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Flexible Approximation of Subjective Expectations using Probability Questions – An Application to the Investment Game –

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Abstract:
We use spline interpolation to approximate the subjective cumulative distribution function of an economic agent over the future realization of a continuous (possibly censored) random variable. The method proposed exploits information collected using a small number of probabillity questions on expectations and requires a weak prior knowledge of the shape of the underlying distribution. We find that eliciting 4 or 5 points on the cumulative distribution function of an agent is sufficient to accurately approximate a wide variety of underlying distributions. We show that estimated moments of general functions of the random variable can be computed analytically and/or using standard simulation techniques. We illustrate the usefulness of the method by estimating a simple method to asses the impact of expectations on investment decisions in a commonly used trust game.

Keywords: Approximation of subjective expectations, spline interpolation, decision making under uncertainty

JEL Classification: Z13, C90, C10, D39

1 Introduction

The measurement of subjective expectations has recently attracted a lot of attention amongst empirical economists. These measurements have proven useful to elicit knowledge of economic agents and experts on the future realization of various economic variables (e.g. Dominitz and Manski (1997), Engelberg, Manski, and Williams (2006), McKenzie, Gibson, and Stillman (2007)). Subjective expectations data can also be used to improve the empirical content of stochastic models of choice under uncertainty. These models typically relate the distribution of choices to the distributions of preferences and expectations in the population. Without placing much structure on the problem, preferences and expectations often cannot both be recovered from the choice distribution alone. The degree of underidentification is often severe (see Magnac and Thesmar (2002) for a discussion of this problem in structural dynamic decision making, and Manski (2002) for experimental games of proposal and response). The combination of subjective expectations and choice data allows empirical researchers to recover preferences under relatively mild assumptions on how agents evaluate the likelihood of future events (see Delavande (2005), Nyarko and Schotter (2002) and Bellemare, Kröger, and van Soest (2005)).

For many reasons it has recently been advocated to infer expectations from subjective probability distributions. First, probabilistic measurement of distributions provide more information on the uncertainty faced by an agent. This additional information can be used by researchers to estimate models of choice under uncertainty (e.g. with risk aversion) where other features of the distribution play an important role. Second, deriving expectations from probability distributions overcomes the problems of interpreting answers given to direct questions on expectations. In particular, there is evidence that agents reveal different features of their subjective distribution (mean, median, or other quantiles) when asked for their best point prediction of a future event (see Manski (2004) for a review of these issues). Whereas elicitation of discrete probability distributions requires asking respondents to place probability mass on each possible realization of the random variable, elicitation of subjective probability distributions of continuous random variables requires some form of approximation. Current approaches typically ask respondents to report several points of their subjective cumulative distribution function and look for the set of parameters of a parametric distribution (e.g. a normal or log-normal distribution) which provides the best fit to these data points (e.g. Dominitz and Manski (1997)). The disadvantages of parametric assumptions are well known (see Pagan and Ullah (1999)). One of the main concerns is that misspecification of the underlying distribution may lead to biased forecasts and inferences.

In this paper, we present a flexible method to approximate the subjective probability distributions of continuous random variables using agents' answers to a small set of probability questions. The method proposed is based on approximating an individual's subjective cumulative distribution function using spline interpolation. The method is simple to apply and does not require that subjective distributions belong to a particular parametric family of distributions. Moreover, the method can accommodate censored distributions in a very natural way. We evaluate the quality of our flexible approximation in relation to the number of data points collected and the degree of the interpolating polynomial. We find that 4 or 5 probability questions are sufficient to provide a very good approximation of symmetric, asymmetric, and bimodal distributions. Moreover, the cubic spline interpolation outperforms both the quadratic and linear spline interpolations.

As mentioned previously, the fitted distributions can be used either to characterize the knowledge of agents, or alternatively to predict behavior using stochastic choice models. In the later case, the estimation of econometric models of choice under uncertainty often involves the computation of $\mathbf{E}(h(x;\boldsymbol{\eta}))$, where $h(\cdot)$ denotes a function which depends on a vector of unknown parameters $\boldsymbol{\eta}$ to be estimated using the choice data. In the estimation of structural dynamic models, $h(x;\boldsymbol{\eta})$ represents the maximum of future value functions and $\boldsymbol{\eta}$ represents a vector of preference parameters. In static models, $h(\cdot)$ can represent the utility function of a risk (or loss) averse agent, and $\boldsymbol{\eta}$ denotes a risk (or loss) aversion parameter. In the important special case where $\mathbf{E}(h(x;\boldsymbol{\eta})) = \mathbf{E}(x^{\eta})$, we show how the expectation can be computed analytically using the estimated spline. However, such analytical solutions do not exist for more general functions $h(\cdot)$. To handle these cases, we present a simple algorithm which allows to generate a sequence of random draws from the approximated distribution. Given a sequence of draws, approximation of $\mathbf{E}(h(x;\boldsymbol{\eta}))$ can be achieved using standard simulation techniques (e.g. Train, 2003).

