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The Information Content of Implied Probabilities to Detect Structural Change

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Abstract:
This paper proposes Pearson-type statistics based on implied probabilities to detect structural change. The class of generalized empirical likelihood estimators (see Smith (1997)) assigns a set of probabilities to each observation such that moment conditions are satisfied. These restricted probabilities are called implied probabilities. Implied probabilities may also be constructed for the standard GMM (see Back and Brown (1993). The proposed test statistics for structural change are based on the information content in these implied probabilities. We consider cases of structural change with unknown breakpoint which can occur in the parameters of interest or in the overidentifying restrictions used to estimate these parameters. The test statistics considered here have good size and power properties.

Keywords: Generalized empirical likelihood, generalized method of moments, parameter instability, structural change

JEL Classification: C12, C32

1 Introduction

This paper proposes structural change tests based on implied probabilities resulting from estimation methods based on unconditional moment restrictions. The Generalized Method of Moments (GMM) is the standard method to estimate parameters of interest through moment restrictions. However, Monte Carlo results reveal that the GMM estimators may be seriously biased in small sample.[1] Recently, a number of alternative estimators to GMM have been proposed. Hansen, Heaton and Yaron (1996) suggested the continuous updated estimator (CUE) which shares same objective function that the GMM but with a weighting matrix depending on the parameters of interest. The empirical likelihood (EL) (see Qin and Lawless (1994)) and the exponential tilting (ET) estimators (see Kitamura and Stutzer (1997)) have also been proposed. Kitamura (2001) showed that tests for the validity of moment restrictions based on EL have optimal properties in terms of large deviations. In particular EL tests are shown to be more powerful than other standard tests. These alternative estimators are special cases of the generalized empirical likelihood (GEL) class considered by Smith (1997) and may be shown to be based on the Cressie and Read (1984) family of power divergence criteria. Newey and Smith (2004) showed (in an i.i.d. setting) that, although estimators based on the GMM, EL, ET or that are CUE have the same first order asymptotic distribution, they have different higher order asymptotic properties. Amongst their findings it is shown that the expression for the second order asymptotic bias of GEL has fewer components than the one of GMM (with EL having the fewest). Anatolyev (2005) extended the Newey and Smith setting to allow for weakly dependent data correlation and show that the asymptotic bias of smoothed GEL estimators is less than the GMM estimators especially with many moment conditions.

GEL estimators assign a probability to each observation such that the moment conditions are satisfied (see Smith (2004)). This resulting empirical measure is called implied probabilities. Implied probabilities may also be constructed for the standard GMM as shown by Back and Brown (1993). The interpretation of the implied probabilities is the following: less weight is assigned to an observation for which the moment restrictions are not satisfied at the estimated values of the parameters and more weight to an observation for which the moment restrictions are satisfied. As suggested by Back and Brown (1993), implied probabilities may then provide a useful diagnostic device for the model specification. In particular, implied probabilities may contain interesting information to detect instability in the sample. Consequently, we propose the use of these weights in detecting an unknown structural change in the model. Antoine, Bonnal and Renault (2007) use the weights given by implied probabilities to propose a three-step estimation procedure asymptotically higher equivalent to empirical likelihood. Schennach (2004) also discusses the use of these weights in the context of model misspecification. Ramalho and Smith (2005) considered Pearson-type test statistics (statistics based on the difference between restricted and unrestricted esti-

[1] See in particular the special issue of *Journal of Business and Economic Statistics*, 1996, volume 14.

mators of the weights) for the validity of moment restrictions and parametric restrictions using implied probabilities.

The proposed test statistics to detect structural change are based on different measures of the discrepancy between these implied probabilities and the unconstrained empirical probabilities $\frac{1}{T}$. These test statistics fall in three categories: 1) general structural change tests to detect instability in the identifying restrictions and in the overidentifying restrictions; 2) structural change tests specially designed to detect instability in the identifying restrictions (see for example Andrews (1993)) and 3) structural change tests to detect instability in the overidentifying restrictions (see for example Hall and Sen (1999)). The asymptotic distribution of the statistic tests is derived for the case of unknown breakpoint under the null and under the alternative hypotheses. In a simulation study, we find that targeted tests based on implied probabilities performed quite well if the structural change corresponds to the target. That is, if instability is present in the identifying restrictions or in the overidentifying restrictions, then the targeted tests have size and power that are at least as good as those of the more standard tests. General tests for structural change have size and power properties that are between those of the targeted tests. Overall the test statistics based on implied probabilities considered in this paper have a nice intuitive appeal, are based on an estimation method that has been shown to have nice higher order asymptotic properties and are relatively easy to compute.

The paper is organized as follows. A discussion on the GMM and GEL estimators are presented in section 2. Section 3 presents formally the full-sample and partial-sample GMM and GEL estimators. Section 4 presents the test statistics proposed based on the implied probabilities. The simulation results are in Section 5 while the proofs are in the Appendix.

2 Discussion of GMM and GEL Estimators

In this section we present the estimators used in this paper. We start with an entropy-based formulation of the problem which puts emphasis on the informational content of the estimated weights. We then move to the more recent GEL formulation (see Newey and Smith (2004) and Smith (2004)).

We consider a random variable $\{x_t : 1 \leq t \leq T, T \geq 1\}$. Suppose we have $q \times 1$ vector function of the data $g(x_t, \theta)$ which depends on some unknown p-vector of parameters $\theta \in \Theta$ with $\Theta \subset R^p$ and that in the population their expected value is 0, namely

$$E[g(x_t, \theta_0)] = 0$$

Along this paper we consider the overidentifying case with $q > p$.

The standard GMM estimators (Hansen (1982)) are obtained as the solution of

$$\tilde{\theta}_T = arg \min_{\theta \in \Theta} g_T(\theta)' W_T g_T(\theta)$$

where W_T is a random positive definite symmetric $q \times q$ matrix and $g_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} g(x_t, \theta)$. The optimal weighting matrix is defined to be the inverse of the covariance matrix of the moment conditions, $W_T = \Omega_T^{-1}$ where Ω_T is a consistent estimator of

$$\Omega = \lim_{T\to\infty} Var\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T} g(x_t, \theta)\right).$$

The optimal weighting matrix can be estimated consistently using methods developed by Gallant (1987), Andrews and Monahan (1992) and Newey and West (1994), among several others.

We note that, in the GMM criterion function, the moment conditions receive equal weight ($1/T$) for each observations. Back and Brown (1993) derive a set of *implicit* weights using the GMM estimators given by

$$\pi_t(\theta) = \frac{1}{T} - \frac{1}{T-p}\left[J_{tT}(\theta) - J_T(\theta)\right]' \hat{\Omega}_T(\theta)^{-1} g_T(\theta) \tag{1}$$

with $J_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} J_{tT}(\theta)$ and

$$J_{tT}(\theta) = \sum_{j=1}^{T} \kappa(|t-j|) g(x_{t-j}, \theta)$$

where $\kappa(|t-j|)$ is a real valued weighting function. The following estimator

$$\hat{\Omega}_T(\theta) = \frac{1}{T-p}\sum_{t=1}^{T} J_{tT}(\theta) g(x_t, \theta)'$$

is a consistent and positive definite estimator of the variance-covariance matrix and it has the usual form of a heteroskedasticity and autocorrelation consistent (HAC) weight matrix for standard kernels $\kappa(|t-s|)$ considered in the vast literature on consistent estimation of variance-covariance matrix (see e.g. Andrews (1991), Newey and West (1987) and Newey and West (1994)). The implied probabilities defined above are the empirical measure that ensures that moment conditions are satisfied in the sample.

Now letting the T-vector of *explicit* weights (explicit because the weights are used directly in estimation) be $\{\pi_t : 1 \le t \le T, T \ge 1\}$ we can recast the population moment conditions as

$$E_\pi[g(x_t, \theta_0)] = 0.$$

The vector π is determined by finding the most probable data distribution of the outcomes given the data. We can think of π as containing information on the content of the moment conditions. Therefore, $g(x, \theta)$ is viewed as a message. That is, when π is small, the message is informative and vice-versa. This relation is summarized by the function $f(\pi) = -\ln \pi$. The average information is then

$$S(\pi) \equiv E_\pi f(\pi) = -\sum_{t=1}^{T} \pi_t \ln \pi_t.$$

In this case, $S(\pi)$ can be interpreted as the entropy measure of Shannon (1948) and it captures the degree of uncertainty in the distribution π with respect to whether or not the distribution is concentrated or dispersed. The vector π is obtained by maximizing entropy

$$\max S(\pi) = -\sum_{t=1}^{T} \pi_t \ln \pi_t,$$

subject to $\sum_{t=1}^{T} \pi_t = 1$ and $\sum_{t=1}^{T} \pi_t g(x_t, \theta) = 0$.

With no constraint, we get $\pi_t = 1/T \quad \forall t$, the maximally uninformative uniform distribution, while with constraints, we want to choose π_t to be as maximally uninformative as the moment conditions will allow. We do not want to assert more about the distribution than is known via the moment conditions. In this sense, the probabilities make use of all the information that is available, and nothing more. In particular, we focus on detecting a structural change in the moment conditions. With no structural change, the weights will fluctuate around $1/T$, otherwise the entropy formulation will attempt to reduce the weight on the observation characterized by the change, and at the same time put more weights on the remaining observations so as to make $S(\pi)$ as large as possible.

We can transpose this formulation using the Kullback and Leibler (1951) information criterion which measures the discrepancy between two distributions p and π. If the subject distribution is π and the reference distribution is $p_t = 1/T, \forall t$ we have

$$KLIC(\pi, p = 1/T) = \sum_{t=1}^{T} \pi_t[\ln(\pi_t) - \ln(1/T)] = \sum_{t=1}^{T} \pi_t \ln(\pi_t) + \ln(T).$$

So that maximizing entropy is equivalent to minimizing the $KLIC(\pi, p = 1/T)$. Estimates of π, given θ, are obtained by maximizing $S(\pi)$ (or by minimizing $KLIC(\pi, p = 1/T)$) subject to the weighted zero functions and the probability constraint. The solution to the Lagrangian yields

$$\pi_t^{ET}(\theta) = \frac{\exp(\gamma' g(x_t, \theta))}{\sum_{t=1}^{T} \exp(\gamma' g(x_t, \theta))}$$

where the q-vector γ contains the Lagrange multipliers and as such measure the degree of departure from zero of the moment conditions and ET stands for exponential tilting (see Kitamura and Stutzer (1997)). Estimates of θ are obtained by substituting π in $S(\pi)$, maximizing it with respect to γ and then with respect to θ (see for example Kitamura and Stutzer (1997)).

If we interchange the subject/reference distributions we get

$$KLIC(p = 1/T, \pi) = \sum_{t=1}^{T} (1/T)[\ln(1/T) - \ln(\pi_t)]$$

and the solution to the optimization problem yields a different set of weights given by

$$\pi_t^{EL}(\theta) = \frac{1}{T\,[1 + \gamma' g(x_t, \theta)]}$$

where EL stands for empirical likelihood (see Qin and Lawless (1994) for example). When we evaluate the weights at some estimators we obtain $\pi_t^{ET}(\tilde{\theta}_T)$ and $\pi_t^{EL}(\tilde{\theta}_T)$. Recently, Schennach (2004) combined ET and EL into the ETEL estimator that combines the advantages of each approach.

We mentioned in the introduction that less weight is assigned to an observation for which the moment conditions are not satisfied. In this section we have seen that the vector π contains all the relevant information with respect to the moment conditions. We now provide a graphical intuition on the use of the weights in the detection of a structural change. We consider a small simulation study that contains three examples that have been studied in the structural change and entropy literature. The first example, which encompasses three cases, is similar to the one used by Imbens, Spady and Johnson (1998) and consists of estimating a single parameter, θ, with 2 moment conditions:

$$E\left[x_t - \theta\right] = 0 \text{ and } E\left[(x_t - \theta)^2 - 4\right] = 0$$

with a sample of 100 observations and $x_t = \theta_t + \epsilon_t$ where $\epsilon_t \sim N(0,4)$. We consider a pulse, a break and a regime shift cases. In the pulse case we have $\theta_t = 5$ for $t = 50$ and $\theta = 10$ otherwise. For the break case we have $\theta_t = 10$ for $t \leq 20$ and $t > 80$ while $\theta_t = 15$ for $21 \leq t \leq 80$. Finally, the regime shift case has $\theta_t = 10$ for $t \leq 20$ and $\theta_t = 15$ otherwise. In these cases, structural change occurs via the parameters and can be tested using procedures proposed by Andrews (1993) and by Andrews and Ploberger (1994).

In contrast, the next two examples consist of structural change through the moment conditions. Following Hall and Horowitz (1996) and Gregory, Lamarche and Smith (2002) we study a simulated environment with CRRA preferences and making a distributional assumption on consumption growth, x_t, with i.i.d data and $T = 100$. In particular, we assume that consumption growth follows a $N(0, \sigma^2 = 0.16)$. There is a single parameter to be estimated, γ, the coefficient of CRRA and two moment conditions are used:

$$\begin{aligned} E_t \exp[-\gamma \ln x_{t+1} - 9\sigma^2/2 + (3-\gamma)z_t] &= 1 \\ E_t z_t \exp[-\gamma \ln x_{t+1} - 9\sigma^2/2 + (3-\gamma)z_t - 1] &= 0 \end{aligned}$$

with $z_t \sim N(0, \sigma^2)$. The moment conditions are satisfied when $\gamma = 3$. The structural break occurs in period 51 and is summarized by a shift in γ from 3 to 4.

Lastly, as in Ghysels, Guay and Hall (1997), we have the estimation of an autoregressive parameter using two moment conditions when the data generating process is an $AR(1)$ ($x_t = \rho x_{t-1} + \epsilon_t$), for $t \leq 50$, and an $ARMA(1,2)$ otherwise ($x_t = \rho x_{t-1} + \epsilon_t + 0.5\epsilon_{t-2}$). There are 100 observations and $\epsilon_t \sim N(0,1)$. The two instruments used are the first and second lags of x_t. The two moment conditions are then:

$$\begin{aligned} E\left[x_{t-1}\left(x_t - \rho x_{t-1}\right)\right] &= 0 \\ E\left[x_{t-2}\left(x_t - \rho x_{t-1}\right)\right] &= 0 \end{aligned}$$

The instability occurs because the second moment condition is violated after $t > 50$.

Figure 1 shows the average of the vector of implied probabilities π over 10,000 replications. The key feature of these panels is that when there is no break, the weights fluctuate around $1/T = 1/100$ (upper right panel). With a structural break in the parameter or in the moment conditions, however, more weight is given to observations (and moment conditions) for which there is no break while less weight is assigned to an observation which violates the moment conditions. This simple simulation study clearly show that implied probabilities contain interesting information to detect structural change. In this paper, we examine the information contained in the estimated implied probabilities to detect structural change and propose test statistics based on some function of implied probabilities.

Now following the recent econometric literature (see Caner (2004), Newey and Smith (2004), Smith (2004), Caner (2005), Guggenberger and Smith (2007) and Ramalho and Smith (2005)) on GEL we let $\rho(\phi)$ be a function of a scalar ϕ that is concave on its domain, an open interval Φ that contains 0. Also, let $\tilde{\Gamma}_T(\theta) = \{\gamma : \gamma' g(x_t, \theta) \in \Phi, t = 1, \ldots, T\}$. Then, the GEL estimator is a solution to the problem

$$\tilde{\theta}_T = \arg\min_{\theta \in \Theta} \sup_{\gamma \in \tilde{\Gamma}_T(\theta)} \sum_{t=1}^{T} \frac{[\rho(\gamma' g(x_t, \theta)) - \rho_0]}{T}$$

where $\rho_j() = \partial^j \rho()/\partial \phi^j$ and $\rho_j = \rho_j(0)$ for $j = 0, 1, 2, \ldots$. Under this formulation a number of estimators can be obtained. First, the ET estimator of θ is found by setting $\rho(\phi) = -\exp(\phi)$. Second, the EL estimator of θ by setting $\rho(\phi) = \ln(1 - \phi)$. Third, the continuously updated estimator, as opposed to the two-step estimator presented above, of Hansen *et al.* (1996) can also be obtained from the GEL formulation by using a quadratic function for $\rho(\phi) = -(1 + \phi)^2/2$.

As in the GMM context an adjustment for the dynamic structure of $g(x_t, \theta)$ can also be made in the GEL context (see Kitamura and Stutzer (1997), Smith (2000), Smith (2004) and Guggenberger and Smith (2007)). The adjustment consists of smoothing the original moment conditions $g(x_t, \theta)$. Defining the smoothed moment conditions as

$$g_{tT}(\theta) = \frac{1}{S_T} \sum_{s=t-T}^{t-1} k\left(\frac{s}{S_T}\right) g(x_{t-s}, \theta)$$

for $t = 1, \ldots, T$ and S_T is a bandwidth parameter, $k(\cdot)$ a kernel function and we define where $k_j = \int_{-\infty}^{\infty} k(a)^j da$. In the time series context, the criteria is then given by:

$$\sum_{t=1}^{T} \frac{[\rho(k\gamma' g_{tT}(\theta)) - \rho_0]}{T}$$

where $k = \frac{k_1}{k_2}$ (see Smith (2004)).

3 Full and Partial-Samples GMM and GEL Estimators

To establish the asymptotic distribution theory of tests for structural change in the parameters based on implied probabilities we need to elaborate on the specification of the parameter vector in our generic setup. We will consider parametric models indexed by parameters (β, δ). With no structural change we define a vector of parameters $(\beta, \delta) \subset B \times \Delta \in R^p$ with $p = r + \nu$. Following Andrews (1993) we make a distinction between pure structural change when no subvector δ appears and the entire parameter vector is subject to structural change under the alternative and partial structural change which corresponds to cases where only a subvector β is subject to structural change under the alternative. The generic null can be written as follows:

$$H_0 : \beta_t = \beta_0 \qquad \forall t = 1, \ldots, T. \tag{2}$$

The tests that we will consider assume as alternative that at some point in the sample there is a single structural break, like for instance:

$$\beta_{tT} = \begin{cases} \beta_1(s) & t = 1, ..., [Ts] \\ \beta_2(s) & t = [Ts] + 1, ..., T \end{cases}$$

where s determines the fraction of the sample before and after the assumed breakpoint and $[.]$ denotes the greatest integer function. The separation $[Ts]$ represents a possible breakpoint which is governed by an unknown parameter s. Hence, we will consider a setup with a parameter vector which encompasses any kind of partial or pure structural change involving a single breakpoint. In particular, we consider a $2r+\nu$ dimensional parameter vector $\theta = (\beta_1', \beta_2', \delta')'$ where β_1 and $\beta_2 \in B \subset R^r$ and $\theta \in \Theta = B \times B \times \Delta \subset R^{2r+\nu}$. The parameters β_1 and β_2 apply to the samples before and after the presumed breakpoint and the null implies that:

$$H_0 : \beta_1 = \beta_2 = \beta_0. \tag{3}$$

We will formulate all our models in terms of θ. Special cases could be considered whenever restrictions are imposed in the general parametric formulation. One such restriction would be that $\theta_0 = (\beta_0', \beta_0')'$, which would correspond to the null of a pure structural change hypothesis. Once we have defined the moment conditions we will also translate this into overidentifying restrictions and relate it to structural change tests, following the analysis of Sowell (1996a) and Hall and Sen (1999).

3.1 Definitions

We need to impose restrictions on the admissible class of functions and processes involved in estimation to guarantee well-behaved asymptotic properties of GMM and GEL estimators using the entire data sample or subsamples of observations. A set of regularity conditions is required to obtain weak convergence of partial-sample GMM and GEL estimators to a function of Brownian motions. To streamline the presentation we provide a detailed description of them in Appendix 7.1. We now formally define the standard GMM estimator using the full sample.

Definition 3.1. *The full-sample General Method of Moments estimator* $\{(\tilde{\beta}_T, \tilde{\delta}_T)\}$ *is a sequence of random vectors such that:*

$$\left(\tilde{\beta}_T', \tilde{\delta}_T'\right)' = arg \min_{(\beta,\delta)\in B\times\Delta} g_T(\beta,\delta)'\hat{W}_T g_T(\beta,\delta)$$

where $\hat{W}_T$ *is a random positive definite symmetric* $q \times q$ *matrix.*

The optimal weighting matrix W is defined to be the inverse of Ω which is defined as:

$$\Omega = \lim_{T\to\infty} Var\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T} g(x_t, \beta_0, \delta_0)\right).$$

The optimal weighting matrix can be estimated consistently using methods developed by Gallant (1987), Andrews and Monahan (1992) and Newey and West (1994), among several others.

To characterize the asymptotic distribution we define the following matrices:

$$G_\beta = \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T} E\partial g(x_t,\beta_0,\delta_0)/\partial\beta' \in R^{q\times r},$$

$$G_\delta = \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T} E\partial g(x_t,\beta_0,\delta_0)/\partial\delta' \in R^{q\times \nu},$$

$$G = [\begin{array}{cc} G_\beta & G_\delta \end{array}] \in R^{q\times p}.$$

where $p = r + \nu$. Finally, let $\tilde{\theta}_T = \left(\tilde{\beta}_T', \tilde{\beta}_T', \tilde{\delta}_T'\right)'$ be a $2r+\nu$-vector. Hereafter, the vector $\tilde{\theta}_T$ is called the full-sample estimator of θ. This restricted estimator is consistent only under the null that $\beta_1 = \beta_2$.

Several tests for structural change involve partial-sample GMM estimators defined by Andrews (1993). We consider again the two subsamples, the first based on observations $t = 1, \dots, [Ts]$ and the second covering $t = [Ts]+1, \dots, T$ where $s \in S \subset (0,1)$. The partial-sample GMM estimators for $s \in S$ based on the first and the second subsamples are formally defined as:

Definition 3.2. *A partial-sample General Method of Moments estimator* $\{\hat{\theta}_T(s)\}$ *is a sequence of random vectors such that:*

$$\hat{\theta}_T(s) = arg \min_{\theta\in\Theta} g_T(\theta,s)'\hat{W}_T(s) g_T(\theta,s)$$

for all $s \in S$. *Define* $g_t(\theta,s) = (g(x_t,\beta_1,\delta)', 0')' \in R^{2q\times 1}$ *for* $t = 1,\dots,[Ts]$ *and* $g_t(\theta,s) = (0', g(x_t,\beta_2,\delta)')' \in R^{2q\times 1}$ *for* $t = [Ts]+1,\dots,T$ *such that*

$$g_T(\theta,s) = \frac{1}{T}\sum_{t=1}^{T} g_t(\theta,s) = \frac{1}{T}\sum_{t=1}^{[Ts]}\left[\begin{array}{c} g(x_t,\beta_1,\delta) \\ 0 \end{array}\right] + \frac{1}{T}\sum_{t=[Ts]+1}^{T}\left[\begin{array}{c} 0 \\ g(x_t,\beta_2,\delta) \end{array}\right]$$

and $\hat{W}_T(s)$ is a random positive definite symmetric $2q \times 2q$ matrix.

According to the definition above $\hat{\theta}_T(s) = \left(\hat{\beta}_{1T}(s)', \hat{\beta}_{2T}(s)', \hat{\delta}_T(s)'\right)'$ is a $2r+\nu$-vector with an estimator $\hat{\beta}_{1T}(s)$ that uses the first subsample $t = 1, \ldots, [Ts]$, an estimator $\hat{\beta}_{2T}(s)$ that uses the second subsample $t = [Ts]+1, \ldots, T$ and an estimator $\hat{\delta}_T(s)$ that uses all the sample.

The partial-sample optimal weighting matrix is defined as the inverse of $\Omega(s)$ where

$$\Omega(s) = \lim_{T\to\infty} Var\left(\frac{1}{\sqrt{T}}\left[\begin{array}{c} \sum_{t=1}^{[Ts]} g(x_t, \beta_0, \delta_0) \\ \sum_{t=[Ts]+1}^{T} g(x_t, \beta_0, \delta_0) \end{array}\right]\right)$$

which under the null (3) is asymptotically equal to

$$\Omega(s) = \left[\begin{array}{cc} s\Omega & 0 \\ 0 & (1-s)\Omega \end{array}\right].$$

We also define

$$G(s) = \left[\begin{array}{ccc} sG_\beta & 0 & sG_\delta \\ 0 & (1-s)G_\beta & (1-s)G_\delta \end{array}\right] \in R^{2q\times(2r+\nu)}.$$

In the GEL setting, the parameter vector is augmented by a vector of auxiliary parameters γ where each element of this vector is associated to an element of the smoothed moment conditions $g_{tT}(\theta)$. Under the null of no structural change relative to the specification of the model, the generic null hypothesis for this vector of auxiliary parameters can be written as follows:

$$H_0 : \gamma_t = \gamma_0 = 0 \qquad \forall t = 1, \ldots, T. \tag{4}$$

As for the parameter vector β, the tests that we will consider assume as alternative that at some point in the sample there is a single structural break, namely:

$$\gamma_t = \left\{\begin{array}{ll} \gamma_1 & t = 1, ..., [Ts] \\ \gamma_2 & t = [Ts]+1, ..., T. \end{array}\right.$$

Thus under the null of no structural change $H_0: \ \gamma_1 = \gamma_2 = \gamma_0 = 0$. Guay and Lamarche (2007) show that a structural change in γ is associated with instability in the overidentifying restrictions.

We now formally define the GEL estimator using the full sample.

Definition 3.3. *Let $\rho(\phi)$ be a function of a scalar ϕ that is concave on its domain, an open interval Φ that contains 0. Also, let $\tilde{\Gamma}_T(\beta,\delta) = \{\gamma : k\gamma' g_{tT}(\beta,\delta) \in \Phi, t = 1, \ldots, T\}$ with $k = \frac{k_1}{k_2}$. Then, the full-sample GEL estimator $\{\tilde{\theta}_T\}$ is a sequence of random vectors such that:*

$$\left(\tilde{\beta}_T', \tilde{\delta}_T'\right)' = \arg \min_{(\beta,\delta)\in B\times\Delta} \sup_{\gamma\in\tilde{\Gamma}_T(\beta,\delta)} \sum_{t=1}^{T} \frac{\left[\rho\left(k\gamma' g_{tT}(\beta,\delta)\right) - \rho_0\right]}{T}$$

where $\rho_j() = \partial^j \rho()/\partial\phi^j$ and $\rho_j = \rho_j(0)$ for $j = 0, 1, 2, \ldots$ ..

The criteria is normalized so that $\rho_1 = \rho_2 = -1$ (see Smith (2004)). As mentioned earlier, the GEL estimator admits a number of special cases recently proposed in the econometrics literature. The CUE of Hansen, Heaton and Yaron (1996) corresponds to the following quadratic function $\rho(\phi) = -(1+\phi)^2/2$. The EL estimator (Qin and Lawless, 1994) is a GEL estimator with $\rho(\phi) = \ln(1-\phi)$. The ET estimator (Kitamura and Stutzer, 1997) is obtained with $\rho(\phi) = -\exp(\phi)$.

