuitively, if banks are connected—in our case through the disclosure policy—an optimal regulation should take into account assets correlations across banks.

In most of the paper we take rollover risk as a given. Banks, however, may attempt to address rollover risk by increasing the liquidity of their balance sheet—for instance, by reducing the maturity of their assets or increasing the maturity of their liabilities. In such a case, the optimal disclosure policy depends on the average quality of banks in the financial system as well as on the liquidity of banks’ balance sheets. In particular, ceteris paribus, an increase in liquidity across the board is associated with less transparency: a liquid balance sheet reduces investors’ incentives to run, which, in turn, allows decreasing transparency and prevents runs on lower-quality banks, without compromising the stability of the whole system.

Given this interplay between the liquidity of the balance sheet and the optimal disclosure policy, we show that regulators can fall into policy traps, and that banks may end up with a balance sheet that is either more or less liquid than is socially desirable. This is due to the fact that the optimal disclosure policy depends on the average liquidity of banks in the financial system, and that each individual bank takes this average liquidity as given when choosing its own liquidity. For instance, consider the case in which a bank expects other banks to have a liquid balance sheet and hence the optimal disclosure policy by the regulator to be one of low transparency. Then this bank has incentives to also have a liquid balance sheet as it may otherwise suffer a run if the average quality turns out to be low and no disclosure takes place.\(^6\)

The previous result relies on investors being able to observe how liquid the balance

\(^6\)Alternatively, if a bank expects other banks to reduce the liquidity of their balance sheet, because, for instance, of risk-shifting incentives, then this bank has incentives to reduce the liquidity of its balance sheet as well.
sheet of each individual bank is. However, in the same way that it is difficult for investors to assess the quality of an individual bank’s balance sheet, it may also be difficult for them to assess its liquidity. In such a case, there is a strong tendency for banks to have a balance sheet that is too illiquid from a social point of view. Intuitively, liquidity allows decreasing transparency which prevents runs on low-quality banks. However, since increasing liquidity is costly for each bank, i.e., it decreases its expected returns, the banking system faces the typical public good problem, in which banks do not internalize the effect that increasing the liquidity of their balance sheets has on the stability of the whole system.\textsuperscript{7}

This paper builds on seminal models by Byrant (1980) and Diamond and Dybvig (1983) where strategic complementarities between depositors may trigger runs and lead to the early liquidation of solvent but illiquid banks. Because these models typically have several equilibria, which makes the impact of public policies difficult to assess, we use the global games approach (Carlsson and van Damme, 1993; Morris and Shin, 1998, 2003) to obtain equilibrium uniqueness. Our paper is therefore related to Morris and Shin (2000) and Goldstein and Pauzner (2005) who use global games techniques to refine models of bank runs.\textsuperscript{8} With respect to this stream of literature, our contribution is to introduce heterogeneity among banks, which, in turn, makes the release of bank-specific information by the regulator a relevant and sensitive issue.

Our paper is also related to the literature on transparency in the banking system.\textsuperscript{9}

\textsuperscript{7}In Bhattacharya and Gale (1987) banks with heterogeneous clienteles of depositors hold less liquidity than would be socially optimal. The reason is that, with a market for liquidity set up after the liquidity needs of the different banks are known, it is privately optimal for banks to free ride on liquidity supply.


\textsuperscript{9}Landier and Thesmar (2011) review the trade-offs related to transparency in financial systems.
Rochet and Vives (2004) show that public information may have destabilizing effects because it reinforces coordination concerns, which generates multiple self-fulfilling equilibria. In contrast, we focus on a setting where the equilibrium is always unique, but opacity may improve the stability of the banking sector by allowing lower-quality banks to be pooled with higher-quality banks, thereby creating mutual insurance in good times.\footnote{This mechanism is reminiscent of Hirshleifer (1971), where the early knowledge of future realizations of uncertainty prevents individuals from sharing risk efficiently through transactions. However, in our model, opacity is only desirable in good times as transparency, which allows depositors to tell banks apart, prevents runs on the most profitable ones in bad economic times.}

He and Manela (2011) explore a different channel through which the release of bank-specific information limits bank runs. In their model, public information on a bank’s solvency crowds out incentives for depositors to acquire private information that could trigger an inefficient run.

Finally, our paper is related to recent contributions on liquidity and systemic risk in the banking system. Kashyap, Rajan, and Stein (2008) and Morris and Shin (2008) claim that the exposure of banks to rollover risk, due to their extensive reliance on short-term financing, is a major source of instability for the entire financial system. Specifically, they argue that banks do not fully internalize the cost that this liquidity risk imposes on the economy, and suggest new forms of intervention by the regulatory authorities. In this paper, we uncover one particular mechanism through which banks exert externalities on each other, that is, the impact of the liquidity of their balance sheets on the choice of a transparency policy by the regulator.

The paper proceeds as follows. Section 2 presents the baseline model. Section 3 analyses the equilibrium and derives the optimal transparency policy. Section 4 studies the commitment problem of the regulator when his disclosure policy signals information about the state of the financial system. Section 5 and 6 introduce the possibilities for banks to diversify idiosyncratic risks and to adjust the liquidity of their balance sheets. Section 7 concludes. All proofs are in the appendix.
2 Model

Consider a risk neutral economy with one consumption good, three periods, $t = 0, 1, 2$, and no discounting. There is a continuum $[0, 1] \times [0, 1]$ of investors, each endowed with one unit of the consumption good. At $t = 0$ investors can invest their unit in a financial institution or store it. Financial institutions, which we will call banks hereafter, have exclusive access to a long-term investment technology that generates a gross return of $1 + r_i$ per unit of consumption good at $t = 2$. Each active bank invests a mass 1 of the consumption good at $t = 0$, and there is free entry in the banking industry. Thus if all investors were to deposit their goods in banks, there would be a continuum $[0, 1]$ of banks each with a continuum $[0, 1]$ of investors.

The net return of the long-term technology, $r_i$, is stochastic. Specifically, $r_i = \mu + \eta_i$ where $\mu$ is a parameter common to all banks that captures the expected return of the banking sector, and $\eta_i$ is a bank-specific component that captures the relative quality of bank $i$. The bank-specific component $\eta_i$ can take values $\Delta^{\eta} > 0$ (for high-quality banks) and $-\Delta^{\eta}$ (for low-quality banks) with probability $p$ and $(1 - p)$, respectively. The proportion $p$ of high-quality banks is drawn from a uniform distribution on $[0, 1]$, so that $E(\eta_i) = 0$ for all $i$. The realization of $p$ is interpreted as an aggregate shock to the expected return of the banking system, $\mu$.

Investors who invest in banks at $t = 0$ can either leave their good in the bank or withdraw it at $t = 1$; thus banks face rollover risk and the possibility of early liquidation. Liquidation is costly because the technology is illiquid: if a proportion $l_i$ of the resources invested in the long-term technology is withdrawn at $t = 1$, the per-unit return at $t = 2$ is reduced to $r_i - cl_i$. For instance, if half of the bank’s investors withdraw, each of them gets one unit of the consumption good back at $t = 1$ and the other half gets $1 + r_i - \frac{c}{2}$ at $t = 2$. This investment technology is similar to the one in Morris and Shin (2001) and, in essence, models rollover risk as a coordination problem among investors.
We assume that early liquidation is efficient \((i.e., \text{the net expected return of the long-term technology is greater than zero})\) and that, when banks’ types are disclosed, low-quality banks face the risk of early liquidation while high-quality banks do not. As we will see in the analysis below (section 3), this boils down to the following assumption on the parameters of the model:\(^{11}\)

\[0 < -\Delta \eta + \mu < \frac{c}{2} < \Delta \eta + \mu. \quad (1)\]

At \(t = 1\) before making their withdrawal decision, investors learn \(p\), the proportion of high-quality banks, in the financial system. While investors do not know the quality of each of the banks in the financial system, the financial system is supervised by a regulator who has access to this information, \(i.e., \{\eta_i\}_{i} \in [0,1]\). In particular, at \(t = 1\) the regulator learns \(\{\eta_i\}_i\) and must decide whether or not to disclose this information to investors. The objective of the regulator is to maximize welfare, which is defined as the total amount of the consumption good available in the economy at \(t = 2\).

We finish the presentation of the model by describing two assumptions that we maintain throughout the analysis. First, we assume that investors have the right to withdraw at \(t = 1\), that is, financial institutions borrow short-term and face rollover risk. A feature of the recent crisis has been the sudden freeze in the credit market which led to the collapse of financial institutions that relied on the rollover of short-term debt in the asset-backed commercial paper and overnight secured repo markets. (See Acharya, Gale, and Yorulmazer [2011], and Gorton and Metrick [2011].) Thus our paper studies the optimal level of transparency in the financial system given the presence of rollover risk, that is, under the implicit assumption that while banks may try to ameliorate rollover risk, they will not be able to eliminate it.\(^{12}\) In section 6, we

\(^{11}\)As discussed in the analysis of the model, this assumption is mainly for expositional reasons; it allows focusing the paper on the more interesting cases.

\(^{12}\)From a theoretical point view, rollover risk can be micro-founded through depositors’ demands for insurance against idiosyncratic liquidity shocks as in Diamond and Dybvig (1983). Stein (2011) proposes
complete the analysis by allowing banks to increase the liquidity of their balance sheet in order to reduce the risk of rollover.

Second, the analysis implicitly assumes that banks cannot credibly disclose their information while the regulator can. That banks cannot credibly disclose soft information is straightforward: low-quality banks do not have any incentives to disclose information that would lead them to cease operations due to a credit run. The regulator, however, who is concerned about the long-term viability of the entire banking system rather than the viability of a single bank, has incentives to disclose information truthfully when he chooses a transparent regime. Notice that there is no aggregate uncertainty at $t = 1$, that is, the number of high-quality banks in the system, $p$, is common knowledge. Therefore, the only way the regulator could misrepresent a low-quality bank as being of high quality without investors immediately knowing that he is not being truthful, is by presenting a high-quality bank as being of low quality at the same time. This, however, would result in a welfare loss, as a high-quality bank would be liquidated instead of a low-quality one.\(^{13}\)

3 Analysis

In this section, we study the optimal disclosure policy in the baseline model. We start by characterizing the rollover decision at $t = 1$ and then proceed to characterize the

\(^{13}\)Chakraborty and Harbaugh (2007, 2010) analyze communication in which a single expert discloses complete or partial rankings of multiple variables in a cheap talk framework. In particular, they show that multiple dimensions increase the scope for communication and that simple rankings are often credible when other forms of cheap talk are not. Intuitively, comparative statements have the property of being positive along one dimension and negative along another dimension at the same time.
regulator’s optimal disclosure policy.