In the second part of the paper we illustrate the usefulness of the method by estimating a simple choice model of investment behavior in a modified version of a well known two players game (Berg, Dickhaut, and McCabe, 1995). In this game, a first player can decide to "invest" or not his endowment. The endowment, if invested, is multiplied by a factor of two and transferred to a second player. This second player must decide how much he will return to the first player. Thus, for the first player, the investment decision involves uncertainty over the amount that the second player will return to him. The investment behavior in this game is often used to infer a player's propensity to trust others. The importance of trust in economics draws on recent evidence suggesting that trust may be an important determinant of economic growth and organizational efficiency (see e.g., Zak and Knack, 2001; La Porta, de Silanes, Schleifer, and Vishny, 1997). Interestingly, trust is generally defined as a person's expectations over the actions taken by others which can affect his or her own well-being.[1] This suggests that it is possible to evaluate the relationship between trust and investments in an experimental context by directly relating expectations of first players concerning the amounts returned (taken as a measure of their trust) and their investment behavior. To proceed, we specify a simple model where expectations and social preferences of first players can both determine investment behavior. We find

[1]For example, the New Oxford Dictionary of English defines trust as "Firm belief in the reliability, truth, ability, or strength of someone or something." The Collins English dictionary states that "If you trust someone, you believe that they are honest and sincere and will not deliberately do anything to harm you."

that players who invest their endowment have significantly higher expected returns than non-investors. Nevertheless, the expected profit from investing is found to be too small to explain the majority of the observed investment decisions. Our model results suggest that social preferences play a significant role in determining investment behavior. This result adds to recent evidence suggesting that other factors determine part of the investment behavior (e.g. Cox (2004), Barr (2003), Karlan (2005)).

The rest of the paper is organized as follows. Section 2 presents the method proposed. Section 3 analyzes the quality of the approximation for several distributions. Section 4 presents the application of the method to the investment game. Section 5 concludes.

2 Flexible Approximation of Distributions and Expectations

2.1 Subjective Distributions

We propose to approximate the subjective cumulative distribution function $F(x|i) = \Pr(X \leq x|i)$ over the realization of the real-valued variable X for a respondent i using cubic spline interpolation. A cubic spline is a piecewise polynomial function defined on n intervals $[x_{j-1}, x_j]$ for $j = 1, \ldots, n$. For each interval, respondent i is asked to state his subjective probability that X will materialize in the corresponding interval. The probability mass in each interval can be used to form a set of knots , denoted by $\{(x_j, F(x_j|i)) : j = 0, 1, \ldots, n\}$ where x_j denote the end points of each of the n intervals, and $0 \leq F(x_j|i) \leq 1$ are the associated points on the subjective cumulative distribution function. The later are derived using the probability mass placed by a respondent in each of the n intervals.

We model $F(x|\, i)$ using cubic spline interpolation which assumes that the function $F(x|\, i)$ is defined by $a_j + b_j x + c_j x^2 + d_j x^3$ on interval j where (a_j, b_j, c_j, d_j) are the interval-specific polynomial coefficients. The spline function is constructed by simply connecting the different polynomials at the relevant knots. The set $\{(a_j, b_j, c_j, d_j) : j = 1, \ldots, n\}$ contains the $4n$ unknown polynomial coefficients to be estimated.[2] The polynomials connect at knots. In order to estimate the set of polynomial coefficients, the interpolation exploits the assumed continuity at the interior knots. This gives rise to $2n$ equations

$$\begin{aligned} F(x_{j-1}|i) &= a_j + b_j x_{j-1} + c_j x_{j-1}^2 + d_j x_{j-1}^3 \qquad \text{for } j = 1, 2, \ldots, n \\ F(x_j|i) &= a_j + b_j x_j + c_j x_j^2 + d_j x_j^3 \qquad \text{for } j = 1, 2, \ldots, n. \end{aligned}$$

Next, the assumption that $F(x|i)$ is twice differentiable at the interior nodes implies the following $2(n-1)$ equations

$$\begin{aligned} b_j + 2c_j x_j + 3d_j x_j^2 &= b_{j+1} + 2c_{j+1} x_j + 3d_{j+1} x_j^2 \qquad \text{for } j = 1, 2 \ldots, (n-1) \\ 2c_j + 6d_j x_j &= 2c_{j+1} + 6d_{j+1} x_j \qquad \text{for } j = 1, 2, \ldots, (n-1). \end{aligned}$$

Two more conditions, so called boundary conditions, are needed in order to estimate the polynomial coefficients of the cubic spline. There is very little guidance in the literature to chose these boundary conditions. Here, we chose to impose that $F''(x_0|i) = F''(x_n|i) = 0$, yielding what is known as a natural cubic spline (see Judd (1998)).[3] This provides a system of $4n$ linear equations in the $4n$ unknown parameters. The estimated parameters $\left\{(\widehat{a}_j, \widehat{b}_j, \widehat{c}_j, \widehat{d}_j) : j = 1, \ldots, n\right\}$ can then be used to compute the estimated cumulative distribution function for a given respondent.[4] Panel a) in Figure 1 illustrates the approximation. The dots in the Figure represent the data points collected from a hypothetical respondent. The bold line represents the cubic spline interpolation.

[2] Of course, it would be possible to fit higher order polynomials. This would require additional assumptions on the degree of smoothness of the function $f(x)$.

[3] We also experimented with a boundary condition restricting the first derivative at x_0. We found that the estimated cubic spline is in many cases very similar.

[4] See Judd (1998) for a more general presentation of splines.

Censoring

The spline interpolation can be further generalized to allow for censoring at the lower end x_0 and upper end x_n of the distribution. Such corner solutions occur frequently in experimental games where players can for example return any amount inside a specified range. Section 4 presents an application with censored distributions. The method proposed can accommodate censoring with only minor modifications. Panel b) in Figure 1 illustrates the approximation of a censored distribution where a hypothetical respondent places subjective probability on values of x_0 and x_n occurring. As we can see, the spline simply starts above 0 at x_0 and ends below 1 at x_n.