More precisely, the GEL estimator is obtained as the solution to a saddle point problem. Firstly, the criterion is maximized for given vector (β, δ). Thus,

$$\tilde{\gamma}_T(\beta,\delta) = \arg \sup_{\gamma \in \tilde{\Gamma}(\beta,\delta)} \sum_{t=1}^{T} \frac{[\rho(k\gamma' g_{tT}(\beta,\delta)) - \rho_0]}{T}.$$

Secondly, the GEL estimator $\left(\tilde{\beta}_T', \tilde{\delta}_T'\right)'$ is given by the following minimization problem:

$$\left(\tilde{\beta}_T', \tilde{\delta}_T'\right)' = \arg \min_{(\beta,\delta)\in B\times\Delta} \sum_{t=1}^{T} \frac{\left[\rho\left(k\tilde{\gamma}_T(\beta,\delta)' g_{tT}(\beta,\delta)\right) - \rho_0\right]}{T}.$$

From now on, following Kitamura and Stutzer (1997) and Guggenberger and Smith (2007) we focus on the truncated kernel defined by

$$k(x) = 1 \text{ if } |x| \leq 1 \text{ and } k(x) = 0 \text{ otherwise}$$

to obtain the following smoothed moment conditions

$$g_{tT}(\beta,\delta) = \frac{1}{2K_T+1} \sum_{j=-K_T}^{K_T} g(x_{t-j},\beta,\delta).$$

To handle the endpoints in the smoothing we use the approach of Smith (2004) and Guggenberger and Smith (2007) which sets

$$g_{tT}(\beta,\delta) = \frac{1}{2K_T+1} \sum_{j=\max\{t-T,-K_T\}}^{\min\{t-1,K_T\}} g(x_{t-j},\beta,\delta).$$

We can easily show for this kernel that $k = \frac{k_1}{k_2} = 1$. A consistent estimator of the long run covariance matrix is then given by:

$$\tilde{\Omega}_T(\beta,\delta) = \frac{2K_T+1}{T} \sum_{t=1}^{T} g_{tT}(\beta,\delta) g_{tT}(\beta,\delta)'.$$

The weighting matrix thus obtained using this type of kernel is similar to the one obtained with the Bartlett kernel estimator of the long run covariance matrix of the moment conditions (see Smith (2004)). Define also the derivatives of the smoothed moment conditions as:

$$G_{tT}(\beta,\delta) = \frac{1}{2K_T+1} \sum_{j=-K_T}^{K_T} \frac{\partial g(x_{t-j},\beta,\delta)}{\partial(\beta',\delta')}.$$

Now consider a possible breakpoint $[Ts]$. Define the vector of auxiliary parameters $\gamma(s) = (\gamma_1(s)', \gamma_2(s)')'$ where γ_1 is the vector of the auxiliary parameters for the first part of the sample e.g. $t = 1, \ldots, [Ts]$ and γ_2 for the second part of the sample; $t = [Ts] + 1, \ldots, T$. The partial-sample GEL estimators for $s \in S$ based on the first and the second subsamples are formally defined as:

Definition 3.4. *Let $\rho(\phi)$ be a function of a scalar ϕ that is concave on its domain, an open interval Φ that contains 0. Also, let $\hat{\Gamma}_T(\theta, s) = \{\gamma(s) = (\gamma_1(s)', \gamma_2(s)')' : k\gamma(s)' g_{tT}(\theta, s)\}$. A partial-sample General Empirical Likelihood (PS-GEL) estimator $\{\hat{\theta}_T(s)\}$ is a sequence of random vectors such that:*

$$
\begin{aligned}
\hat{\theta}_T(s) &= \arg\min_{\theta \in \Theta} \sup_{\gamma(s) \in \hat{\Gamma}_T(\theta, s)} \sum_{t=1}^{T} \frac{[\rho(k\gamma(s)' g_{tT}(\theta, s)) - \rho_0]}{T} \\
&= \arg\min_{\theta \in \Theta} \sup_{\gamma(s) \in \hat{\Gamma}_T(\theta, s)} \left(\sum_{t=1}^{[Ts]} \frac{[\rho(k\gamma_1' g_{tT}(\beta_1, \delta)) - \rho_0]}{T} + \sum_{t=[Ts]+1}^{T} \frac{[\rho(k\gamma_2' g_{tT}(\beta_2, \delta)) - \rho_0]}{T} \right)
\end{aligned}
$$

for all $s \in S$, where $g_{tT}(\theta, s) = (g_{tT}(\beta_1, \delta)', 0')' \in R^{2q \times 1}$ for $t = 1, \ldots, [Ts]$ and $g_{tT}(\theta, s) = (0', g_{tT}(\beta_2, \delta)')' \in R^{2q \times 1}$ for $t = [Ts] + 1, \ldots, T$ with $\gamma(s) = (\gamma_1(s)', \gamma_2(s)')' \in R^{2q \times 1}$.

To be more precise, the first order conditions corresponding to the Lagrange multiplier γ are obtained from the maximization of the partial-sample GEL criterion for a given β_1, β_2, δ. Thus,

$$
\begin{aligned}
\hat{\gamma}_{1T}(\beta_1, \delta) &= \arg \sup_{\gamma_1 \in \hat{\Gamma}_{1T}(\beta_1, \delta)} \sum_{t=1}^{[Ts]} \frac{[\rho(k\gamma_1(\beta_1, \delta)' g_{tT}(\beta_1, \delta)) - \rho_0]}{T}, \\
\hat{\gamma}_{2T}(\beta_2, \delta) &= \arg \sup_{\gamma_2 \in \hat{\Gamma}_{2T}(\beta_2, \delta)} \sum_{t=[Ts]+1}^{T} \frac{[\rho(k\gamma_2(\beta_2, \delta)' g_{tT}(\beta_2, \delta)) - \rho_0]}{T}.
\end{aligned}
$$

with $\hat{\Gamma}_{1T}(\beta_1, \delta) = \{\gamma_1 : k\gamma_1' g_{tT}(\beta_1, \delta) \in \Phi, t = 1, \ldots, [Ts]\}$ and $\hat{\Gamma}_{2T}(\beta_2, \delta) = \{\gamma_2 : k\gamma_2' g_{tT}(\beta_2, \delta) \in \Phi, t = [Ts] + 1, \ldots, T\}$.

We now present the corresponding implied probabilities defined by Back and Brown (1993) and Smith (2004) for the most commonly used full and partial-sample estimators. Following Back and Brown (1993), the full-sample GMM implied probabilities are defined as:

$$
\pi_t(\tilde{\theta}_T) = \frac{1}{T} - \frac{1}{T} \left[J_{tT}(\tilde{\theta}_T) - J_T(\tilde{\theta}_T) \right]' \hat{\Omega}_T(\tilde{\theta}_T)^{-1} g_T(\tilde{\theta}_T)
$$

with $J_T(\tilde{\theta}_T) = \frac{1}{T} \sum_{t=1}^{T} J_{tT}(\tilde{\theta}_T)$ and

$$
J_{tT}(\tilde{\theta}_T) = \sum_{j=1}^{T} \kappa(|t-j|) g(x_{t-j}, \tilde{\theta}_T)
$$

where $\kappa(|t-j|)$ is a real valued weighting function. In practice, however, some of the estimated probabilities may be negative in finite sample although these probabilities are asymptotically positive. Antoine, Bonnal, and Renault (2007) proposed a shrinkage procedure defined as a weighted average of the standard 2S-GMM's implied probabilities ($1/T$) and the computed implied probabilities to guarantee the non-negativity of these implied probabilities in finite sample.

The general formula of the implied probabilities for the full-sample GEL estimator is defined by the following ratio (see Smith, 2004):

$$\pi_t^{GEL}(\tilde{\theta}_T) = \frac{\rho_1\left(\tilde{\gamma}_T' g_{tT}(\tilde{\theta}_T)\right)}{\sum_{t=1}^{T} \rho_1\left(\tilde{\gamma}_T' g_{tT}(\tilde{\theta}_T)\right)}.$$

Implied probabilities for the full-sample ET, EL and CUE estimators with the smoothed moment conditions are respectively given by:

$$\pi_t^{ET}(\tilde{\theta}_T) = \frac{\exp[\tilde{\gamma}_T{}' g_{tT}(\tilde{\theta}_T)]}{\sum_{t=1}^{T} \exp[\tilde{\gamma}_T' g_{tT}(\tilde{\theta}_T)]},$$

$$\pi_t^{EL}(\tilde{\theta}_T) = \frac{1}{T[1 + \tilde{\gamma}_T' g_{tT}(\tilde{\theta}_T)]},$$

and

$$\pi_t^{CUE}(\tilde{\theta}_T) = \frac{1}{T} - \frac{1}{T} g_{tT}(\tilde{\theta}_T)\left[\frac{1}{T}\sum_{t-1}^{T} g_{tT}(\tilde{\theta}_T) g_{tT}(\tilde{\theta}_T)'\right]^{-1} \frac{1}{T}\sum_{t=1}^{T} g_{tT}(\tilde{\theta}_T).$$

Note that $\frac{2K+1}{T}\sum_{t-1}^{T} g_{tT}(\tilde{\theta}_T) g_{tT}(\tilde{\theta}_T)'$ is a consistent estimator of Ω.

The corresponding unrestricted partial-sample GMM implied probabilities are defined for $s \in S$ as:

$$\pi_t(\hat{\theta}_T(s), s) = \frac{1}{T} - \frac{1}{T}\left[J_{tT}(\hat{\theta}_T(s), s) - J_T(\hat{\theta}_T(s), s)\right]' \hat{\Omega}_T(s)^{-1} g_T(\hat{\theta}_T(s), s)$$

with $J_T(\hat{\theta}_T(s), s) = \frac{1}{T}\sum_{t=1}^{T} J_{tT}(\hat{\theta}_T(s), s)$ and

$$J_{tT}(\hat{\theta}_T(s), s) = \sum_{j=1}^{T} \kappa(|t-j|) g_{t-j}(\hat{\theta}_T(s), s)$$

where $\kappa(|t-j|)$ is a real valued weighting function and $g_T(\hat{\theta}_T(s), s) = \frac{1}{T}\sum_{t=1}^{T} g_t(\hat{\theta}_T(s), s)$. The general formula for the unrestricted partial-sample implied probabilities for the GEL are defined for $s \in S$ as:

$$\pi_t^{GEL}(\hat{\theta}_T(s), s) = \frac{\rho_1\left(\hat{\gamma}_T(s)' g_{tT}(\hat{\theta}_T(s), s)\right)}{\sum_{t=1}^{T} \rho_1\left(\hat{\gamma}_T(s)' g_{tT}(\hat{\theta}_T(s), s)\right)}. \quad (5)$$

For example, in the case of t between observations 1 and $[Ts]$, we get for the unrestricted implied probabilities at t:

$$\pi_t^{GEL}(\hat{\beta}_{1T}(s), \hat{\delta}_T(s), s) = \frac{\rho_1\left(\hat{\gamma}_{1T}(s)' g_{tT}(\hat{\beta}_{1T}(s), \hat{\delta}_T(s))\right)}{\sum_{t=1}^{T} \rho_1\left(\hat{\gamma}_{1T}(s)' g_{tT}(\hat{\beta}_{1T}(s), \hat{\beta}_{2T}(s), \hat{\delta}_T(s), s)\right)}.$$

For the commonly used GEL partial-sample estimators, we get

$$\pi_t^{ET}(\hat{\theta}_T(s), s) = \frac{\exp[\hat{\gamma}_T(s)' g_{tT}(\hat{\theta}_T(s), s)]}{\sum_{t=1}^{T} \exp[\hat{\gamma}_T(s)' g_{tT}(\hat{\theta}_T(s), s)]},$$

$$\pi_t^{EL}(\hat{\theta}_T(s), s) = \frac{1}{T[1 + \hat{\gamma}_T(s)' g_{tT}(\hat{\theta}_T(s), s)]},$$

$$\pi_t^{CUE}(\hat{\theta}_T(s), s) = \frac{1}{T} - \frac{1}{T} g_{tT}(\hat{\theta}_T(s), s) \left[\frac{1}{T}\sum_{t-1}^{T} g_{tT}(\hat{\theta}_T(s), s) g_{tT}(\hat{\theta}_T(s), s)'\right]^{-1} \frac{1}{T}\sum_{t=1}^{T} g_{tT}(\hat{\theta}_T(s), s).$$

The purpose of the next subsection is to refine the null hypothesis of no structural change. Such a refinement will enable us to construct various tests for structural change in the spirit of Sowell (1996a) and Hall and Sen (1999).

3.2 Refining the Null Hypothesis

The moment conditions for the full sample under the null can be written as:

$$Eg(x_t, \theta_0) = 0, \quad \forall t = 1, \ldots, T. \tag{6}$$

Following Sowell (1996b), we can project the moment conditions on the subspace identifying the parameters and the subspace of overidentifying restrictions. In particular, considering the (standardized) moment conditions for the full-sample GMM estimator, such a decomposition corresponds to:

$$\Omega^{-1/2} Eg(x_t, \theta_0) = P_G \Omega^{-1/2} Eg(x_t, \theta_0) + (I_q - P_G)\Omega^{-1/2} Eg(x_t, \theta_0), \tag{7}$$

where $P_G = \Omega^{-1/2} G \left[G'\Omega{-}1G\right]^{-1} G'\Omega^{-1/2}$. The first term is the projection identifying the parameter vector and the second term is the projection for the overidentifying restrictions. The projection argument enables us to refine the null hypothesis (3). For instance, following Hall and Sen (1999) we can consider the null, for the case of a single possible breakpoint s, which separates the identifying restrictions across the two subsamples:

$$H_0^I(s) = \begin{cases} P_G \Omega^{-1/2} E[g(x_t, \theta_0)] = 0 & \forall t = 1, \ldots, [Ts] \\ P_G \Omega^{-1/2} E[g(x_t, \theta_0)] = 0 & \forall t = [Ts] + 1, \ldots, T. \end{cases}$$

Moreover, the overidentifying restrictions are stable if they hold before and after the breakpoint. This is formally stated as $H_0^O(s) = H_0^{O1}(s) \cap H_0^{O2}(s)$ with:

$$\begin{aligned} H_0^{O1}(s) : (I_q - P_G)\Omega^{-1/2} E[g(x_t, \theta_0)] &= 0 \qquad \forall t = 1, \ldots, [Ts] \\ H_0^{O2}(s) : (I_q - P_G)\Omega^{-1/2} E[g(x_t, \theta_0)] &= 0 \qquad \forall t = [Ts] + 1, \ldots, T. \end{aligned}$$

The projection reveals that instability must be a result of a violation of at least one of the three hypotheses: $H_0^I(s), H_0^{O1}(s)$ or $H_0^{O2}(s)$. Various tests can be constructed with local power properties against any particular one of these three null hypotheses (and typically no power against the others). To elaborate further on this we consider a sequence of Pitman local alternatives based on the moment conditions:

Assumption 3.1. *A sequence of local alternatives is specified as:*

$$Eg(x_t, \theta_0) = \frac{h(\eta, \tau, \frac{t}{T})}{\sqrt{T}} \tag{8}$$

where $h(\eta, \tau, s)$, for $r \in [0, 1]$, is a q-dimensional function. The parameter τ locates structural changes as a fraction of the sample size and the vector η defines the local alternatives.[2] These local alternatives are chosen to show that the structural change tests presented in this paper have non trivial power against a large class of alternatives. Also, our asymptotic results can be compared with Sowell's results for the GMM framework.

Accordingly with the decomposition in equation (7) , the sequence of alternatives can be rewritten as:

$$\Omega^{-1/2} Eg(x_t, \theta_0) = P_G \Omega^{-1/2} \frac{h(\eta, \tau, \frac{t}{T})}{\sqrt{T}} + (I_q - P_G)\, \Omega^{-1/2} \frac{h(\eta, \tau, \frac{t}{T})}{\sqrt{T}} \tag{9}$$

where the first component is the local alternative on the identifying moments and the second is the local alternative on the overidentifying restrictions.

For instability in the parameter vector, consider a general local alternative of the form (see Sowell (1996a))

$$\beta_{tT} = \beta_0 + \frac{f(\eta, \tau, \frac{t}{T})}{\sqrt{T}}$$

for $t = 1, \ldots, T$. A Taylor expansion of $g_{tT}(x_t, \theta_{tT})$ yields

$$Eg(x_t, \beta_0) = -G_\beta \frac{f(\eta, \tau, \frac{t}{T})}{\sqrt{T}} + o_p(1)$$

and by substituting this expression into (9) this shows that the expression above is orthogonal to the second component of (9) and puts restrictions on the first component (the identifying restrictions). In the case of pure structural $P_G = P_{G_\beta} = \Omega^{-1/2} G_\beta \left[G'_\beta \Omega^{-1} G_\beta \right]^{-1} G'_\beta \Omega^{-1/2}$. The alternative that at some point there is a single structural break at τ, $H_A^I(\tau)$, is represented as:

$$\beta_{tT} = \begin{cases} \beta_0 & t = 1, ..., [T\tau] \\ \beta_0 + \frac{\eta}{\sqrt{T}} & t = [T\tau] + 1, ..., T \end{cases}$$

which corresponds to a specific form for $f(\eta, \tau, \frac{t}{T})$. Note that the true structural change breakpoint τ is allowed to differ than the possible breakpoint s chosen by the researcher.

[2]The function $h(\cdot)$ allows for a wide range of alternative hypotheses (see Sowell (1996b)). In its generic form it can be expressed as the uniform limit of step functions, $\eta \in R^i$, $\tau \in R^j$ such that $0 < \tau_1 < \tau_2 < \ldots < \tau_j < 1$ and θ^* is in the interior of Θ. Therefore it can accommodate multiple breaks.

For instability of overidentifying restrictions at a single breakpoint τ occurring before and/or after the breakpoint, this is formally stated as $H_A^O(\tau) = H_A^{O1}(\tau) \cap H_A^{O2}(\tau)$ with:

$$\begin{aligned} H_A^{O1}(\tau) : (I_q - P_G)\Omega^{-1/2}E[g(x_t, \theta_0)] &= \frac{\eta_1}{\sqrt{T}} \qquad \forall t = 1, \ldots, [T\tau] \\ H_A^{O2}(\tau) : (I_q - P_G)\Omega^{-1/2}E[g(x_t, \theta_0)] &= \frac{\eta_2}{\sqrt{T}} \qquad \forall t = [T\tau] + 1, \ldots, T. \end{aligned}$$

and $\eta_1 \neq \eta_2$.[3] This formulation of the alternative for a single breakpoint corresponds to a specific form of $h(\eta, \tau, \frac{t}{T})$ in (9).

4 Tests for a structural change based on implied probabilities

Ramalho and Smith (2005) introduced in i.i.d. setting a specification test for moment conditions based on implied probabilities similar in spirit to the classical Pearson Chi-Square goodness-of-fit test. The test is based on the following statistic:

$$\sum_{t=1}^{T} \left(T\pi_t^{GEL}(\tilde{\theta}_T) - 1 \right)^2 .$$

They showed that such statistic is asymptotically equivalent to the overidentifying moment restrictions J-test proposed by Hansen (1982). Guay and Pelgrin (2007) and Guggenberger, Ramalho and Smith (2007) also used this statistic in the time series context and showed that:

$$\frac{1}{2K+1} \sum_{t=1}^{T} \left(T\pi_t(\tilde{\theta}_T) - 1 \right)^2 . \tag{10}$$

is asymptotically first order equivalent to the overidentifying moment restrictions J-test. However, as shown by Ghysels and Hall (1990), the J-test has no power to detect structural change in parameter values, a property that is shared by the specification tests above proposed by those authors as we demonstrate below.

In the same spirit, we first consider a statistic test based on the partial-sample implied probabilities evaluated at the restricted estimator for GEL. The implied probabilities structural change (IPSC) test statistic proposed to detect instability is given by the following partial sum:

$$IPSC_T^{GEL}(s) = \frac{s}{2K_T + 1} \sum_{t=1}^{[Ts]} \left(T\pi_t^{GEL}(\tilde{\theta}_T, s) - 1 \right)^2 \tag{11}$$

with

$$\pi_t^{GEL}(\tilde{\theta}_T, s) = \frac{\rho_1\left(\tilde{\gamma}_{1T}(s)' g_{tT}(\widetilde{\theta}_T, s)\right)}{\sum_{t=1}^{T} \rho_1\left(\tilde{\gamma}_T(s)' g_{tT}(\widetilde{\theta}_T, s)\right)}.$$

[3] This specification allows for the overidentifying restrictions to be violated just after the breakpoint ($\eta_1 = 0$ and $\eta_2 \neq 0$), just before the breakpoint ($\eta_1 \neq 0$ and $\eta_2 = 0$) or both ($\eta_1 \neq 0$, $\eta_2 \neq 0$ and $\eta_1 \neq \eta 2$).

where for the numerator $\tilde{\gamma}_{1T}(s)$ is the solution of the following maximization problem:

$$\gamma_{1T}(\beta,\delta) \quad = \quad \arg \sup_{\gamma_1 \in \hat{\Gamma}_T(\beta,\delta)} \sum_{t=1}^{[Ts]} \frac{[\rho(\gamma_1(\beta,\delta)' g_{tT}(\beta,\delta)) - \rho_0]}{T} \tag{12}$$

evaluated at the restricted estimator $\tilde{\theta}_T$ and for the denominator $\tilde{\gamma}_T(s) = (\tilde{\gamma}_{1T}(s)', \tilde{\gamma}_{2T}(s)')'$ with $\tilde{\gamma}_{1T}(s)$ defined as above and

$$\gamma_{2T}(\beta,\delta) \quad = \quad \arg \sup_{\gamma_2 \in \hat{\Gamma}_T(\beta,\delta)} \sum_{t=[Ts]+1}^{T} \frac{[\rho(\gamma_2(\beta,\delta)' g_{tT}(\beta,\delta)) - \rho_0]}{T}. \tag{13}$$

It is crucial to note that this statistic is based on the **unrestricted** implied probabilities evaluated at the **restricted** estimator of θ.

The next theorem establishes the asymptotic distribution for this general test of a structural change under the null and the sequence of alternatives defined in (8).

Theorem 4.1. *Under Assumptions (7.1) to (7.10), the following processes indexed by s for $s \in [0,1]$ satisfy, under the null (6),*

$$IPSC_T^{GEL}(s) \Rightarrow BB_p(s)'BB_p(s) + B'_{(q-p)}(s)B_{(q-p)}(s)$$

and under the alternative (8)

$$\begin{aligned} IPSC_T^{GEL}(s) \quad \Rightarrow \quad & BB_p(s)'BB_p(s) + (H(s) - sH(1))'\,\Omega^{-1/2}P_G\Omega^{-1/2}\,(H(s) - sH(1)) \\ & + B_{q-p}(s)'B_{q-p}(s) + H(s)'\Omega^{-1/2}\,(I - P_G)\,\Omega^{-1/2}H(s), \end{aligned}$$

where $B_{(q-p)}(s)$ is a $(q-p)$-vector of standard Brownian motion, $BB_p(s) = B_p(s) - sB_p(1)$ is a p-vector of Brownian bridge with $p = r + \nu$ and $H(s) = \int_0^s h(\eta,\tau,r)dr$.

Proof: See the Appendix.

The Theorem shows that the structural change test based on this quadratic form of the partial-sample sum of the implied probabilities evaluated at the full-sample estimator combines two components. The first component of the limiting distribution is a function of the Brownian bridges corresponds to a parameter stability tests for the whole set of parameters (β and δ) and the second component to a stability of overidentifying restrictions. This test statistic, based on implied probabilities, can be viewed as a more general form of misspecification due to instability than just a test for parameter variation. The predictive tests proposed by Ghysels, Guay and Hall (1997) shares the same properties. In the Appendix we show that the $IPSC$ test statistic is asymptotically equivalent to the test statistic proposed by Sowell (1996b). He showed that his test statistic is optimal for a one time jump in all moment conditions where the location of the jump is unknown and consistent for arbitrary alternatives. These properties are then shared by our test. Note that the limiting distribution exists for $s = 0$ which is trivially equal to 0. For $s = 1$, the test statistic corresponds to the specification test for moment conditions developed by Guay and Pelgrin (2007) and Guggenberger *et al.* (2007) and by the above theorem the limiting distribution is given by:

$$B_{q-p}(1)' B_{q-p}(1) + H(1)' \Omega^{-1/2} \left(I - P_G\right) \Omega^{-1/2} H(1).$$

This limiting distribution shows that this test statistic has a chi-square distribution with $q-p$ degrees of freedom under the null and that has local power equal to the size to detect instability in parameter values as the J-test proposed by Hansen (1982). Moreover, the test statistic (10) can not detect asymptotically instability in the overidentifying restrictions for which $\left(I - P_G\right) \Omega^{-1/2} H(1) = 0$.

When the breakpoint is unknown, one can construct statistics across $s \in S$. In the context of maximum likelihood estimation, Andrews and Ploberger (1994) derive asymptotic optimal tests for a gaussian a priori of the amplitude of the structural change based on the Neyman-Pearson approach which are characterized by an average exponential form. The Sowell (1996a) optimal tests are a generalization of the Andrews and Ploberger approach to the case of two measures that do not admit densities. The most powerful test is given by the Radon-Nikodym derivative of the probability measure implied by the local alternative with respect to the probability measure implied by the null hypothesis.

The optimal average exponential form applied to a statistic $Q_T(s)$ for $s \in S$ has the following form:

$$Exp - Q_T = (1+c)^{-q/2} \int \exp\left(\frac{1}{2}\frac{c}{1+c} Q_T(s)\right) dJ(s),$$

where various choices of c determine power against close or more distant alternatives and $J(\cdot)$ is the weight function over the value of $s \in S$. In the case of close alternatives ($c = 0$), the optimal test statistic takes the average form, $aveQ_T = \int_S Q_T(s) dJ(s)$. For a distant alternative ($c = \infty$), the optimal test statistics takes the exponential form, $expQ_T = log\left(\int_S \exp[\frac{1}{2} Q_T(s)] dJ(s)\right)$. The supremum form often used in the literature corresponds to the case where $\frac{c}{(1+c)} \rightarrow \infty$. The sup test is given by $\sup Q_T = \sup_{s \in S} Q_T(s)$.

The following Theorem gives the asymptotic distribution for the average exponential mapping for $Q_T^{IPSC}(s)$ where $Q_T^{IPSC}(s)$ corresponds to the statistic presented above based on the implied probabilities.

Theorem 4.2. *Under the null hypothesis H_0 in (6) and Assumptions 7.1 to 7.10, the following processes indexed by s for a given set S whose closure lies in [0,1] satisfy:*

$$\sup Q_T^{IPSC} \Rightarrow \sup_{s \in S} Q_{p,q-p}(s), \quad aveQ_T^{IPSC} \Rightarrow \int_S Q_{p,q-p}(s) dJ(s), \quad expQ_T^{IPSC} \Rightarrow log\left(\int_S \exp[\frac{1}{2} Q_{p,q-p}(s)] dJ(s)\right),$$

with

$$Q_{p,q-p}(s) = BB_p(s)' BB_p(s) + B_{(q-p)}(s)' B_{(q-p)}(s)$$

and $J(s)$ is the weighting distribution function for the location of the instability s.

This result is obtained through the application of the continuous mapping theorem (see Pollard (1984)). The asymptotic critical values where obtained using simulated Brownian motions and Brownian bridges over 10,000 replications for maximum values of p and $q-p$ set at 10. The critical values appear in Tables 1 to 3 for symmetric intervals $S=[s_0, 1-s_0]$. Critical values for the entire sample appear at $s_0=0$.

An asymptotically equivalent modified statistic to (11) in the spirit of the Neyman-modified chi-square is given by:

$$IPSCM_T^{GEL}(s) = \frac{s}{2K_T+1} \sum_{t=1}^{[Ts]} \frac{\left(T\pi_t^{GEL}(\tilde{\theta}_T, s) - 1\right)^2}{T\pi_t^{GEL}(\tilde{\theta}_T, s)}$$

since $T\pi_t^{GEL}(\tilde{\theta}_T, s) = 1 + o_p(1)$ under the null.

In the sequel, we propose structural change tests based on implied probabilities specially design to detect instability in the parameters of interest or in the overidentifying restrictions.