### 3.1 Rollover Equilibrium

Consider an investor who invests in bank $i$ at $t = 0$. At $t = 1$, this investor can either withdraw his unit of the consumption good or roll over his investment. An investor who rolls over his investment at $t = 1$ receives a random payoff of $1 + r_i - c l_i$ at $t = 2$. Hence, an investor’s willingness to withdraw financing depends on the withdrawal decisions of all the other investors in the bank, that is, depends on $l_i$. As Diamond and Dybvig (1983) point out, these strategic complementarities typically lead to multiple equilibria. In one equilibrium, investors roll over their investment and banks’ assets pay off at $t = 2$. Another equilibrium, however, involves a bank run in which all investors demand early withdrawal causing healthy banks to recall loans, to terminate productive investments, and eventually to fail.

This multiplicity of equilibria leads to a lack of empirical predictions and policy implications in terms of the probability of a banking crisis. Multiplicity is a by-product of the common knowledge assumption which allows perfect coordination among investors. Following the global games literature (see Carlsson and van Damme [1993] and Morris
and Shin [1998] for two seminal contributions), we relax this assumption. Specifically, instead of considering that the common component in the return of the banking industry is a known parameter $\mu$, we assume that it is equal to a random variable $\tilde{\mu}$, which is normally distributed with mean $\mu$ and precision $h_\mu$. Between $t = 0$ and $t = 1$, each investor $j$ receives a noisy signal $s_j = \tilde{\mu} + \varepsilon_j$, where $\varepsilon_j$ is normally distributed with mean 0 and precision $h_\varepsilon$, and independent across investors. When the precision of individual signals, $h_\varepsilon$, is sufficiently high, this information structure allows to obtain a unique equilibrium as it makes perfect coordination among investors impossible. In this unique equilibrium, an investor withdraws his investment at $t = 1$ if and only if the signal he receives is below a threshold.

We draw on Morris and Shin (2003) in solving for the threshold equilibrium. Since $\tilde{\mu}$ and $\varepsilon_j$ are normally distributed, the expectation of $\tilde{\mu}$ conditional on $s_j$, which we denote $\rho_j$, is

$$\rho_j = \frac{h_\mu \mu + h_\varepsilon s_j}{h_\mu + h_\varepsilon}. \quad (2)$$

An investor $j$ who invests in bank $i$ at $t = 0$ withdraws his investment at $t = 1$ if $\rho_j$ is below some threshold $\rho_i^*$. At the threshold, the investor must be indifferent between rolling over and withdrawing his investment, and hence, $\rho_i^*$ is determined by the following condition:

$$1 + E[r_i - cl_i|\rho_i^*, p, a] = 1, \quad (3)$$

where $a \in \{T, O\}$ is the disclosure policy of the regulator, which can be transparent, T, or opaque, O.

Under this information structure, this threshold equilibrium is the unique equilibrium provided that the private signal is sufficiently precise relative to the public signal (i.e., that $h_\varepsilon$ is large relative to $h_\mu$). In particular, let

$$\gamma \equiv \frac{h_\mu^2 (h_\mu + h_\varepsilon)}{h_\varepsilon (h_\mu + 2h_\varepsilon)}, \quad (4)$$

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then based on Morris and Shin (2003) the next proposition obtains:¹⁴

**Proposition 1.** If \( c^2 \gamma \leq 2\pi \), there is a unique equilibrium. In this equilibrium, at \( t = 1 \), every investor \( j \) in bank \( i \) rolls over his investment if and only if \( \rho_j \geq \rho^*_i \), where \( \rho^*_i \) is the unique solution to

\[
\rho^*_i = c \times \Phi \left( \sqrt{\gamma} (\rho^*_i - \mu) \right) - E(\eta_i | p, a),
\]

and \( \Phi(\cdot) \) is the standard normal cumulative distribution function.

The uniqueness of the equilibrium allows to perform comparative statics in terms of the probability of a bank run and hence to draw policy implications for the stability of the financial system. In particular, in equilibrium, the probability of a bank run is related to the bank’s underlying fundamentals. (Thus, bank runs are not sun-spot phenomena.) The following corollary characterizes the likelihood of bank runs.

**Corollary 1.** The probability that any given investor withdraws his investment at \( t = 1 \) is decreasing in the two components of the bank’s expected return, i.e., \( \mu \) and \( E(\eta_i | p, a) \), and increasing in its liquidation costs, i.e., \( c \).

As expected, on the one hand, if the long-term investment is very profitable, either because the average quality \( \mu \) is high or because the bank is expected to be of high quality (i.e., \( E(\eta_i | p, a) \) is high), investors are more willing to refinance the investment at \( t = 1 \). On the other hand, if early liquidation is costly, that is, if a few withdrawals have a large impact on the expected return at \( t = 2 \), investors are more likely to withdraw their investment at \( t = 1 \).

Consider next what happens when the underlying uncertainty on \( \hat{\mu} \), becomes very small. In particular, let both \( h_\mu \) and \( h_\epsilon \) go to \( +\infty \) and the ratio \( \frac{h_\mu^2}{h_\epsilon} \) go to zero, that is,

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¹⁴To simplify the exposition we assume that if an investor is indifferent between rolling over and withdrawing his investment, he will roll it over.
let the prior belief on $\bar{\mu}$ become very precise but the private signals on $\bar{\mu}$ become even more precise.\textsuperscript{15} In the limit, if $\frac{h_\mu^2}{h_\varepsilon}$ goes to zero so does $\gamma$, the threshold becomes

$$\rho_i^* = \frac{c}{2} - E(\eta_i | p, a), \quad (6)$$

and the next corollary follows:

**Corollary 2.** Let $h_\mu \to +\infty$ and let $\frac{h_\mu^2}{h_\varepsilon} \to 0$ then, at $t = 1$, every investor in bank $i$ rolls over his investment if and only if

$$\mu + E(\eta_i | p, a) \geq \frac{c}{2}. \quad (7)$$

Notice that this limit equilibrium is well-defined and allows to avoid some of the technical complexities of the non-limit case while preserving its appealing economic properties.\textsuperscript{16} That is, the equilibrium is unique, and in equilibrium, investors are more willing to roll over their investment the higher its expected return, $\mu + E(\eta_i | p, a)$, and the lower its liquidation costs, $c$. The aggregate return of banks’ investments depends on two components: $\mu$, which is known at $t = 0$, and the proportion $p$ of high-quality banks, which is realized at $t = 1$. In the rest of the paper, we will focus on this limit case. Thus, the global games approach is essentially used here as a refinement to obtain equilibrium uniqueness.

### 3.2 Disclosure Policy

At $t = 1$, before the rollover decision is made, investors learn $p$, the proportion of high-quality banks in the financial system. Investors, however, cannot distinguish on their own between high- and low-quality banks, i.e., between $\eta_i = \Delta_\eta$ and $\eta_i = -\Delta_\eta$. The regulator has access to this information and must decide whether disclose it or not to

\textsuperscript{15}Notice that if one takes the limit of $h_\mu$ to $+\infty$ without taking the limit of $h_\varepsilon$ to $+\infty$ at a fast enough rate, the model would revert to multiple equilibria.

\textsuperscript{16}In a model of credit risk, Morris and Shin (2004) consider a similar case where both the precision of the prior distribution and the precision of the private signals go to infinity.
investors. Consider first the case in which the regulator decides to be transparent, \( a = T \), and discloses the quality of each individual bank. In that case, bank \( i \)'s investors roll over their investment if and only if \( \mu + \eta_i \geq \frac{c}{2} \), which, given the assumption in (1), implies that investors roll over their investment if the bank is of high quality, \( \eta_i = \Delta_\eta \). Hence, the aggregate net returns generated by the banking system under a policy of transparency are:

\[
\pi_T(p) = p(\mu + \Delta_\eta).
\] (8)

Alternatively, consider the case in which the regulator decides to be opaque, \( a = O \), and does not disclose information at \( t = 1 \). Then, \( E(\eta_i | p, o) = (2p - 1) \Delta_\eta \) and banks’ investors roll over their investment if and only if \( \mu + (2p - 1) \Delta_\eta \geq \frac{c}{2} \). Hence, the net returns of the banking system under a policy of opacity are:

\[
\pi_o(p) = \begin{cases} 
\mu + (2p - 1) \Delta_\eta & \text{if } \mu + (2p - 1) \Delta_\eta \geq \frac{c}{2} \\
0 & \text{if } \mu + (2p - 1) \Delta_\eta < \frac{c}{2} 
\end{cases}.
\]

From the above discussion the next proposition follows:

**Proposition 2.** The regulator follows a policy of transparency if and only if \( p < p^* \) where

\[
p^* \equiv \frac{1}{2 \Delta_\eta} \left( \frac{c}{2} - \mu \right) + \frac{1}{2}.
\]

As proposition 2 points out, the optimal disclosure policy is contingent on \( p \), the proportion of high-quality banks in the financial system. If the financial system suffers a negative shock and \( p \) falls below some threshold \( p^* \), then it is optimal to disclose information. Otherwise, it is optimal for the regulator to be opaque and not disclose information about the quality of each individual bank. The result relies on the threshold nature of the equilibrium: If a bank’s quality is well above the threshold, a small change in its perceived quality does not have a significant effect on the probability of suffering a bank run, and hence, pooling high-quality banks with a few low-quality banks can avert

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*To simplify the exposition we assume that if the regulator is indifferent between disclosing and not disclosing information, then he does not disclose information.*

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runs on low-quality banks without inducing runs on high-quality ones.\footnote{The threshold nature of the equilibrium is a general feature of global games. For instance, in Goldstein and Pauzner (2005), early withdrawals by patient depositors occur if and only if the bank’s fundamentals fall below a threshold. (See also, \textit{e.g.}, Morris and Shin, 1998.)} This pooling equilibrium without bank runs is sustainable as long as the proportion of high-quality banks in the financial system, \( p \), is large enough. Otherwise, if the proportion of low-quality banks becomes too large, pooling leads to a run on all banks (both high- and low-quality) in the financial system.

This result is consistent with the general demand for increased transparency during the 2008-09 financial crisis. For instance, Governor D. Tarullo from the Federal Reserve Board of Governors argued that the publication of stress tests performed on US banks helped stabilizing the financial system during the crisis: “\textit{This departure from the standard practice of keeping examination information confidential was based on the belief that greater transparency of the process and findings would help restore confidence in U.S. banks at a time of great uncertainty.}” (Keynote speech at the Federal Reserve Board International Research Forum on Monetary Policy, Washington, D.C., 26 March 2010.)