2.2 Subjective Expectations

Summary statistics of an uncensored variable X can be directly estimated from the fitted subjective cumulative distribution function. It is simple to show that the t-th moment of X can be computed analytically using

$$\widehat{\mathbf{E}}^t(X) = \sum_{j=1}^{n} \left[\frac{\widehat{b}_j x^{t+1}}{t+1} + \frac{2\widehat{c}_j x^{t+2}}{t+2} + \frac{3\widehat{d}_j x^{t+3}}{t+3} \Bigg|_{x_{j-1}}^{x_j} \right]. \tag{1}$$

In the case of censoring from below at x_0 and from above at x_n, a slight modification of (1) yields

$$\widehat{\mathbf{E}}^t(X) = x_0 \widehat{F}(x_0) + \sum_{j=1}^{n} \left[\frac{\widehat{b}_j x^{t+1}}{t+1} + \frac{2\widehat{c}_j x^{t+2}}{t+2} + \frac{3\widehat{d}_j x^{t+3}}{t+3} \Bigg|_{x_{j-1}}^{x_j} \right] + x_n(\widehat{\Pr}(X = x_n)). \tag{2}$$

A slightly more difficult task consists of approximating the subjective expected value $\mathbf{E}(h(X;\boldsymbol{\eta})) = \int h(x;\boldsymbol{\eta})dF(x|i)$ of any function $h(\cdot)$ known up to some known finite vector of parameters $\boldsymbol{\eta}$ when no closed form expression exist. One example of $h(\cdot)$ is a utility function of an agent with constant absolute risk aversion, such that $\boldsymbol{\eta}$ denotes the risk aversion parameter. In structural dynamic econometric models,

$h(\cdot)$ denotes the maximum of future value functions, and $\boldsymbol{\eta}$ a vector of preference parameters (see Rust (1994)).

In such cases, numerical integration will need to be performed. In the recent years, simulation methods have proven useful to approximate expectations in different econometric models (see Train (2003)). As long as the estimated piecewise polynomial function $\widehat{F}(x|i)$ is monotonically increasing, an arbitrary number of random draws can be taken from the fitted cumulative distribution function and used to approximate $\mathbf{E}(h(X;\boldsymbol{\eta}))$. In line with existing simulation based methods, we propose to generate draws by simply inverting the estimated piecewise polynomial function $\widehat{F}(x|i)$.[5] In particular, a sequence of draws $\{\widehat{x}_s : s = 1, 2, ..., S\}$ from the estimated subjective distribution of subject i is obtained using $\{\widehat{x}_s = \widehat{F}^{-1}(\mu_s|i) : s = 1, 2, ..., S\}$ where $\{\mu_s|s = 1, 2, ..., S\}$ represents a sequence of i.i.d draws taken from a uniform [0,1] distribution. The approximated expectations are obtained using

$$\widehat{\mathbf{E}}(h(X;\boldsymbol{\eta})) \approx \frac{1}{S}\sum_{s=1}^{S} h(\widehat{x}_s;\boldsymbol{\eta}). \tag{3}$$

As is well known, the precision of the approximation increases with the number of draws S. See Train (2003) for a detailed overview of simulation techniques and their overall efficiency.

2.3 Monotonicity

The cubic spline approximation does not generally guarantee that the approximated cumulative distribution function is monotonically increasing. This is particularly problematic when drawing from an approximated distribution, as generating $\{\widehat{x}_s : s = 1, 2, ..., S\}$ requires that the inverse of the approximated function $F(\cdot|i)$ be unique. Non-monotonicity is likely to occur when respondents place no probability mass in

[5] We also experimented with quadrature techniques to integrate over $h(x;\boldsymbol{\eta})$ (see Judd (1998)). We found that the simulation approach was computationally faster and more stable than quadrature.

one or more intervals. Such a problem is represented in panel (a) of Figure 2. There, the spline approximation is fitted to data such that no probability mass is placed between $\widehat{x}_1$ and $\widehat{x}_3$. As can be seen, the requirement that the spline be continuous at the knots results in a non-monotone approximation. As a result, inverting draw μ (or any draw in the same neighborhood) does not produce a unique solution.

Perhaps the simplest way to correct for non-monotonicity is to use the Hyman filter (Hyman, 1983). This filter works in two steps. In a first step, define $\widehat{f}'(x_i)$ as the estimated value of the first derivative of the spline function at the point x_i. Next, define $S_{i-1/2} = (\widehat{F}(x_i|i) - \widehat{F}(x_{i-1}|i))/(x_i - x_{i-1})$ and $S_{i+1/2} = (\widehat{F}(x_{i+1}|i) - \widehat{F}(x_i|i))/(x_{i+1} - x_i)$ respectively as the left-hand side slope connecting with the previous knot $(\widehat{F}(x_{i-1}|i), x_{i-1})$ and right-hand side slope connecting with the following knot $(\widehat{F}(x_{i+1}|i), x_{i+1})$. Boor and Schwarz (1977) have shown that if an estimated function satisfies the following criteria

$$0 \leq \widehat{f}'(x_i) \leq 3\min(S_{i-1/2}, S_{i+1/2}), \tag{4}$$

then it is monotone on the interval $[x_i, x_{i+1}]$. The criteria (4) can thus be used to identify all points where monotonicity is violated. In a second step, the condition of the equality of the second derivatives at each of the knots where monotonicity is violated is replaced by

$$\widehat{f}'_{x_i} = \min\left[\max\left(0, \widehat{f}'(x_i)\right), 3\min\left(S_{i-1/2}, S_{i+1/2}\right)\right].$$

Applying the two steps to the example in Figure 2 produces the spline interpolation presented in panel (b). As we can see, the Hyman filter effectively corrects the non-monotonicity present in panel (a). The problem of invertibility is reduced to a single point on the vertical axis, where the cumulative distribution function levels off. In the (unlikely) event that a draw μ is taken at that point of the vertical axis, we propose to select with equal probability either the lower bound $\widehat{x}_1$ or upper bound $\widehat{x}_3$ of the interval as the corresponding draw from the fitted distribution. As long as enough draws are taken from the fitted distribution, this randomization is unlikely to have an important effect on the approximation.