4.1 Tests for a structural change in the parameters based on implied probabilities

The test statistics proposed to specifically detect parameter instability are based on the difference between the partial sum of unrestricted implied probabilities evaluated at the unrestricted estimator $\hat{\theta}_T(s)$ with the corresponding partial sum of unrestricted implied probability but at the restricted estimator $\tilde{\theta}_T$. More precisely the test statistic is defined as:

$$IPSC_T^{I,GEL}(s) = \frac{1}{2K_T+1} \sum_{t=1}^{T} \left(T\pi_t^{GEL}(\hat{\theta}_T(s), s) - T\pi_t^{GEL}(\tilde{\theta}_T, s)\right)^2$$

with

$$\pi_t^{GEL}(\tilde{\theta}_T, s) = \frac{\rho_1\left(\tilde{\gamma}_T(s)' g_{tT}(\tilde{\theta}_T, s)\right)}{\sum_{t=1}^{T} \rho_1\left(\tilde{\gamma}_T(s)' g_{tT}(\tilde{\theta}_T, s)\right)}.$$

where $\tilde{\gamma}_T(s) = (\tilde{\gamma}_{1T}(s)'\tilde{\gamma}_2(s)')'$ is the solution of the respective maximization problems defined in equations (12) and (13) and $\pi_t^{GEL}(\hat{\theta}_T(s), s)$ is defined in equation (5).

The next theorem establishes the asymptotic distribution for this test of a structural change in the parameter values under the null that the vector β is constant throughout the sample.

Theorem 4.3. *Under the null hypothesis H_0 in (3) and Assumptions (7.1) to (7.10), the following processes indexed by s for a given set S whose closure lies in $(0,1)$ satisfy:*

$$IPSC_T^{I,GEL}(s) \Rightarrow Q_r(s) = \frac{BB_r(s)'BB_r(s)}{s(1-s)}$$

and under the alternative (8)

$$Q_r(s) = \frac{BB_r(s)'BB_r(s)}{s(1-s)} + \frac{\left(H(s) - sH(1)\right)' \Omega^{-1/2} P_{G_\beta} \Omega^{-1/2} \left(H(s) - sH(1)\right)}{s(1-s)},$$

where $BB_r(s) = B_r(s) - sB_r(1)$ is a Brownian bridge, B_r is r-vector of independent Brownian motions and $P_{G_\beta} = \Omega^{-1/2} G_\beta \left[(G_\beta)' \Omega^{-1} G_\beta\right]^{-1} (G_\beta)' \Omega^{-1/2}$. Moreover,

$$\sup IPSC_T^{I,GEL} \Rightarrow \sup_{s \in S} Q_r(s), \quad aveIPSC_T^{I,GEL} \Rightarrow \int_S Q_r(s) dJ(s),$$
$$expIPSC_T^{I,GEL} \Rightarrow log\left(\int_S \exp[\frac{1}{2} Q_r(s)] dJ(s)\right).$$

Proof: See the Appendix.

The Theorem shows that the asymptotic distribution of the test based on implied probabilities is asymptotically equivalent under the null and the alternative to the $Wald$, LM and LR tests for parameter instability (see Andrews 1993). More precisely, the limiting distribution is function of a r-vector of Brownian bridge with the same dimension than the parameter vector β. However, the small sample properties can differ compared to those more standard tests. Note that, in contrast to the preceding Theorem, the limiting distribution in Theorem 4.3 is valid only for S in the open interval $(0, 1)$.

The two following modified statistics are asymptotically equivalent to the one defined above:

$$IPSCM1_T^{I,GEL}(s) = \frac{1}{2K_T + 1} \sum_{t=1}^{T} \frac{\left(T\pi_t^{GEL}(\hat{\theta}_T(s), s) - T\pi_t^{GEL}(\tilde{\theta}_T, s)\right)^2}{T\pi_t^{GEL}(\hat{\theta}_T(s), s)}$$

and

$$IPSCM2_T^{I,GEL}(s) = \frac{1}{2K_T + 1} \sum_{t=1}^{T} \frac{\left(T\pi_t^{GEL}(\hat{\theta}_T(s), s) - T\pi_t^{GEL}(\tilde{\theta}_T, s)\right)^2}{T\pi_t^{GEL}(\tilde{\theta}_T, s)}.$$

4.2 Tests for a structural change for overidentifying restrictions based on implied probabilities

Now, we propose a test statistic designed specially to detect instability in the overidentifying restrictions based on implied probabilities. The statistic is powerful against violation of $H_0^{O1}(s)$ and $H_0^{O2}(s)$. The statistic is asymptotically equivalent to the ones proposed by Hall and Sen (1999) and thus shares its asymptotic properties. As previously, the sample is split in two subamples with a single breakpoint at $[Ts]$. An estimator of the parameter vector is obtained with the first subsample (for $t = 1, \ldots, [Ts]$) and with the second subsample (for $t = [Ts] + 1, \ldots, T$). The entire parameter vector is allowed to vary for both subsamples. The proposed statistic specifically designed to detect instability for overidentifying restrictions is based on the specification test statistic for moment conditions given in equation (10) for the first and the second subsamples.

More precisely the statistic is defined as:

$$IPSC_T^{O,GEL}(s) = \frac{1}{2K_T+1}\sum_{t=1}^{[Ts]}\Big([Ts]\pi_t^{GEL}(\hat{\theta}_{1T}(s),s)-1\Big)^2 \tag{14}$$

$$+\frac{1}{2K_T+1}\sum_{t=[Ts]+1}^{T}\Big((T-[Ts])\pi_t^{GEL}(\hat{\theta}_{2T}(s),s)-1\Big)^2 \tag{15}$$

where $\hat{\theta}_{1T}(s)=\hat{\beta}_{1T}(s)$ and $\hat{\theta}_{2T}(s)=\hat{\beta}_{2T}(s)$. The statistic is the sum of the overidentifying restrictions statistics (10) but for the first and the second parts of the sample evaluated at the unrestricted estimator.

The next Theorem establishes the asymptotic distribution of this statistic and the corresponding average mappings.

Theorem 4.4. *Under Assumptions (7.1) to (7.10), the following processes indexed by s for a given set S whose closure lies in* $(0,1)$ *satisfy*

$$IPSC_T^{O,GEL}(s) \Rightarrow Q_{q-r}(s)$$

with under the null of no structural change

$$Q_{q-r}(s) \Rightarrow \frac{B_{q-r}(s)'B_{q-r}(s)}{s} + \frac{[B_{q-r}(1)-B_{q-r}(s)]'[B_{q-r}(1)-B_{q-r}(s)]}{1-s}$$

and under the alternative (8)

$$\begin{aligned} Q_{q-r}(s) \quad \Rightarrow \quad & \frac{B_{q-r}(s)'B_{q-r}(s)}{s} + \frac{H(s)'\Omega^{-1/2}(I-P_G)\,\Omega^{-1/2}H(s)}{s} \\ & \frac{[B_{q-r}(1)-B_{q-r}(s)]'[B_{q-r}(1)-B_{q-r}(s)]}{1-s} + \frac{[H(1)-H(s)]'\,\Omega^{-1/2}(I-P_G)\,\Omega^{-1/2}[H(1)-H(s)]}{(1-s)}, \end{aligned}$$

where $B_{q-r}(s)$ *is a* $q-r$*-vector of standard Brownian motion and* $P_G=P_{G_\beta}$ *with* $P_{G_\beta}=\Omega^{-1/2}G_\beta\left(G_\beta'\Omega^{-1}G_\beta\right)^{-1}G'\Omega^{-1/2}$. *Moreover,*

$$\sup IPSC_T^{O,GEL} \Rightarrow \sup_{s\in S} Q_{q-r}(s), \quad aveIPSC_T^{O,GEL} \Rightarrow \int_S Q_{q-r}(s)dJ(s),$$

$$expIPSC_T^{O,GEL} \Rightarrow log\left(\int_S \exp[\frac{1}{2}Q_{q-r}(s)]dJ(s)\right).$$

Proof: See the Appendix.

The Theorem shows that the proposed test statistics can detect instability occurring in overidentifying restrictions before and after the breakpoint. Indeed, the term $\frac{H(s)'\Omega^{-1/2}(I-P_G)\Omega^{-1/2}H(s)}{s}$ corresponds to a structural change in the moment conditions before the breakpoint s while the term $\frac{[H(1)-H(s)]'\Omega^{-1/2}(I-P_G)\Omega^{-1/2}[H(1)-H(s)]}{(1-s)}$ to a structural change after the breakpoint s.

The asymptotic critical values for the interval $S=[.15,.85]$ can be found in Hall and Sen (1999). For other symmetric interval $[s_0,1-s_0]$, critical values can be obtained in Guay (2003), Tables 1 to 3 for a number of overidentifying restrictions divided by 2 (in those Tables). To see this, note that the critical values for the supremum, the average and the log exponential mappings applied to $\frac{B_{2q-2r}(s)'B_{2q-2r}(s)}{s}$ are equivalent to ones corresponding to $\frac{B_{q-r}(s)'B_{q-r}(s)}{s} + \frac{(B_{q-r}(1)-B_{q-r}(s))'(B_{q-r}(1)-B_{q-r}(s))}{1-s}$ for a symmetric interval S.[4]

An asymptotic equivalent statistic to (14) in the spirit of the Neyman-modified chi-square is given by:

$$
\begin{aligned}
IPSCM_T^{O,GEL}(s) \quad &= \quad \frac{1}{2K_T+1}\sum_{t=1}^{[Ts]} \frac{\left([Ts]\pi_t^{GEL}(\hat{\theta}_{1T}(s),s)-1\right)^2}{[Ts]\pi_t^{GEL}(\hat{\theta}_{1T}(s),s)} \\
&\quad + \frac{1}{2K_T+1}\sum_{t=[Ts]+1}^{T} \frac{\left((T-[Ts])\pi_t^{GEL}(\hat{\theta}_{2T}(s),s)-1\right)^2}{(T-[Ts])\pi_t^{GEL}(\hat{\theta}_{2T}(s),s)}.
\end{aligned}
$$

5 Simulation Evidence

To evaluate the performance of the test statistics we use the data generating process in Ghysels *et al.* (1997) and in Hall and Sen (1999). The time series model used is an AR(1) process for the variable x_t. One parameter is estimated, the autoregressive parameter (denoted by θ in the expression below), using two moment conditions formed with the lagged values of x_t.

The data generating process is given by

$$x_t = \theta_i x_{t-1} + u_t \tag{16}$$

for $t = 1, \ldots, T$. Structural change in the identifying restrictions (in the parameter) is studied by considering different values of θ_i where the index $i = 1, 2$ denotes the first or second subsamples. Structural stability in the overidentifying restrictions is studied by allowing for an ARMA(1,2) model

$$x_t = \theta_i x_{t-1} + u_t + \alpha u_{t-2} \tag{17}$$

and considering nonzero values of α in the second subsample. The change is set at $T/2$. In the above, $u_t \sim N(0,1)$. The sample size was set to 200 observations and the number of Monte Carlo replications was limited to 500 since the time required to estimate by exponential tilting, the method used in these simulations experiments.

Table 7 summarizes the different parametrization and is taken from Hall and Sen (1999). The null hypothesis of structural stability is denoted by H_0^{SS} (DGP 1 to 3). For those DGPs we vary the magnitude of the autoregressive parameter θ. The alternatives of instability in the parameters or in the overidentifying restrictions are denoted by H_A^I (DGP 4 to 6), where we vary the magnitude of the change in the autoregressive parameter, and H_A^O (DGP 7 to 10) where we consider various values of the moving average

[4] This is verified by comparing the critical values in Hall and Sen (1999) and Guay (2003). The critical values in Table 1 in Hall and Sen for $q - r$ in our notation are the same than the critical values in Guay (2003) but for $2q - 2r$.

parameter, respectively. In this situation only one parameter is estimated using two moment conditions created with the first two lags of x_t. Under H_0^{SS}, where $\alpha = 0$, the instruments are appropriate. Under the first class of alternative hypothesis (H_A^I) the two instruments are also valid while they no longer are for the second part of the sample with the second class of alternative hypothesis (H_A^O).

Smoothing the moment conditions is done via an appropriate choice of K_T. In a GMM setting this is equivalent to using some form of estimate of the long run covariance matrix of the moment conditions (for example using the Newey-West estimator as the weight matrix in quadratic form). Most of the previous simulation work considered a fixed degree of smoothing (see for example Gregory *et al.* (2002) and Guggenberger and Smith (2007)). Otsu (2006) did also look at fixed smoothing but also looked at applying the automatic bandwidth selection rule of Newey and West (1994).

Our Monte Carlo study is not specially designed to investigate smoothing because under the null hypothesis the optimal value of K_T is 0 while it is not 0 under the alternative hypothesis H_A^O. For this reason we set $K_T = 0$ for all DGPs except DGPs 7, 8, 9 and 10. For these DGPs we select a bandwidth parameter chosen via the automatic, data-driven, procedure which chooses a bandwidth m_T which is then transformed as $K_T = [(m_T - 1)/2]$. The average, taken over Monte Carlo replications, K_T was found to vary between 1.6 and 2.3, increasing with the moving average component. A complete analysis of the effects of smoothing is left for future work. Lastly, a trimming rule of 0.15 was used, namely $S = [.15, .85]$.

Table 5 contains the results for the general specification tests $IPSC$ and $IPSCM$ (the supremum, exponential and average version of the test statistics are presented). Table 6 contains the rejection frequencies for the test statistics designed to have power against a structural change in the parameters while Table 7 presents the results for test statistics which are designed to have power against a structural change in the overidentifying restrictions. All the test statistics were all computed in the GEL setting. The tests used for comparison appear in Guay and Lamarche (2008). For completeness we report them also here:

$$Wald_T(s) = T\left(\hat{\beta}_{1T}(s) - \hat{\beta}_{2T}(s)\right)' (\hat{V}_\Omega(s))^{-1}\left(\hat{\beta}_{1T}(s) - \hat{\beta}_{2T}(s)\right),$$

where $\hat{V}_\Omega(s) = \left(\hat{V}_1(s)/s + \hat{V}_2(s)/(1-s)\right)$ and $\hat{V}_i(s) = \left(\hat{G}^\beta_{i,tT}(s)'\hat{\Omega}^{-1}_{i,T}(s)\hat{G}^\beta_i(s)\right)^{-1}$ for $i = 1, 2$ corresponding to the first and the second part of the sample. For the first part of the sample:

$$\begin{aligned} \hat{G}^\beta_{1,tT}(s) &= \frac{1}{[Ts]}\sum_{t=1}^{[Ts]} \frac{\partial g_{tT}(\hat{\beta}_1(s), \hat{\delta}(s))}{\partial \beta_1'} \\ \widehat{\Omega}_{1T}(s) &= \frac{2K_T + 1}{[Ts]}\sum_{t=1}^{[Ts]} g_{tT}(\beta_1(s), \delta) g_{tT}(\hat{\beta}_1(s), \hat{\delta}(s))' \end{aligned}$$

and for the second part of the sample:

$$\begin{aligned}
\widehat{G}^{\beta}_{2,tT} &= \frac{1}{T-[Ts]}\sum_{t=[Ts]+1}^{T}\frac{\partial g_{tT}(\hat{\beta}_2(s),\hat{\delta}(s))}{\partial \beta_2'} \\
\widehat{\Omega}_{2T}(s) &= \frac{2K_T+1}{T-[Ts]}\sum_{t=[Ts]+1}^{T} g_{tT}(\hat{\beta}_2(s),\hat{\delta}(s))g_{tT}(\hat{\beta}_2(s),\hat{\delta}(s))'.
\end{aligned}$$

The Lagrange Multiplier statistic is given by:

$$LM_T(s) = \frac{T}{s(1-s)} g_{1T}(\tilde{\theta}_T,s)'\hat{\Omega}_T^{-1}\hat{G}^{\beta}_{tT}\left[(\hat{G}^{\beta}_{tT})'\hat{\Omega}_T^{-1}\hat{G}^{\beta}_{tT}\right]^{-1}(\hat{G}^{\beta}_{tT})'\hat{\Omega}_T^{-1}g_{1T}(\tilde{\theta}_T,s).$$

where

$$\begin{aligned}
g_{1T}(\tilde{\theta}_T,s) &= \frac{1}{T}\sum_{t=1}^{[Ts]} g_{tT}(\tilde{\theta}_T), \\
\hat{G}^{\beta}_{tT} &= \frac{1}{T}\sum_{t=1}^{T}\frac{\partial g_{tT}(\tilde{\beta},\tilde{\delta})}{\partial \beta'}, \\
\widehat{\Omega}_T &= \frac{2K_T+1}{T}\sum_{t=1}^{T} g_{tT}(\tilde{\beta},\tilde{\delta})g_{tT}(\tilde{\beta},\tilde{\delta})'.
\end{aligned}$$

The LR-like statistic is defined as:

$$LR_T(s) = \frac{2T}{2K+1}\left[\sum_{t=1}^{T}\frac{\left[\rho(\tilde{\gamma}(s)'g_{tT}(\tilde{\theta},s))-\rho_0\right]}{T} - \sum_{t=1}^{T}\frac{\left[\rho(\hat{\gamma}(s)'g_{tT}(\hat{\theta}(s),s))-\rho_0\right]}{T}\right]$$

An finally, the test statistic proposed by Hall and Sen (1999) is

$$O_T(s) = O1_T(s) + O2_T(s)$$

where

$$O1_T(s) = \left[\frac{1}{\sqrt{[Ts]}}\sum_{t=1}^{[Ts]} g_{tT}(\hat{\beta}_1(s),\hat{\delta}(s))\right]'\hat{\Omega}_{1T}^{-1}(s)\left[\frac{1}{\sqrt{[Ts]}}\sum_{t=1}^{[Ts]} g_{tT}(\hat{\beta}_1(s),\hat{\delta}(s))\right]$$

and

$$O2_T(s) = \left[\frac{1}{\sqrt{(T-[Ts])}}\sum_{t=[Ts]+1}^{[T]} g_{tT}(\hat{\beta}_2(s),\hat{\delta}(s))\right]'\hat{\Omega}_{2T}^{-1}(s)\left[\frac{1}{\sqrt{(T-[Ts])}}\sum_{t=[Ts]+1} g_{tT}(\hat{\beta}_2(s),\hat{\delta}(s))\right]$$

Focusing first on size we find that the modified tests ($IPSCM1^I$, $IPSCM2^I$ and $IPSCM^O$) based on implied probabilities have rejection frequencies that are much too large. The intuition for this is that the modified test statistics contain a more volatile term in the denominator which can inflate the value of the tests and hence increase the rejection frequencies. So for brevity, we don't report those results here.[5]

[5]Obviously, the results can be obtained upon request.

On the other hand, all other tests have rejection frequencies that are quite good, with some overrejection except for the O test statistics which slightly underreject . The sup$IPSC^I$ has a nominal size close to the true one while the average and exponential forms for this statistic slightly overreject, a property shared by the $Wald$ and LR tests. The LM test statistics (supremum, average and exponential mappings) significantly underreject for all DGPs (1 to 3) under the null and have poor power for other DGPs. For these reasons, the rejection frequencies for the LM test statistics are not reported in the tables here.

The study of power is divided into two cases. In case 1 structural change occurs in the parameter values while in case 2 structural change occurs in the overidentifying restrictions. Under the alternative of instability in the parameter, H_A^I (DGP 4 to 6), we see that the newly proposed test statistics based on implied probabilities have good rejection frequencies. The power of these test statistics is equal or larger than the power of the standard $Wald$ and LR tests. The $IPSC^O$ and O test statistics, geared toward instability in the overidentifying restrictions, have not useful power while the more general specification test, $IPSC$, has some reasonable power.

Under the alternative of instability in the overidentifying restrictions, e.g. H_A^O, (DGP 7 to 10), we see that the test statistics specially designed to detect a change in the parameter $IPSC^I$, and the standard $Wald$ and LR tests have little power while the targeted $IPSC^O$ and O tests have very good power. Importantly the tests based on implied probabilities are seen to have significantly higher power than O tests for all cases. In some cases, the gain in power can be twice as important. As expected, the general specification tests based on implied probabilities have rejection frequencies that fall between those of $IPSC^O$ and $IPSC^I$.

The increase in the autoregressive coefficient from 0 to 0.8 does not impact greatly on the rejection frequencies under the null hypothesis but under the alternative hypotheses the magnitude of the change is important. Under H_A^I, for example, we see that power is close to unity when the change in the autoregressive parameter is quite extreme (0 to 0.8). Under H_A^O, which captures a change in the overidentifying restrictions, an increase (in absolute terms) in the moving average coefficient increases power.

6 Conclusion

As noted by Back and Brown (1993) implied probabilities obtained from estimation of models using estimating equations could be used as an additional tool for model specification in the researcher's tool kit. An important specification test that has received considerable attention in the econometric literature has been a test for structural stability of either the underlying key parameters composing the estimation equations and or the stability of the additional equations (the overidentifying restrictions) that are often used in estimation. In this paper we have focused on the class of estimators based on the Generalized Empirical Likelihood approach. This approach is appealing intuitively because it requires the search for a vector of weights, one for each observation, that yields the most probable data distribution. We view these weights as potentially containing information on the content of the moment conditions (the estimating equations). In a pure entropy setting with no estimating equations the vector of weights is found to be maximally uninformative. That is we obtain weights fluctuating around $1/T$ where T is the sample size. With constrains, the weights are chosen to be as maximally uninformative as the constraints will allow.

In this sense the weights make use of the available information contained in the sample. In particular we use the weights to detect an unknown structural change. We suggest three types of testing procedures for the detection of a structural change each based on different measures of the discrepancy between the estimated weights and the unconstrained weights $\frac{1}{T}$. Specifically, we propose general structural change tests to detect instability both in the identifying restrictions and in the overidentifying restrictions, instability in the identifying restrictions and finally instability in the overidentifying restrictions. This type of classification is similar to the one found in classical papers on structural change in time series as proposed by Andrews (1993) and Hall and Sen (1999) for example. We found that tests based on these implied probabilities have good finite sample size and power properties. An issue that was not investigated in length in this paper was the impact of smoothing (to take into account serial dependence) on the performance of the tests. This interesting avenue of research is left for future work. The important question of structural change tests for GEL robust to weak identification is also left for future investigation.

7 Appendix

7.1 Assumptions

Assumption 7.1. *The process* $\{x_t\}_{t=1}^{\infty}$ *is a finite dimensional stationary and strong mixing coefficients* $\sum_{j=1}^{\infty} \alpha(j)^{(\nu-1)/\nu} < \infty$ *for some* $\nu > 1$.

Assumption 7.2. ***(a)*** $(\beta_0, \delta_0) \in B \times \Delta$ *is the unique solution to* $E[g(x_t, \beta, \delta)] = 0$ *where* $B \times \Delta$ *is compact,* $g(x_t, \beta, \delta)$ *is continuous at each* $(\beta, \delta) \in B \times \Delta$ *with probability one;* ***(b)*** $E\left[\sup_{(\beta,\delta)\in B\times\Delta} \|g(x_t, \beta, \delta)\|^{\alpha}\right] < \infty$ *for some* $\alpha > \max\left(4\nu, \frac{1}{\eta}\right)$; ***(c)*** $\Omega(\beta, \delta)$ *is finite and positive definite for all* $(\beta, \delta) \in B \times \Delta$.

Define the smoothed moment conditions as:

$$g_{tT}(\beta, \delta) = \frac{1}{S} \sum_{s=t-T}^{t-1} k\left(\frac{s}{S_T}\right) g_{t-s}(x_t, \beta, \delta)$$

for an appropriate kernel. From now on, we consider the uniform kernel proposed by Kitamura and Stutzer (1997):

$$g_{tT}(\beta, \delta) = \frac{1}{2K_T + 1} \sum_{s=-K_T}^{K_T} g_{t-s}(x_t, \beta, \delta)$$

Assumption 7.3. $K_T^2/T \to 0$ *and* $K_T \to \infty$ *as* $T \to \infty$.

Assumption 7.4. ***(a)*** $\rho(\cdot)$ *is twice continuously differential and concave on its domain, an open interval* Φ *containing 0,* $\rho_1 = \rho_2 = -1$; ***(b)*** $\gamma \in \Gamma$ *where* $\Gamma = \{\gamma : \|\gamma\| \leq D\left(T/K_T^2\right)^{-\zeta}\}$ *for some* $D > 0$ *with* $\frac{1}{2} > \zeta > \frac{1}{2\alpha\eta}$.

Under Assumptions 7.1 to 7.4, Smith (2004) shows that for the full-sample estimators $\tilde{\theta}_T \xrightarrow{p} \theta_0$, $\tilde{\gamma}_T \xrightarrow{p} 0$, $\|\tilde{\gamma}_T\| = O_p\left(\left(T/2K_T + 1)^2\right)\right)^{-1/2}$ and $\|\frac{1}{T}\sum_{t=1}^{T} g_{tT}(\tilde{\theta}_T)\| = O_p(T^{-1/2})$. Now, Let $G(\beta, \delta) = E\left[\partial g(x_t, \beta, \delta)/\partial(\beta', \delta')\right]$ and $G = G(\beta_0, \delta_0)$.

Assumption 7.5. $g(\cdot, \beta, \delta)$ *is differentiable in* $(\beta_0, \delta_0) \in B_0 \times \Delta_0$ *where* $B_0 \times \Delta_0$ *is a neighborhood of* (β_0, δ_0), $\sup_{1\leq t\leq T} E\left[\sup_{(\beta,\delta)\in B_0\times\Delta_0} \|\partial g(x_t, \beta, \delta)/\partial(\beta', \delta')\|^{\alpha/(\alpha-1)}\right] < \infty$ *and rank*$(G) = p + \nu$.

Assumptions 7.1 to 7.5 yield the asymptotic distribution of $T^{1/2}\left(\tilde{\theta}_T - \theta_0\right)$ and $\left(T/(2K_T + 1)^2\right)^{1/2} \tilde{\gamma}_T$ (see Smith, 2004). Also, under these assumptions:

$$\frac{1}{T} \sum_{t=1}^{T} G_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) \xrightarrow{p} G$$

and

$$\tilde{\Omega}_T(\tilde{\theta}_T) = \frac{2K + 1}{T} \sum_{t=1}^{T} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T)' \xrightarrow{p} \Omega.$$

The following high level assumptions are sufficient to derive the weak convergence under the null of the PS-GEL estimators $\hat{\theta}_T(s)$ and $\hat{\gamma}_T(s)$ (see Guay and Lamarche, 2008). These assumptions are similar to the ones in Andrews (1993).

Assumption 7.6. $Var\left(\frac{1}{\sqrt{T}} \sum_{t=1}^{Ts} g(x_t, \theta)\right) \to s\Omega$ $\forall s \in [0, 1]$ *for some positive definite matrix* Ω *where* $\Omega = \lim_{T\to\infty} Var\left(\frac{1}{\sqrt{T}} \sum_{t=1}^{T} g(x_t, \theta)\right)$.

Assumption 7.7. $\sup_{s\in S}\|\hat{\Omega}_T(s)-\Omega(s)\| \xrightarrow{p} 0$ *where* $\Omega(s)$ *is defined in Section 3.1 and* S *whose closure lies in* $(0,1)$.

Assumption 7.8. $\sup_{s\in S}\|\hat{\theta}_T(s)-\theta_0\| \xrightarrow{p} 0$ *for some* θ_0 *in the interior of* Θ *and* $\sup_{s\in S}\|\hat{\gamma}_T(s)-0\| \xrightarrow{p} 0$ *and* S *whose closure lies in* $(0,1)$.

Assumption 7.9. $\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{Ts}E\partial g(x_t,\beta_0,\delta_0)/\partial(\beta',\delta')$ *exists uniformly over* $s\in S$ *and equals* sG $\forall s\in S$ *and* S *whose closure lies in* $(0,1)$.

Assumption 7.10. $G(s)'\Omega(s)^{-1}G(s)$ *is nonsingular* $\forall s\in S$ *and has eigenvalues bounded away from zero* $\forall s\in S$ *and* S *whose closure lies in* $(0,1)$.