The result is also consistent with the recent debate among European countries regarding the convenience of disclosing information. On the one hand, Spain, a country particularly affected by the crisis and whose banks were having difficulties accessing the capital markets, was one of the main advocates of publishing the results of stress tests.\footnote{For instance, the chairman of BBVA, Spain’s second-largest bank, argued that capital markets were closed to most Spanish banks and advocated “doing and publishing” stress tests. (Bloomberg \textit{Businessweek}, June 17, 2010)} On the other hand, Germany, the strongest economy in the region, was reluctant to disclose information and warned that the publication of stress test results could backfire. In the words of the German Bundesbank President Axel Weber “\textit{Any stress test only makes sense if it is accompanied by a corresponding commitment by the respective gov-}
The disclosure threshold $p^*$ in proposition 2 depends on the underlying fundamentals of the economy. In particular, $p^*$ decreases with the expected return of the banking industry, $\mu$. That is, the regulator is more likely to implement a policy of opacity at $t = 1$ when investors anticipate high long-term returns at $t = 0$. Indeed, in that case, only a large negative aggregate shock at $t = 1$, (i.e., a very low realization of $p$) could trigger a run on the entire system in the absence of disclosure. Notice also that $p^*$ is increasing in the cost of liquidation, $c$. Intuitively, a higher liquidation cost makes each investor more sensitive to other investors’ roll-over decisions, which exacerbates the coordination problem. As a result, when $c$ increases, an opaque financial system becomes more fragile, that is, investors run on a bank perceived as average for higher realizations of $p$.

We conclude this section by briefly discussing two assumptions on which the previous analysis relies. First, the analysis assumes that disclosure policy is one of full transparency or full opacity. One could consider however a policy in which the regulator only discloses information on a subset of banks. That is, if the proportion of high-quality banks is below $p^*$, the regulator could disclose information (and cause a run) on a subset of low-quality banks only, rather than disclose information on all banks and cause a run on every low-quality bank. Notice that under this alternative disclosure rule, there would still be an increase in disclosure during financial crises and disclosure would again be an attempt to prevent a run on the whole banking system. However, implementing a policy of selective disclosure can be difficult in situations in which the regulator has private information about the average quality of banks in the financial system. As the analysis in section 4 shows, the regulator faces then a commitment problem when setting its disclosure policy. This makes disclosure rules that give extensive discretion to the regulator more difficult to implement.
Second, a word on the role that the assumption in (1) plays in the optimal disclosure policy derived in proposition 2. In the current set up, the optimal disclosure policy minimizes bank runs. This relies on the assumption that all bank runs are inefficient, which is implied by $\mu - \Delta \eta > 0$. If we were to also allow for $\mu$ to be low enough for the liquidation of low-quality banks to become efficient, there would be yet another reason to disclose information during crises, namely, to cleanse the financial system.\textsuperscript{20} Nonetheless, the message would remain qualitatively the same: it is optimal to increase transparency following a negative shock to quality because it allows saving high-quality banks.\textsuperscript{21}

4 Asymmetric Information and Ex-Ante Policy

In previous sections, we have assumed that at $t = 1$, the proportion of high-quality banks in the financial system, $p$, was common knowledge. That is, while the regulator had private information about the quality of each bank, investors learnt the realization of aggregate uncertainty about the state of the banking system, which is likely to be related to general economic conditions. While it is reasonable to assume that information asymmetries are particularly acute at the individual bank level, there are situations in which the regulator may also have an informational advantage over investors regarding an aggregate shock to the quality of the banking system, \textit{i.e.}, the realization of $p$. In this section we consider the optimal disclosure policy when the regulator also has private information about $p$.

\textsuperscript{20}The problems of rollover risk and of liquidation of inefficient banks are, however, different in nature. The later can be viewed as an internal governance problem: if a bank produces negative long-term returns, it is in the best interest of its investors to liquidate it.

\textsuperscript{21}We also restrict attention to the case where only low-quality banks face the risk of early liquidation. Indeed, in the case where even banks that are known to be of high quality suffer runs, disclosure policy becomes irrelevant. Notice that the intense debate on transparency in the banking system suggests that information disclosure does matter, and hence, that the latter case is not the most relevant one.
The following proposition characterizes the optimal disclosure policy if investors do not observe $p$ and the regulator can commit to a disclosure policy as a function of $p$.

**Proposition 3.** If the regulator can commit to a disclosure policy as a function of $p$, then, he discloses information if and only if $p$ is below some threshold $p^C$ such that

- if $\mu \geq \frac{c}{2}$, $p^C = 0$, that is, information is never disclosed,
- if $\mu < \frac{c}{2}$, $p^C$ is the unique solution to

$$\mu + E(2p - 1|p > p^C) \Delta \eta = \frac{c}{2}.$$ 

As in the case in which investors observe $p$, the optimal disclosure policy minimizes bank runs. The comparative statics on the disclosure threshold $p^C$ is similar to the comparative statics on $p^*$, that is, $p^C$ increases with $c$ and decreases with $\mu$. Asymmetric information, however, leads to less information disclosure by the regulator since $p^C < p^*$. Intuitively, pooling can now take place not only between high- and low-quality banks, but also across different states of the world (i.e., realizations of $p$) which allows lowering the disclosure threshold $p^C$ below $p^*$.

The equilibrium in proposition 3 relies on the ability of the regulator to commit to a disclosure policy as a function of his private information on the state of the banking system, $p$. While factors such as reputation could help sustaining the optimal policy with commitment as an equilibrium, it is also plausible that the regulator may have incentives to renege on his commitment in some instances. Note that, according to proposition 3, it is optimal (to commit ex-ante) to disclose information when the average quality $p$ is low. Therefore, an increase in transparency leads investors to revise their expectation of $p$ downwards. This creates an incentive for the regulator to refrain from disclosing information in order to boost investors’ beliefs. In particular, if the regulator cannot commit to a disclosure policy contingent on $p$ and expected returns are low, $\mu < \frac{c}{2}$, the strategy in proposition 3, that is, disclosing information only if $p$ is lower than $p^C$, cannot be an equilibrium policy. Indeed, if investors believed that the transparency policy was
as in proposition 3, the regulator would deviate ex-post and choose not to disclose, even when $p < p^C$, as this would raise investor’s expectations and save the entire system. As a result, the only equilibrium when $\mu < \frac{c}{2}$ is one where the regulator always discloses information.\textsuperscript{22} Therefore, the lack of commitment ability would prevent the regulator from implementing a disclosure policy that is contingent on his private information on the state of the banking system $p$.\textsuperscript{23}

The above discussion is related to the broader issue of the credibility of the regulator’s actions in times of crisis. For instance, doubts have been raised on the informativeness of stress tests conducted on Europeans banks. In particular, financial analysts were concerned that sovereign-debt default was not among the worst-case scenarios under consideration in the July 2011 stress tests, even though credit-default swaps already indicated a 87 percent chance that Greece would not be able to repay its debts.\textsuperscript{24} In the same line, Antonio Borges, director of the International Monetary Fund for Europe, advised Spain in September 2011 to have an external auditor assess the situation of its banking system in order to restore the investors’ confidence.

\begin{quote}
\textsuperscript{22}In the case where $\mu \geq \frac{c}{2}$, two regimes can be sustained in (perfect Bayesian) equilibrium. One in which the regulator never discloses information, and another one in which the regulator always discloses information. (This last equilibrium relies on out-of-equilibrium beliefs from investors that $p < p^\ast$ if the regulator chooses opacity.).

\textsuperscript{23}Notice that despite the lack of commitment ability to implement the optimal $p$-contingent disclosure policy, we are implicitly assuming that the regulator’s reputational incentives are still sufficiently strong to prevent him from falsifying information, that is, from claiming that a bank is of high quality when it actually is of bad quality. In the alternative case, in which the regulator could not even commit not to falsify information, information transmission would not be feasible and the regime would be one of opacity. In either case, information asymmetry about $p$ makes it difficult to implement a disclosure policy which is contingent on $p$.

\textsuperscript{24}“Instead, European authorities looked at the ability of banks to endure an economic contraction of 0.5 percent – in other words, a mild recession – as well as a 15 percent stock-market decline, rising unemployment, a drop in housing prices and trading losses on government debt.” (Bloomberg-Businessweek, July 15, 2011).
\end{quote}
The difficulties of implementing the optimal state-contingent disclosure policy are not only linked to the presence of asymmetric information but also to the fact that such a policy requires the regulator to react rapidly to shocks to the banking system. (For instance, the optimal disclosure policy may involve some transparency requirements from banks that can be difficult to implement on a very short notice.) Consequently, it is also interesting to consider the optimal disclosure policy if the regulator must choose the level of transparency before $p$ is realized. Clearly, this choice will depend on the quality of the information that investors have on the state of the financial system at $t = 1$. To take this into account, we let $q$ be the probability that investors learn $p$ at $t = 1$ and consider the case in which the regulator must decide his disclosure policy—transparency or opacity— at $t = 0$. Then, the next proposition characterizes the optimal policy.

**Proposition 4.** If the regulator must decide his disclosure policy at $t = 0$ and the policy cannot be made contingent on $p$, then he chooses a policy of transparency if and only if

$$\frac{\mu + \Delta \eta}{2} > q(1 - p^\star) [\mu + E (2p - 1|p > p^\star) \Delta \eta] + (1 - q)\mu 1_{\mu \geq \frac{c}{2}}$$

where $1_{\mu \geq \frac{c}{2}}$ is a dummy variable equal to one if $\mu \geq \frac{c}{2}$.

As in previous cases (see propositions 2 and 3), increases in $c$ and decreases in $\mu$ lead to more transparency. Moreover, if $\mu \geq \frac{c}{2}$, which implies that a bank perceived as average when $p$ is unknown would not suffer a run, the tendency to disclose information increases with the probability that investors learn $p$ at $t = 1$, $q$. That is, if investors have timely information about shocks to banks’ expected return then a policy of transparency is more likely to be optimal. This suggests that regulators in countries where information is processed faster and spreads more widely should also set a higher level of transparency.