3 Goodness of fit

3.1 Approximated Distributions

To be effective, the approach proposed should be able to fit a wide range of distributions with a reasonably small set of questions (knots). In this section, we evaluate how well the approach proposed fits three different distributions: a symmetric distribution, an asymmetric distribution, and a bimodal distribution. The symmetric distribution is chosen to be a standard normal N(0,1). The asymmetric distribution is chosen to be a chi-square distribution with 3 degrees of freedom. This distribution is severely skewed to the left and is unimodal. For the bimodal distribution, the density $f(x)$ was chosen to be $\frac{sin(x)+1}{A}$ defined over the [0,3π] interval, where $A = 2 + 3\pi$ insures that the function integrates to 1 over its domain. This density function has two modes: the first at $\pi/2$ and the second at $5\pi/2$.

We fitted each cumulative distribution function using 4 to 6 knots. With 4 knots, the domain of the random variable is split into 3 intervals, requiring 3 probability questions. For 6 knots, the domain is split into 5 intervals requiring as many probability questions. Boundary knots (the first and last knots) are assumed to be known by the analysts (either by experimental design or prior information on the range of possible values).[6] Boundary knots of the normal distribution were chosen to be 3 and -3, those of the chi-square distribution are chosen to be 0 and 15, while those of the sinus function are chosen to be 0 and 3π. All knots are equally spaced between the boundary knots. Using these knots, we fit the underlying distributions using three different forms of interpolations: linear interpolation, quadratic splines, and cubic splines. Quadratic splines are similar to cubic splines but exploit a lower degree of differentiability in the underlying function. Details on the computation of quadratic splines are similar to those of cubic splines and can be found in the appendix.

[6]In the case where X is income, a natural interval is $[0, \max(X)]$, where $\max(X)$ denotes an empirically relevant maximal income level.

Results for the standard normal distribution are reported in Figure 3. We find that the linear and quadratic spline interpolation have difficulties capturing the curvature of the function with only 4 knots. The cubic spline on the other end already provides a reasonable fit. As expected, the goodness of fit increases with the number of knots for all three interpolations. With 6 knots, the cubic spline sill outperforms the other two interpolations at the lower and upper tails of the distribution.

Figure 4 reports results for the chi-square distribution. While the cubic spline clearly outperforms the other two interpolations with 4 knots, all three interpolations have some problems fitting the lower end of the distribution. Slight problems remain in the lower hand of the distribution when we increase the number of knots from 4 to 6.

Finally, Figure 5 presents the fitted bimodal sinus distribution. We find that the cubic spline interpolation has more difficulties fitting this bimodal distribution than the other two distributions. All three interpolations provide poor fits with 4 and 5 knots. With 6 knots, the cubic spline interpolation clearly outperforms the other two interpolations, and manages to provide a very good fit.

3.2 Approximated Expectations

We next asses the bias of the expected value inferred from the approximated cumulative distributions. To simplify the comparisons across the three distributions, we computed the bias of each approximated distribution relative to the theoretical variance of the underlying true distribution. The approximated expectation is computed using (1). Table 1 presents the results. As expected, the (standardized) bias of the approximated means diminish with the number of knots. We do not find that the approximation systematically overestimates or underestimates the true mean of any of the three distributions. Furthermore, the bias is relatively small in all three cases when 5 and 6 knots are used. This is especially true of the cubic spline interpolation.

4 Empirical illustration : the investment game

4.1 Experimental design and procedure

Our experimental design is a modified version of the two player investment game of Berg, Dickhaut and McCabe (1995). In our experiment, a first player and a second player were both endowed with 6$US.[7] Contrary to Berg, Dickhaut and McCabe (1995), we restricted the decision space of first players to two choices: investing all or none of the endowment. If the first player invested his endowment, that amount was doubled and added to the endowment of the second player. In turn, the second player had the opportunity to return any amount from his augmented endowment tot he first player. By doubling investments, a surplus is generated, opening up the possibility that second players reward first players who invested. We used a binary version of the game because we wanted to elicit the first players' subjective distribution functions about second player behavior for every possible choice they could make. Expending the choice set of first players is in principle possible, but this will require asking each participant to answer many more questions on their beliefs (see below).

Before observing the investment decision, second players had to decide how much to return to the first player if that player invested his endowment, and how much to return if the first player did not invest his endowment. The decision which corresponded to the actual choice of the first player was chosen to be the effective action and determined the payoff of both participants. This method of eliciting the complete strategy of a player has the advantage of gathering choice data of all decisions which may occur.[8] After all participants made their decisions, first and second play-

[7]The complete content of the computer screens can be downloaded from http://w3.ecn.ulaval.ca/∼cbellemare.

[8]There is no clear evidence that the possibility of direct reaction by a second player to a choice of a first player ("hot" environment) triggers stronger responses in various two player games (see McLeish and Oxoby (2004) and Brandts and Charness (2000)).

ers were randomly matched and payoffs were computed based on the decisions of the pair. Participants were informed about the outcome only after completion of the experiment.

After making their decisions, all first players were asked to answer questions concerning their subjective beliefs. Players were not rewarded for the accuracy of their beliefs.[9] Before stating their beliefs, they were further reminded of the decision tasks and given examples to clarify the belief elicitation procedure. First players could go forward and backward between the screens used to elicit beliefs, but they could not go back to change their decisions.

All first players had to state their subjective beliefs in two scenarios. We first asked them to consider what would happen if they did not invest their endowment. For that scenario, second players could return any amount between 0$ and their personal endowment of 6$. To elicit beliefs, first players had to state how many of 100 second players would return 0$,[10] and how many would return amounts in the following intervals $\{(0,1], (1,2], (2,3], (3,4], (4,5], (5,6]\}$.[11,12] By allowing first players to place a positive probability on getting back 0, we allow their subjective distribution functions to be censored from below. The elicitation of distribution functions conditional on first players investing their endowment followed a similar line. In this case, second players receive 12$ from investors (the investment of 6$ multiplied by 2) which is added to their own personal endowment of 6$. Hence, they could choose to return any amount between 0$ and 18$. To elicit beliefs, first players had to state how many

[9]Friedman and Massaro (1998) and Sonnemans and Offerman (2001) find insignificant differences between elicited beliefs of paid and unpaid subjects.