7.2 Lemmas

The following Lemmas are necessary to establish the proofs of the Theorems:

Lemma 7.1. *We denote* $\{B(s): s\in[0,1]\}$ *as q-dimensional vectors of mutually independent Brownian motion on* $[0,1]$ *and define*

$$J(s)=\begin{bmatrix}\Omega^{1/2}B(s)\\ \Omega^{1/2}(B(1)-B(s))\end{bmatrix}$$

where $B(\pi)$ *is a q-dimensional vector of standard Brownian motion. Under Assumptions 7.1 to 7.10, every sequence of PS-GEL estimators* $\{\hat{\theta}(\cdot),\hat{\gamma}(\cdot),T\geq 1\}$ *satisfies*

$$\begin{aligned}\sqrt{T}\left(\hat{\theta}_T(\cdot)-\theta_0\right) &\Rightarrow -\left(G(\cdot)'\Omega(\cdot)^{-1}G(\cdot)\right)^{-1}G(\cdot)'\Omega(\cdot)^{-1}J(\cdot),\\ \frac{\sqrt{T}}{2K_T+1}\hat{\gamma}_T(\cdot) &\Rightarrow -\left(\Omega(\cdot)^{-1}-\Omega(\cdot)^{-1}\left(G(\cdot)'\Omega(\cdot)^{-1}G(\cdot)\right)^{-1}G(\cdot)'\Omega(\cdot)^{-1}\right)J(\cdot)\end{aligned}$$

as a process indexed by $s\in S$*, where* S *has closure in (0,1). Further, the sequence GEL estimators* $\hat{\theta}_T(\cdot)$ *and the sequence of auxiliary estimators* $\hat{\gamma}_T(\cdot)$ *are asymptotically uncorrelated. Under the alternative (8), the same results hold except that:*

$$J(s)=\begin{bmatrix}\Omega^{1/2}B(s)+H(s)\\ \Omega^{1/2}(B(1)-B(s))+H(1)-H(s)\end{bmatrix}$$

where $H(s)=\int_0^s h(r)dr$ *with* $h(r)=h(\eta,\tau,r)$ *to simplify the notation.*

Proof of Lemma 7.1: see Guay and Lamarche (2008).

Lemma 7.2. *Under Assumptions 7.1 to 7.10, for the unrestricted implied probabilities evaluated at the restricted estimator, we get*

$$\pi_t^{GEL}(\tilde{\theta}_T,s)=\frac{1}{T}+\frac{1}{T}g_{tT}(\tilde{\theta}_T,s)'\widetilde{\gamma}_T(s)(1+o_p(1))+O_p\left(K_T/T^2\right)$$

and $\pi_t^{GEL}(\tilde{\theta}_T)=\frac{1}{T}+o_p(1)$ *uniformly in* $t=1,\ldots,T$*. According to notation in Definition 3.4, for* $t=1,\ldots,[Ts]$,

$$T\pi_t^{GEL}(\tilde{\theta}_T,s)-1=g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)'\widetilde{\gamma}_{1T}(1+o_p(1))+O_p\left(K_T/T\right)$$

and for $t=[Ts]+1,\ldots,T$,

$$T\pi_t^{GEL}(\tilde{\theta}_T,s)-1=g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)'\widetilde{\gamma}_{2T}(1+o_p(1))+O_p\left(K_T/T\right).$$

Proof of Lemma 7.2

We need to derive the asymptotic distributions of the partial-sample implied probabilities evaluated at the full-sample estimator, namely:

$$\pi_t^{GEL}(\tilde{\theta}_T, s) = \frac{\rho_1\left(\widetilde{\gamma}_T(s)' g_{tT}(\widetilde{\theta}_T, s)\right)}{\sum_{t=1}^T \rho_1\left(\widetilde{\gamma}_T(s)' g_{tT}(\widetilde{\theta}_T, s)\right)}.$$

A mean value expansion for $\pi_t^{GEL}(\tilde{\theta}_T, s)$ around $(\tilde{\theta}_T, \tilde{\gamma}_T) = (\tilde{\theta}_T, 0)$ yields:

$$\begin{aligned}\pi_t^{GEL}(\tilde{\theta}_T, s) &= \frac{1}{T} + \frac{\rho_2\left(\bar{\gamma}_T(s)' g_{tT}(\widetilde{\theta}_T, s)\right)}{\sum_{t=1}^T \rho_1\left(\widetilde{\gamma}_T(s)' g_{tT}(\widetilde{\theta}_T, s)\right)} g_{tT}(\widetilde{\theta}_T, s)'\widetilde{\gamma}_T(s) - \\ &\quad \frac{\rho_1\left(\widetilde{\gamma}_T(s)' g_{tT}(\widetilde{\theta}_T, s)\right)}{\left[\sum_{t=1}^T \rho_1\left(\bar{\gamma}_T(s)' g_{tT}(\widetilde{\theta}_T, s)\right)\right]^2} \sum_{t=1}^T \rho_2\left(\bar{\gamma}_T(s)' g_{tT}(\widetilde{\theta}_T, s)\right) g_{tT}(\widetilde{\theta}_T, s)'\widetilde{\gamma}_T(s)\end{aligned}$$

where $\bar{\gamma}_T(s)$ lies on the line segment joining $\widetilde{\gamma}_T(s)$ and 0 and may differ from row to row.

Since $\widetilde{\gamma}_T(s)$ converges in probability to 0 and using Lemma A.4 in Smith (2004), this yields that $\rho_j\left(\bar{\gamma}_T(s)' g_{tT}(\widetilde{\theta}_T, s)\right) = \rho_j(0) + o_p(1) = -1 + o_p(1)$, for $j = 1, 2$, $\forall t$. Thus, we get:

$$\pi_t^{GEL}(\tilde{\theta}_T, s) = \frac{1}{T} + \frac{1}{T}\left[g_{tT}(\tilde{\theta}_T, s)'\widetilde{\gamma}_T(s)\left(1 + o_p(1)\right)\right] - \frac{1}{T^2}\left[\sum_{t=1}^T g_{tT}(\tilde{\theta}_T, s)'\widetilde{\gamma}_T(s)\left(1 + o_p(1)\right)\right].$$

As $\sum_{t=1}^T g(\tilde{\theta}_T) = \sum_{t=1}^T g_{tT}(\tilde{\theta}_T) + O_p\left(K^2/(2K+1)\right) = O_p(T^{1/2})$ which implies $\sum_{t=1}^T g_{tT}(\tilde{\theta}_T, s) = O_p(T^{1/2})$, $g_{tT}(\tilde{\theta}_T, s) = O_p((2K_T+1)^{-1/2})$ and $\tilde{\gamma}_T(s) = O_p(2K_T + 1/\sqrt{T})$ [6]yields:

$$\pi_t^{GEL}(\tilde{\theta}_T, s) = \frac{1}{T} + \frac{1}{T}\left[g_{tT}(\tilde{\theta}_T, s)'\widetilde{\gamma}_T(s)\left(1 + o_p(1)\right)\right] + O_p\left(K_T/T^2\right)$$

and

$$\pi_t^{GEL}(\tilde{\theta}_T, s) = \frac{1}{T}\left(1 + o_p(1)\right).$$

uniformly in $t = 1, \ldots, T$. Thus, we get:

$$T\pi_t^{GEL}(\tilde{\theta}_T, s) - 1 = g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T)'\tilde{\gamma}_{1T}(1 + o_p(1)) + O_p\left(K_T/T\right). \tag{18}$$

uniformly in $t = 1, \ldots, [Ts]$ and

$$T\pi_t^{GEL}(\tilde{\theta}_T, s) - 1 = g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T)'\tilde{\gamma}_{2T}(1 + o_p(1)) + O_p\left(K_T/T\right). \tag{19}$$

uniformly in $t = [Ts] + 1, \ldots, T$.

[6]See Kitamura and Stutzer (1997) and Guay and Lamarche (2008).

7.3 Proofs of Theorems

Proof of Theorem 4.1

We expand the FOC of the Lagrange multiplier vector for the partial-sample GEL evaluated at the restricted estimator in a Taylor series about 0 for the first part of the sample $t = 1, \ldots, [Ts]$. Thus

$$\begin{aligned}\frac{1}{T}\sum_{t=1}^{[Ts]}\rho_1\left(\tilde{\gamma}_{1T}'g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)\right)g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T) &= -\frac{1}{T}\sum_{t=1}^{[Ts]}g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)-\frac{1}{T}\sum_{t=1}^{[Ts]}g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)'\tilde{\gamma}_{1T}\\ &+ \frac{1}{T}\sum_{t=1}^{[Ts]}g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)\sum_{j=2}^{\infty}\frac{1}{j!}\rho_{j+1}(0)\left(g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)'\tilde{\gamma}_{1T}\right)^j\end{aligned}$$

where $\tilde{\gamma}_{1T}$ is the partial-sample Lagrange multiplier evaluated for the restricted estimator for $t = 1, \ldots, [Ts]$ and $\rho_1(0) = \rho_2(0) = -1$. By the fact that $g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T) = O_p\left((2K_T+1)^{-1/2}\right)$, this yields

$$\begin{aligned}\frac{1}{T}\sum_{t=1}^{[Ts]}\rho_1\left(\tilde{\gamma}_{1T}'g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)\right)g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T) &= -\frac{1}{T}\sum_{t=1}^{[Ts]}g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)-\frac{1}{T}\sum_{t=1}^{[Ts]}g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)'\tilde{\gamma}_{1T}\\ &+ O_p\left((2K_T+1)^{-3/2}\|\tilde{\gamma}_{1T}\|^2\right).\end{aligned}$$

By using a consistent estimator $\tilde{\Omega}_T = \frac{2K+1}{[Ts]}\sum_{t=1}^{[Ts]}g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)'$ and $\tilde{\gamma}_{1T} = O_p\left((2K_T+1)/\sqrt{T}\right)$, we get under the null:

$$0 = \frac{1}{T}\sum_{t=1}^{[Ts]}g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T) + \frac{s}{2K+1}\tilde{\Omega}_T\tilde{\gamma}_{1T} + O_p\left((2K_T+1)^{-3/2}\|\tilde{\gamma}_{1T}\|^2\right)$$

which yields

$$\frac{s}{2K_T+1}\tilde{\gamma}_{1T} = -\tilde{\Omega}_T^{-1}\frac{1}{T}\sum_{t=1}^{[Ts]}g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T) + O_p\left(K_T^{1/2}/T\right). \quad (20)$$

Now, expanding this expression around β_0 and δ_0 gives:

$$\frac{s\sqrt{T}}{2K_T+1}\tilde{\gamma}_{1T} = -\tilde{\Omega}_T^{-1}\frac{1}{\sqrt{T}}\sum_{t=1}^{[Ts]}g_{tT}(\beta_0,\delta_0) - s\tilde{\Omega}_T^{-1}G\sqrt{T}\begin{pmatrix}\tilde{\beta}_T-\beta_0\\ \tilde{\delta}_T-\delta_0\end{pmatrix} + o_p(1).$$

We can easily show for the restricted estimators under the null that (see also Smith, 2004):

$$\sqrt{T}\begin{pmatrix}\tilde{\beta}_T-\beta_0\\ \tilde{\delta}_T-\delta_0\end{pmatrix} = -\left(G'\Omega^{-1}G\right)^{-1}\ G'\Omega^{-1}\frac{1}{\sqrt{T}}\sum_{t=1}^{T}g_{tT}(\beta_0,\delta_0) + o_p(1). \quad (21)$$

Combining the two preceding results, we obtain

$$\frac{s\sqrt{T}}{2K_T+1}\tilde{\gamma}_{1T} = -\Omega^{-1}\frac{1}{\sqrt{T}}\sum_{t=1}^{[Ts]}g_{tT}(\beta_0,\delta_0) + s\Omega^{-1}G(G'\Omega^{-1}G)^{-1}G'\Omega^{-1}\frac{1}{\sqrt{T}}\sum_{t=1}^{T}g_{tT}(\beta_0,\delta_0) + o_p\ (1).$$

By Lemma 5.1 in Guay and Lamarche (2008), under the null of no structural change, we have

$$\Omega^{-1/2}\frac{1}{\sqrt{T}}\sum_{t=1}^{[Ts]} g_{tT}(\beta_0,\delta_0) \Rightarrow B(s) \tag{22}$$

and under the generic alternative (8),

$$\Omega^{-1/2}\frac{1}{\sqrt{T}}\sum_{t=1}^{[Ts]} g_{tT}(\beta_0,\delta_0) \Rightarrow B(s)+\Omega^{-1/2}H(s) \tag{23}$$

where $B(s)$ is a q-vector of standard Brownian motions and $H(s)=\int_0^s h(r)dr$. Note also that:

$$\frac{1}{T}\sum_{t=1}^{[Ts]} g_{tT}(\beta_0,\delta_0) = \frac{1}{T}\sum_{t=1}^{[Ts]} g_{t}(\beta_0,\delta_0) + o_p(1).$$

Using Lemma 7.2 and the derivation above,

$$\begin{aligned} T\pi_t^{GEL}(\tilde{\theta}_T,s)-1 \quad &= \quad -\left(\frac{2K+1}{s}\tilde{\Omega}_T^{-1}\frac{1}{T}\sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)+O_p\left(K_T^{3/2}/T\right)\right)' \times &(24)\\ &\qquad g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)(1+o_p(1))+O_p(K_T/T). &(25)\end{aligned}$$

Let us now examine the asymptotic distribution of the expression on the right-hand side. First, we show the following asymptotic result for the partial sums of the moments conditions evaluated at $\tilde{\theta}_T$:

$$\Omega^{-1/2}\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)\right] \Rightarrow B(s)-s\Omega^{-1/2}G[G'\Omega^{-1}G]^{-1}G'\Omega^{-1/2}B(1).$$

By a mean value expansion:

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T) = \frac{1}{\sqrt{T}}\sum_{t=1}^{[Ts]} g_{tT}(\beta_0,\delta_0)+\frac{1}{T}\sum_{t=1}^{[Ts]}\frac{\partial g_{tT}(\bar{\beta}_T,\bar{\delta}_T)}{\partial(\beta',\delta')}\begin{pmatrix}\tilde{\beta}_T-\beta_0\\ \tilde{\delta}_T-\delta_0\end{pmatrix}$$

where $\bar{\beta}_T$ lies on the line segment joining $\tilde{\beta}_T$ and β_0 and may differ from row to row and respectively for $\bar{\delta}_T$. By applying eq. (21), this gives

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T) = \frac{1}{\sqrt{T}}\sum_{t=1}^{[Ts]} g_{tT}(\beta_0,\delta_0) - sG[G'\Omega^{-1}G]^{-1}G'\Omega^{-1}\frac{1}{T}\sum_{t=1}^{T} g_{tT}(\beta_0,\delta_0)+o_p(1). \tag{26}$$

It follows by (22),

$$\Omega^{-1/2}\frac{1}{\sqrt{T}}\sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T) \Rightarrow B(s)-s\Omega^{-1/2}G[G'\Omega^{-1}G]^{-1}G'\Omega^{-1/2}B(1). \tag{27}$$

By (24), the following partial sum can be shown to be:

$$\begin{aligned}\sum_{t=1}^{[Ts]}[T\ \pi_t^{GEL}(\tilde{\theta}_T,s)-1]^2 \quad &= \quad \left(\frac{2K+1}{s}\tilde{\Omega}_T^{-1}\frac{1}{T}\sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)\right)'\sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)'\\ &\qquad \times\left(\frac{2K+1}{s}\tilde{\Omega}_T^{-1}\frac{1}{T}\sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)\right)+o_p(1).\end{aligned}$$

Considering that $\tilde{\Omega}_T = \frac{2K+1}{[Ts]}\sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)'$, the expression above gives:

$$\sum_{t=1}^{[Ts]}[T\ \pi_t^{GEL}(\tilde{\theta}_T,s)-1]^2 = \frac{2K+1}{s}\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)\right)'\tilde{\Omega}_T^{-1}\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T,\tilde{\delta}_T)\right)+o_p(1).$$

By the result above, the partial sum of interest yields by equations (27) and the consistency of $\tilde{\Omega}_T$:

$$\frac{s}{2K+1}\sum_{t=1}^{[Ts]}[T\pi_t^{GEL}(\tilde{\theta}_T,s)-1]^2 \quad\Rightarrow\quad \Big[B(s)-s\Omega^{-1/2}G[G'\Omega^{-1}G]^{-1}G'\Omega^{-1/2}B(1)\Big]'\times \tag{28}$$

$$\Big[B(s)-s\Omega^{-1/2}G[G'\Omega^{-1}G]^{-1}G'\Omega^{-1/2}B(1)\Big]. \tag{29}$$

The expression $\big[B(s)-s\Omega^{-1/2}G[G'\Omega^{-1}G]^{-1}G'\Omega^{-1/2}B(1)\big]$ can be rewritten as:

$$[I-P_G]\,B(s)+P_G\,[B(s)-sB(1)]$$

with $P_G=\Omega^{-1/2}G[G'\Omega^{-1}G]^{-1}G'\Omega^{-1/2}$. We can now decompose:

$$\Omega^{-1/2}G(G'\Omega^{-1}G)^{-1}G'\Omega^{-1/2}=C'\Lambda C \tag{30}$$

where $CC'=I$ and

$$\Lambda=\begin{bmatrix} I_p & 0\\ 0 & 0\end{bmatrix}$$

and I_p is an identity matrix with dimension $p\times p$ where $p=r+\nu$ with r and ν the respective dimension the vectors β and δ. Note also that $CB(s)$ has the same asymptotic distribution as $B(s)$. The RHS of expression (29) can then be rewritten as:

$$B(s)'\,[I-C'\Lambda C]\,B(s)+[B(s)-sB(1)]'\,C'\Lambda C\,[B(s)-sB(1)]$$

which is equal in distribution to

$$B_{(q-p)}(s)'B_{(q-p)}(s)+BB_p(s)'BB_p(s)$$

where $B_{(q-p)}(s)$ is a $(q-p)$-vector of standard Brownian motion and $BB_p(s)=B_p(s)-sB_p(1)$ is a p-vector of Brownian bridge. The result follows. The asymptotic distribution under the alternative (8) can be easily obtained similarly by using (23).

Proof of Theorem 4.3

By Lemma 7.2 applied to the difference between the partial-sample implied probabilities evaluated at the unrestricted and at the restricted estimators we get:

$$T\Big[\hat{\pi}_t(\hat{\theta}_T(s),s)-\tilde{\pi}_t(\tilde{\theta}_T,s)\Big]=g_{tT}(\hat{\theta}_T(s),s)'\hat{\gamma}_T(s)-g_{tT}(\tilde{\theta}_T,s)'\tilde{\gamma}_T(s)+o_p(T^{-1/2}).$$

Let us define the following selection matrices:

$$H_1=\begin{bmatrix} I_{r\times r} & 0_{r\times r} & 0_{r\times\nu}\\ 0_{\nu\times r} & 0_{\nu\times r} & I_{\nu\times\nu}\end{bmatrix}$$

and

$$H_2 = \begin{bmatrix} 0_{r\times r} & I_{r\times r} & 0_{r\times\nu} \\ 0_{\nu\times r} & 0_{\nu\times r} & I_{\nu\times\nu} \end{bmatrix}.$$

and the corresponding estimator: $\hat{\theta}_{1T}(s) = H_1\hat{\theta}_T(s) = \left(\hat{\beta}_{1T}(s)', \hat{\delta}_T(s)'\right)'$ and $\hat{\theta}_{2T}(s) = H_2\hat{\theta}_T(s) = \left(\hat{\beta}_{2T}(s)', \hat{\delta}_T(s)'\right)'$. Similarly, for the restricted estimator $\tilde{\theta}_{1T}(s) = H_1\tilde{\theta}_T(s) = \left(\tilde{\beta}_T(s)', \tilde{\delta}_T(s)'\right)'$ and $\tilde{\theta}_{2T}(s) = H_2\tilde{\theta}_T(s) = \left(\tilde{\beta}_T(s)', \tilde{\delta}_T(s)'\right)'$ where $\tilde{\theta}_T(s) = \left(\tilde{\beta}_T(s)', \tilde{\beta}_T(s)', \tilde{\delta}_T(s)'\right)'$. Accordingly, we define $\theta_{1,0} = H_1\theta_0 = (\beta_0', \delta_0')'$ and $\theta_{2,0} = H_2\theta_0 = (\beta_0', \delta_0')'$.

Let us examine the expression for the first part of the sample, namely $t = 1, \ldots, [Ts]$. Replacing the unrestricted and the restricted estimators of γ by the corresponding expression (20), we get:

$$\begin{aligned} T\left[\pi_t(\hat{\theta}_T(s), s) - \pi_t(\tilde{\theta}_T, s)\right] &= -\left[(2K_T+1)\Omega(s)^{-1}\frac{1}{[Ts]}\sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_T(s), s) + o_p(1)\right]' g_{tT}(\hat{\theta}_T(s), s)(1+o_p(1)) \\ &\quad + \left[(2K_T+1)\Omega(s)^{-1}\frac{1}{[Ts]}\sum_{t=1}^{[Ts]} g_{tT}(\tilde{\theta}_T, s) + o_p(1)\right]' g_{tT}(\tilde{\theta}_T, s)(1+o_p(1)) \\ &\quad + O_p(K_T/T) \end{aligned}$$

which equals

$$T\left[\pi_t(\hat{\theta}_T(s), s) - \pi_t(\tilde{\theta}_T, s)\right] = -\left[\frac{1}{[Ts]}\sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_{1T}(s)) - \frac{1}{[Ts]}\sum_{t=1}^{[Ts]} g_{tT}(\tilde{\theta}_{1T}) + o_p(1)\right]' \tag{31}$$

$$\times\Omega^{-1}(2K_T+1)g_{tT}(\hat{\theta}_{1T}(s))(1+o_p(1)) + O_p(K_T/T) \tag{32}$$

since $G_{tT}(\cdot) = O_p((2K_T+1)^{-1/2})$ and $\hat{\theta}_{1T}(s) - \tilde{\theta}_{1T} = O_p(T^{-1/2})$.

Now, consider the following mean value expansion:

$$\frac{1}{[Ts]}\sum_{t=1}^{[Ts]} g_{tT}(\tilde{\theta}_{1T}) = \frac{1}{[Ts]}\sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_{1T}(s)) + \frac{1}{[Ts]}\sum_{t=1}^{[Ts]} \frac{\partial g_{tT}(\bar{\theta}_{1T}(s))}{\partial\theta_1'}\left(\tilde{\theta}_{1T} - \hat{\theta}_{1T}(s)\right). \tag{33}$$

where $\bar{\theta}_{1T}(s)$ lies on the line segment joining $\hat{\theta}_{1T}(s)$ and $\tilde{\theta}_{1T}$ and may differ from row to row. Thus,

$$\begin{aligned} T\left[\pi_t(\hat{\theta}_T(s), s) - \pi_t(\tilde{\theta}_T, s)\right] &= \left[\frac{1}{[Ts]}\sum_{t=1}^{[Ts]} \frac{\partial g_{tT}(\bar{\theta}_{1T}(s))}{\partial\theta_1'}\left(\tilde{\theta}_{1T} - \hat{\theta}_{1T}(s)\right) + o_p(1)\right]' \\ &\quad \times\Omega^{-1}(2K_T+1)g_{tT}(\hat{\theta}_{1T}(s))(1+o_p(1)) + O_p(K_T/T). \end{aligned}$$

The partial sum over the first subsample for the square of the LHS expression above yields:

$$\frac{1}{2K+1}\sum_{t=1}^{[Ts]}\left[T\pi_t(\hat{\theta}_T(s), s) - T\pi_t(\tilde{\theta}_T, s)\right]^2 = [Ts]\left(\tilde{\theta}_{1T} - \hat{\theta}_{1T}(s)\right)' G'\Omega^{-1}G\left(\tilde{\theta}_{1T} - \hat{\theta}_{1T}(s)\right) + o_p(1)$$

as $(2K_T+1)\frac{1}{[Ts]}\sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_{1T}(s), s)g_{tT}(\hat{\theta}_{1T}(s), s)'$ is a consistent estimator of Ω.

Similarly for the second subsample $t = [Ts]+1, \ldots, T$, we obtain

$$\frac{1}{2K+1} \sum_{t=[Ts]+1}^{T} \left[T\pi_t(\hat{\theta}_T(s), s) - T\pi_t(\tilde{\theta}_T, s)\right]^2 = (T - [Ts]) \left(\tilde{\theta}_{2T} - \hat{\theta}_{2T}(s)\right)' G'\Omega^{-1} G \left(\tilde{\theta}_{2T} - \hat{\theta}_{2T}(s)\right) + o_p(1)$$

By results derived above and some calculations, this yields

$$\frac{1}{2K+1} \sum_{t=1}^{T} \left[T\pi_t(\hat{\theta}_T(s), s) - T\pi_t(\tilde{\theta}_T, s)\right]^2 = T \left(\tilde{\theta}_T - \hat{\theta}_T(s)\right)' G(\theta_0, s)'\Omega^{-1}(s) G(\theta_0, s) \left(\tilde{\theta}_T - \hat{\theta}_T(s)\right) + o_p(1).$$

Andrews (1993, p. 851-852) shows that this expression is asymptotically equivalent to the $LM_T(s)$ statistic for parameters instability. This gives the result follows under the null. Under the alternative, Guay and Lamarche (2008) show that the $LM_T(s)$ statistic for the restricted GEL estimator has the same asymptotic distribution as in Theorem 4.3. Since the asymptotic equivalence between the expression above and the $LM_T(s)$ statistic, the result follows.