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25 The execution of the Troubled Asset Relief Program illustrates the difficulty of switching quickly from a regime of opacity to one of transparency. Indeed, one of the main obstacles to the implementation of the initial program was that opacity had made most “toxic assets” impossible to value given the time constraints and the short-term threat to the survival of the banking system. (See “Why Toxic Assets Are So Hard to Clean Up?”, *Wall Street Journal*, July 20, 2009.)
in their banking system. Proposition 4 also suggests that in the initial stages of a crisis, when information about the deterioration of economic fundamentals is unlikely to be widespread (when $q$ is low), the regulator has incentives to hide information from investors hoping for a prompt recovery. However, if the deterioration persists and it becomes more likely that investors learn about the crisis (when $q$ is high), then the regulator has incentives to increase transparency.

Proposition 4 emphasizes one of the key differences between a transparent and an opaque disclosure policy. Regulators face a trade-off between a transparent regime with frequent runs that affect a reduced number of banks, and a less transparent regime in which runs are less frequent but can affect the entire banking system. While proposition 4 suggests that when the expected return in the banking system, $\mu$, deteriorates, it is optimal, if feasible, to increase transparency, it also suggests that this increase may not need to be permanent. (This will depend on whether this decline is more or less temporary.) These two predictions of the model are in line with the increase in transparency during the recent crisis as well as with the voices that warn against making this increase permanent: “In more normal economic times, when market participants are not fearing the worst and when banks do not have access to government capital injections as a backstop, the revelation that some major banks may have capital needs under a stress scenario might be unnecessarily destabilizing.” (Governor D. Tarullo, Keynote speech at the Federal Reserve Board International Research Forum on Monetary Policy, Washington, D.C., 26 March 2010.)

5 Diversification

In previous sections, we considered the risk exposure of each bank as given. However, banks do have some control over their risk profiles. For instance, they may choose to diversify their risk by investing across different industries, regions, or asset classes. In this section, we endogenize banks' exposure to idiosyncratic risk. In particular, we
assume that the return $r_i$, as defined in section 2, now characterizes a class of assets of type $i$, instead of a bank. We assume also that banks can invest into different asset classes at $t = 0$ (hence, before knowing $r_i$ or the realization of $p$). Specifically, each bank $i$ can invest a fraction $\alpha_i \in (0, 1]$ of its resources in assets of type $i$, and the rest of its resources, $1 - \alpha_i$, into each other type of assets $j \neq i$ in equal proportion. We consider symmetric equilibria where $\alpha_i = \alpha$ for all $i$. Notice that under this specification, independently of $\alpha$, there is always a mass 1 of each asset type in the economy, which implies that diversification affects only the extent to which banks are exposed to asset-idiosyncratic risk. Notice also that the model studied in previous sections corresponds to the particular case where $\alpha = 1$, that is, the case in which each bank chooses to be fully invested in one asset type. Using a natural extension of the terminology introduced in previous sections, we refer to an asset with idiosyncratic component $\eta_i = \Delta \eta$, as a high-quality asset, and its corresponding bank as a high-quality bank.

Since there is a continuum of assets, the payoff of bank $i$ from the share $1 - \alpha$ invested in assets $j \neq i$ is equal to $(1 - \alpha)[\mu + (2p - 1)\Delta \eta]$. In this payoff, the asset-idiosyncratic risk is perfectly diversified and the only source of uncertainty comes from the aggregate component $p$. Hence the long-term return of bank $i$ is now $\mu + (1 - \alpha)(2p - 1)\Delta \eta + \alpha \eta_i$ where $\alpha \eta_i$ determines bank $i$ exposure to asset-idiosyncratic risk.

In order to make more apparent the impact that the optimal disclosure policy has on banks’ diversification choices, we start with a benchmark case in which the banking system is always transparent.\footnote{When the disclosure policy cannot be made contingent on the realization of $p$ there is no loss of generality in assuming that the system is transparent. Indeed, by choosing $\alpha$ arbitrarily close to 0 – and hence, by fully diversifying idiosyncratic risks – banks can achieve the same expected payoff as if the system were opaque.} The following result obtains.

**Proposition 5.** If the banking system is transparent, banks optimally choose to diversify, i.e., $\alpha < 1$, provided $\mu$ is sufficiently high.
Intuitively, when the system is always transparent, diversification can prevent runs on low-quality banks for high realizations of $p$ (that is, when the banking system experiences a positive aggregate shock). Diversification, however, also entails costs. First, it may cause runs on high-quality banks for low realizations of $p$. Second, when only low-quality banks suffer runs, that is, for intermediate realizations of $p$, diversification decreases the asset value of banks which turn out to be of high quality, and hence, that are not liquidated, while it increases the asset value of low-quality banks, which are liquidated. In other words, while diversification may reduce the frequency of bank runs, it makes them costlier as those diversified banks that are liquidated hold some high-quality assets. As proposition 5 indicates, when the expected return of the financial system, $\mu$, is high, the probability that $p$ falls into the upper region where all banks are saved is sufficiently large to outweigh the costs of diversification.

Turn now to the case where the regulator can set a $p$-contingent disclosure policy. The following proposition characterizes the optimal disclosure policy and level of diversification.

**Proposition 6.** The regulator follows a policy of transparency if and only if $p < p^*$ where $p^*$ is defined as in proposition 2. Under this disclosure policy, each bank concentrates its investments in one asset class at $t = 0$, i.e., $\alpha^* = 1$.

As proposition 6 states, the optimal disclosure policy is independent of $\alpha$. Indeed, the decision to disclose depends only on the aggregate expected return of the banking system, which is unaffected by the degree of diversification $\alpha$.27 Therefore, the regulator will disclose information if and only if $p < p^*$ where $p^*$ is defined as in the case in which each bank’s risk exposure is considered exogenous. (See Proposition 2).

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27 Notice that the regulator discloses only if a bank that is perceived as average suffers a run. Since the set of assets in the economy is given, the average expected return of the banking sector does not depend on banks’ portfolio choices, i.e., $\alpha$. 23
Proposition 6 also states that under the optimal disclosure policy, each bank chooses to be fully invested in one asset class, that is, diversification is suboptimal. To see this, notice that when \( p \geq p^* \), the regulator does not disclose information and the expected return of a bank is simply \( \mu + (2p - 1)\Delta_\eta \), which is independent from \( \alpha \). Alternatively, when the regulator does disclose information (when \( p < p^* \)) the expected return of a bank is \( p[\mu + \alpha \Delta_\eta + (1 - \alpha)(2p - 1)\Delta_\eta] \) which is strictly increasing in \( \alpha \). It is therefore strictly optimal to choose \( \alpha^* = 1 \). Intuitively, diversification has benefits in our model only if the system is transparent and if \( p \) is high. However, when \( p \) is high, opacity allows to reach the same outcome as diversification, that is, saving all banks, and makes individual risk choices by banks irrelevant. For lower realizations of \( p \), however, diversification only has costs. As discussed earlier, when the regulator switches to a transparent regime, diversification prevents from concentrating assets that turn out to be of high quality in certain banks, which increases the value destroyed in liquidations.

Some of the emergency measures that were discussed at the height of the 2008 credit crisis are in line with this idea that within a diversified bank, assets that turn out to be of low quality may contaminate high-quality assets. In particular, the Trouble Asset Relief Program (TARP) was originally designed as a vehicle that would buy troubled and unsalable mortgages and mortgage-backed securities. This attempt can be viewed as a way to concentrate the exposure to one class of risk (real estate) into a dedicated recapitalized entity, clearly distinct from other banks, so as to avoid runs on the entire financial system.

Finally, proposition 6 shows that the optimal transparency policy in proposition 2 is robust to a specification where the risk profile of each bank is endogenously determined. Indeed, anticipating an optimal response of the regulator to economic conditions, banks would choose to concentrate their investments in one type of assets (\( \alpha = 1 \)).

\(^{28}\)Notice that banks may have other incentives take correlated risks, for instance because government bailouts are conditional on a sufficiently high number of banks defaulting (see, e.g., Acharya and Yorulmazer [2008] and Farhi and Tirole [2010]).
Overall, propositions 5 and 6 indicate that, in our model, the optimal contingent disclosure policy can achieve the benefits of diversification while avoiding some of its costs. The results in this section also emphasize the importance of distinguishing between measures of risk at the individual bank level (for instance, based on the volatility of a bank’s assets) and measures of risks at the level of the entire banking system. This distinction between individual risk and systemic risk is at the core of recent proposals for the reform of the financial system (Kashyap, Rajan, and Stein, 2008; Morris and Shin, 2008). In particular, Morris and Shin (2008) argue that the traditional approach to banking regulation, which limits the risk that each bank may take on its asset side, is of little help in preventing liquidity crises and their systemic effects. In line with this argument, our analysis shows that minimizing the volatility of banks’ assets through diversification, that is, pushing $\alpha$ towards 0, may actually increase the risk to the entire system. In fact, in our model, when banks are fully diversified, a negative aggregate shock (a low realization of $p$) has particularly severe consequences since it may cause a run on the entire system.

6 Liquidity

So far we have studied the optimal disclosure policy in the presence of rollover risk.\(^{29}\) Banks, however, in an attempt to deal with rollover risk, may increase the liquidity of their balance sheets by either increasing the maturity of their liabilities or decreasing the maturity of their assets. This section endogenizes the liquidity of banks’ balance sheets to study how their attempt to cope with rollover risk affects the regulator’s optimal disclosure policy.

\(^{29}\)This is consistent with the view that a major source of fragility of the banking sector in the 2008 credit crisis was, on the liability side of balance sheets, the reliance of banks on short-term financing from some categories of investors, such as hedge funds or money market funds. (See, for instance, Morris and Shin, 2008.)
Consider the following change to the long-term investment technology of the basic model. At $t = 0$ each bank chooses the liquidity of its investment technology, $L_i$. Specifically, an investor who rolls over his investment at $t = 1$ obtains a random payoff of $1 + r_i - \tau \frac{L_i^2}{2} - c \max\{l_i - L_i, 0\}$ at $t = 2$, where $L_i \geq 0$. Thus, $L_i$ affects the return at $t = 2$ through two different channels. On the one hand, if no liquidations take place at $t = 1$, that is, if $l_i = 0$, a more liquid technology is associated with a lower expected return, which is captured by $-\tau \frac{L_i^2}{2}$. One interpretation of this convex cost is that banks sacrifice first their less profitable long-term investments in order to retain liquid assets on their balance sheet. On the other hand, a more liquid technology makes liquidations less costly, which is captured by $-c \max\{l_i - L_i, 0\}$.