[10]This follows Hoffrage, Lindsey, Hertwig, and Gigerenzer (2000) who find that people are better at working with natural frequencies than with percent probabilities.

[11]Winkler (1967) found that respondents had more difficulties stating beliefs in cdf rather than in pdf form. We tested direct elicitation of cdfs in a pilot session with 22 subjects and come to the same conclusion. The level of noise appeared lower when participants formulated their beliefs in terms of intervals of the density function rather than as points on the cumulative distribution function.

[12]If the probability mass entered exceeded 100, players where automatically instructing to go back and adjust their answers.

of 100 second players would return 0$, and how many would return amounts in the following intervals $\{(0,3], (3,6], (6,9], (9,12], (12,15], (15,18]\}$.

Another issue concerns the order in which the decisions and belief questions were presented to players. Asking beliefs after the actual choice raises the concern that decision makers state beliefs to rationalize their decisions. In order to detect these potential problems, we randomized approximately one third of all participants in our experiment to a group of "observers," who did not make any decisions but who answered the belief questions after having read the same instructions as all other participants. Comparing the answers of observers and first players thus provides an indication of the influence decision making has on stated beliefs.[13] Observers received each 6$ for their participation.

At the end of the experiment, all participants were informed about the outcome of the experiment and their final payoffs. They were also asked to fill a post-experimental questionnaire gathering information on basic background characteristics and their personal comments. Before leaving the laboratory, all participants received their payoff from the experiment in a sealed envelope with their ID number. The experiment was conducted in May 2005 at the Economic Science Laboratory at the University of Arizona using the software zTree (Fischbacher (2007)). Participants were recruited via email and were mainly students in finance, business administration, economics, and engineering. Participants received a 5$ show-up fee upon arrival at the laboratory. In total 122 participants interacted in the 9 sessions of the experiment. Roles were assigned randomly. We observed 38 pairs of first-second players as well as 46 observers. An experimental session lasted on average 60 minutes, and, including their show up fee, participants earned on average 12.18$ (9.92$ as first players, 15.87$ as second players, and 11.00$ as observers).

[13] Examples of past research using observers are Dawes, McTavish and Shaklee (1977) and Offerman, Sonnemans and Schram (1996).

4.2 Data

24 of the 38 first players (63%) invested their endowment. This proportion is in line with existing studies on the investment game with binary proposals and a multiplier of two (e.g. see Dufwenberg and Gneezy (2000)).

Figure 6 presents the estimated cumulative distribution functions of subjective beliefs for four sending players considering the event of investing. This figure highlights the flexibility of our method to adapt to different sets of beliefs. The dark points in each graph represent the knots used to compute the cubic spline interpolation. These knots are derived from the answers each first player gave to the 7 probability questions. We present below each graph a histogram of the reported probability mass placed on receiving nothing and in each of the subsequent 6 intervals. We see that the first probability distribution appears symmetric and could be well approximated by some known distribution (e.g. normal). The probability distributions of the other players are clearly non standard and exhibit a mixture of discrete (with a significant probability on getting back nothing) and continuous distributions. The second probability distribution is bimodal, placing a high probability on either getting back nothing or getting back between 6$ and 12$. The third distribution is unimodal but skewed to the left. The fourth distribution is also skewed to the left but places close to 70% probability on getting back nothing in the event of investing. Despite these differences, the cubic spline approximation has no problem accommodating any of the four sets of data points. Finally, the violations of monotonicity of the fitted cumulative distribution functions of first players are relatively minor. Of the 76 fitted distributions (two for each of the 38 first players), only 12 present small visible violations of monotonicity on subsets of the distributions.[14]

[14] Graphs of all the fitted distributions are available upon request.

First Players vs. Observers

We first investigate whether stated beliefs of first players appear to have been influenced by their decisions. To proceed, we compare the stated beliefs of first players when considering investing and not investing with the corresponding beliefs of participants in the observer treatment. In the later case, participants stated their beliefs without making any prior or subsequent decisions. Hence, a comparison of the beliefs of both groups provides an indication of how decision making can affect stated beliefs. Figure 7 presents the average beliefs of observers (light bars, $N = 46$) along side those of first players (dark bars, $N = 38$). We find that beliefs of all first players and observers are remarkably similar, both when they consider investing and when they consider not investing. Chi-square tests do not reject the null hypothesis that distributions are the same between treatments, both when investing (p-value = 0.549) and when not investing (p-value = 0.218). These results suggest that the stated beliefs of first players where not significantly affected by their decisions.[15]

Subjective beliefs vs. realizations : aggregate differences

Figure 8 presents the discrete subjective probability distributions averaged over all first players (dark bars, $N = 38$) and actual return decisions averaged over all second players (light bars, $N = 38$), contingent on investing and not investing.[16]. To compare with the distribution of observed responses, we discretized the amounts second players return to the same intervals used to elicit beliefs of first players. The left graph of Figure 8 presents the comparison in the event that first players invest. The right hand graph presents the comparison in the event that first players do not invest. Qualitatively, the subjective and realized distributions are very similar. On average, first players placed a 68.4% probability of getting nothing from the second players when

[15]Similar results are obtained by Offerman, Sonnemans, and Schram (1996).