Proof of Theorem 4.4

The two subsamples are now evaluated separately. A direct application of Lemma 7.2 but for the estimation of the subsample with the first $[Ts]$ observations gives

$$[Ts]\pi_t^{GEL}(\hat{\theta}_{1T}(s), s) - 1 = g_{tT}(\hat{\theta}_{1T}(s))'\hat{\gamma}_{1T}(s) + o_p([Ts]^{-1/2})$$

uniformly in $t = 1, \ldots, [Ts]$. This is obtained with a proof similar to the one of Lemma 7.2 but only for the first part of the sample. This yields

$$\left([Ts]\pi_t^{GEL}(\hat{\theta}_{1T}(s), s) - 1\right)^2 = \hat{\gamma}_{1T}(s)' g_{tT}(\hat{\theta}_{1T}(s)) g_{tT}(\hat{\theta}_{1T}(s)'\hat{\gamma}_{1T}(s)) + o_p([Ts]^{-1}).$$

By summing the expression above to $t = \ldots, [Ts]$, and by (20) for the unrestricted estimator applied to the sample $t = 1, \ldots, [Ts]$, this yields

$$\begin{aligned}\sum_{t=1}^{[Ts]} \left([Ts]\ \pi_t^{GEL}(\hat{\theta}_{1T}(s), s) - 1\right)^2 \quad = \quad & (2K_T+1)^2 \frac{1}{\sqrt{[Ts]}} \sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_{1T}(s))'\Omega^{-1} \frac{1}{[Ts]} \sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_{1T}(s)) \\ & \times \left(g_{tT}(\hat{\theta}_{1T}(s))\right)' \Omega^{-1} \frac{1}{\sqrt{[Ts]}} \sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_{1T}(s)) + o_p(1).\end{aligned}$$

As $(2K_T+1)\frac{1}{[Ts]}\sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_{1T}(s)) g_{tT}(\hat{\theta}_{1T}(s))'$ is a consistent estimator of Ω, this gives

$$\frac{1}{2K_T+1} \sum_{t=1}^{[Ts]} \left([Ts]\ \pi_t^{GEL}(\hat{\theta}_{1T}(s), s) - 1\right)^2 = \frac{1}{\sqrt{[Ts]}} \sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_{1T}(s))'\Omega^{-1} \frac{1}{\sqrt{[Ts]}} \sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_{1T}(s)) + o_p(1). \quad (34)$$

Similarly, we can easily obtain that

$$\frac{1}{2K_T+1} \sum_{t=[Ts]+1}^{T} \left((T - [Ts])\ \pi_t^{GEL}(\hat{\theta}_{2T}(s), s) - 1\right)^2 =$$

$$\frac{1}{\sqrt{T-[Ts]}}\sum_{t=[Ts]+1}^{T} g_{tT}(\hat{\theta}_{2T}(s))'\Omega^{-1}\frac{1}{\sqrt{T-[Ts]}}\sum_{t=[Ts]+1}^{T} g_{tT}(\hat{\theta}_{2T}(s))+o_p(1). \quad (35)$$

Now, by a mean value expansion and the consistency of $\hat{\theta}_T(s)$,

$$\Omega(s)^{-1/2}\frac{1}{\sqrt{T}}\sum_{t=1}^{T} g_{tT}(\hat{\theta}_T(s),s)=\Omega(s)^{-1/2}\left[\begin{array}{c}\frac{1}{\sqrt{T}}\sum_{t=1}^{[Ts]} g_{tT}(\beta_0,\delta_0)\\ \frac{1}{\sqrt{T}}\sum_{t=[Ts]+1}^{T} g_{tT}(\beta_0,\delta_0)\end{array}\right]+\Omega(s)^{-1/2}G(s)\sqrt{T}\left(\hat{\theta}_T(s)-\theta_0\right)+o_p(1).$$

By Lemma 7.1 and (22),

$$\begin{aligned}\Omega(s)^{-1/2}\frac{1}{\sqrt{T}}\sum_{t=1}^{T} g_{tT}(\hat{\theta}_T(s),s) \quad \Rightarrow \quad & \left[\begin{array}{c}\frac{1}{\sqrt{s}}B(s)\\ \frac{1}{\sqrt{(1-s)}}(B(1)-B(s))\end{array}\right]-\Omega(s)^{-1/2}G(s)\left(G(s)'\Omega(s)^{-1}G(s)\right)^{-1}\\ & \times G(s)'\Omega(s)^{-1/2}\left[\begin{array}{c}B(s)\\ (B(1)-B(s))\end{array}\right]\end{aligned}$$

The RHS can be rewritten as

$$\left(I-\Omega(s)^{-1/2}G(s)\left(G(s)'\Omega(s)^{-1}G(s)\right)^{-1}G(s)'\Omega(s)^{-1/2}\right)\left[\begin{array}{c}\frac{1}{\sqrt{s}}B(s)\\ \frac{1}{\sqrt{(1-s)}}(B(1)-B(s))\end{array}\right]$$

Since the entire parameter vector is estimated for both subsample, e.g. $\hat{\theta}_{1T}(s)=\hat{\beta}_{1T}(s)$ and $\hat{\theta}_{2T}(s)=\hat{\beta}_{2T}(s)$, the matrices $\Omega(s)$ and $G(s)$ are block-diagonal and by the definition of $g_{tT}(\hat{\theta}_T(s),s)$, we get:

$$\Omega^{-1/2}\frac{1}{\sqrt{T}}\sum_{t=1}^{[Ts]} g_{tT}(\hat{\beta}_{1T}(s)) \quad \Rightarrow \left(I_q-\Omega^{-1/2}G_\beta\left(G'_\beta\Omega^{-1}G_\beta\right)^{-1}G'_\beta\Omega^{-1/2}\right)B(s) \quad (36)$$

and

$$\Omega^{-1/2}\frac{1}{\sqrt{T}}\sum_{t=[Ts]+1}^{T} g_{tT}(\hat{\beta}_{2T}(s)) \quad \Rightarrow \left(I_q-\Omega^{-1/2}G_\beta\left(G'_\beta\Omega^{-1}G_\beta\right)^{-1}G'_\beta\Omega^{-1/2}\right)(B(1)-B(s)).$$

We can now decompose:

$$\left(I-\Omega^{-1/2}G_\beta\left(G'_\beta\Omega^{-1}G_\beta\right)^{-1}G'\Omega^{-1/2}\right)=(I-C'\Lambda C)$$

where $CC'=I$ and

$$\Lambda=\left[\begin{array}{cc}I_r & 0\\ 0 & 0\end{array}\right]$$

and I_r is an identity matrix with dimension $r\times r+$ with r is the dimension of the vectors β. By noting that $C\left(I-C'\Lambda C\right)B(s)=\left(I-\Lambda\right)CB(s)$ and $CB(s)$ is also a r-vector of Brownian motion. Now consider the multiplication of the RHS term of equation (36) by the matrix C, this yields

$$C\Omega^{-1/2}\frac{1}{\sqrt{T}}\sum_{t=1}^{[Ts]} g_{tT}(\hat{\beta}_{1T}(s))\Rightarrow\left(I_q-\Lambda\right)B(s)$$

By using this result, we obtain directly the asymptotic distribution of the statistic for the first subsample by equation (34) considering that $C'C=I_q$. The proof is similar for the second subsample (eq. 35), the asymptotic distribution under the null is then derived.

Under the alternative (8) by using Lemma 7.1 it is straightforward to show the asymptotic distribution by proceeding in the same manner as above.

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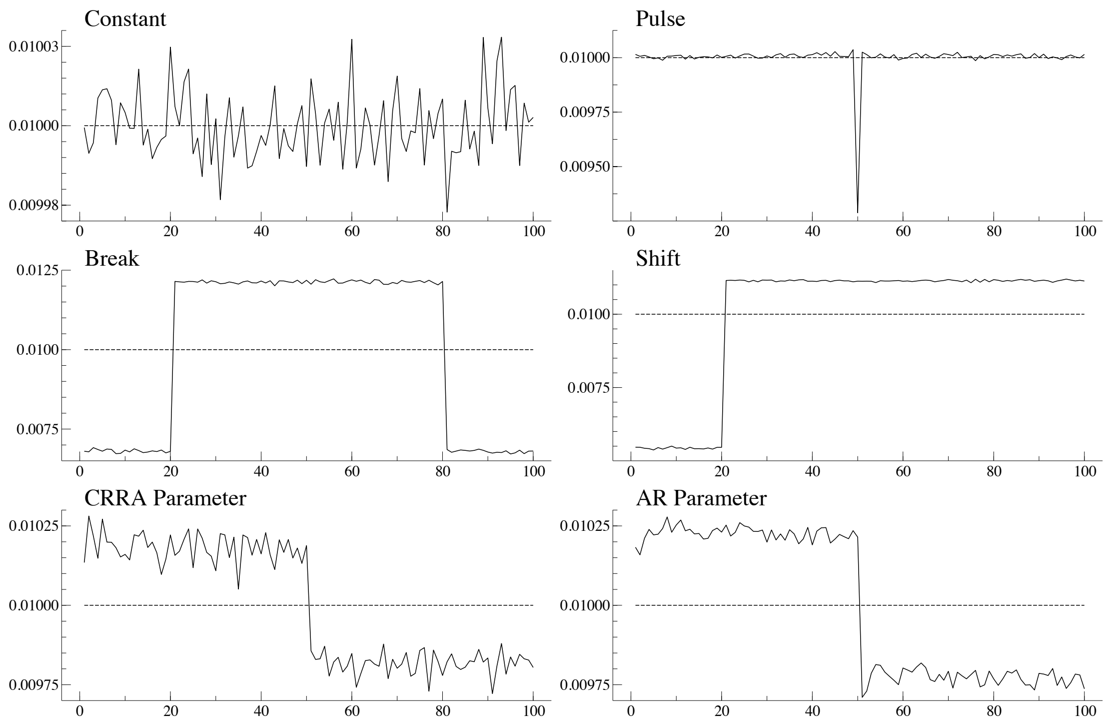
Figure 1: Simulated Implied Probabilities



Notes: The *constant* case refers to no break, the *pulse* case to a one-time temporary jump at observation 50, the *break* case to a change in the mean for 60 time periods and the *shift* to a permanent change in the mean at observation 20. For the CRRA case, a preference parameter is estimated using two moments. The moment conditions are violated at observation 51. For the AR case a autoregressive parameter is estimated. The data generating process is represented by an $AR(1)$ process for $t \leq 50$ and by an $ARMA(1,2)$ otherwise. The sample is of size 100 and in all cases the horizontal lines correspond to the empirical weights of 1/100.

Table 1: Asymptotic critical values for the supremum mapping

p	s_0	$q-p=1$			$q-p=2$			$q-p=3$			$q-p=4$			$q-p=5$		
		10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
1	0	3.9888	5.1459	8.0097	5.9222	7.3909	10.7028	7.5042	9.0923	12.5084	9.1585	10.9007	14.7139	10.6181	12.3841	16.3002
	0.15	3.5598	4.4989	6.7781	5.1057	6.4632	9.1665	6.4753	7.8475	10.6851	7.8827	9.3669	12.4983	9.1560	10.5950	13.8459
	0.20	3.4055	4.2824	6.5025	4.8564	6.0832	8.6644	6.1452	7.3453	10.1741	7.4470	8.7885	11.8919	8.6225	9.9858	13.0468
	0.25	3.2517	4.1119	6.2268	4.5899	5.7037	8.1342	5.7880	6.9920	9.5275	7.0098	8.3281	11.2309	8.1160	9.4629	12.2113
	0.30	3.0853	3.9137	5.8164	4.2987	5.3301	7.7153	5.4848	6.5937	8.9652	6.5836	7.7919	10.7135	7.6634	8.8448	11.4426
	0.35	2.9074	3.6581	5.3887	4.0309	4.9595	7.2013	5.1259	6.1131	8.3882	6.1680	7.3314	9.7638	7.1775	8.3143	10.7923
	0.40	2.7003	3.4070	5.0319	3.7413	4.6458	6.7155	4.7551	5.6817	7.7017	5.7432	6.7812	8.9728	6.6324	7.7381	9.8909
	0.50	1.7537	2.3678	3.6877	2.6251	3.3202	4.9903	3.4158	4.1890	5.8005	4.2173	5.1175	6.8987	4.9638	5.9035	7.8275
2	0	4.2480	5.3017	7.8258	5.9254	7.2540	10.5377	7.7273	9.3159	12.9666	9.1699	10.7556	14.5338	10.6678	12.3656	16.4260
	0.15	3.8657	4.7129	7.0572	5.3043	6.3333	8.9072	6.8405	8.2420	11.4170	8.0922	9.3975	12.7004	9.2630	10.7878	14.0015
	0.20	3.7332	4.5581	6.6550	5.0703	6.0490	8.4544	6.4925	7.8445	10.8480	7.7305	8.9416	12.0481	8.7697	10.2657	13.2560
	0.25	3.6042	4.3931	6.3764	4.8574	5.7918	7.9881	6.1750	7.4695	10.1321	7.3096	8.4436	11.3044	8.2951	9.7409	12.4754
	0.30	3.4317	4.2211	6.0408	4.6347	5.4757	7.5702	5.8322	7.0421	9.3802	6.9278	7.9902	10.8169	7.8608	9.1332	11.7222
	0.35	3.2796	4.0195	5.6562	4.3566	5.1283	7.1421	5.5018	6.6101	8.9341	6.4902	7.5748	10.0346	7.3583	8.4945	10.9429
	0.40	3.0958	3.7443	5.3817	4.0648	4.8572	6.6124	5.1097	6.1299	8.2836	6.0254	7.0663	9.3381	6.8720	7.8437	10.2758
	0.50	2.1461	2.7002	4.1888	2.8910	3.6042	5.1756	3.7579	4.5814	6.2840	4.5367	5.4341	7.4211	5.1289	6.1212	8.1392
3	0	4.4578	5.4526	8.2041	6.2780	7.5130	10.4686	7.7581	9.2314	12.5700	9.5083	11.2721	14.7783	10.7982	12.5502	16.2030
	0.15	4.1966	5.0796	7.4813	5.7102	6.7819	9.0515	6.9921	8.1506	10.9953	8.4598	9.8842	13.0589	9.6082	11.1174	14.2474
	0.20	4.0982	4.9731	7.0698	5.4584	6.4819	8.7509	6.6817	7.8029	10.3022	8.0642	9.4424	12.5643	9.1728	10.5453	13.7357
	0.25	3.9891	4.8340	6.8950	5.2838	6.2342	8.4686	6.3771	7.4652	9.7166	7.6663	8.9733	11.8721	8.7095	9.9964	13.0777
	0.30	3.8303	4.6556	6.6164	5.0507	5.9179	7.9869	6.1029	7.1186	9.2371	7.3242	8.4344	11.0122	8.2271	9.5233	12.2145
	0.35	3.6811	4.4607	6.2600	4.7737	5.6768	7.5211	5.7421	6.6974	8.7307	6.9213	8.0071	10.4071	7.7482	8.9566	11.3836
	0.40	3.4913	4.2037	5.8003	4.5069	5.3370	7.0691	5.4383	6.3226	8.2805	6.4375	7.3987	9.6220	7.2384	8.3599	10.7141
	0.50	2.4795	3.0584	4.5287	3.2717	4.0045	5.4395	4.0619	4.8012	6.6910	4.8417	5.7980	7.5840	5.5105	6.4626	8.5755
4	0	4.7102	5.6098	7.9403	6.4099	7.6955	10.6530	8.0975	9.6400	13.1705	9.3735	11.1432	14.7414	11.0087	12.8406	16.6290
	0.15	4.4791	5.3213	7.5146	5.9130	7.0466	9.5622	7.4217	8.7222	11.8068	8.4519	9.8924	12.8766	9.8230	11.3544	14.8027
	0.20	4.3821	5.1529	7.3700	5.7441	6.7339	9.1763	7.1182	8.4327	11.2224	8.1314	9.4403	12.4099	9.4069	10.8109	14.0859
	0.25	4.2817	5.0525	6.8920	5.5296	6.5148	8.6910	6.8621	7.9993	10.5962	7.8150	8.9913	11.6675	8.9997	10.3500	13.4356
	0.30	4.1500	4.9047	6.6751	5.3222	6.2174	8.2964	6.5563	7.6151	10.0787	7.4506	8.5308	10.8886	8.5127	9.7961	12.6252
	0.35	4.0004	4.7418	6.3716	5.0738	5.9322	7.8744	6.2469	7.2830	9.4134	7.0567	8.0532	10.3600	8.0551	9.2432	11.9023
	0.40	3.7889	4.5123	6.0544	4.8106	5.5700	7.5241	5.8914	6.8800	8.9899	6.6127	7.5826	9.7203	7.5462	8.6217	10.9837
	0.50	2.7578	3.2962	4.6682	3.5654	4.2494	5.9371	4.4298	5.2684	7.0296	5.0782	5.9168	7.8511	5.8486	6.7420	8.7734
5	0	4.9258	5.9115	8.4116	6.6846	7.9159	10.7495	8.0738	9.5334	12.7477	9.4938	11.0872	14.6923	10.9727	12.5834	16.7246
	0.15	4.7316	5.6427	7.6194	6.2801	7.3558	9.7626	7.5050	8.7814	11.6794	8.6806	9.9412	13.2613	9.9381	11.3811	14.7100
	0.20	4.6736	5.5562	7.3635	6.0742	7.0949	9.5339	7.2984	8.4761	11.2599	8.3809	9.5544	12.8611	9.5711	10.9525	14.0793
	0.25	4.5900	5.4313	7.1466	5.8962	6.8915	9.1417	7.0399	8.1509	10.7401	8.0531	9.2264	12.1551	9.2154	10.5557	13.2710
	0.30	4.4855	5.2507	7.0561	5.7191	6.6745	8.5607	6.8420	7.8913	10.3629	7.7360	8.8301	11.4653	8.7309	10.0271	12.7417
	0.35	4.3686	5.1091	6.8385	5.4985	6.4150	8.1981	6.5474	7.5363	9.6881	7.3383	8.4577	10.7531	8.3035	9.4383	12.1146
	0.40	4.1741	4.8491	6.4766	5.2588	6.0720	7.8490	6.1612	7.1378	9.1692	6.9715	8.0099	10.2287	7.8630	8.8944	11.2032
	0.50	3.1106	3.7128	5.1431	3.9263	4.7029	6.2149	4.7524	5.5693	7.4078	5.4184	6.2904	8.1737	6.0948	7.0481	8.9452
6	0	5.2921	6.2280	8.5476	6.8171	8.1093	10.9911	8.2765	9.6858	13.1398	9.7646	11.3573	14.9386	11.2064	13.1177	16.9244
	0.15	5.1847	6.0092	8.0495	6.5145	7.5728	10.0654	7.7349	9.0582	11.8748	8.9849	10.3537	13.4073	10.2789	11.8365	15.1786
	0.20	5.0938	5.9131	7.9148	6.3719	7.3778	9.7802	7.5703	8.7074	11.4632	8.7486	10.0729	13.0329	9.9140	11.3381	14.4318
	0.25	5.0177	5.7969	7.6575	6.2130	7.2100	9.4884	7.3041	8.4234	10.8584	8.4346	9.7305	12.3824	9.5518	10.9861	13.7696
	0.30	4.9443	5.6931	7.4947	6.0389	7.0045	9.0382	7.0510	8.0700	10.2751	8.1441	9.2898	11.8951	9.1647	10.4575	13.2134
	0.35	4.8201	5.5166	7.3392	5.8548	6.7740	8.5586	6.7319	7.7034	9.8845	7.8282	8.8985	11.1744	8.7657	9.9195	12.6195
	0.40	4.6142	5.2764	7.0877	5.5714	6.4399	8.1975	6.4217	7.3678	9.4167	7.3851	8.3894	10.5071	8.2239	9.4039	11.7964
	0.50	3.4666	4.0997	5.5934	4.2441	4.9965	6.6345	4.9757	5.8062	7.4461	5.7484	6.6971	8.5722	6.4637	7.4586	9.5551
7	0	5.6856	6.6622	8.9875	7.1132	8.3574	11.1899	8.4921	9.7947	13.0079	9.9407	11.3249	14.5903	11.3624	13.1096	16.5698
	0.15	5.5517	6.4628	8.5799	6.8503	7.9715	10.5575	8.0361	9.2375	11.9229	9.2001	10.5302	13.4982	10.4601	11.9690	15.1505
	0.20	5.5002	6.3782	8.3830	6.6881	7.8009	10.0668	7.8074	8.9833	11.4653	8.9116	10.2028	12.9757	10.1427	11.5317	14.5941
	0.25	5.4302	6.2639	8.2015	6.5308	7.5437	9.7759	7.6025	8.7599	11.1715	8.6454	9.8658	12.5170	9.7711	11.0874	13.9765
	0.30	5.3219	6.1663	8.0456	6.3449	7.3553	9.5496	7.3519	8.4373	10.7061	8.3220	9.5303	12.1156	9.3341	10.5981	13.1834
	0.35	5.1981	5.9912	7.7122	6.1724	7.0908	9.0590	7.0952	8.0957	10.4705	7.9968	9.1392	11.5310	8.9452	10.1039	12.5803
	0.40	4.9727	5.7738	7.3688	5.8689	6.7848	8.6387	6.7994	7.7507	9.8695	7.6178	8.7100	10.9807	8.5040	9.6427	11.8860
	0.50	3.8208	4.4784	5.9504	4.5647	5.3171	7.0590	5.2995	6.1557	7.9341	6.0208	6.9484	8.9678	6.7197	7.7247	9.6328
8	0	5.9538	6.8218	9.2027	7.3724	8.5981	11.4308	8.7480	10.1628	13.3083	10.0604	11.5412	15.1155	11.3612	13.0791	16.7310
	0.15	5.8619	6.6857	8.7029	7.1203	8.2320	10.7199	8.3113	9.5514	12.4705	9.4856	10.8120	13.8942	10.6607	12.1225	15.3768
	0.20	5.8117	6.6225	8.5898	6.9943	8.0618	10.4275	8.1360	9.3507	12.0043	9.2237	10.5281	13.3558	10.3287	11.7905	14.7645
	0.25	5.7658	6.5508	8.4004	6.8680	7.9168	10.0472	7.9945	9.0406	11.7531	9.0183	10.2201	12.9679	10.0066	11.3559	14.0737
	0.30	5.6893	6.4365	8.2180	6.6872	7.6851	9.7292	7.7124	8.7378	11.1483	8.7228	9.8629	12.4153	9.5929	10.9545	13.4411
	0.35	5.5430	6.2752	8.0612	6.4761	7.4471	9.4032	7.4437	8.4098	10.6454	8.4231	9.4925	11.7495	9.1644	10.3741	12.8120
	0.40	5.3275	6.0772	7.6688	6.2024	7.1298	8.9635	7.1009	8.0354	10.0347	8.0160	9.0986	11.2270	8.7405	9.8456	12.2135
	0.50	4.1161	4.7596	6.3306	4.8852	5.6085	7.3019	5.5787	6.4421	8.2252	6.3509	7.3032	9.2788	6.9766	7.9240	10.0840
9	0	6.2631	7.1389	9.1523	7.5814	8.7388	11.4959	8.9095	10.2185	13.2170	10.2774	11.8159	15.1942	11.5373	13.1556	17.0805
	0.15	6.1773	7.0149	8.8224	7.3536	8.3967	10.9917	8.5494	9.7486	12.2807	9.7810	11.0871	14.1818	10.7929	12.2739	15.5924
	0.20	6.1294	6.9642	8.6929	7.2676	8.2583	10.6443	8.3951	9.5152	11.9317	9.5997	10.8113	13.5369	10.5423	11.9775	15.0552
	0.25	6.0735	6.8744	8.5370	7.1434	8.0752	10.3620	8.2044	9.2672	11.7128	9.3654	10.5310	13.1694	10.2192	11.6144	14.5251
	0.30	5.9731	6.7574	8.4020	7.0071	7.9107	10.0835	7.9890	9.0262	11.4042	9.0574	10.1757	12.7885	9.8686	11.0579	14.0424
	0.35	5.8586	6.6413	8.2492	6.8143	7.6871	9.9102	7.7241	8.6793	10.8906	8.6595	9.7680	12.3532	9.4366	10.6016	13.3856
	0.40	5.6464	6.4081	8.0726	6.5533	7.4161	9.4690	7.3839	8.3459	10.4584	8.2960	9.3091	11.6011	8.9990	10.0444	12.4772
	0.50	4.4464	5.1016	6.5752	5.1528	5.8757	7.6693	5.8735	6.7287	8.6121	6.6405	7.5797	9.5602	7.1927	8.1507	10.4038
10	0	6.6176	7.4914	9.5063	7.8477	8.8745	11.6600	9.1615	10.4507	13.7900	10.4457	11.9950	15.7040	11.7928	13.3244	16.8277
	0.15	6.5454	7.3796	9.2409	7.6692	8.6245	11.1726	8.8433	10.0080	12.7778	9.9978	11.3520	14.2820	11.1151	12.3546	15.4803
	0.20	6.5075	7.3278	9.1643	7.5633	8.5439	10.9134	8.6946	9.7975	12.5732	9.7542	11.0671	13.9288	10.7932	12.0950	15.0682
	0.25	6.4572	7.2417	9.0526	7.4501	8.3825	10.5690	8.5000	9.6149	12.1579	9.5356	10.7332	13.5526	10.4814	11.8587	14.5012
	0.30	6.3884	7.1498	8.7679	7.3089	8.2522	10.3865	8.3154	9.4182	11.8060	9.2918	10.4187	13.2551	10.1566	11.4271	13.9510
	0.35	6.2654	7.0096	8.6433	7.1415	7.9905	10.0299	8.0555	9.0774	11.4300	8.9963	10.0513	12.6878	9.7907	10.9305	13.4471
	0.40	6.0705	6.7826	8.4336	6.8542	7.7637	9.7175	7.7266	8.7372	11.0369	8.6329	9.6284	11.9171	9.3206	10.3392	12.8186
	0.50	4.7948	5.5210	7.0740	5.4541	6.2782	8.0810	6.2025	7.0750	9.0192	6.8658	7.8535	10.0305	7.4750	8.4549	10.4821

Table 1: Asymptotic critical values for the supremum mapping (continued)