We restrict attention to the case where the marginal cost of a liquid balance sheet, $\tau$, is sufficiently large, which ensures an interior solution when solving for the optimal level of liquidity,

$$\frac{c}{\tau} \leq \min\left\{1 - \frac{2(\mu + \Delta)}{c} ; \frac{2(\mu - \Delta)}{c}\right\}. \quad (9)$$

As will become clear below, this condition also implies that rollover risk cannot be completely eliminated.

The following proposition, which corresponds to proposition 2 of the basic model, characterizes the optimal disclosure policy at $t = 1$.

**Proposition 7.** Assume that all banks choose the same level of liquidity $L \leq \frac{c}{\tau}$, the regulator follows a policy of transparency if and only if $p < p^*(L)$ where

$$p^*(L) = \frac{1}{2\Delta} \left[ \frac{c}{2} - \mu + \tau \frac{L^2}{2} - cL \right] + \frac{1}{2}.$$  

As in proposition 2, the optimal disclosure policy is contingent on the proportion of high-quality banks in the banking system, $p$. Proposition 7 says that it is optimal for the regulator to be opaque as long as $p$ is above some threshold $p^*(L)$.

The disclosure threshold depends on how liquid the banks’ balance sheets are: an increase in $L$ decreases $p^*(L)$. Intuitively, increasing $L$ lowers long-term expected returns but makes investors who rollover their investments less sensitive to early withdrawals by
other investors. The first effect increases investors’ incentives to run while the second one diminishes them. As long as $L \leq \frac{c}{\tau}$, the second effect dominates so that a higher $L$ lowers the threshold below which investors run on an average bank. This, in turn, allows decreasing the disclosure threshold. In other words, banks face the following trade-off when choosing the liquidity of their balance sheet: a more liquid balance sheet has a lower expected return in the absence of withdrawals but makes investors less likely to withdraw, i.e., to run. The implication of this increase in stability (i.e., lower incentives to run) is that the optimal disclosure policy becomes more opaque since it is now feasible to pool high-quality banks with more low-quality banks (i.e., to lower $p^*(L)$) without causing a run on the whole banking system. Finally, independently of the disclosure policy, increasing $L$ beyond $\frac{c}{\tau}$ is suboptimal for any bank, since it both decreases long-term expected returns in the absence of early liquidation and increases the probability of a run at the interim date. As a result, $p^*(L)$ reaches a minimum for $L = \frac{c}{\tau}$, and the assumption in (9) implies that this minimum is strictly positive. In words, even if banks try to minimize the probability of a run, regardless of the impact on long-term returns, there still are realizations of $p$ which are sufficiently low to trigger information disclosure by the regulator and a run on low-quality banks.\footnote{“Granted, better managed and capitalized institutions are less likely to encounter a run - it is not a surprise that it was Bear and Stearns rather than JP Morgan that went under a few weeks back - but no institution is immune to panics, as long as it is providing its socially useful liquidity transformation and intermediation role.” Ricardo Caballero, \textit{XI Angelo Costa Lecture}, March 23, 2010.}

Let us now turn to each bank’s choice of liquidity at $t = 0$ given the regulator’s optimal disclosure policy at $t = 1$. The optimal disclosure policy does not depend on the liquidity of a single bank but on the liquidity of all banks in the system. Since banks are infinitesimally small, each bank will choose its optimal liquidity taking as given the liquidity choice of all other banks, that is, taking the regulator’s disclosure policy as fixed. This can lead to multiple self-fulfilling equilibria in which banks may collectively end up with an investment technology (and hence, a balance sheet) that is either too
liquid or too illiquid from the social point of view. For instance, consider the case in which a bank expects all other banks to choose a very liquid technology, and hence the regulator’s disclosure policy to be rather opaque (i.e., $p^*(L)$ to be low). In that case, unless this bank chooses a technology that is as liquid as the one of the other banks, it will suffer a run when $p$ is low enough (yet larger than $p^*(L)$). Hence, if a bank expects other banks to choose a very liquid technology, then it may have incentives to choose a very liquid technology as well. The following proposition states the possibility of multiple symmetric equilibria in which banks hold more or less liquidity than is socially optimal.

**Proposition 8.** There are multiple self-fulfilling equilibria in which banks may choose an investment technology that is too liquid or too illiquid from a social point of view.

The above proposition suggests that there might be instances in which the financial system is trapped into investments that are either too liquid or too illiquid. In such cases, it may be valuable for the regulator to commit to a disclosure policy as a function of $p$.$^{31}$ However, in order for this commitment to allow implementing the first-best, investors must be able to observe the liquidity choice of each individual bank (rather than just the average liquidity in the banking system). Intuitively, a liquid balance sheet is valuable because it prevents runs. But if a bank’s liquidity choice is not observed by investors, it will not help preventing runs, and hence, banks will have little incentives to hold a liquid balance sheet in the first place. In the same way that we assumed that the quality of each individual bank is not observed by investors, one can argue that the liquidity of a bank’s balance sheet may also be difficult to observe under a policy of opacity. In this case, banks tend to have balance sheets that are too illiquid from the social point of view. This equilibrium outcome points towards the need of having liquidity requirements as a way of helping banks resolve this commitment problem. This recommendation is in line with Kashyap, Rajan, and Stein (2008), who suggest introducing mandatory holdings

$^{31}$As discussed in section 4, commitment can be difficult to achieve in the presence of asymmetric information on $p$. 
of Treasury bills for banks.

7 Conclusion

One of the reactions to the recent financial crisis has been the demand for an increase in the transparency of financial institutions. In fact, regulation authorities in Europe and the United States have tried to improve the quality of public information on individual banks by performing stress tests, and more importantly, by releasing their results to investors. One stated objective of these tests is to prevent a contagion of investors’ distrust to the entire banking system by providing information on the specific risk exposure of each financial institution. This is consistent with the view that, partly, the banking crisis was a run on the liability side of banks’ balance sheets.\footnote{The fate of Bear Stearns was the result of a lack of confidence, not a lack of capital. When the tumult began last week, and at all times until its agreement to be acquired by JP Morgan Chase during the weekend, the firm had a capital cushion well above what is required to meet supervisory standards calculated using the Basel II standard. Specifically, even at the time of its sale on Sunday, Bear Stearns’ capital, and its broker-dealers’ capital, exceeded supervisory standards. Counterparty withdrawals and credit denials, resulting in a loss of liquidity - not inadequate capital - caused Bear’s demise. (Letter to the Chairman of the Basel Committee on Banking Supervision, dated March 20th 2008, posted on the SEC website on: http://www.sec.gov/news/press/2008/2008-48.htm)} This paper studies the optimal level of transparency in the banking system when banks have short-term liabilities and are exposed to rollover risk. In particular, it shows that increasing transparency during crises increases the stability of the banking system by reducing the number of bank runs. The paper, nonetheless, cautions against a permanent increase in transparency as it may have a destabilizing effect on the financial system during normal economic times. Thus, the optimal disclosure policy is one contingent on the state of the economy in which transparency is increased in times of crises. Implementing this optimal policy, however, can sometimes be difficult. Under such policy, an increase in transparency signals a deterioration of the economy’s fundamentals, and hence, the regulator has ex-post incentives to hide this deterioration from investors by not disclosing
information. In that case, the regulator may be forced to choose a disclosure policy that is not state contingent, facing a trade-off between the frequency and the magnitude of banks runs.

Finally, the analysis was extended to allow for the possibility that banks, in an attempt to deal with rollover risk, either diversify their risks or increase the liquidity of their balance sheets. An increase in the liquidity of banks’ balance sheets decreases their investors’ incentives to run and allows to decrease transparency without compromising the stability of higher-quality banks. Moreover, given that the optimal disclosure policy depends on the average liquidity of banks, and that each individual bank takes this average liquidity as given when choosing its own liquidity, regulators can fall into policy traps. In that case, banks may end up with a balance sheet that is either more or less liquid than is socially optimal.

While this paper focuses on a regulatory measure that has been central in the recent debate on the reform of the financial system, namely, transparency, regulators combine several instruments to cope with liquidity crises. Among them, the provision of liquidity by central banks or governments, acting as lenders of last resort, has been an emergency recourse for financial institutions during the recent credit crisis.33 There can be interesting interactions between public provision of liquidity to the banking system and transparency. In particular, to the extent that the regulator faces a trade-off between the size and frequency of banks runs when choosing a transparency regime, disclosure policy is likely to have an effect on the magnitude of the liquidity shock that a government

33The provision of liquidity by the central bank or the government, however, has limitations. Indeed, it creates a well-known moral hazard problem for banks (Freixas and Rochet, 2004). Also, banks are typically reluctant to use the discount window of the central bank as this signals their fragility and may eventually worsen the liquidity dry-up they face. Furthermore, institutions that suffer from a liquidity shortage are not only banks in the strict sense, but also investment vehicles such as conduits and asset-backed securities (the “shadow banking system”), which do not have a direct access to public provision of liquidity. (Investment banks did not have access to the discount window in the 2008 financial crisis.)
or a central bank would have to withstand in times of crisis in order to maintain the financial system afloat. We believe that this is an interesting avenue for future research in the task of building a more stable financial system.

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Appendix

Proof of Proposition 1

The first part of the Proof follows roughly the same lines as in Morris and Shin (2000).

At the threshold $\rho^*_i$, any investor must be indifferent between rolling over his investment and withdrawing, so that $\rho^*_i$ is determined by the following condition,

$$1 + E[r_i - c_i | \rho^*_i, p, a] = 1,$$

which may be rewritten as

$$\rho^*_i + E[\eta_i | p, a] = c E[l_i | \rho^*_i, p, a]. \tag{A.1}$$

To compute the right-hand side of (A.1), we use the fact that the expectation of withdrawals conditional on $\rho^*_i$ is equal to the probability that any investor $j$ withdraws given $\rho^*_i$. Using (2), this probability is

$$Pr(\rho_{ij} < \rho^*_i | \rho^*_i) = Pr(x_{ij} < \rho^*_i + \frac{h\mu}{h\epsilon}(\rho^*_i - \mu) | \rho^*_i).$$

Furthermore, the variance of $x_{ij}$ conditional on $\rho^*_i$ is equal to $\frac{h\epsilon(h\mu + h\epsilon)}{h\mu + 2h\epsilon}$. It follows that

$$Pr(\rho_{ij} < \rho^*_i | \rho^*_i) = \Phi\left(\sqrt{\frac{h\epsilon(h\mu + h\epsilon)}{h\mu + 2h\epsilon}} \left(\rho^*_i + \frac{h\mu}{h\epsilon}(\rho^*_i - \mu) - \rho^*_i\right)\right).$$

Rearranging,

$$Pr(\rho_{ij} < \rho^*_i | \rho^*_i) = \Phi\left(\sqrt{\gamma(\rho^*_i - \mu)}\right), \quad \text{where} \quad \gamma = \frac{h^2\epsilon(h\mu + h\epsilon)}{h\epsilon(h\mu + 2h\epsilon)}.$$

Combining with (A.1), the equation that determines the threshold equilibrium is

$$\rho^*_i + E[\eta_i | p, a] = c \Phi\left[\sqrt{\gamma(\rho^*_i - \mu)}\right]. \tag{A.2}$$

For $c^2\gamma < 2\pi$, there is only one solution to this equation.