[16]Because of the strategy method, we observe two decisions for each second player: how much they return if first players invest and how much to return if first players do not invest.

they consider not investing their endowment. In comparison, the objective probability of this event is 89.6%. When first players consider investing their endowment, the average subjective and realized distributions are similar for amounts returned of 3$ or less. On the other hand, first players on average underpredict the probability of receiving between 3$ and 9$ (30.4% versus an objective 47.9%), but overpredict the probability of receiving amounts above 9$ (8.5% versus an objective 2.1%).

Subjective beliefs vs. realizations : individual differences

To explore the heterogeneity in beliefs, we computed for each first player the difference between their subjective expectations (computed using equation (2)) and the average amount returned by second players both when first players invest and do not invest their endowment. Figure 9 presents the distribution of those differences in the event where first players invest (left graph) and not invest (right graph). Positive (negative) values on the horizontal axis represent first players who have subjective expectations exceeding (falling short of) the observed average behavior of second players.

In the event that first players invest, we find substantial dispersion in the expectations across players, with an important fraction of first players having expectations which are both below and above the observed average responses. In the event that first players do not invest, we find lower dispersion in expectations, with a substantial part of the distribution centered around the average observed second player behavior. Hence, a substantial fraction of players correctly anticipated that they can expect to receive very little in the event of not investing, whereas there is much more dispersion in individual beliefs in the event of investing relative to what actually occurred.

Investors vs. Non-investors

To gain some insights on whether investors and non-investors trusted the second player differently, we compare the distributions of beliefs of investors with the distributions of beliefs of non-investors ($N = 14$). Figure 10 presents the average subjective belief distributions of investors (light bars, $N = 24$) and non investors (dark bars, $N = 14$). We find that both groups had similar beliefs about second players' behavior if they consider not investing their endowment. In particular, both investors and non investors place a very high probability of getting nothing back from second players. Differences between both groups emerge when we look at their beliefs in the event of investing their endowment. There, non investors placed a 48.3% probability on getting nothing back from second players, substantially less than the 24.6% probability placed by investors as a whole. Moreover, investors placed significantly more probability than non investors on getting back any positive amount. These results indicate that investors believed they would get higher returns when investing their endowment than non-investors, evoking the idea that they trusted relatively more that second players would not behave in a purely self-interested manner. In fact, the decision of investors could be motivated by pure expected money maximization if the expected returns in the event of investing are greater than 6$ plus the (low) expected return in the event of not investing. The belief data reveals a very different picture: 81.5% of investors have expected final payoffs which are lower when they invest than when they do not invest. These numbers are comparable to those found elsewhere in the literature (see Dufwenberg and Gneezy, 2000; Ashraf, Bohnet and Piankov, 2006). This suggests that a model where agents maximize expected earnings would not be sufficient to explain the proportion of investments observed.

4.3 A Simple Model of Choice

The results of the previous section suggest that differences in expectations alone are not sufficient to explain investment behavior. To formally test this hypothesis, we next specify and estimate a simple structural model of investment behavior.

We start by assuming that the utility of not investing for player i is given by $u_i^{keep} = \beta(w + r_i^{keep})$, where r_i^{keep} denotes the amount the second player returns to first player

i when i does not invest, w denotes the monetary endowment that the first player keeps when he does not invest, and β measures the marginal utility of income.[17] The amount returned when not investing r_i^{keep} can vary between 0 and the endowment w of the second player.

When first players invest, they forego and amount w which is then doubled and transferred to the second player they are matched with. As a result, a surplus of w is created when investing. We model the utility of investing for player i as $u_i^{invest} = \beta r_i^{invest} + \theta$, where r_i^{invest} denotes the amount returned to the investor, and θ denotes the first players' utility attached to creating a surplus of w when investing. Recent studies suggest that concerns for social efficiency may be particularly important (see Charness and Rabin (2002), Engelmann and Strobel (2004)).[18] In terms of our model, this would imply that $\theta > 0$. The amount returned r_i^{invest} can vary between 0 and the wealth of the second player. The later is given by the sum of his endowment w and $2w$, the investment of sending players doubled. Thus, second players can return an amount between 0 and $3w$ (see Section 4 on the experimental design).

We next assume that first players make their decisions by comparing their subjective expected utilities of investing and not investing. The expected utilities of not investing and investing are given by

$$\begin{aligned} \mathbf{E}\left(u_i^{keep}\right) &= \beta\left(w + \mathbf{E}\left(r_i^{keep}\right)\right) && (5) \\ \mathbf{E}\left(u_i^{invest}\right) &= \beta \mathbf{E}\left(r_i^{invest}\right) + \theta, && (6) \end{aligned}$$

[17]To investigate whether the risk neutrality hypothesis is reasonable, we asked participants to play at the end of the experiment a sequence of lotteries similar to that proposed by Holt and Laury (2002). In particular, each participant was asked to choose 10 times between two binary lotteries. Because the risk of winning low and high payoffs varied across the 10 choices, the decisions of an individual identifies a tight range around his coefficient of risk aversion (see Holt and Laury (2002) for details). We found no significant relationship between measured risk preferences and investment behavior. Similar results have been found by Eckel and Wilson (2004) and Houser, Schunk, and Winter (2006).

[18]The preferences estimated are equivalent to preferences with linear altruism $u_i = \gamma x_i + \alpha x_j$ with $\gamma = \beta + \alpha$, $\alpha = \theta / w$, and where x_i and x_j respectively denote income of player i and j.

where the expectations are computed with respect to the subjective distribution functions of first player i. To allow for the fact that some first players will make sub-optimal choices, we add standard normal error terms ϵ_i^{invest} and ϵ_i^{keep} to the true expected utilities $\mathbf{E}(u_i^{invest})$ and $\mathbf{E}(u_i^{keep})$, and assume that first player i chooses the option $j \in \{keep, invest\}$ that maximizes $\mathbf{E}(u_i^j) + \epsilon_i^j$ rather than $\mathbf{E}(u_i^j)$.