p	s_0	$q-p=6$			$q-p=7$			$q-p=8$			$q-p=9$			$q-p=10$		
		10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
1	0	12.0278	13.8867	18.0133	13.2941	15.3660	19.5146	14.7705	16.7431	21.3467	16.2942	18.4361	23.2096	17.2716	19.5937	24.4227
	0.15	10.2618	11.8700	15.3363	11.4475	13.0633	16.7531	12.6292	14.4055	18.1061	13.7891	15.6698	19.7162	14.6798	16.6948	21.0456
	0.20	9.7258	11.1535	14.4465	10.8202	12.3476	15.9321	11.9507	13.5566	17.0725	13.0071	14.7212	18.9150	13.9251	15.7367	19.6192
	0.25	9.1520	10.5289	13.7446	10.1828	11.6944	14.8675	11.1823	12.7893	16.1834	12.1920	13.8027	17.6524	13.0683	14.7320	18.4698
	0.30	8.5962	9.8653	12.7897	9.5768	10.9758	13.8087	10.4769	12.0933	15.0683	11.4059	12.9377	16.4094	12.2906	13.8731	17.2115
	0.35	8.0420	9.2507	11.7145	8.9408	10.2211	13.0719	9.8146	11.1558	14.0743	10.5748	12.0922	15.2929	11.4620	12.9159	15.9006
	0.40	7.4106	8.5280	10.8256	8.3133	9.4554	12.0847	9.0758	10.3146	13.0910	9.7829	11.1977	13.9101	10.6206	11.8755	14.7214
	0.50	5.5870	6.5443	8.4677	6.2815	7.3228	9.3475	6.9936	8.0327	10.3220	7.5676	8.6857	10.9264	8.2851	9.3546	11.7818
2	0	12.1783	14.0338	17.7614	13.5227	15.4638	19.3457	14.8762	17.1151	21.7348	16.1164	18.4063	22.8377	17.5500	19.8245	24.5089
	0.15	10.5992	12.2581	15.6667	11.6712	13.3125	16.5353	12.9480	14.7049	18.8480	13.9045	15.8285	19.6310	15.1162	17.2408	21.2349
	0.20	10.0941	11.5740	14.7388	11.0890	12.5405	15.5944	12.2434	13.9518	18.0828	13.1453	15.0623	18.5805	14.3561	16.3021	20.1236
	0.25	9.5638	10.9504	13.9430	10.4580	11.8245	14.7332	11.5289	13.2229	16.8425	12.4390	14.0680	17.3094	13.5757	15.3016	18.8450
	0.30	8.9885	10.4183	13.2262	9.7725	11.1230	13.8627	10.8125	12.3345	15.8206	11.6232	13.1853	16.2476	12.7051	14.3511	17.6917
	0.35	8.4465	9.7479	12.3385	9.1612	10.3626	12.9294	10.1052	11.4942	14.5855	10.8639	12.3779	15.1240	11.8667	13.3668	16.5297
	0.40	7.8784	9.0498	11.5393	8.5438	9.6600	12.0763	9.3962	10.6999	13.5546	10.0869	11.5182	14.3335	10.9748	12.3708	15.1592
	0.50	5.9924	6.9679	9.0283	6.5719	7.6472	9.8905	7.2549	8.3649	10.7143	7.8916	8.9628	11.1892	8.5534	9.7538	12.0103
3	0	12.2016	14.0378	18.1663	13.5166	15.5604	19.6497	14.8760	16.9702	21.1017	16.0357	18.1834	22.7354	17.5377	19.8695	24.6699
	0.15	10.7894	12.4086	15.8315	11.8979	13.6525	17.1819	12.9918	14.8153	18.2547	13.9631	15.7635	19.5960	15.2731	17.2296	21.3597
	0.20	10.2514	11.6780	15.2206	11.3005	12.8714	16.1727	12.4162	14.0377	17.2296	13.2805	15.0370	18.3727	14.4875	16.3329	20.3354
	0.25	9.7294	11.0939	14.4625	10.7303	12.2770	15.2659	11.7464	13.2583	16.5010	12.6737	14.2321	17.2803	13.7033	15.4070	18.8600
	0.30	9.2376	10.5265	13.4401	10.0464	11.4962	14.5172	11.0803	12.4945	15.4073	11.9129	13.4625	16.4026	12.8519	14.4196	17.7168
	0.35	8.6439	9.9841	12.5033	9.4357	10.8313	13.5073	10.3881	11.7270	14.5432	11.2031	12.6467	15.5456	12.0740	13.4559	16.6381
	0.40	8.0785	9.2844	11.4712	8.8835	10.0594	12.5576	9.7065	11.0214	13.8246	10.4550	11.7473	14.4474	11.2128	12.5961	15.5129
	0.50	6.1995	7.2206	9.3303	6.8034	7.8485	9.9770	7.5572	8.7084	11.0113	8.1545	9.3088	11.6688	8.8419	10.0548	12.4878
4	0	12.2440	14.1438	18.1693	13.7027	15.7624	19.5969	14.8432	17.1170	21.5417	16.2990	18.4002	22.8265	17.5738	19.6919	24.1938
	0.15	10.8999	12.4622	16.1025	12.1678	13.8619	17.4412	13.2382	15.0889	18.9937	14.3746	16.1395	19.9357	15.4791	17.3734	21.0158
	0.20	10.4203	11.8271	15.3209	11.6039	13.1619	16.5628	12.5784	14.3178	17.9871	13.6445	15.3606	18.8872	14.7523	16.5577	19.8196
	0.25	9.9096	11.2881	14.4624	11.0080	12.5125	15.9964	11.9739	13.6508	16.9680	12.9680	14.5592	17.7345	13.9350	15.7166	18.9826
	0.30	9.5048	10.7686	13.7096	10.3825	11.7788	15.0876	11.2956	12.7820	15.9878	12.3058	13.7342	16.9555	13.1354	14.7888	18.0269
	0.35	8.9577	10.1585	12.8458	9.7978	11.1133	14.2034	10.6735	12.0731	15.1174	11.6156	12.9971	16.1429	12.3755	13.9327	16.9583
	0.40	8.3372	9.5935	11.9212	9.1632	10.4374	13.0717	9.9948	11.2505	14.0355	10.7953	12.1818	14.8897	11.5293	12.9419	15.6255
	0.50	6.5154	7.5454	9.7837	7.2090	8.2843	10.4114	7.8970	8.9283	11.1231	8.5033	9.6631	12.0274	9.1042	10.2993	12.7738
5	0	12.2549	14.0767	18.0027	13.7269	15.6476	19.9182	15.0712	17.0842	21.5448	16.3173	18.3222	22.5418	17.6397	19.9305	25.0414
	0.15	10.9802	12.4731	16.1522	12.2289	14.0540	17.3490	13.3350	15.0219	18.8731	14.4284	16.2570	20.0212	15.7481	17.6760	22.0673
	0.20	10.5647	12.0413	15.3859	11.6804	13.2889	16.5905	12.7412	14.4791	17.8656	13.7616	15.4777	19.0649	15.0072	16.9259	20.9362
	0.25	10.0820	11.5598	14.6661	11.1508	12.6871	15.8169	12.2078	13.6993	17.2306	13.0833	14.6705	18.2270	14.2727	15.9883	19.9278
	0.30	9.5887	10.9278	13.7668	10.6791	12.0636	15.0998	11.6518	13.0068	16.1871	12.3683	14.0305	17.0411	13.4912	15.0896	18.6144
	0.35	9.1108	10.3052	13.0563	10.1307	11.4288	14.2855	10.9768	12.2736	15.1675	11.7196	13.2204	15.9662	12.6764	14.2479	17.5415
	0.40	8.5691	9.6764	12.2485	9.4985	10.7134	13.2862	10.2934	11.5416	14.1519	11.0268	12.3958	15.0019	11.8227	13.2813	16.4711
	0.50	6.6776	7.7027	9.7974	7.4804	8.5855	10.8101	8.1193	9.2255	11.6007	8.6698	9.8379	12.1125	9.3686	10.6165	13.3072
6	0	12.5192	14.3935	18.6810	13.8152	15.8230	19.7867	15.1144	17.1702	21.5552	16.5512	18.8435	23.5056	17.8608	20.0350	24.8286
	0.15	11.3261	12.9387	16.4931	12.4830	14.1981	17.8361	13.5744	15.2843	19.1369	14.8140	16.7926	21.1222	15.9181	17.8557	21.8899
	0.20	10.8656	12.4132	15.5687	11.9979	13.5929	17.0117	13.0372	14.6010	18.1127	14.1775	16.0034	20.0213	15.2498	16.9708	20.9355
	0.25	10.4763	11.8393	15.1038	11.5071	13.0465	15.9691	12.4765	13.8814	17.2978	13.4970	15.2692	19.2015	14.5229	16.0711	19.8417
	0.30	10.0870	11.3207	14.2060	11.0009	12.4308	15.1674	11.8638	13.2473	16.3101	12.8365	14.4819	18.2548	13.7726	15.2514	18.7135
	0.35	9.5726	10.7485	13.3709	10.3889	11.6679	14.2666	11.2343	12.5752	15.4237	12.1504	13.6987	17.0258	12.9927	14.4638	17.6659
	0.40	9.0587	10.2475	12.5197	9.7917	10.9796	13.4887	10.6099	11.9479	14.7801	11.4001	12.8010	15.8373	12.1845	13.4718	16.3539
	0.50	7.1285	8.2005	10.2534	7.7370	8.7470	10.9698	8.4322	9.5374	11.9654	9.1102	10.3449	12.8433	9.7945	10.9895	13.3438
7	0	12.6872	14.5641	18.4356	14.0473	16.0118	20.3080	15.2084	17.2676	21.4486	16.5417	18.6581	23.3009	17.8822	20.0102	24.8730
	0.15	11.6471	13.2581	16.9070	12.7855	14.5450	17.8669	13.8581	15.4371	19.2307	15.0302	16.8084	20.6614	16.0047	17.9240	22.0112
	0.20	11.2758	12.7952	16.3664	12.3232	13.9137	17.1129	13.3588	14.8338	18.3043	14.3483	16.0420	19.9419	15.3880	17.1725	20.9000
	0.25	10.7987	12.2741	15.3541	11.7673	13.2611	16.4756	12.7755	14.3047	17.4292	13.6666	15.3029	18.6949	14.7182	16.3997	20.0244
	0.30	10.3135	11.6595	14.5113	11.2205	12.6447	15.6945	12.1940	13.5531	16.6075	13.0553	14.6387	17.7501	14.0334	15.6275	18.9591
	0.35	9.7890	11.0964	13.7555	10.7071	11.9417	14.7813	11.4774	12.7928	15.7581	12.4167	13.7754	16.9756	13.3075	14.7989	17.9971
	0.40	9.2670	10.5613	13.0050	10.1245	11.3430	13.8432	10.8445	12.1222	14.9613	11.5839	12.9154	15.9646	12.4414	13.8606	16.7176
	0.50	7.3610	8.3947	10.6747	8.1037	9.1478	11.3589	8.6866	9.7399	12.0833	9.3047	10.4749	12.8484	9.9540	11.1850	13.8647
8	0	12.7966	14.5863	18.3079	14.0732	16.0377	20.3706	15.2789	17.1821	21.5353	16.6496	18.7782	22.8802	18.1188	20.2700	25.2943
	0.15	11.8261	13.4793	16.6083	12.9114	14.5724	18.3533	13.8943	15.6105	19.3111	15.1117	16.7721	20.9204	16.3637	18.3395	21.9960
	0.20	11.4935	12.9457	15.9568	12.4848	14.0740	17.5810	13.4026	15.0663	18.6681	14.5514	16.1890	20.2139	15.7119	17.5274	21.0741
	0.25	11.0156	12.4451	15.4461	11.9669	13.5513	16.8890	12.9131	14.4481	17.5469	13.9728	15.4839	19.0600	15.0218	16.7194	20.0143
	0.30	10.5680	11.9463	14.7062	11.4929	12.9820	15.9148	12.3618	13.6765	16.9143	13.3557	14.8063	18.0014	14.3121	15.9411	19.1575
	0.35	10.1118	11.3888	13.9201	10.9842	12.2840	15.1251	11.7833	13.1276	15.8012	12.7136	14.1413	17.1162	13.5193	15.0704	18.1635
	0.40	9.5835	10.7833	13.1875	10.4043	11.6563	14.2525	11.1446	12.4168	14.9879	11.9776	13.2975	15.8554	12.7516	14.0917	17.1562
	0.50	7.6491	8.7468	10.9457	8.3082	9.4531	11.7782	8.9311	10.0989	12.3543	9.6289	10.8028	13.2364	10.2765	11.3882	13.7629
9	0	12.8637	14.5416	18.4652	14.2514	16.2440	20.5182	15.5302	17.4051	21.5472	16.7408	18.9121	23.6552	18.0499	20.2402	25.0841
	0.15	11.9505	13.4126	16.7727	13.1286	14.9561	18.7724	14.2656	15.9164	19.2912	15.3079	17.1150	21.2066	16.4994	18.4123	22.2551
	0.20	11.6278	13.0257	16.0392	12.7502	14.3585	17.8597	13.7209	15.3751	18.5979	14.7680	16.6438	20.4024	15.9442	17.7513	21.4420
	0.25	11.1971	12.6439	15.6003	12.2537	13.7942	16.9870	13.1965	14.7540	18.0646	14.1841	15.8642	19.4797	15.2509	17.0367	20.4494
	0.30	10.8279	12.1221	15.0235	11.7407	13.1548	16.2892	12.6649	14.1524	17.0812	13.6444	15.1387	18.4806	14.5380	16.2581	19.7701
	0.35	10.3776	11.5560	14.3045	11.2318	12.5257	15.3526	12.0909	13.4473	16.2909	12.9474	14.4295	17.7007	13.8228	15.3610	18.6156
	0.40	9.8345	10.9176	13.5024	10.6689	11.8771	14.3867	11.4656	12.7091	15.4550	12.3070	13.6408	16.6975	13.0643	14.3844	17.6064
	0.50	7.8392	8.8189	11.1361	8.5719	9.6446	11.7929	9.2520	10.3483	12.4563	9.9647	11.1390	13.5845	10.5706	11.8283	14.3659
10	0	13.0759	14.8635	18.1336	14.4165	16.2218	20.5995	15.8079	17.7145	21.9581	17.0738	19.2076	23.1358	17.8815	20.1171	24.4278
	0.15	12.2789	13.8498	16.9923	13.3565	15.0097	18.5659	14.6306	16.3298	19.9454	15.6394	17.4548	21.2010	16.3852	18.3502	22.3689
	0.20	11.9291	13.4720	16.6387	12.9559	14.5317	17.9465	14.1630	15.7859	19.0951	15.0028	16.7935	20.2892	15.8499	17.7352	21.6110
	0.25	11.5856	12.9968	15.9801	12.4890	14.1057	17.3481	13.6656	15.1328	18.3016	14.4604	16.0756	19.4643	15.2958	16.9854	20.8025
	0.30	11.2017	12.4927	15.3458	12.0228	13.4460	16.6663	13.0876	14.5052	17.4440	13.8958	15.4540	18.5856	14.6646	16.2717	19.5789
	0.35	10.7254	11.9989	14.5328	11.5034	12.8037	15.6884	12.5107	13.9020	16.6525	13.2031	14.6477	17.3920	14.0167	15.4847	18.6285
	0.40	10.2511	11.4554	13.9318	10.9511	12.1339	14.8627	11.8253	13.2041	15.7823	12.5562	13.8458	16.6407	13.2792	14.6810	17.5179
	0.50	8.3265	9.3384	11.6032	8.8419	9.9409	12.3731	9.5944	10.6847	12.9892	10.2269	11.4140	13.7764	10.9110	12.0759	14.6294

Table 2: Asymptotic critical values for the exponential mapping

p	s_0	$q-p=1$			$q-p=2$			$q-p=3$			$q-p=4$			$q-p=5$		
		10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
1	0	0.8342	1.1957	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184
	0.15	0.8379	1.1546	1.9415	1.3543	1.7758	2.7809	1.8344	2.3208	3.4365	2.3752	2.9141	4.2105	2.8596	3.4031	4.7544
	0.20	0.8385	1.1486	1.8834	1.3372	1.7275	2.6528	1.8015	2.2507	3.3235	2.2941	2.8410	4.0170	2.7551	3.2636	4.5235
	0.25	0.8491	1.1587	1.8387	1.3219	1.6854	2.6456	1.7742	2.1852	3.2144	2.2359	2.7452	3.8538	2.6720	3.1632	4.2894
	0.30	0.8585	1.1723	1.8146	1.3066	1.6663	2.6216	1.7512	2.1483	3.0883	2.1788	2.6900	3.7167	2.6079	3.0920	4.1122
	0.35	0.8676	1.1696	1.8048	1.3042	1.6620	2.5712	1.7168	2.1103	3.0351	2.1580	2.6348	3.5636	2.5538	3.0180	4.0444
	0.40	0.8757	1.1722	1.8095	1.3064	1.6548	2.4791	1.7062	2.0836	2.9636	2.1428	2.6009	3.4739	2.5243	2.9695	3.9511
	0.50	0.8768	1.1839	1.8439	1.3126	1.6601	2.4952	1.7079	2.0945	2.9003	2.1087	2.5588	3.4494	2.4819	2.9518	3.9137
2	0	0.9578	1.2598	2.1343	1.4994	1.9330	3.1166	2.1388	2.7414	4.1997	2.7060	3.3159	4.7715	3.2466	3.9556	5.5217
	0.15	0.9777	1.2582	2.0291	1.4615	1.8288	2.7674	2.0108	2.5077	3.6876	2.4871	2.9515	4.2438	2.9295	3.5027	4.8123
	0.20	0.9918	1.2615	2.0075	1.4483	1.7909	2.6273	1.9802	2.4569	3.5250	2.4197	2.8787	4.1032	2.8402	3.3969	4.6098
	0.25	1.0012	1.2761	1.9971	1.4410	1.7646	2.6001	1.9514	2.3893	3.4421	2.3545	2.8151	3.9598	2.7766	3.2873	4.4235
	0.30	1.0155	1.2870	1.9662	1.4369	1.7589	2.5414	1.9169	2.3685	3.3243	2.3169	2.7823	3.8352	2.7132	3.1755	4.2809
	0.35	1.0326	1.2954	1.9664	1.4365	1.7554	2.5192	1.9005	2.3233	3.2285	2.2901	2.7440	3.7739	2.6693	3.0898	4.1580
	0.40	1.0505	1.3174	1.9999	1.4375	1.7644	2.5302	1.8884	2.3008	3.1396	2.2782	2.7160	3.6899	2.6339	3.0293	4.0512
	0.50	1.0731	1.3501	2.0944	1.4455	1.8021	2.5878	1.8789	2.2907	3.1420	2.2684	2.7171	3.7105	2.5645	3.0606	4.0696
3	0	1.0553	1.3942	2.2999	1.6539	2.0587	3.1116	2.1948	2.7097	4.0457	2.8531	3.5273	4.9643	3.3735	4.0583	5.6029
	0.15	1.1224	1.4241	2.1979	1.6330	1.9896	2.9023	2.0732	2.5182	3.5580	2.6503	3.1988	4.4272	3.1017	3.7025	5.0346
	0.20	1.1470	1.4418	2.1749	1.6210	1.9716	2.8205	2.0441	2.4808	3.4027	2.6016	3.1140	4.3050	2.9993	3.5892	4.8263
	0.25	1.1686	1.4573	2.1720	1.6273	1.9633	2.7418	2.0394	2.4285	3.3022	2.5495	3.0254	4.1382	2.9448	3.4851	4.6456
	0.30	1.1932	1.4682	2.1503	1.6218	1.9655	2.7336	2.0409	2.4063	3.2412	2.4977	2.9424	3.9683	2.8888	3.3843	4.4007
	0.35	1.2145	1.4935	2.1414	1.6208	1.9603	2.7195	2.0272	2.3882	3.2353	2.4662	2.9059	3.8526	2.8378	3.2947	4.3243
	0.40	1.2234	1.5134	2.1541	1.6316	1.9695	2.7183	2.0302	2.3850	3.2270	2.4425	2.8871	3.7855	2.7981	3.2617	4.2302
	0.50	1.2397	1.5292	2.2644	1.6359	2.0022	2.7198	2.0309	2.4006	3.3455	2.4208	2.8990	3.7920	2.7552	3.2313	4.2877
4	0	1.1518	1.4518	2.2657	1.7297	2.1693	3.2758	2.3685	2.9440	4.2831	2.8440	3.4817	4.9303	3.4660	4.1792	5.8806
	0.15	1.2349	1.5324	2.2484	1.7447	2.1066	3.0549	2.2883	2.7774	3.9052	2.6887	3.1974	4.3784	3.2076	3.8078	5.1973
	0.20	1.2576	1.5475	2.2341	1.7469	2.0818	2.9614	2.2674	2.7315	3.7777	2.6473	3.1282	4.2182	3.1467	3.7071	5.0243
	0.25	1.2844	1.5727	2.2456	1.7479	2.0797	2.9083	2.2516	2.6865	3.6476	2.6082	3.0657	4.0761	3.0944	3.6050	4.8549
	0.30	1.3110	1.5999	2.2442	1.7564	2.0848	2.9015	2.2553	2.6562	3.5525	2.5781	3.0181	3.9848	3.0309	3.5152	4.6790
	0.35	1.3305	1.6206	2.2876	1.7583	2.0971	2.9071	2.2497	2.6490	3.5423	2.5523	2.9732	3.9342	2.9786	3.4250	4.5008
	0.40	1.3481	1.6449	2.2870	1.7678	2.0928	2.9210	2.2277	2.6239	3.5001	2.5456	2.9463	3.9149	2.9526	3.3914	4.4537
	0.50	1.3789	1.6481	2.3341	1.7827	2.1247	2.9685	2.2149	2.6342	3.5148	2.5391	2.9584	3.9255	2.9243	3.3710	4.3867
5	0	1.2498	1.5596	2.3774	1.8510	2.3043	3.2659	2.3936	2.8944	4.2408	2.9309	3.5258	4.9943	3.5025	4.1622	5.8794
	0.15	1.3533	1.6605	2.3635	1.8853	2.2884	3.1305	2.3585	2.8171	3.9706	2.7983	3.2952	4.6497	3.3052	3.8525	5.2123
	0.20	1.3899	1.6803	2.3827	1.9000	2.2878	3.1349	2.3637	2.8084	3.8767	2.7643	3.2268	4.5082	3.2402	3.7992	4.9896
	0.25	1.4274	1.7092	2.4031	1.9195	2.2790	3.0720	2.3576	2.7979	3.8198	2.7313	3.1935	4.3834	3.1862	3.6994	4.7857
	0.30	1.4586	1.7465	2.4348	1.9355	2.2865	3.0756	2.3635	2.7808	3.7089	2.7262	3.1717	4.2070	3.1339	3.6245	4.7499
	0.35	1.4971	1.7728	2.4633	1.9500	2.3058	3.0787	2.3631	2.7761	3.6832	2.7133	3.1362	4.0530	3.0909	3.5695	4.6276
	0.40	1.5277	1.8009	2.4806	1.9607	2.3228	3.0503	2.3662	2.7771	3.6663	2.7086	3.1365	4.0747	3.0741	3.5525	4.5451
	0.50	1.5553	1.8564	2.5715	1.9631	2.3515	3.1075	2.3762	2.7847	3.7039	2.7092	3.1452	4.0868	3.0474	3.5241	4.4726
6	0	1.3840	1.6913	2.5068	1.9421	2.3924	3.4879	2.4789	2.9772	4.3705	3.0626	3.6564	5.1509	3.6522	4.3545	5.8617
	0.15	1.5124	1.8010	2.5464	2.0078	2.3995	3.3772	2.4572	2.9097	4.0222	2.9616	3.4994	4.7226	3.4587	4.0845	5.4314
	0.20	1.5513	1.8440	2.5539	2.0372	2.4227	3.3019	2.4527	2.8832	3.9459	2.9407	3.4457	4.5994	3.4070	4.0051	5.2755
	0.25	1.5923	1.8755	2.5875	2.0659	2.4209	3.2663	2.4471	2.8685	3.8803	2.9360	3.3963	4.4754	3.3600	3.9481	5.1084
	0.30	1.6411	1.9153	2.6425	2.0875	2.4551	3.2546	2.4504	2.8820	3.8126	2.9200	3.3728	4.4071	3.3275	3.8564	4.9778
	0.35	1.6672	1.9541	2.6916	2.0973	2.4768	3.2198	2.4630	2.8768	3.7699	2.9108	3.3500	4.2909	3.2835	3.7920	4.8460
	0.40	1.6911	2.0042	2.7378	2.1179	2.4763	3.2388	2.4734	2.8741	3.7449	2.8951	3.3331	4.2545	3.2667	3.7487	4.7593
	0.50	1.7333	2.0499	2.7967	2.1220	2.4982	3.3173	2.4878	2.9031	3.7230	2.8742	3.3486	4.2861	3.2318	3.7293	4.7775
7	0	1.5271	1.8666	2.7316	2.0600	2.5453	3.5862	2.5770	3.0885	4.3393	3.1346	3.7010	5.0892	3.7098	4.4286	5.8817
	0.15	1.6769	1.9789	2.7193	2.1239	2.5446	3.5143	2.5869	3.0401	4.0849	3.0260	3.5828	4.7224	3.5493	4.1444	5.3909
	0.20	1.7280	2.0223	2.7414	2.1577	2.5535	3.4960	2.6009	3.0336	4.0619	3.0099	3.5351	4.6557	3.5132	4.0713	5.2961
	0.25	1.7680	2.0761	2.7471	2.1778	2.5717	3.4497	2.6072	3.0386	4.0116	3.0039	3.4871	4.5949	3.4667	3.9874	5.1241
	0.30	1.8035	2.1231	2.7955	2.2088	2.5864	3.4079	2.6133	3.0419	3.9842	3.0013	3.4501	4.5286	3.4310	3.9037	5.0088
	0.35	1.8466	2.1669	2.8197	2.2303	2.5956	3.3881	2.6313	3.0348	3.9666	3.0043	3.4486	4.5094	3.3960	3.8803	4.9349
	0.40	1.8754	2.1920	2.8378	2.2388	2.6028	3.4214	2.6403	3.0495	3.9582	3.0128	3.4506	4.4639	3.3873	3.8757	4.7984
	0.50	1.9104	2.2392	2.9752	2.2823	2.6586	3.5295	2.6498	3.0779	3.9671	3.0104	3.4742	4.4839	3.3599	3.8623	4.8164
8	0	1.6289	1.9538	2.8305	2.1581	2.6079	3.6726	2.6942	3.2489	4.5327	3.2347	3.8457	5.1969	3.7399	4.4165	5.9687
	0.15	1.7960	2.1068	2.8384	2.2616	2.6796	3.6337	2.7242	3.2170	4.3514	3.2017	3.7177	4.8983	3.6269	4.2330	5.4898
	0.20	1.8526	2.1582	2.8582	2.2885	2.7074	3.5542	2.7378	3.2090	4.2995	3.1969	3.6872	4.8280	3.5930	4.1635	5.4005
	0.25	1.9048	2.2111	2.8887	2.3212	2.7261	3.5949	2.7524	3.1836	4.1968	3.1837	3.6720	4.7669	3.5554	4.0950	5.2190
	0.30	1.9536	2.2559	2.9093	2.3480	2.7513	3.5832	2.7677	3.1757	4.1297	3.1842	3.6626	4.6355	3.5342	4.0457	5.1112
	0.35	1.9907	2.2975	2.9684	2.3660	2.7645	3.5704	2.7822	3.1719	4.1280	3.1801	3.6612	4.5951	3.5096	3.9936	5.0498
	0.40	2.0182	2.3533	3.0391	2.3935	2.7819	3.5532	2.7826	3.1845	4.0880	3.1935	3.6625	4.6012	3.5104	3.9606	5.0031
	0.50	2.0580	2.3798	3.1653	2.4426	2.8042	3.6510	2.7894	3.2210	4.1126	3.1755	3.6516	4.6394	3.4883	3.9620	5.0420
9	0	1.7464	2.0885	2.8257	2.2575	2.6883	3.7524	2.7811	3.2886	4.4879	3.3494	3.9135	5.3830	3.8221	4.5051	6.0611
	0.15	1.9229	2.2486	2.9224	2.3809	2.7564	3.7307	2.8372	3.2775	4.4127	3.3337	3.8340	5.0105	3.7337	4.3141	5.7374
	0.20	1.9882	2.2960	2.9553	2.4275	2.7825	3.7210	2.8574	3.2733	4.3227	3.3452	3.8091	4.9463	3.7131	4.2679	5.5839
	0.25	2.0406	2.3459	2.9911	2.4672	2.8232	3.7123	2.8661	3.3071	4.2701	3.3379	3.7865	4.9190	3.6825	4.1872	5.4478
	0.30	2.0919	2.3931	3.0471	2.4912	2.8581	3.7564	2.8879	3.3195	4.2690	3.3351	3.7732	4.8421	3.6581	4.1304	5.3232
	0.35	2.1357	2.4492	3.1130	2.5266	2.9011	3.7667	2.9055	3.3363	4.2723	3.3221	3.7720	4.7674	3.6385	4.1120	5.2713
	0.40	2.1696	2.4820	3.1852	2.5458	2.9146	3.7689	2.9274	3.3451	4.3010	3.3331	3.7706	4.7359	3.6164	4.0605	5.2064
	0.50	2.2232	2.5508	3.2876	2.5764	2.9379	3.8346	2.9367	3.3643	4.3060	3.3202	3.7899	4.7801	3.5964	4.0753	5.2019
10	0	1.8844	2.1768	3.0026	2.3614	2.7707	3.8702	2.8841	3.3875	4.7244	3.4237	4.0121	5.5098	3.9474	4.5674	5.9862
	0.15	2.0760	2.3627	3.1031	2.4916	2.8747	3.9221	2.9591	3.4519	4.5251	3.4319	3.9679	5.2167	3.8672	4.4126	5.5970
	0.20	2.1369	2.4259	3.1526	2.5410	2.9216	3.9188	2.9904	3.4480	4.5320	3.4362	3.9368	5.1725	3.8338	4.3773	5.5609
	0.25	2.2031	2.4876	3.1852	2.5978	2.9701	3.9218	3.0090	3.4541	4.4781	3.4464	3.9087	5.1297	3.8259	4.3301	5.4477
	0.30	2.2670	2.5576	3.2418	2.6286	3.0060	3.9345	3.0381	3.4661	4.4332	3.4517	3.8879	5.0254	3.8076	4.2590	5.3888
	0.35	2.3141	2.6189	3.2948	2.6707	3.0325	3.9288	3.0613	3.4667	4.4511	3.4558	3.8828	4.9629	3.7871	4.2154	5.2820
	0.40	2.3516	2.6867	3.3557	2.7117	3.0665	3.9681	3.0828	3.4864	4.4390	3.4515	3.8787	4.9539	3.7694	4.2018	5.2732
	0.50	2.3974	2.7605	3.5370	2.7271	3.1391	4.0405	3.1013	3.5375	4.5096	3.4329	3.9267	5.0153	3.7375	4.2275	5.2410