In the second part of the Proof, we show that if there is a unique equilibrium in which any investor $j$ withdraws if and only if $\rho_{ij} < \rho^*_i$, then this is the only equilibrium. We follow roughly the same lines as in Morris and Shin (2004).

For any investor $j$, we define by $u(\rho_{ij}, \hat{\rho})$ the expected payoff obtained by investor $j$ at $t = 2$ if he does not withdraw at $t = 1$ given $\rho_{ij}$ (as defined in (2)) and given that the strategy of other investors is to withdraw if and only if their conditional expectation of $\hat{\rho}$ is below a given $\hat{\rho}$. We have

$$u(\rho_{ij}, \hat{\rho}) = 1 + \rho_{ij} + E[\eta_i] - c \Phi\left[\sqrt{\gamma} \left(\hat{\rho} - \mu + \frac{h\epsilon}{h\mu}(\hat{\rho} - \rho_{ij})\right)\right].$$

32
Since \( \Phi \) is strictly increasing, \( u \) is strictly increasing in its first argument, and strictly decreasing in its second argument. As in Morris and Shin (2004), we refer to this property as the monotonicity of \( u \). In addition,

\[
\lim_{\rho_{ij} \to -\infty} u(\rho_{ij}, \hat{\rho}) = -\infty \quad \text{and} \quad \lim_{\rho_{ij} \to \infty} u(\rho_{ij}, \hat{\rho}) = \infty
\]

Thus, for any investor \( j \), withdrawing is a dominant strategy when \( \rho_{ij} \) is low enough, and rolling over is a dominant strategy when \( \rho_{ij} \) is high enough.

We define two sequence of numbers. First, the sequence \( \{\rho_1, \rho_2, \ldots, \rho_k, \ldots\} \) solves

\[
u(\rho_1, -\infty) = 1 \quad \text{and} \quad u(\rho_{k+1}, \hat{\rho}_k) = 1 \quad \text{for any} \quad k \geq 1. \tag{A.3}\]

Second, the sequence \( \{\bar{\rho}_1, \bar{\rho}_2, \ldots, \bar{\rho}_k, \ldots\} \) solves

\[
u(\bar{\rho}_1, \infty) = 1 \quad \text{and} \quad u(\bar{\rho}_{k+1}, \bar{\rho}_k) = 1 \quad \text{for any} \quad k \geq 1. \tag{A.4}\]

We also let \( \rho \) solve \( u(\rho, \rho) = 1 \).

Applying (A.3) to \( k = 1 \) gives \( u(\rho_1, -\infty) = u(\rho_2, \rho_1) \). Since \( u \) is monotonic, this implies that \( \rho_2 > \rho_1 \).

Likewise, we find that \( \rho_{k+1} > \rho_k \) for any \( k \geq 1 \). In addition, since \( u(\rho_{k+1}, \rho_k) = u(\rho, \rho) \) (both are equal to 1), the monotonicity of \( u \) implies that \( \rho_k < \rho \) for any \( k \). To summarize, we have

\[
\rho_1 < \rho_2 < \cdots < \rho_k < \cdots < \rho. \tag{A.5}\]

Denote by \( \underline{\rho} \) the smallest solution to \( u(\rho, \rho) = 1 \). The monotonicity of \( u \) and (A.5) imply that \( \underline{\rho} \) is the least upper bound for the sequence \( \{\rho_k\} \). Finally, since this sequence is increasing and bounded (see (A.5)), it converges to its smallest upper bound: \( \lim_{k \to \infty} \rho_k = \underline{\rho} \). Similarly, if we define \( \bar{\rho} \) as the largest solution to \( u(\rho, \rho) = 1 \), then we can show that \( \lim_{k \to \infty} \bar{\rho}_k = \bar{\rho} \).

For any investor \( j \), we define a strategy \( \sigma(\rho_{ij}) \) as a mapping of \( \rho_{ij} \) into the investor’s action at \( t = 1 \), namely withdraw (\( W \)) or roll over (\( R \)). We are going to show by iterative induction that if the strategy \( \sigma \) survives \( k \) rounds of elimination of dominated strategies, then it is such that

\[
\sigma(\rho_{ij}) = \begin{cases} 
W & \text{if } \rho_{ij} < \rho_k \\
R & \text{if } \rho_{ij} > \bar{\rho}_k
\end{cases} \tag{A.6}
\]

Notice that (A.6) does not necessarily fully characterize a strategy since the investor’s action remains undefined for \( \rho_{ij} \in [\rho_k, \bar{\rho}_k] \). Denote by \( \sigma^{-j} \) the strategy used by all investors other than \( j \), and by \( \hat{u}(\rho_{ij}, \sigma^{-j}) \) the (random) payoff obtained by investor \( j \) if he does not withdraw conditional on \( \rho_{ij} \) and on other investors using the strategy \( \sigma^{-j} \).

We first show that (A.6) holds for \( k = 1 \). Because \( u \) is strictly decreasing in its second argument, we have for any \( \rho_{ij} \) and any \( \sigma^{-j} \)

\[
u(\rho_{ij}, \infty) \leq \hat{u}(\rho_{ij}, \sigma^{-j}) \leq u(\rho_{ij}, -\infty). \tag{A.7}
\]
To start with, suppose that $\rho_{ij} < \rho_1$. Using successively (A.3), the fact that $u$ is strictly increasing in its first argument and $\rho_{ij} < \rho_1$, and (A.7), we have

$$1 = u(\rho_1, -\infty) > u(\rho_{ij}, -\infty) \geq \tilde{u}(\rho_{ij}, \sigma^{-j}).$$

That is, if $\rho_{ij} < \rho_1$, then withdrawing is a dominant strategy. Now suppose that $\rho_{ij} > \bar{\rho}_1$. Using successively (A.4), the fact that $u$ is strictly increasing in its first argument and $\rho_{ij} > \bar{\rho}_1$, and (A.7), we have

$$1 = u(\bar{\rho}_1, \infty) < u(\rho_{ij}, \infty) \leq \tilde{u}(\rho_{ij}, \sigma^{-j}).$$

That is, if $\rho_{ij} > \bar{\rho}_1$, then rolling over is a dominant strategy. We have shown that (A.6) holds for $k = 1$.

We now show that if (A.6) holds for any $k \geq 1$, then it holds for $k + 1$. We denote by $\Sigma^k$ the set of strategies which satisfy (A.6) for a given $k \geq 1$, i.e., the set of strategies which survive $k$ rounds of elimination of dominated strategies. We must show that, given that other investors only use strategies in $\Sigma^k$, any strategy which is not in the set $\Sigma^{k+1}$ is dominated. Accordingly, suppose that investor $j$ believes that $\sigma^{-j}$ is in $\Sigma^k$. Then the smallest withdrawal threshold potentially used by other investors is $\rho_k$, and the largest withdrawal threshold potentially used by other investors is $\bar{\rho}_k$. Given that $u$ is decreasing in its second argument, for any strategy $\sigma^{-j}$ in $\Sigma^k$ and any $\rho_{ij}$, we have

$$u(\rho_{ij}, \rho_k) \leq \tilde{u}(\rho_{ij}, \sigma^{-j}) \leq u(\rho_{ij}, \bar{\rho}_k).$$

(A.8)

To start with, suppose that $\rho_{ij} < \rho_{k+1}$. As above, using successively (A.3), the fact that $u$ is strictly increasing in its first argument and $\rho_{ij} < \rho_{k+1}$, and (A.8), we have

$$1 = u(\rho_{k+1}, \rho_k) > u(\rho_{ij}, \rho_k) \geq \tilde{u}(\rho_{ij}, \sigma^{-j}).$$

That is, if $\rho_{ij} < \rho_{k+1}$, and other investors only use strategies in $\Sigma^k$, then withdrawing is a dominant strategy. Now, suppose that $\rho_{ij} > \bar{\rho}_{k+1}$. As above, using successively (A.3), the fact that $u$ is strictly increasing in its first argument and $\rho_{ij} > \bar{\rho}_{k+1}$, and (A.8), we have

$$1 = u(\bar{\rho}_{k+1}, \rho_k) < u(\rho_{ij}, \rho_k) \leq \tilde{u}(\rho_{ij}, \sigma^{-j}).$$

That is, if $\rho_{ij} > \bar{\rho}_{k+1}$, and other investors only use strategies in $\Sigma^k$, then rolling over is a dominant strategy. We have shown that if (A.6) holds for $k \geq 1$, then it holds for $k + 1$, and the demonstration by iterative induction is complete.

We can now conclude the proof of proposition 1. If all other investors withdraw if and only if their signal is smaller than $\rho$ (remember that $\rho$ solves $u(\rho, \rho) = 1$), then, for $\rho_{ij} = \rho$, investor $j$ is indifferent between withdrawing and rolling over (since he gets a payoff of 1 from withdrawing). In addition, the utility function $u$ being strictly increasing in its first argument,

for any $\rho_-$ and $\rho_+$ s.t. $\rho_- < \rho < \rho_+$, we have $u(\rho_-, \rho) < 1 < u(\rho_+, \rho)$.  

(A.9)
That is, for any $\rho_{ij} < \rho$ it is optimal for investor $j$ to withdraw, and for any $\rho_{ij} > \rho$ it is optimal for investor $j$ to roll over. Since $\underline{\rho}$ is by definition the smallest solution to $u(\rho, \rho) = 1$ and $\bar{\rho}$ is the largest solution to $u(\rho, \rho) = 1$, (A.9) implies that $\underline{\rho} = \rho = \bar{\rho}$. It follows that the only strategy which survives the iterated elimination of dominated strategies is the strategy which consists in withdrawing if and only if $\rho_{ij} < \rho$. This notably implies that the equilibrium in which each investor follows this strategy is the unique equilibrium.