4.4 Inference without Information on Subjective Expectations

Before estimating $\boldsymbol{\eta} = (\beta, \theta)$ using both the choice and expectations data, we first estimate the region of (β, θ) that can be identified using only the choice data alone. The following is based on the analysis of the binary choice model with linear utilities discussed in Manski (2007). The idea is to characterize the set of values of (β, θ) which are consistent with the observed choice distribution. In the special case of the binary choice model with linear utilities, it can be shown that this set is convex (Manski, 2007). To estimate this set, we first consider the extreme case where all first players expect to receive with probability 1 the highest possible amount when investing ($r^{invest} = 3w$) and the lowest possible amount when not investing ($r^{keep} = 0$). This gives rise to the largest payoff difference between investing and not investing. In this case, the decision rule is to invest when

$$\beta(2w) + \theta + \epsilon_i > 0. \tag{7}$$

A second extreme case occurs when all first players expect to receive with probability 1 the lowest amount possible when investing ($r^{invest} = 0$), and the highest possible amount when they do not invest ($r^{keep} = w$). This gives rise to the smallest payoff difference between investing and not investing. In this case, the decision rule is to invest when

$$\beta(-2w) + \theta + \epsilon_i > 0. \tag{8}$$

Assuming that errors ϵ_i are statistically independent of each other and follow a standard normal distribution, aggregating inequalities (7) and (8) across the population yields the following set of inequalities relating the population probability of investing to the model parameters

$$\Phi\left(\beta(-2w) + \theta\right) \leq \Pr(invest) \leq \Phi\left(\beta(2w) + \theta\right) \tag{9}$$

where $\Phi(\cdot)$ denotes the standard normal cumulative distribution. The identification region for (β, θ) is the set of parameter values that satisfy inequalities (9).

The shaded area in Figure 11 represents the identified region estimated by replacing $\Pr(invest)$ with the proportion of investments observed in our sample. It is immediate from (9) that θ is point-identified and equal to $\Phi^{-1}(\Pr(invest))$ when expectations have no influence on the decision process ($\beta = 0$). Otherwise, the observed proportion of investments is compatible with any combination of $\beta > 0$ and θ within the shaded area. We can easily see that the identified range of the social preference parameter θ increases with β, the strength of the effect of expectations on investment behavior. This suggests that a wide range of social preferences are possibly consistent with the choice data.

It is possible to reduce the size of the identified region by making further assumptions. For instance, it is plausible to assume that $\mathbf{E}(r^{invest}) \geq \mathbf{E}(r^{keep})$, which says that players do not expect to receive less when they invest as opposed to when they do not invest. Under this assumption, inequality (7) is unchanged, but inequality (8) is replaced by

$$\beta(-w) + \theta + \epsilon_i > 0. \tag{10}$$

Aggregating inequalities (7) and (10) across the population produces a new set of inequalities relating the population probability of investing and the model parameters

$$\Phi\left(\beta(-w) + \theta\right) \leq \Pr(invest) \leq \Phi\left(\beta(2w) + \theta\right). \tag{11}$$

The smaller identified region derived from (11) is given by the dark shaded area in the Figure (11). As expected, the new area is a strict subset of the area derived previously.

Nevertheless, a wide range of values of β and θ remain consistent with the observed choice behavior.

Another way to reduce the size of the identification region is to assume that first players have objectively correct (rational) expectations. This would imply that $\mathbf{E}\left(r_i^{invest}\right)$ and $\mathbf{E}\left(r_i^{keep}\right)$ both coincide with observed average second player behavior, $\bar{r}^{invest}$ and $\bar{r}^{keep}$, and are common for all players. Then, the identification region is a line, connecting all values of β and θ which solve

$$\Phi\left(\beta(\bar{r}^{invest} - \bar{r}^{keep}) - \beta w + \theta\right) = \Pr(invest). \tag{12}$$

The dashed straight line in Figure (11) represents the estimated identified region assuming rational expectations, estimated by replacing $\bar{r}^{invest}$ and $\bar{r}^{keep}$ with the corresponding sample averages. While the rational expectations assumption effectively reduces the identified region to a single line, this assumption is unlikely to hold in our experiment since the beliefs of first players differ in important ways from the observed average behavior of second players (see Section 4.2).

4.5 Estimation results

The upper part of Table 2 presents the model parameters estimated by maximum likelihood using linear, quadratic, and cubic spline interpolation.[19] For all interpolations, the subjective expectations are computed analytically (see Section 2.2). For the cubic spline interpolation, we compare our results both when we impose and when we do not impose monotonicity of the estimated cumulative distribution functions.

Results are relatively similar across the different interpolations. This is consistent with the analysis of section 3 which indicated that, with 6 knots, linear, quadratic, and cubic interpolations provide similar fits to all the distributions considered. Since

[19] We also estimated models including measures of dispersion of the individual subjective distributions (subjective variances, interquartile ranges). These models produced insignificant increases in the log-likelihood.

7 knots are used in the experiment, differences across the three forms of interpolations are possibly even smaller. Accordingly, we will discuss the estimates obtained using the cubic spline approximation which imposes monotonicity using the Hyman filter.

Because β measures the marginal utility of earnings, it is reasonable to assume that it can take only non-negative values. We find that the estimated value of β is 0.117 and is significant at the 5% level against the one-sided alternative that $\beta > 0$. This suggests that differences in expectations between investing and not investing have a significant impact on the propensity to invest. We further find that the social preference parameter θ is 0.569 and significant at the 5% level against a two-sided alternative.[20] Others have reached similar conclusions by comparing the investment behavior across multiple treatments where expectations are manipulated by the experimenters (e.g. Cox (2004)). Our approach requires a single treatment, at the expense of having to collect subjective probability data.