Table 2: Asymptotic critical values for the exponential mapping (continued)

p	s_0	$q-p=6$			$q-p=7$			$q-p=8$			$q-p=9$			$q-p=10$		
		10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
1	0	3.7762	4.5583	6.3487	4.3340	5.1767	7.0518	4.9683	5.8478	7.8408	5.5956	6.5069	8.7303	6.0392	7.0746	9.3698
	0.15	3.3055	3.9354	5.3786	3.7837	4.4698	5.9843	4.2980	5.0138	6.6367	4.7683	5.5502	7.4177	5.2009	6.0159	7.9159
	0.20	3.1784	3.7586	5.1454	3.6258	4.2753	5.6838	4.0703	4.7998	6.3073	4.5261	5.2552	7.0249	4.9512	5.7043	7.3760
	0.25	3.0776	3.6349	4.8546	3.4979	4.1077	5.4523	3.9124	4.6038	5.9681	4.3278	5.0166	6.5803	4.7380	5.4165	6.9628
	0.30	2.9975	3.5235	4.6185	3.3826	3.9434	5.1907	3.7676	4.3926	5.6591	4.1475	4.7932	6.2854	4.5390	5.1837	6.5745
	0.35	2.9233	3.4078	4.4056	3.2789	3.8452	5.0520	3.6574	4.2400	5.4641	3.9983	4.5958	5.8736	4.3802	4.9946	6.2324
	0.40	2.8436	3.3218	4.3063	3.2165	3.7344	4.8895	3.5704	4.1074	5.4003	3.8630	4.4461	5.6614	4.2478	4.8106	5.9643
	0.50	2.7935	3.2722	4.2339	3.1407	3.6614	4.6738	3.4968	4.0163	5.1610	3.7838	4.3429	5.4632	4.1426	4.6773	5.8909
2	0	3.9239	4.6685	6.3144	4.4299	5.2562	6.9617	5.0387	5.9450	8.0641	5.6026	6.5437	8.5147	6.2232	7.2134	9.2988
	0.15	3.5027	4.1331	5.5161	3.9207	4.5622	5.9713	4.4149	5.1972	7.0981	4.8660	5.6573	7.2814	5.4193	6.2707	8.0354
	0.20	3.3733	3.9749	5.2478	3.7699	4.3565	5.6783	4.2379	4.9688	6.7441	4.6449	5.3928	6.9286	5.1631	5.9546	7.6662
	0.25	3.2757	3.8610	5.1199	3.6226	4.1886	5.4058	4.0684	4.7186	6.3130	4.4424	5.1378	6.5980	4.9388	5.6253	7.2090
	0.30	3.1771	3.7318	4.8967	3.5014	4.0283	5.1785	3.9302	4.5297	6.0136	4.2635	4.9360	6.2958	4.7325	5.4065	6.8207
	0.35	3.1262	3.6307	4.7521	3.4160	3.9099	5.0172	3.7943	4.4008	5.6776	4.1365	4.7815	6.0043	4.5551	5.1721	6.5545
	0.40	3.0610	3.5172	4.5629	3.3560	3.8512	4.8955	3.7044	4.2690	5.5372	4.0517	4.6054	5.8041	4.4141	5.0078	6.2373
	0.50	2.9962	3.4839	4.5141	3.2860	3.8236	4.9453	3.6275	4.1824	5.3572	3.9458	4.4814	5.5946	4.2767	4.8769	6.0051
3	0	3.9366	4.6934	6.4621	4.4874	5.3322	7.0521	5.0634	5.9673	7.7719	5.5956	6.5070	8.5295	6.2443	7.2462	9.3070
	0.15	3.5666	4.2047	5.6580	4.0116	4.7196	6.2354	4.5063	5.2456	6.7316	4.9366	5.6842	7.3500	5.4747	6.3027	8.1192
	0.20	3.4696	4.0395	5.4325	3.8766	4.5383	5.9579	4.3522	5.0165	6.4065	4.7597	5.4874	6.8989	5.2366	6.0079	7.6842
	0.25	3.3687	3.9194	5.1265	3.7476	4.3591	5.7117	4.1817	4.8060	6.1478	4.5686	5.2611	6.6218	5.0222	5.7146	7.2019
	0.30	3.2768	3.8216	4.9135	3.6384	4.2260	5.4725	4.0561	4.6375	5.9132	4.4186	5.0763	6.3665	4.8312	5.4640	6.9070
	0.35	3.2135	3.7483	4.7177	3.5540	4.0938	5.2310	3.9470	4.5381	5.7747	4.2893	4.8700	6.1829	4.6564	5.2913	6.6536
	0.40	3.1526	3.6559	4.6425	3.4937	4.0192	5.0897	3.8737	4.4594	5.6494	4.1857	4.7466	5.9630	4.5279	5.1473	6.3944
	0.50	3.0998	3.6103	4.6651	3.4017	3.9242	4.9885	3.7786	4.3542	5.5056	4.0773	4.6544	5.8344	4.4210	5.0274	6.2439
4	0	3.9989	4.7570	6.4031	4.6042	5.4268	7.1840	5.1302	6.0453	8.0613	5.7632	6.6577	8.5940	6.2609	7.2503	9.2492
	0.15	3.6464	4.2451	5.7763	4.1836	4.8531	6.4642	4.5954	5.3678	7.0734	5.1170	5.8881	7.4168	5.5825	6.4115	7.9587
	0.20	3.5513	4.1365	5.5716	4.0301	4.6624	6.1596	4.4547	5.1513	6.6793	4.9076	5.6459	7.1824	5.3669	6.1391	7.5941
	0.25	3.4785	4.0140	5.3464	3.9014	4.4928	5.9900	4.2913	4.9621	6.3884	4.7413	5.4036	6.8963	5.1549	5.8730	7.3680
	0.30	3.4028	3.9147	5.1233	3.7839	4.3625	5.7340	4.1891	4.7900	6.1528	4.6003	5.2351	6.6699	4.9625	5.6519	7.0364
	0.35	3.3458	3.8567	4.9640	3.7070	4.2648	5.4549	4.0861	4.6642	5.9021	4.4846	5.0760	6.4495	4.7853	5.4420	6.7061
	0.40	3.3001	3.8177	4.8875	3.6408	4.2130	5.3037	4.0191	4.5335	5.7494	4.3651	4.9400	6.2766	4.6477	5.2968	6.5407
	0.50	3.2577	3.7727	4.8919	3.6045	4.1421	5.2057	3.9485	4.4642	5.5616	4.2516	4.8316	6.0137	4.5521	5.1497	6.3869
5	0	4.0124	4.7317	6.5212	4.6698	5.4611	7.1668	5.1880	6.0932	7.8910	5.7546	6.5837	8.4830	6.3433	7.3135	9.6401
	0.15	3.7258	4.3382	5.8340	4.2248	4.9411	6.4027	4.7168	5.4197	7.0776	5.1446	5.9417	7.5819	5.7187	6.5901	8.4446
	0.20	3.6346	4.2253	5.5830	4.1100	4.7593	6.1609	4.5782	5.2259	6.8304	4.9498	5.7110	7.2602	5.4999	6.3162	8.0204
	0.25	3.5672	4.1154	5.3664	4.0068	4.6229	5.9188	4.4433	5.0722	6.5384	4.8006	5.5222	6.9536	5.3016	6.0640	7.7496
	0.30	3.4978	4.0106	5.1695	3.9374	4.5073	5.7133	4.3179	4.9156	6.3160	4.6711	5.3420	6.6044	5.0961	5.8409	7.3339
	0.35	3.4262	3.9448	5.0489	3.8707	4.4135	5.5742	4.2299	4.8113	6.0431	4.5745	5.1602	6.3638	4.9438	5.6188	6.9522
	0.40	3.3909	3.8850	4.9597	3.8109	4.3192	5.4764	4.1406	4.6918	5.9267	4.4615	5.0500	6.1853	4.8224	5.4562	6.7306
	0.50	3.3388	3.8513	4.8987	3.7402	4.2928	5.4050	4.0596	4.6127	5.8003	4.3349	4.9189	6.0563	4.6843	5.3083	6.6536
6	0	4.1744	4.9586	6.6719	4.7143	5.5264	7.2461	5.2539	6.1710	8.1273	5.9232	6.8366	9.0118	6.4464	7.4006	9.6205
	0.15	3.8957	4.5539	5.9327	4.3709	5.0678	6.5228	4.8224	5.4930	7.0695	5.3493	6.1673	8.0777	5.8297	6.5880	8.4945
	0.20	3.8330	4.4119	5.7575	4.2491	4.9114	6.2285	4.6817	5.3338	6.8202	5.1548	5.9473	7.6611	5.6071	6.3377	8.1311
	0.25	3.7684	4.3053	5.5727	4.1638	4.7729	6.0036	4.5606	5.1775	6.5992	4.9813	5.7447	7.4279	5.4348	6.1120	7.6734
	0.30	3.6963	4.2352	5.3909	4.0630	4.6497	5.7664	4.4345	5.0445	6.3796	4.8545	5.5569	7.1360	5.2510	5.9162	7.3681
	0.35	3.6653	4.1892	5.2287	3.9847	4.5249	5.6124	4.3514	4.9420	6.1518	4.7329	5.4046	6.7940	5.1135	5.7315	7.0898
	0.40	3.6235	4.1407	5.1417	3.9201	4.4517	5.5413	4.2721	4.8389	6.0634	4.6207	5.2828	6.6067	4.9945	5.5807	6.8750
	0.50	3.5643	4.1002	5.1267	3.8685	4.3735	5.4849	4.2161	4.7687	5.9827	4.5551	5.1725	6.4217	4.8972	5.4948	6.6719
7	0	4.2453	5.0332	6.6737	4.8395	5.6413	7.4693	5.3535	6.1876	8.0324	5.9001	6.8126	8.9114	6.4700	7.4053	9.6017
	0.15	4.0289	4.6849	6.1438	4.4926	5.2061	6.6417	4.9527	5.6185	7.2818	5.4348	6.1779	7.8598	5.9109	6.7219	8.4534
	0.20	3.9357	4.5646	5.9596	4.3819	5.0503	6.4869	4.8285	5.4604	6.9706	5.2166	5.9954	7.5704	5.7396	6.4773	8.1072
	0.25	3.8693	4.4469	5.7457	4.2708	4.8865	6.2866	4.7097	5.3033	6.7116	5.0963	5.8121	7.2451	5.5528	6.2843	7.7897
	0.30	3.7998	4.3427	5.5879	4.2028	4.7558	6.1034	4.5668	5.1501	6.4421	4.9915	5.5754	6.9632	5.4082	6.0873	7.5430
	0.35	3.7448	4.2677	5.3830	4.1485	4.6806	5.8613	4.4632	5.0653	6.3279	4.8521	5.4366	6.7853	5.2349	5.8980	7.2501
	0.40	3.7086	4.2423	5.3750	4.0896	4.6343	5.7043	4.4059	4.9543	6.2270	4.7634	5.3492	6.6435	5.1097	5.7372	7.0419
	0.50	3.6805	4.1973	5.3373	4.0519	4.5739	5.6795	4.3433	4.8699	6.0417	4.6524	5.2374	6.4242	4.9770	5.5925	6.9323
8	0	4.3282	5.0981	6.5849	4.8602	5.6808	7.5076	5.3588	6.2091	8.1384	5.9865	6.8107	8.7520	6.6155	7.5659	9.7197
	0.15	4.1490	4.7502	6.1333	4.5786	5.2879	6.9112	4.9900	5.6994	7.3236	5.5136	6.2734	7.9991	6.0537	6.8784	8.5663
	0.20	4.0775	4.6610	5.9577	4.4911	5.1649	6.6696	4.8808	5.5807	7.0541	5.3802	6.0812	7.6715	5.8545	6.6341	8.1969
	0.25	4.0010	4.5712	5.8182	4.4150	5.0440	6.4467	4.7864	5.4443	6.7867	5.2374	5.9181	7.3129	5.6739	6.4275	7.8707
	0.30	3.9353	4.5101	5.6412	4.3405	4.9225	6.2267	4.7206	5.2609	6.5698	5.1046	5.7394	7.0634	5.5032	6.2134	7.6116
	0.35	3.8969	4.4539	5.5335	4.2602	4.8291	6.0268	4.6169	5.2078	6.3910	5.0318	5.6143	6.9029	5.3579	6.0086	7.3672
	0.40	3.8731	4.3969	5.4409	4.2208	4.7739	5.9081	4.5559	5.1227	6.2771	4.9116	5.4822	6.6451	5.2641	5.8492	7.1085
	0.50	3.8246	4.3734	5.4728	4.1541	4.7265	5.8891	4.4656	5.0495	6.1772	4.8144	5.4014	6.6182	5.1382	5.6941	6.8814
9	0	4.3711	5.1111	6.7196	4.9650	5.8561	7.5928	5.5089	6.3219	8.0094	6.0606	6.9647	9.0408	6.6302	7.5887	9.6699
	0.15	4.1883	4.8421	6.1646	4.7090	5.4034	6.9096	5.1630	5.8780	7.4080	5.6407	6.4389	8.0850	6.1656	6.9695	8.6595
	0.20	4.1292	4.7491	6.0030	4.6107	5.2633	6.6971	5.0420	5.7385	7.1663	5.5064	6.2359	7.8685	5.9719	6.7577	8.4482
	0.25	4.1061	4.6679	5.9280	4.5228	5.1081	6.4638	4.9310	5.5744	6.9450	5.3671	6.0531	7.5904	5.8015	6.5717	8.1730
	0.30	4.0518	4.5657	5.7311	4.4576	5.0193	6.3263	4.8446	5.4479	6.7569	5.2454	5.9059	7.3668	5.6435	6.4025	7.8624
	0.35	4.0071	4.5130	5.6425	4.4024	4.9276	6.1072	4.7700	5.3416	6.5355	5.1415	5.7810	7.0973	5.5250	6.1630	7.6121
	0.40	3.9687	4.4560	5.6008	4.3515	4.8651	5.9995	4.7126	5.2558	6.3595	5.0799	5.6691	6.9073	5.4139	6.0264	7.3398
	0.50	3.9196	4.4094	5.5680	4.2860	4.8223	5.8965	4.6260	5.1741	6.2281	4.9823	5.5695	6.7923	5.2853	5.9141	7.1829
10	0	4.5080	5.2545	6.6941	5.0512	5.8037	7.6700	5.6858	6.4687	8.3182	6.1742	7.1168	8.8438	6.5796	7.5058	9.6262
	0.15	4.3759	5.0007	6.3896	4.8230	5.4739	7.0306	5.3460	6.0631	7.5284	5.7653	6.5271	8.1161	6.1284	6.9207	8.7560
	0.20	4.3407	4.9468	6.2357	4.7475	5.3704	6.8253	5.2446	5.9191	7.2549	5.6186	6.3290	7.8585	6.0006	6.7474	8.4453
	0.25	4.2834	4.8705	6.0915	4.6586	5.2835	6.6320	5.1146	5.7709	7.1176	5.4836	6.1457	7.6028	5.8519	6.5796	8.1201
	0.30	4.2406	4.7974	5.9739	4.5869	5.1658	6.4483	5.0248	5.6537	6.8459	5.3633	6.0078	7.3689	5.7276	6.4023	7.8976
	0.35	4.2038	4.7421	5.8465	4.5397	5.0619	6.3642	4.9351	5.5283	6.6926	5.2665	5.8885	7.1212	5.6264	6.2562	7.6568
	0.40	4.2022	4.6906	5.7653	4.4959	4.9995	6.2252	4.8682	5.4581	6.5759	5.1695	5.7962	6.9920	5.5197	6.1530	7.4758
	0.50	4.1632	4.6692	5.8016	4.4210	4.9704	6.1866	4.7972	5.3424	6.4946	5.1134	5.7070	6.8882	5.4555	6.0380	7.3147

Table 3: Asymptotic critical values for the average mapping

p	s_0	$q-p=1$			$q-p=2$			$q-p=3$			$q-p=4$			$q-p=5$		
		10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
1	0	1.3855	1.8811	3.0387	2.2209	2.8120	4.2081	2.9541	3.6043	4.9969	3.7560	4.4533	5.9559	4.4303	5.1393	6.6764
	0.15	1.4691	1.9944	3.0965	2.2958	2.8970	4.2977	3.0354	3.6743	5.1468	3.8245	4.5642	6.1265	4.5015	5.2550	6.8338
	0.20	1.5087	2.0292	3.1642	2.3415	2.9249	4.4277	3.0795	3.7524	5.2302	3.8551	4.6409	6.1638	4.5553	5.3286	6.9269
	0.25	1.5577	2.0932	3.1768	2.3754	2.9771	4.5237	3.1239	3.7991	5.3386	3.8955	4.7241	6.3088	4.6192	5.3775	7.0261
	0.30	1.5934	2.1320	3.2965	2.4048	3.0484	4.6350	3.1707	3.8715	5.4273	3.9669	4.7744	6.3762	4.6676	5.4613	7.1244
	0.35	1.6427	2.1873	3.3369	2.4692	3.1050	4.7207	3.2161	3.9387	5.4776	4.0306	4.8561	6.5156	4.7405	5.5801	7.2541
	0.40	1.6934	2.2536	3.4014	2.5164	3.1792	4.7887	3.2884	3.9976	5.5535	4.1052	4.9419	6.5545	4.8129	5.6580	7.3917
	0.50	1.7537	2.3678	3.6877	2.6251	3.3202	4.9903	3.4158	4.1890	5.8005	4.2173	5.1175	6.8987	4.9638	5.9035	7.8275
2	0	1.5896	1.9941	3.1470	2.3773	2.9046	4.1651	3.1955	3.8680	5.3218	3.9026	4.5962	6.1384	4.5338	5.2583	6.8338
	0.15	1.7402	2.1770	3.3180	2.4984	3.0293	4.3251	3.3483	4.0337	5.4609	4.0451	4.7846	6.3516	4.6937	5.4251	6.9494
	0.20	1.8018	2.2502	3.4012	2.5496	3.0909	4.4259	3.3917	4.1084	5.5956	4.1022	4.8515	6.4292	4.7556	5.5005	7.0784
	0.25	1.8599	2.3223	3.4730	2.6062	3.1640	4.4495	3.4442	4.1713	5.7219	4.1619	4.9355	6.5469	4.8037	5.5317	7.2595
	0.30	1.9183	2.3919	3.5877	2.6566	3.2435	4.5796	3.4986	4.2634	5.8012	4.2051	5.0031	6.6750	4.8955	5.6186	7.4170
	0.35	1.9720	2.4638	3.7232	2.7119	3.3111	4.7496	3.5794	4.3340	5.8784	4.2760	5.1030	6.7692	4.9651	5.7036	7.5222
	0.40	2.0292	2.5380	3.8378	2.7832	3.4028	4.9134	3.6375	4.4238	6.0335	4.3602	5.1960	6.9698	5.0369	5.8014	7.6602
	0.50	2.1461	2.7002	4.1888	2.8910	3.6042	5.1756	3.7579	4.5814	6.2840	4.5367	5.4341	7.4211	5.1289	6.1212	8.1392
3	0	1.7766	2.2437	3.4156	2.6144	3.1548	4.3288	3.3178	3.9484	5.2177	4.1517	4.8667	6.3940	4.7706	5.5140	7.0221
	0.15	2.0094	2.5054	3.6023	2.8048	3.3375	4.5886	3.5017	4.1084	5.4509	4.3329	5.0312	6.6109	4.9529	5.7210	7.2000
	0.20	2.0850	2.5791	3.6948	2.8732	3.4323	4.6654	3.5787	4.1871	5.5712	4.4072	5.1003	6.6991	5.0327	5.8093	7.3661
	0.25	2.1553	2.6639	3.8145	2.9348	3.5257	4.8081	3.6710	4.2748	5.7020	4.4829	5.2027	6.7911	5.1146	5.8957	7.4966
	0.30	2.2425	2.7517	3.8915	3.0012	3.6160	4.9238	3.7326	4.3838	5.8175	4.5523	5.3132	6.9672	5.2018	5.9948	7.6965
	0.35	2.3116	2.8453	3.9818	3.0755	3.7053	5.0342	3.8163	4.4989	6.0090	4.6378	5.4138	7.0960	5.2959	6.1033	7.8959
	0.40	2.3670	2.9249	4.1241	3.1461	3.7955	5.1905	3.8883	4.5993	6.2165	4.7009	5.5246	7.2305	5.3662	6.2162	8.0793
	0.50	2.4795	3.0584	4.5287	3.2717	4.0045	5.4395	4.0619	4.8012	6.6910	4.8417	5.7980	7.5840	5.5105	6.4626	8.5755
4	0	1.9557	2.3840	3.4152	2.7881	3.3166	4.5548	3.6184	4.3121	5.6559	4.2304	4.9241	6.4190	4.9543	5.6788	7.3049
	0.15	2.2150	2.6826	3.6921	3.0178	3.5578	4.8804	3.8514	4.5516	5.9570	4.4692	5.1492	6.7046	5.1897	5.9392	7.6356
	0.20	2.3043	2.7911	3.8675	3.1077	3.6533	4.9713	3.9376	4.6321	6.0628	4.5371	5.2429	6.8049	5.2870	6.0659	7.8245
	0.25	2.3855	2.8937	3.9987	3.1851	3.7450	5.0977	4.0443	4.7611	6.2459	4.6383	5.3444	6.9116	5.3813	6.1477	8.0089
	0.30	2.4575	3.0089	4.1200	3.2723	3.8543	5.2839	4.1408	4.8713	6.3371	4.7179	5.4414	7.0595	5.4588	6.2456	8.1524
	0.35	2.5271	3.1020	4.2665	3.3390	3.9631	5.4304	4.2369	4.9446	6.5191	4.8115	5.5282	7.2649	5.5587	6.3554	8.2703
	0.40	2.6154	3.1686	4.4398	3.4187	4.0420	5.5691	4.2892	5.0520	6.6200	4.8996	5.6460	7.4807	5.6649	6.4899	8.3969
	0.50	2.7578	3.2962	4.6682	3.5654	4.2494	5.9371	4.4298	5.2684	7.0296	5.0782	5.9168	7.8511	5.8486	6.7420	8.7734
5	0	2.1178	2.5550	3.6413	3.0186	3.5530	4.7459	3.7275	4.3513	5.7522	4.4074	5.0706	6.5887	5.1182	5.8719	7.3872
	0.15	2.4390	2.8923	4.0354	3.3074	3.8841	5.0734	4.0419	4.6717	6.2273	4.6782	5.3741	6.9397	5.3868	6.1805	7.7946
	0.20	2.5561	3.0428	4.1444	3.4170	4.0126	5.2302	4.1615	4.8079	6.3537	4.7639	5.4706	7.0568	5.4802	6.3200	7.9393
	0.25	2.6478	3.1574	4.3085	3.5253	4.1164	5.4562	4.2719	4.9553	6.5634	4.8780	5.5936	7.1897	5.5947	6.4319	8.1232
	0.30	2.7540	3.2750	4.4547	3.6089	4.2590	5.5657	4.3696	5.0897	6.6832	4.9802	5.7202	7.3591	5.7046	6.5434	8.3303
	0.35	2.8627	3.3772	4.6595	3.6813	4.3673	5.7225	4.4671	5.2135	6.7929	5.0864	5.8467	7.5294	5.8127	6.6815	8.5116
	0.40	2.9674	3.4799	4.8170	3.7624	4.4807	5.9020	4.5594	5.3499	7.0678	5.2052	6.0109	7.7151	5.9042	6.8346	8.6953
	0.50	3.1106	3.7128	5.1431	3.9263	4.7029	6.2149	4.7524	5.5693	7.4078	5.4184	6.2904	8.1737	6.0948	7.0481	8.9452
6	0	2.3398	2.7618	3.8478	3.1770	3.7088	5.0051	3.8806	4.5166	6.0241	4.6509	5.3009	6.9012	5.3352	6.1271	7.7769
	0.15	2.7078	3.1786	4.2777	3.5369	4.1226	5.3944	4.2057	4.8591	6.3765	5.0044	5.7004	7.2352	5.6771	6.5249	8.1968
	0.20	2.8451	3.3192	4.4841	3.6686	4.2573	5.6200	4.3245	4.9726	6.5077	5.1283	5.8356	7.4120	5.7978	6.6467	8.3349
	0.25	2.9558	3.4489	4.6465	3.7749	4.3827	5.7667	4.4434	5.1022	6.6482	5.2365	5.9890	7.5445	5.9130	6.7877	8.5129
	0.30	3.0732	3.5888	4.8751	3.8921	4.5395	5.9281	4.5532	5.2326	6.8414	5.3502	6.1263	7.6813	6.0513	6.8954	8.6956
	0.35	3.1820	3.7189	5.0759	3.9757	4.6617	6.1092	4.6778	5.4053	6.9920	5.4622	6.2546	7.9258	6.1557	7.0306	8.8829
	0.40	3.2676	3.8651	5.2625	4.0817	4.7771	6.2751	4.7948	5.5267	7.1243	5.5483	6.4037	8.1283	6.2683	7.1674	9.0807
	0.50	3.4666	4.0997	5.5934	4.2441	4.9965	6.6345	4.9757	5.8062	7.4461	5.7484	6.6971	8.5722	6.4637	7.4586	9.5551
7	0	2.5703	3.0540	4.1017	3.3382	3.9516	5.2357	4.0601	4.6927	5.9908	4.7335	5.4678	6.8950	5.4778	6.2708	7.7983
	0.15	3.0069	3.5064	4.5554	3.7631	4.3833	5.6737	4.4721	5.1135	6.5275	5.1373	5.8805	7.4296	5.8800	6.6582	8.2573
	0.20	3.1541	3.6552	4.7651	3.8863	4.5211	5.8308	4.6016	5.2558	6.6882	5.2990	6.0103	7.6703	6.0222	6.7948	8.4855
	0.25	3.2756	3.8137	4.9277	4.0096	4.6576	5.9871	4.7289	5.4024	6.9249	5.4309	6.1647	7.8959	6.1532	6.9568	8.6647
	0.30	3.3966	3.9594	5.1001	4.1288	4.7955	6.1497	4.8517	5.5542	7.1017	5.5665	6.3111	8.0711	6.2509	7.0921	8.8549
	0.35	3.5364	4.1223	5.3450	4.2348	4.9280	6.3439	4.9679	5.7309	7.3185	5.6749	6.4438	8.3122	6.3752	7.2566	9.0401
	0.40	3.6400	4.2435	5.4251	4.3400	5.0368	6.5600	5.0845	5.8591	7.5344	5.7902	6.5826	8.5066	6.4946	7.4350	9.1448
	0.50	3.8208	4.4784	5.9504	4.5647	5.3171	7.0590	5.2995	6.1557	7.9341	6.0208	6.9484	8.9678	6.7197	7.7247	9.6328
8	0	2.7437	3.2121	4.3589	3.5332	4.1264	5.4828	4.2700	4.9157	6.4420	4.9791	5.7136	7.2725	5.6058	6.3412	7.9185
	0.15	3.2573	3.7370	4.8311	3.9915	4.5854	5.9428	4.7251	5.4011	6.9213	5.4562	6.1978	7.7786	6.0467	6.8217	8.4926
	0.20	3.4033	3.9019	5.0427	4.1419	4.7432	6.1366	4.8551	5.5445	7.0862	5.5941	6.3400	7.8647	6.1796	6.9626	8.7216
	0.25	3.5502	4.0732	5.3376	4.2814	4.9012	6.2823	5.0047	5.6579	7.2852	5.7316	6.4985	8.1284	6.3361	7.1424	8.9249
	0.30	3.6896	4.2194	5.4724	4.4145	5.0827	6.4424	5.1447	5.8202	7.4663	5.8950	6.6832	8.2789	6.4623	7.3026	9.1804
	0.35	3.8127	4.3844	5.6960	4.5160	5.2313	6.6195	5.2785	5.9833	7.6937	6.0178	6.8434	8.5086	6.5923	7.4740	9.3738
	0.40	3.9124	4.5639	5.9010	4.6439	5.3744	6.8453	5.3699	6.1462	7.9278	6.1333	7.0259	8.8206	6.7395	7.6012	9.6121
	0.50	4.1161	4.7596	6.3306	4.8852	5.6085	7.3019	5.5787	6.4421	8.2252	6.3509	7.3032	9.2788	6.9766	7.9240	10.0840
9	0	2.9222	3.3842	4.4364	3.6874	4.2780	5.5543	4.4395	5.0665	6.4842	5.1785	5.8514	7.4577	5.7832	6.5624	8.2144
	0.15	3.4681	3.9892	5.0436	4.2265	4.8038	6.0891	4.9134	5.6222	7.0554	5.6833	6.3764	7.9381	6.2891	7.0393	8.7882
	0.20	3.6373	4.1787	5.2594	4.3827	4.9881	6.2960	5.0753	5.7979	7.2555	5.8359	6.5778	8.1341	6.4651	7.2053	8.9748
	0.25	3.8037	4.3526	5.4720	4.5448	5.1686	6.5513	5.2201	5.9886	7.4616	5.9873	6.7622	8.3510	6.6073	7.3823	9.2429
	0.30	3.9524	4.5237	5.6869	4.6785	5.3449	6.7720	5.3759	6.1400	7.7431	6.1563	6.9068	8.5621	6.7367	7.5243	9.3970
	0.35	4.0871	4.6844	5.9298	4.7986	5.5191	7.0892	5.5117	6.2960	7.9518	6.3060	7.0753	8.7573	6.8682	7.6747	9.6851
	0.40	4.2136	4.8150	6.1598	4.9227	5.6427	7.2372	5.6459	6.4573	8.2190	6.4084	7.2253	9.0515	6.9547	7.8106	9.8566
	0.50	4.4464	5.1016	6.5752	5.1528	5.8757	7.6693	5.8735	6.7287	8.6121	6.6405	7.5797	9.5602	7.1927	8.1507	10.4038
10	0	3.1156	3.5507	4.6520	3.8414	4.4367	5.7757	4.6045	5.2641	6.6907	5.3315	6.0015	7.5562	5.9660	6.6704	8.1212
	0.15	3.7281	4.2170	5.2849	4.4295	5.0238	6.4074	5.1596	5.8279	7.3826	5.8848	6.5876	8.2326	6.4847	7.2401	8.8108
	0.20	3.9247	4.4348	5.5381	4.6229	5.2169	6.6791	5.3340	6.0352	7.5913	6.0513	6.7630	8.4745	6.6687	7.4119	9.0249
	0.25	4.1047	4.6228	5.7572	4.7825	5.4094	6.9464	5.5033	6.1859	7.7942	6.2178	6.9648	8.6798	6.8366	7.5848	9.3223
	0.30	4.2692	4.8113	6.0106	4.9475	5.6016	7.1926	5.6632	6.3750	8.0805	6.3736	7.1066	8.9489	6.9839	7.7741	9.5236
	0.35	4.4281	5.0045	6.2784	5.1002	5.7583	7.4629	5.8347	6.5355	8.3551	6.5227	7.2861	9.1518	7.1064	7.9325	9.7837
	0.40	4.5661	5.1802	6.5227	5.2529	5.9204	7.6775	5.9603	6.6949	8.5945	6.6613	7.4664	9.4526	7.2385	8.0767	10.0598
	0.50	4.7948	5.5210	7.0740	5.4541	6.2782	8.0810	6.2025	7.0750	9.0192	6.8658	7.8535	10.0305	7.4750	8.4549	10.4821