Proof of Proposition 3

Notice first that the equilibrium threshold in Corollary 2 is a function of $p$, which becomes a (potentially unknown) random variable in the current section. However, it suffices to use investors’ expectation of $p$ in lieu of the actual $p$ in (7).

We assume that investors do not observe $p$ but that the regulator can commit to a disclosure policy conditional on $p$. Consider first the case where $\mu \geq \frac{c}{2}$. Then $\mu + E[2p - 1|p > p^C]\Delta = \mu - \frac{c}{2}$, which is positive. In words, investors never run if they can’t distinguish between banks and don’t update their beliefs on $p$. Since a policy of unconditional opacity does not convey any information on $p$, it prevents bank runs and is therefore optimal.

Turn to the case where $\mu < \frac{c}{2}$ and suppose that the regulator commits to disclose information if and only if $p < \hat{p}$. Let $p^C$ be implicitly defined by

$$\mu + E[2p - 1|p > p^C]\Delta = \frac{c}{2}.$$  

$E[2p - 1|p > \hat{p}]$ is decreasing in $\hat{p}$, and $\mu < \frac{c}{2}$ implies $\mu + E[2p - 1|p > 0]\Delta < \frac{c}{2}$. Therefore $p^C$ is well defined and strictly positive.

Notice first that if $\hat{p} = p^C$, investors never run when the regime is opaque, that is, when $p \geq p^C$ and run on low-quality banks when the regime is transparent, that is, when $p < p^C$. Notice next, that if $\hat{p} < p^C$ investors run on all banks when the regime is opaque, that is, when $p \geq \hat{p}$. This is dominated by unconditional transparency where high-quality banks never suffer runs. Notice finally that if $\hat{p} > p^C$ investors run on low-quality bank when $p < \hat{p}$. This is dominated by $\hat{p} = p^C$ where low-quality banks face run only if $p < p^C$. Therefore $\hat{p} = p^C$ is optimal.

Proof of Proposition 4

We begin by deriving the expected utility of an investor in three different cases, depending on the disclosure policy set at $t = 0$, and on whether investors learn $p$ at $t = 1$ or not.
First, with disclosure, whether investors learn \( p \) at \( t = 1 \) does not matter, and the expected utility of an investor is

\[
\int_0^1 [p(1 + \mu + \Delta_\eta) + (1 - p)1] \, dp = 1 + \frac{\mu + \Delta_\eta}{2}.
\]

(A.10)

Second, without disclosure and if investors learn \( p \) at \( t = 1 \), the expected utility of an investor is

\[
\int_0^{p^*} 1 \, dp + \int_{p^*}^1 (1 + \mu + (2p - 1)\Delta_\eta) \, dp = 1 + (1 - p^*) \left[ \mu + E[(2p - 1)|p > p^*]\Delta_\eta \right].
\]

Third, without disclosure and if investors do not learn \( p \) at \( t = 1 \), then investors withdraw from any bank at \( t = 1 \) if and only if

\[
\mu < \frac{c}{2} - \int_0^1 (2p - 1)\Delta_\eta \, dp \iff \mu < \frac{c}{2}.
\]

Accordingly, we distinguish between two cases. First, if \( \mu < \frac{c}{2} \), then investors withdraw from any bank at \( t = 1 \), and the expected utility of any investor is equal to 1. Second, if \( \mu > \frac{c}{2} \), then investors roll over all their investments at \( t = 1 \), and the expected utility of any investor is

\[
\int_0^1 (1 + \mu + (2p - 1)\Delta_\eta) \, dp = 1 + \mu.
\]

Assuming that investors learn \( p \) at \( t = 1 \) with probability \( q \), we can now compare the expected utility of any investor with and without disclosure. With disclosure, the expected utility is as in (A.10). Without disclosure, and if \( \mu < \frac{c}{2} \), the expected utility is

\[
q \left\{ 1 + (1 - p^*) \left[ \mu + E[(2p - 1)|p > p^*]\Delta_\eta \right] \right\} + (1 - q).
\]

Without disclosure, and if \( \mu > \frac{c}{2} \), the expected utility is

\[
q \left\{ 1 + (1 - p^*) \left[ \mu + E[(2p - 1)|p > p^*]\Delta_\eta \right] \right\} + (1 - q)(1 + \mu).
\]

The condition in (4) immediately follows from the definition of the optimal disclosure policy.

Proof of Proposition 5

Depending on the choice of \( \alpha \) and the realization of \( p \), there are three possible cases.

1. If \( \mu - \alpha\Delta_\eta + (1 - \alpha)(2p - 1)\Delta_\eta > \frac{c}{2} \iff p > \frac{1}{2} + \frac{\xi - \mu + \alpha\Delta_\eta}{2(1 - \alpha)\Delta_\eta} \equiv \overline{\beta}(\alpha) \), there is no run on any bank in the system. This happens with a strictly positive probability if \( \alpha < \frac{1}{2} + \frac{1}{2\Delta_\eta} \left( \mu - \frac{c}{2} \right) \equiv \overline{\alpha} \).

2. If \( \mu + \alpha\Delta_\eta + (1 - \alpha)(2p - 1)\Delta_\eta < \frac{c}{2} \iff p < \frac{1}{2} + \frac{\xi - \mu - \alpha\Delta_\eta}{2(1 - \alpha)\Delta_\eta} \equiv \underline{\beta}(\alpha) \), there is a run on every bank in the system. This happens with a strictly positive probability if \( \alpha > \frac{1}{2} + \frac{1}{2\Delta_\eta} \left( \frac{c}{2} - \mu \right) \equiv \underline{\alpha} \).
3. If $p(\alpha) < p < \overline{p}(\alpha)$, only low-quality banks suffer from a run.

Notice first that if $\alpha \geq \max(\overline{\alpha}, \alpha)$, diversification does not affect the probability of a bank run. Thus, the expected return of a bank is

$$\int_0^1 p[\mu + \alpha \Delta_\eta + (1 - \alpha)(2p - 1)\Delta_\eta]dp.$$  \hspace{1cm} (A.11)

(A.11) is increasing in $\alpha$, therefore $\alpha = 1$ strictly dominates any $\alpha \in [\max(\overline{\alpha}, \alpha), 1)$. Intuitively, when $\alpha = 1$, high-quality assets are always safe because they all belong to high-quality banks, while all low-quality assets, held by low-quality banks, are liquidated. On the contrary, when banks diversify but $\alpha$ stays above $\max(\overline{\alpha}, \alpha)$, low-quality banks are still liquidated, but they now hold a fraction $(1 - \alpha)p$ of high-quality assets. As a result, a quantity $(1 - p)(1 - \alpha)p$ of high-quality assets is liquidated in lieu of low-quality assets. Therefore, diversification destroys value in the range $[\max(\overline{\alpha}, \alpha), 1)$.

However, for values of $\alpha$ smaller than $\max(\overline{\alpha}, \alpha)$, diversification brings one benefit: it prevents runs of low-quality banks for high realizations of $p$. Consider the case where $\mu \geq \frac{\Delta_\eta}{2}$, so that $\overline{\alpha} < \overline{\pi}$. Banks can then choose $\alpha$ in $(\overline{\alpha}, \overline{\pi})$, that is, such that diversification prevents runs on low-quality banks when $p$ is high, but never causes a run on the entire system when $p$ is low. Such an $\alpha$ dominates $\alpha = 1$ if the following condition holds

$$\int_0^1 p[\mu + \Delta_\eta]dp < \int_0^{\overline{\pi}(\alpha)} p[\mu + \alpha \Delta_\eta + (1 - \alpha)(2p - 1)\Delta_\eta]dp + \int_{\overline{\pi}(\alpha)}^1 \mu + (2p - 1)\Delta_\eta dp.$$  \hspace{1cm} (A.12)

Rearranging,

$$\int_0^\overline{\pi}(\alpha) 2\Delta_\eta(1 - p)(1 - \alpha)pdp < \int_{\overline{\pi}(\alpha)}^1 (1 - p)(\mu - \Delta_\eta)dp.$$  \hspace{1cm} (A.12)

The LHS of (A.12) is the cost of diversification: when $p$ is low, a quantity $(1 - p)(1 - \alpha)p$ of high-quality assets is liquidated in lieu of low-quality assets, which has a net cost of $2\Delta_\eta$ per asset. The RHS is the benefit of diversification: when $p$ is high, even low-quality assets can be brought to maturity. Notice that $\overline{\pi}(\alpha)$ is decreasing in $\mu$ and tends to 0 when $\mu \to \frac{\Delta_\eta}{2} + \Delta_\eta$. Therefore, there exists a threshold for $\mu$ above which both the initial assumption $\mu - \Delta_\eta < \frac{\Delta_\eta}{2}$ and (A.12) hold. Thus, in good economic times, that is, if $\mu$ is sufficiently high, banks will choose to diversify.

**Proof of Proposition 6**

Notice first that the optimal disclosure policy is independent of $\alpha$. Indeed, if $p \geq \overline{\pi}(\alpha)$ or $p \leq \underline{p}(\alpha)$, the disclosure policy is irrelevant. If $p(\alpha) < p < \overline{p}(\alpha)$, it is optimal to disclose if there is a run on a bank perceived as average, that is, if $p < p^*$. 

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Notice next that for all \( p \geq p^* \), the expected return of a bank is simply \( \mu + (2p - 1)\Delta \eta \) which is independent from \( \alpha \). Notice finally that for any \( p < p^* \), the expected return of a bank is \( p[\mu + \alpha \Delta \eta + (1 - \alpha)(2p - 1)\Delta \eta] \) which is strictly increasing in \( \alpha \). It is therefore strictly optimal to choose \( \alpha = 1 \).

**Proof of Proposition 7**

The proof follows the lines of proposition 2. The regulator chooses to disclose \( p \) if it is such that there is a run on the average bank, that is, if

\[
\mu + (2p - 1)\Delta \eta - \frac{\tau L^2}{2} < c \left( \frac{1}{2} - L \right) \Leftrightarrow p < \frac{1}{2} + \frac{1}{2\Delta} \left[ \frac{c}{2} - \mu + \frac{\tau L^2}{2} - cL \right].
\]

**Proof of Proposition 8**

Notice first \( p^*(L) > 0 \) reaches a minimum for \( L = \frac{c}{\tau} \), and that given (9), \( p^*(\frac{c}{\tau}) > 0 \). This implies (a) that there are always runs on banks that are revealed to be of low quality, whatever their choice of \( L_i \); (b) that increasing \( L_i \) beyond \( \frac{c}{\tau} \) is strictly dominated regardless of the disclosure policy since it does not prevent a run if the bank is revealed to be of poor quality, and it increases the probability of a run if the regulator chooses opacity. We can therefore restrict attention to liquidity choices \( L_i \) in the interval \([0, \frac{c}{\tau}] \). On that interval, \( p^*(.) \) is strictly decreasing.

We derive first the socially optimal liquidity of banks. We start by computing the optimal level, \( L^* \), restricting attention to a subset of choices where all banks have the same level of liquidity, that is, \( L_i = L \) for all \( i \). We will later check that this property must be true at optimum.

Let \( g(L) \) denote the aggregate return of the banking sector, given the optimal disclosure policy \( p^*(L) \),

\[
g(L) = \int_0^{p^*(L)} p \left( \mu + \Delta - \frac{\tau L^2}{2} \right) dp + \int_{p^*(L)}^1 \left[ \mu + (2p - 1)\Delta - \frac{\tau L^2}{2} \right] dp. \tag{A.13}
\]

Differentiating,

\[
g'(L) = p'^*(L) \left\{ p^*(L) \left( \mu + \Delta - \frac{\tau L^2}{2} \right) - \mu - [2p^*(L) - 1]\Delta + \frac{\tau L^2}{2} \right\} - \tau L \left\{ \frac{[p^*(L)]^2}{2} + 1 - p^*(L) \right\} = p'^*(L)[p^*(L) - 1] \left( \mu - \Delta - \frac{\tau L^2}{2} \right) - \tau L \left\{ \frac{[p^*(L)]^2}{2} + 1 - p^*(L) \right\}
\]

Notice that

\[
g'(0) > 0 \text{ and } g' \left( \frac{c}{\tau} \right) < 0. \tag{A.14}
\]

Therefore \( g(.) \) admits at least one maximum \( L^* \) in \((0, \frac{c}{\tau})\), and \( L^* \) is such that \( g'(L^*) = 0 \).
The next step consists in showing that \( g'(L) = 0 \) implies \( g''(L) < 0 \), which, together with (A.14), implies that \( g'(L) = 0 \) has a unique solution. Suppose \( g'(L) = 0 \).

\[
g''(L) = p''(L)[p^*(L) - 1] \left( \mu - \Delta - \tau \frac{L^2}{2} \right) + \frac{p''(L)^2}{2} \left( \mu - \Delta - \tau \frac{L^2}{2} \right) - 2\tau L p''(L)[p^*(L) - 1] - \tau \left\{ \frac{[p^*(L)]^2}{2} + 1 - p^*(L) \right\} < [p''(L)]^2 \left( \mu - \Delta - \tau \frac{L^2}{2} \right) - \tau \left\{ \frac{[p^*(L)]^2}{2} + 1 - p^*(L) \right\},
\]

where the last inequality stems from \( p''(L) > 0 \), \( p^*(L) < 0 \) for \( L \in [0, \frac{\tau}{p}] \), and, from (9), \( \mu - \Delta - \tau \frac{L^2}{2} > 0 \) for \( L \in [0, \frac{\tau}{p}] \). Using \( g'(L) = 0 \) to substitute,

\[
g''(L) < [p''(L)]^2 \left( \mu - \Delta - \tau \frac{L^2}{2} \right) - p''(L) \frac{p^*(L) - 1}{L} \left( \mu - \Delta - \tau \frac{L^2}{2} \right) < p''(L) \left( \mu - \Delta - \tau \frac{L^2}{2} \right) \left( p''(L) - \frac{p^*(L) - 1}{L} \right).
\]

Consider the function \( f(L) = L p''(L) - p^*(L) + 1 \). \( f'(L) = L p'''(L) > 0 \). Therefore \( f(L) > f(0) = 1 - p(0) > 0 \). Using again \( \mu - \Delta - \tau \frac{L^2}{2} > 0 \) and \( p'(L) < 0 \), this, in turn, implies that \( g''(L) < 0 \).

Therefore, \( L^* \) is uniquely defined by \( g'(L^*) = 0 \), and \( L^* \in (0, \frac{\tau}{p}) \).

In the derivation of \( L^* \), we imposed that all banks choose the same \( L_i \). We show now banks must hold the same liquidity level at optimum.

Consider the disclosure threshold \( \hat{p} \) as given. Notice first that for any set of liquidity choices, \( \hat{p} > p^*(0) \) is strictly dominated by \( \hat{p} = p^*(0) \). In words, it cannot be optimal to increase transparency above the level that is optimal when \( L = 0 \). Notice also that for any set of liquidity choices, \( \hat{p} < p^* \left( \frac{\tau}{p} \right) \) is dominated by \( \hat{p} = p^* \left( \frac{\tau}{p} \right) \). In words, it cannot be optimal to decrease the level of transparency below the level where a bank perceived as average fails, whatever its level of liquidity. We therefore restrict attention to the case where \( p^* \left( \frac{\tau}{p} \right) \leq \hat{p} \leq p^*(0) \). The individual return of a bank is then

\[
h(L_i) = \int_0^{\hat{p}} p \left( \mu + \Delta - \tau \frac{L^2}{2} \right) dp + \int_{\max(\hat{p}, p^*(L_i))}^{1} \left( \mu + (2p - 1)\Delta - \tau \frac{L^2}{2} \right) dp. \tag{A.15}
\]

For a given \( \hat{p} \), optimal liquidity levels solve

\[
\max_{\{L_i, i \in [0, 1]\}} \int_0^{1} h(L_i, \hat{p}) di.
\]

Notice that this optimization problem boils down to maximizing \( h(L_i) \) for each bank \( i \). Notice also that \( L_i > p^{*-1}(\hat{p}) \) is dominated by \( L_i = p^{*-1}(\hat{p}) < \frac{\tau}{p} \). Finally, for any \( L_i \in [0, p^{*-1}(\hat{p})] \),

\[
h'(L_i) = -\tau L_i \left[ \frac{\hat{p}^2}{2} + 1 - p^*(L_i) \right] - c \left( \frac{1}{2} - L_i \right) \frac{\tau L_i - c}{2\Delta}.
\]

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\( h'(0) > 0, \ h' \left( \frac{c}{\tau} \right) < 0, \lim_{L \to -\infty} h'(L) = -\infty \) and \( \lim_{L \to +\infty} h'(L) = +\infty \). Therefore, since \( h'(\cdot) \) is a polynomial of order three, it has exactly one root, \( L^*_i(\hat{p}) \), in \( [0, \frac{c}{\tau}] \). Thus, either \( h'[p^{*^{-1}}(\hat{p})] \geq 0 \), and it is then optimal for each bank \( i \) to choose \( L_i = p^{*^{-1}}(\hat{p}) \), or \( h'[p^{*^{-1}}(\hat{p})] < 0 \) and it is then optimal for each bank \( i \) to choose \( L_i = L^*_i(\hat{p}) \). In either cases, the optimization problem has a unique solution, such that every bank chooses the same level of liquidity. Since this property holds for any \( p \) that can be part of an optimum, it must be true at the optimum of the full optimization program (jointly over \( p \) and \( \{L_i\}_{i \in [0,1]} \)).

We turn now to the equilibrium level of liquidity and focus on symmetric equilibria. Let \( L^E \) be a candidate equilibrium.

Notice first that, given \( L^E \), there is no incentive for a single bank to deviate by choosing \( L_i > L^E \). Indeed, the regulator chooses opacity if and only if it prevents runs on the entire system, that is, if \( p \geq p^*(L^E) \). In this case, \( L_i > L^E \) is dominated by \( L_i = L^E \) (since liquidity is costly). On the other hand, if the regulator chooses transparency, \( L_i \) does not affect the outcome, that is, a low-quality bank suffers a run, while a high-quality bank is safe. Therefore, for any candidate equilibrium \( L^E \) we only need to check individual incentives to deviate from below, that is, by choosing \( L_i < L^E \).

Given a candidate equilibrium \( L^E \leq \frac{c}{\tau} \) a bank chooses the level of liquidity \( L_i \in [0, L^E] \) to maximize

\[
\int_0^{p^*(L^E)} p \left[ \mu + \Delta - \tau \frac{L^2}{2} \right] dp + \int_{p^*(L_i)}^{1} \left[ \mu + (2p - 1)\Delta - \tau \frac{L^2}{2} \right] dp.
\]

Using (A.13), this best-response function can be rewritten as

\[
g(L_i) - \int_0^{p^*(L_i)} p \left[ \mu + \Delta - \tau \frac{L^2}{2} \right] dp + \int_0^{p^*(L^E)} p \left[ \mu + \Delta - \tau \frac{L^2}{2} \right] dp. \tag{A.16}
\]

Taking the first derivative of (A.16) with respect to \( L_i \) yields

\[
g'(L_i) + \frac{c - \tau L_i}{2\Delta}p^*(L_i) \left( \mu + \Delta - \tau \frac{L^2}{2} \right) + \int_{p^*(L_i)}^{p^*(L^E)} p\tau L_i dp. \tag{A.17}
\]

Notice first that \( g'(L_i) \geq 0 \) on \([0, L^*]\), since \( g \) is single-peaked in \( L^* \). Notice then that the second term in (A.17) is strictly positive for \( L_i < L^E \), since \( L^E \leq \frac{c}{\tau} \) and (9) holds. Notice finally that the third term is also strictly positive for \( L_i < L^E \), since \( p^* \) is strictly decreasing in \( L_i \) on \([0, \frac{c}{\tau}] \).

Consider first the case where \( L^E \in [0, L^*] \). It follows from the previous paragraph that (A.17) is then strictly positive for any \( L_i < L^E \). Since \( L_i > L^E \) is dominated by \( L_i = L^E \), the best response of a bank to any \( L^E \in [0, L^*] \) is to choose \( L_i = L^E \). Therefore, any non-negative level of liquidity \( L^E \leq L^* \) is an equilibrium.

Next, consider the case where \( L^E > L^* \). For \( L_i \in [0, L^*] \),

\[
g'(L_i) + \frac{c - \tau L_i}{2\Delta}p^*(L_i) \left( \mu + \Delta - \tau \frac{L^2}{2} \right) > 0. \tag{A.18}
\]
By continuity, there exists $L^E > L^*$ such that (A.18) is strictly positive for $L_i \in [0, L^E)$. Therefore, there exists $L^E > L^*$ such that (A.17) is strictly positive for $L \in [0, L^E)$. Since $L_i > L^E$ is dominated by $L_i = L^E$, there exists a continuum of equilibrium levels of liquidity $L^E$, such that $L^E > L^*$.
References