Table 3: Asymptotic critical values for the average mapping (continued)

p	s_0	$q-p=6$			$q-p=7$			$q-p=8$			$q-p=9$			$q-p=10$		
		10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
1	0	4.9999	5.7918	7.6041	5.6832	6.4950	8.1553	6.3454	7.2283	9.1097	7.0170	7.8794	9.9146	7.5764	8.5370	10.3239
	0.15	5.1091	5.9204	7.6727	5.7628	6.6743	8.4253	6.4245	7.3505	9.2117	7.0666	7.9337	9.9721	7.7008	8.6224	10.5086
	0.20	5.1659	5.9710	7.6812	5.8356	6.7113	8.5041	6.4779	7.4291	9.3235	7.0875	7.9893	10.0564	7.7555	8.6893	10.5667
	0.25	5.2491	6.0555	7.7780	5.8890	6.7852	8.5720	6.5542	7.4932	9.3912	7.1284	8.0907	10.1670	7.8323	8.7536	10.7103
	0.30	5.3212	6.1110	7.8902	5.9458	6.8846	8.6676	6.6139	7.5579	9.4689	7.1898	8.1744	10.3300	7.8894	8.8826	10.9240
	0.35	5.3654	6.2239	7.9817	6.0103	6.9861	8.8104	6.6856	7.6417	9.7388	7.2615	8.3052	10.3920	7.9739	8.9710	11.0535
	0.40	5.4203	6.3216	8.1332	6.1036	7.1163	9.0419	6.7680	7.7701	9.9631	7.3545	8.4090	10.5639	8.0537	9.1021	11.2539
	0.50	5.5870	6.5443	8.4677	6.2815	7.3228	9.3475	6.9936	8.0327	10.3220	7.5676	8.6857	10.9264	8.2851	9.3546	11.7818
2	0	5.3282	6.1266	7.7153	5.9009	6.6401	8.3560	6.5336	7.4280	9.4354	7.1573	8.0874	9.8486	7.8536	8.8420	10.8022
	0.15	5.4666	6.2967	8.0277	6.0160	6.8176	8.5768	6.6579	7.6042	9.5810	7.2574	8.1829	10.0297	8.0178	9.0057	11.0106
	0.20	5.5426	6.3699	8.1197	6.0817	6.9231	8.6649	6.7186	7.6839	9.6585	7.3288	8.3083	10.1949	8.0921	9.0795	11.1139
	0.25	5.6240	6.4545	8.2768	6.1558	6.9744	8.7248	6.8004	7.7597	9.8109	7.4264	8.3994	10.3781	8.1522	9.1510	11.2153
	0.30	5.6857	6.5575	8.3870	6.2355	7.0768	8.8372	6.8790	7.8680	9.9261	7.5140	8.4748	10.5559	8.2329	9.2668	11.3462
	0.35	5.7600	6.6368	8.4693	6.3201	7.1657	8.9503	6.9664	7.9468	10.1361	7.6139	8.5481	10.6874	8.3132	9.3683	11.4907
	0.40	5.8396	6.7163	8.6114	6.3929	7.3062	9.1669	7.0507	8.0669	10.3546	7.6944	8.6804	10.8626	8.3724	9.4550	11.6588
	0.50	5.9924	6.9679	9.0283	6.5719	7.6472	9.8905	7.2549	8.3649	10.7143	7.8916	8.9628	11.1892	8.5534	9.7538	12.0103
3	0	5.4536	6.2616	7.8542	6.0429	6.8894	8.5578	6.6933	7.5613	9.3226	7.3064	8.2305	10.0184	8.0079	8.9481	10.7991
	0.15	5.6591	6.4487	8.0829	6.2165	7.0729	8.9220	6.8744	7.7586	9.6620	7.4894	8.3943	10.2819	8.1456	9.1330	11.1304
	0.20	5.7264	6.5724	8.1700	6.2974	7.1475	9.0896	6.9413	7.8961	9.8329	7.5746	8.4906	10.5297	8.2446	9.2386	11.1975
	0.25	5.7733	6.6590	8.3000	6.3620	7.2481	9.2143	7.0312	8.0343	10.0293	7.6764	8.5996	10.6523	8.3470	9.3087	11.3801
	0.30	5.8509	6.7513	8.4222	6.4581	7.3886	9.3548	7.1364	8.1402	10.1978	7.7758	8.7044	10.8369	8.4549	9.4784	11.6045
	0.35	5.9476	6.8978	8.6090	6.5518	7.5064	9.3881	7.2738	8.2996	10.4136	7.8655	8.8660	10.9418	8.5499	9.6014	11.8582
	0.40	6.0519	6.9541	8.8716	6.6522	7.6180	9.6041	7.3923	8.4548	10.5793	7.9428	9.0195	11.2009	8.6439	9.7501	12.0487
	0.50	6.1995	7.2206	9.3303	6.8034	7.8485	9.9770	7.5572	8.7084	11.0113	8.1545	9.3088	11.6688	8.8419	10.0548	12.4878
4	0	5.5885	6.3937	8.0978	6.2702	7.0615	8.8157	6.8690	7.7539	9.6595	7.5811	8.4556	10.2630	8.1091	9.0612	10.9623
	0.15	5.8518	6.6279	8.4757	6.4995	7.3411	9.1055	7.1387	7.9726	9.9495	7.8176	8.7203	10.6558	8.3590	9.3249	11.2799
	0.20	5.9426	6.7562	8.6073	6.5625	7.4294	9.2761	7.2497	8.0890	10.0933	7.8955	8.8317	10.8489	8.4496	9.4641	11.4767
	0.25	6.0409	6.8579	8.6806	6.6600	7.5599	9.5107	7.3422	8.2108	10.2211	7.9974	8.9636	10.9940	8.5302	9.5841	11.6338
	0.30	6.1255	6.9665	8.8933	6.7452	7.7374	9.6835	7.4560	8.3208	10.3786	8.1131	9.1110	11.2060	8.6304	9.7234	11.8577
	0.35	6.2123	7.1241	8.9968	6.8458	7.8511	9.8444	7.5629	8.5014	10.5616	8.1951	9.2703	11.3789	8.7440	9.8729	11.9890
	0.40	6.3347	7.2421	9.2084	6.9617	8.0020	9.9561	7.6632	8.6216	10.6945	8.2974	9.4346	11.6614	8.8292	10.0050	12.2164
	0.50	6.5154	7.5454	9.7837	7.2090	8.2843	10.4114	7.8970	8.9283	11.1231	8.5033	9.6631	12.0274	9.1042	10.2993	12.7738
5	0	5.7167	6.4415	8.1901	6.4202	7.2475	9.0459	7.0579	7.9065	9.8347	7.6023	8.5787	10.3919	8.3691	9.3242	11.4077
	0.15	5.9840	6.7959	8.4981	6.6924	7.5792	9.4336	7.3603	8.2653	10.2654	7.9309	8.8720	10.6681	8.6307	9.6614	11.7466
	0.20	6.0774	6.9051	8.7112	6.8259	7.7124	9.5811	7.4849	8.3810	10.3614	8.0539	9.0227	10.8348	8.7435	9.7979	11.9552
	0.25	6.1976	7.0256	8.8804	6.9430	7.8331	9.7775	7.6164	8.5103	10.4660	8.1656	9.1396	10.9835	8.8130	9.9177	12.1140
	0.30	6.2797	7.1625	9.0819	7.0496	7.9958	10.0397	7.7272	8.6752	10.6970	8.2670	9.2828	11.1396	8.9350	10.0552	12.2507
	0.35	6.3784	7.2920	9.2152	7.1522	8.0861	10.1919	7.8170	8.7921	10.9523	8.3746	9.4353	11.3991	9.0281	10.1972	12.3644
	0.40	6.5059	7.4508	9.3630	7.2780	8.2515	10.4105	7.9159	8.9306	11.0899	8.4704	9.5447	11.6517	9.1290	10.3207	12.5709
	0.50	6.6776	7.7027	9.7974	7.4804	8.5855	10.8101	8.1193	9.2255	11.6007	8.6698	9.8379	12.1125	9.3686	10.6165	13.3072
6	0	5.9984	6.7607	8.4046	6.6189	7.4571	9.0308	7.2493	8.1171	10.0110	7.9491	8.8822	11.0231	8.5198	9.4593	11.4693
	0.15	6.3307	7.1892	8.9378	6.9508	7.7970	9.4306	7.5695	8.4866	10.3779	8.2416	9.2359	11.4177	8.8689	9.7912	11.9780
	0.20	6.4601	7.3645	9.0211	7.0697	7.9279	9.6128	7.6604	8.6383	10.5111	8.3734	9.3823	11.6167	8.9908	9.9561	12.1668
	0.25	6.5724	7.4928	9.1730	7.1819	8.0541	9.8164	7.8045	8.7645	10.6493	8.5024	9.5361	11.8149	9.1246	10.1440	12.2768
	0.30	6.7157	7.5999	9.3400	7.2998	8.2031	9.9527	7.9350	8.9082	10.9792	8.6126	9.7258	12.0618	9.2505	10.3163	12.5419
	0.35	6.8459	7.7688	9.5577	7.4025	8.3275	10.1807	8.0157	9.0693	11.2346	8.6916	9.8998	12.1846	9.3716	10.4654	12.6776
	0.40	6.9459	7.9142	9.8294	7.5157	8.4721	10.4611	8.1795	9.2052	11.5356	8.8248	10.0162	12.4277	9.5031	10.6303	13.0010
	0.50	7.1285	8.2005	10.2534	7.7370	8.7470	10.9698	8.4322	9.5374	11.9654	9.1102	10.3449	12.8433	9.7945	10.9895	13.3438
7	0	6.1450	6.9244	8.6771	6.8068	7.6602	9.4216	7.4102	8.1859	9.9572	8.0138	8.9367	10.7578	8.6432	9.6013	11.6597
	0.15	6.5362	7.3187	9.1606	7.1562	8.0333	9.8921	7.7966	8.5954	10.3835	8.4101	9.3579	11.2036	9.0824	10.0752	12.1603
	0.20	6.6532	7.4505	9.2895	7.3195	8.1541	10.0536	7.9139	8.7746	10.6054	8.5654	9.4898	11.4298	9.2364	10.2313	12.3642
	0.25	6.7705	7.5941	9.4823	7.4531	8.3071	10.2771	8.0392	8.9294	10.8269	8.7296	9.6433	11.6594	9.3705	10.3932	12.5494
	0.30	6.8793	7.7542	9.6827	7.5701	8.5044	10.4914	8.1575	9.0878	11.0815	8.8423	9.7790	11.8183	9.4998	10.5952	12.7992
	0.35	6.9804	7.9460	9.9626	7.6983	8.6928	10.6578	8.2746	9.2862	11.4270	8.9541	9.9536	12.1653	9.6405	10.7647	12.9958
	0.40	7.1250	8.1047	10.2376	7.8512	8.8390	10.8444	8.4127	9.4128	11.7039	9.1079	10.1415	12.5291	9.7417	10.9065	13.1880
	0.50	7.3610	8.3947	10.6747	8.1037	9.1478	11.3589	8.6866	9.7399	12.0833	9.3047	10.4749	12.8484	9.9540	11.1850	13.8647
8	0	6.3396	7.1324	8.8003	6.9274	7.8565	9.5728	7.5246	8.3706	10.2221	8.2425	9.1547	10.9687	8.8947	9.8279	11.7448
	0.15	6.7493	7.6250	9.3041	7.3930	8.2758	10.1245	7.9980	8.8795	10.6898	8.6652	9.6419	11.5363	9.2858	10.2708	12.1699
	0.20	6.8801	7.7672	9.5054	7.5514	8.4122	10.3677	8.1571	9.0513	10.8900	8.8176	9.8055	11.7408	9.4355	10.4439	12.3279
	0.25	7.0027	7.9188	9.7404	7.7138	8.5955	10.5788	8.3115	9.2386	11.0920	8.9718	9.9874	11.9064	9.5951	10.6651	12.5902
	0.30	7.1417	8.0939	10.0219	7.8353	8.7459	10.7585	8.4575	9.4020	11.4145	9.1096	10.1056	12.1540	9.7437	10.8047	12.8734
	0.35	7.2649	8.2471	10.2222	7.9513	8.9248	10.9593	8.5622	9.6092	11.6327	9.2385	10.2949	12.3797	9.8589	10.9540	13.0599
	0.40	7.4182	8.3733	10.4575	8.0835	9.1078	11.2351	8.7124	9.7702	11.9125	9.3653	10.4334	12.6645	10.0129	11.1251	13.2375
	0.50	7.6491	8.7468	10.9457	8.3082	9.4531	11.7782	8.9311	10.0989	12.3543	9.6289	10.8028	13.2364	10.2765	11.3882	13.7629
9	0	6.4222	7.2170	8.8841	7.1435	8.0080	9.7992	7.7630	8.6007	10.3317	8.4070	9.3749	11.2578	9.0582	9.9941	12.0760
	0.15	6.9092	7.7165	9.5003	7.5821	8.4812	10.2342	8.2245	9.0953	10.9899	8.9138	9.9072	11.8174	9.5485	10.5231	12.7607
	0.20	7.0730	7.8622	9.6955	7.7384	8.6360	10.4365	8.4027	9.2583	11.1969	9.0854	10.1064	11.9621	9.7239	10.7600	13.0404
	0.25	7.2316	8.0383	9.8706	7.8947	8.7973	10.6707	8.5460	9.4174	11.4302	9.2480	10.3047	12.1976	9.9069	10.9617	13.2635
	0.30	7.3728	8.2270	10.0789	8.0456	8.9728	10.9344	8.7070	9.6205	11.5981	9.4034	10.4270	12.4636	10.0503	11.1506	13.4612
	0.35	7.4976	8.4102	10.4171	8.2060	9.1365	11.1611	8.8804	9.8765	11.8010	9.5649	10.5817	12.7353	10.1580	11.3081	13.5658
	0.40	7.6184	8.5479	10.6182	8.3349	9.3203	11.3375	8.9976	10.0181	12.0583	9.6948	10.7748	13.0256	10.3261	11.4979	13.8065
	0.50	7.8392	8.8189	11.1361	8.5719	9.6446	11.7929	9.2520	10.3483	12.4563	9.9647	11.1390	13.5845	10.5706	11.8283	14.3659
10	0	6.7158	7.4741	9.0993	7.3043	8.1265	10.0227	8.0381	8.8455	10.6073	8.6189	9.5022	11.2285	9.1158	10.0249	12.0253
	0.15	7.2562	8.0889	9.8133	7.8401	8.7133	10.6442	8.5430	9.4389	11.2658	9.0927	10.0547	11.9666	9.6742	10.6200	12.6359
	0.20	7.4136	8.2888	9.9533	8.0034	8.9189	10.7802	8.6905	9.6450	11.4492	9.2585	10.2395	12.2225	9.8509	10.8504	12.8974
	0.25	7.5951	8.4711	10.2147	8.1642	9.0740	11.0580	8.8303	9.8232	11.7210	9.4355	10.4104	12.4862	10.0127	11.0651	13.2028
	0.30	7.7388	8.6634	10.4854	8.3361	9.2580	11.3154	9.0225	10.0037	11.9485	9.5829	10.5910	12.7267	10.2015	11.2183	13.4786
	0.35	7.9149	8.8407	10.7077	8.4829	9.4022	11.4654	9.1671	10.1896	12.1822	9.7445	10.8054	12.9132	10.3788	11.4284	13.7568
	0.40	8.0616	9.0325	10.9602	8.6565	9.5762	11.8180	9.3394	10.3652	12.4176	9.8901	10.9891	13.2418	10.5387	11.6805	14.0197
	0.50	8.3265	9.3384	11.6032	8.8419	9.9409	12.3731	9.5944	10.6847	12.9892	10.2269	11.4140	13.7764	10.9110	12.0759	14.6294

Table 4: Data Generating Processes

	H_0^{SS}	H_A^I	H_A^O
DGP1	$\theta_1 = \theta_2 = 0$		$\alpha = 0$
DGP2	$\theta_1 = \theta_2 = 0.4$		$\alpha = 0$
DGP3	$\theta_1 = \theta_2 = 0.8$		$\alpha = 0$
DGP4		$\theta_1 = 0, \theta_2 = 0.4$	$\alpha = 0$
DGP5		$\theta_1 = 0, \theta_2 = 0.8$	$\alpha = 0$
DGP6		$\theta_1 = 0.4, \theta_2 = 0.8$	$\alpha = 0$
DGP7	$\theta_1 = \theta_2 = 0.4$		$\alpha = 0.5$
DGP8	$\theta_1 = \theta_2 = 0.4$		$\alpha = 0.9$
DGP9	$\theta_1 = \theta_2 = 0.4$		$\alpha = -0.5$
DGP10	$\theta_1 = \theta_2 = 0.4$		$\alpha = -0.9$

Table 5: Rejection Frequencies for General Tests of Structural Change

DGP	Size (%)	$supIPSC$	$aveIPSC$	$expIPSC$	$supIPSCM$	$aveIPSCM$	$expIPSCM$
DGP1	1	0.0200	0.0200	0.0180	0.0720	0.0420	0.0560
	5	0.0620	0.0660	0.0720	0.1380	0.0960	0.1180
	10	0.1160	0.1140	0.1140	0.2020	0.1620	0.1640
DGP2	1	0.0140	0.0140	0.0140	0.0680	0.0340	0.0520
	5	0.0660	0.0420	0.0480	0.1520	0.0900	0.1100
	10	0.1300	0.1080	0.1240	0.2160	0.1660	0.1840
DGP3	1	0.0340	0.0160	0.0220	0.0860	0.0540	0.0660
	5	0.0760	0.0720	0.0740	0.1560	0.1020	0.1140
	10	0.1120	0.1240	0.1120	0.2140	0.1680	0.1820
DGP4	1	0.1420	0.1680	0.1680	0.3620	0.2820	0.3280
	5	0.3720	0.3780	0.3760	0.5820	0.5260	0.5460
	10	0.5640	0.5820	0.5820	0.6920	0.6700	0.6720
DGP5	1	0.9820	0.9640	0.9780	1.0000	0.9940	0.9980
	5	0.9980	1.0000	1.0000	1.0000	1.0000	1.0000
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
DGP6	1	0.4000	0.3960	0.4180	0.6800	0.5860	0.6440
	5	0.7080	0.6640	0.6840	0.8460	0.7840	0.8100
	10	0.8260	0.7860	0.8020	0.9000	0.8620	0.8820
DGP7	1	0.2920	0.1780	0.2480	0.5340	0.3520	0.4700
	5	0.5360	0.3780	0.4560	0.6920	0.5240	0.5960
	10	0.6680	0.5480	0.5980	0.7840	0.6260	0.7000
DGP8	1	0.6000	0.4420	0.5460	0.7980	0.6460	0.7480
	5	0.8140	0.6920	0.7540	0.9100	0.7720	0.8420
	10	0.8980	0.8220	0.8620	0.9500	0.8560	0.8980
DGP9	1	0.4060	0.2420	0.3400	0.5620	0.3240	0.4640
	5	0.6220	0.5040	0.5920	0.7180	0.5140	0.6400
	10	0.7540	0.6480	0.7020	0.8040	0.6840	0.7360
DGP10	1	0.7860	0.6640	0.7400	0.8340	0.6780	0.8060
	5	0.9340	0.8680	0.9080	0.9280	0.8520	0.8960
	10	0.9660	0.9520	0.9540	0.9580	0.9240	0.9520

Table 6: Rejection Frequencies for Tests of Structural Change in the Parameters

DGP	Size (%)	$supIPSC^I$	$aveIPSC^I$	$expIPSC^I$	$supW$	$aveW$	$expW$	$supLR$	$aveLR$	$expLR$
DGP1	1	0.0140	0.0120	0.0200	0.0160	0.0120	0.0200	0.0180	0.0140	0.0240
	5	0.0540	0.0660	0.0700	0.0640	0.0620	0.0680	0.0600	0.0600	0.0680
	10	0.1300	0.1260	0.1280	0.1200	0.1140	0.1240	0.1300	0.1260	0.1360
DGP2	1	0.0160	0.0120	0.0140	0.0180	0.0140	0.0180	0.0160	0.0140	0.0140
	5	0.0600	0.0720	0.0800	0.0720	0.0660	0.0780	0.0660	0.0680	0.0760
	10	0.1260	0.1400	0.1480	0.1220	0.1160	0.1400	0.1280	0.1340	0.1420
DGP3	1	0.0060	0.0120	0.0120	0.0300	0.0140	0.0180	0.0100	0.0120	0.0120
	5	0.0600	0.0680	0.0760	0.0860	0.0640	0.0800	0.0700	0.0720	0.0800
	10	0.1400	0.1260	0.1360	0.1380	0.1300	0.1380	0.1460	0.1280	0.1360
DGP4	1	0.4720	0.5100	0.5360	0.4740	0.4900	0.5240	0.4620	0.4980	0.5340
	5	0.6960	0.7540	0.7600	0.6920	0.7380	0.7460	0.6840	0.7400	0.7540
	10	0.7880	0.8440	0.8400	0.7880	0.8360	0.8340	0.7820	0.8440	0.8360
DGP5	1	1.0000	1.0000	1.0000	1.0000	0.9980	1.0000	1.0000	1.0000	1.0000
	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
DGP6	1	0.6840	0.7460	0.7620	0.6960	0.7460	0.7600	0.6820	0.7320	0.7580
	5	0.8680	0.8940	0.9000	0.8660	0.8920	0.9020	0.8700	0.8980	0.9060
	10	0.9120	0.9340	0.9360	0.9080	0.9300	0.9360	0.9060	0.9300	0.9320
DGP7	1	0.3100	0.2720	0.3220	0.2800	0.2220	0.2920	0.1720	0.1820	0.1980
	5	0.4560	0.4580	0.4860	0.4220	0.3820	0.4340	0.3400	0.3620	0.3800
	10	0.5660	0.5480	0.5860	0.5100	0.4860	0.5300	0.4460	0.4700	0.4780
DGP8	1	0.5100	0.4820	0.5340	0.4200	0.3700	0.4480	0.3460	0.3560	0.3740
	5	0.6500	0.6320	0.6640	0.5640	0.5480	0.6000	0.5220	0.5420	0.5640
	10	0.7400	0.7280	0.7700	0.6720	0.6560	0.6900	0.6180	0.6320	0.6460
DGP9	1	0.3060	0.1640	0.2640	0.2280	0.1300	0.2140	0.1240	0.1040	0.1280
	5	0.4800	0.3620	0.4500	0.3740	0.2840	0.3660	0.2660	0.2400	0.2740
	10	0.5840	0.4700	0.5560	0.4760	0.4200	0.4720	0.3760	0.3880	0.4060
DGP10	1	0.5400	0.2740	0.5040	0.4240	0.2560	0.4080	0.1860	0.1180	0.1680
	5	0.7200	0.5340	0.6740	0.5840	0.4540	0.5500	0.3820	0.3060	0.4020
	10	0.8080	0.6720	0.7600	0.6520	0.5820	0.6540	0.5420	0.4660	0.5200

Table 7: Rejection Frequencies for Tests of Structural Change in the Overidentifying Restrictions

DGP	Size (%)	$supIPSC^O$	$aveIPSC^O$	$expIPSC^O$	$supO$	$aveO$	$expO$
DGP1	1	0.0200	0.0200	0.0160	0.0040	0.0100	0.0100
	5	0.0720	0.0720	0.0620	0.0200	0.0500	0.0380
	10	0.1200	0.1280	0.1400	0.0600	0.1040	0.0960
DGP2	1	0.0380	0.0160	0.0300	0.0060	0.0120	0.0120
	5	0.0840	0.0620	0.0860	0.0420	0.0480	0.0600
	10	0.1300	0.1260	0.1340	0.0680	0.0980	0.0860
DGP3	1	0.0240	0.0140	0.0220	0.0020	0.0080	0.0040
	5	0.0780	0.0760	0.0880	0.0240	0.0580	0.0540
	10	0.1320	0.1220	0.1360	0.0700	0.1120	0.1000
DGP4	1	0.0340	0.0240	0.0320	0.0060	0.0100	0.0120
	5	0.1100	0.0740	0.1040	0.0380	0.0520	0.0520
	10	0.1740	0.1520	0.1780	0.0960	0.1060	0.1200
DGP5	1	0.3060	0.1400	0.3000	0.1560	0.0940	0.1800
	5	0.5120	0.3500	0.4860	0.3660	0.2660	0.4000
	10	0.6420	0.5060	0.6220	0.5260	0.4400	0.5360
DGP6	1	0.0480	0.0240	0.0520	0.0100	0.0120	0.0140
	5	0.1120	0.0960	0.1120	0.0580	0.0720	0.0800
	10	0.1900	0.1540	0.1740	0.1040	0.1300	0.1300
DGP7	1	0.6180	0.5840	0.6440	0.0440	0.1920	0.1580
	5	0.8060	0.8020	0.8200	0.2860	0.5660	0.5100
	10	0.8540	0.8740	0.8760	0.4780	0.7380	0.7220
DGP8	1	0.8120	0.8140	0.8340	0.0840	0.3560	0.2620
	5	0.8960	0.9140	0.9120	0.4060	0.7800	0.6940
	10	0.9260	0.9600	0.9560	0.6240	0.8840	0.8580
DGP9	1	0.8660	0.8900	0.9060	0.0940	0.4120	0.2880
	5	0.9580	0.9560	0.9660	0.4340	0.8200	0.7800
	10	0.9840	0.9880	0.9840	0.7080	0.9340	0.9220
DGP10	1	0.9920	0.9900	0.9980	0.1000	0.6620	0.4900
	5	0.9960	1.0000	0.9980	0.6160	0.9660	0.9400
	10	1.0000	1.0000	1.0000	0.8660	0.9960	0.9880