It is interesting to highlight that we obtain similar conclusions when using the cubic spline approximation without restricting the approximated distribution functions to be monotonic. This is consistent with our previous observation (see Section 4.2) that monotonicity is not severely violated in our data.

5 Conclusion

In this paper we proposed a flexible and simple method to approximate subjective probability distributions using answers to a small set of probability questions. We showed that 4 or 5 probability questions provide sufficient information to approximate well a wide variety of distributions. This suggests that the approach proposed is relatively inexpensive to implement in practice in areas where data on subjective

[20] The standard errors reported in the table are possibly a little conservative as they do do not account for noise in the approximated expectations. Furthermore, both parameters remain significant when we perform our tests using bootstrapped critical values derived from the empirical distribution functions of both test statistics (500 bootstrap repetitions).

probability distributions are either already available or easily collectable (e.g. in experimental games of proposal and response).

We also showed how to compute subjective expectations of functionals of the random variable using the approximated distributions. While we focused on expectations of such functionals, the approach presented here can be easily extended to approximate any other points of these functionals (e.g. quantiles).

We illustrated the usefulness of the approach by measuring beliefs of first players concerning second player behavior in the investment game. In this way, we obtained a direct measure of first players' trust independently of their investment decision, allowing us to investigate whether trust is a significant determinant of investments. We found that differences in trust do have a significant effect on investment decisions. However, trust alone is insufficient to explain the majority of investments, suggesting that other factors – altruism or efficiency concerns – also affect investments in the game.

One limitation of the approach proposed is that the data collected is analyzed as if it were free of measurement error. Future research should explore ways to control for possible measurement error in the reported probabilities before using the spline interpolation. Furthermore, the choice of equidistant knots used in the paper was made for simplicity and is in no way optimal. Future work should also look at methods to chose more efficiently these knots in order to provide accurate approximations requiring possibly less probability questions.

A Quadratic Splines

The quadratic spline interpolation is based on the assumption that the function $F\left(x|\, i\right)$ is, on interval j, defined by $a_j + b_j x + c_j x^2$ where (a_j, b_j, c_j) are the interval polynomial coefficients. The set $\left\{(a_j, b_j, c_j) : j = 1, 2, \ldots, n\right\}$ contains the $3n$ unknown polynomial coefficients to be estimated. In order to estimate the set of polynomial coefficients, the interpolation exploits the continuity at the interior knots. This gives rise to $2n$ equations

$$\begin{aligned} F(x_{j-1}|i) &= a_j + b_j x_{j-1} + c_j x_{j-1}^2 \qquad \text{for } j = 1, 2, \ldots, n \\ F(x_j|i) &= a_j + b_j x_j + c_j x_j^2 \qquad \text{for } j = 1, 2, \ldots, n. \end{aligned}$$

Next, the assumption that $F(x|i)$ is continuous at the interior nodes implies the following $n-1$ equations

$$b_j + 2c_j x_j + 3d_j x_j^2 = b_{j+1} + 2c_{j+1} x_j \qquad \text{for } j = 1, 2 \ldots, (n-1)$$

One boundary condition is needed in order to estimate the polynomial coefficients of the quadratic spline. We tried many boundary conditions and found that the best results were obtained when imposing $F''(x_n|i) = 0$. This boundary condition restricts the interpolation to be linear on the last segment of the spline.

As with cubic spline, summary statistics can be directly estimated from the fitted subjective cumulative distribution function. For instance, the t-th moment of X, in the case of censoring from below at x_0 and from above at x_n, can be computed analytically using

$$\widehat{\mathbf{E}}^t(X) = x_0 \widehat{F}(x_0) + \sum_{j=1}^{n} \left[\frac{\widehat{b}_j x^{t+1}}{t+1} + \frac{2\widehat{c}_j x^{t+2}}{t+2} \Bigg|_{x_{j-1}}^{x_j} \right] + x_n(1 - \widehat{F}(x_n)) \tag{13}$$

B Figures

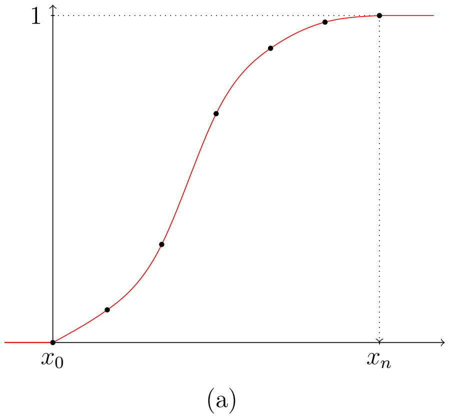

Figure 1: Fitted uncensored (Panel (a)) and censored below at x_0 and above at x_n (Panel (b)) cumulative distribution of a hypothetical respondent. Dots represent data points collected using answers to probability questions. Bold line represents the fitted distribution.

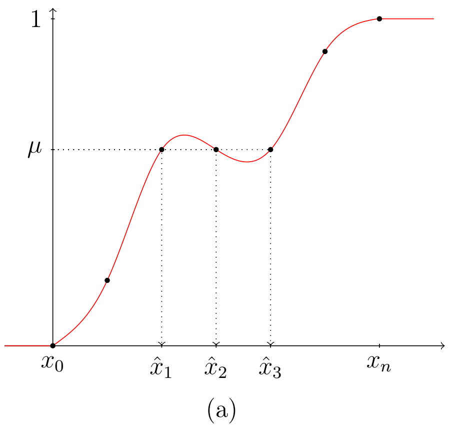

Figure 2: Drawing from an approximated subjective distribution without monotonicity correction (Panel (a)) and with monotonicity correction (Panel (b)).

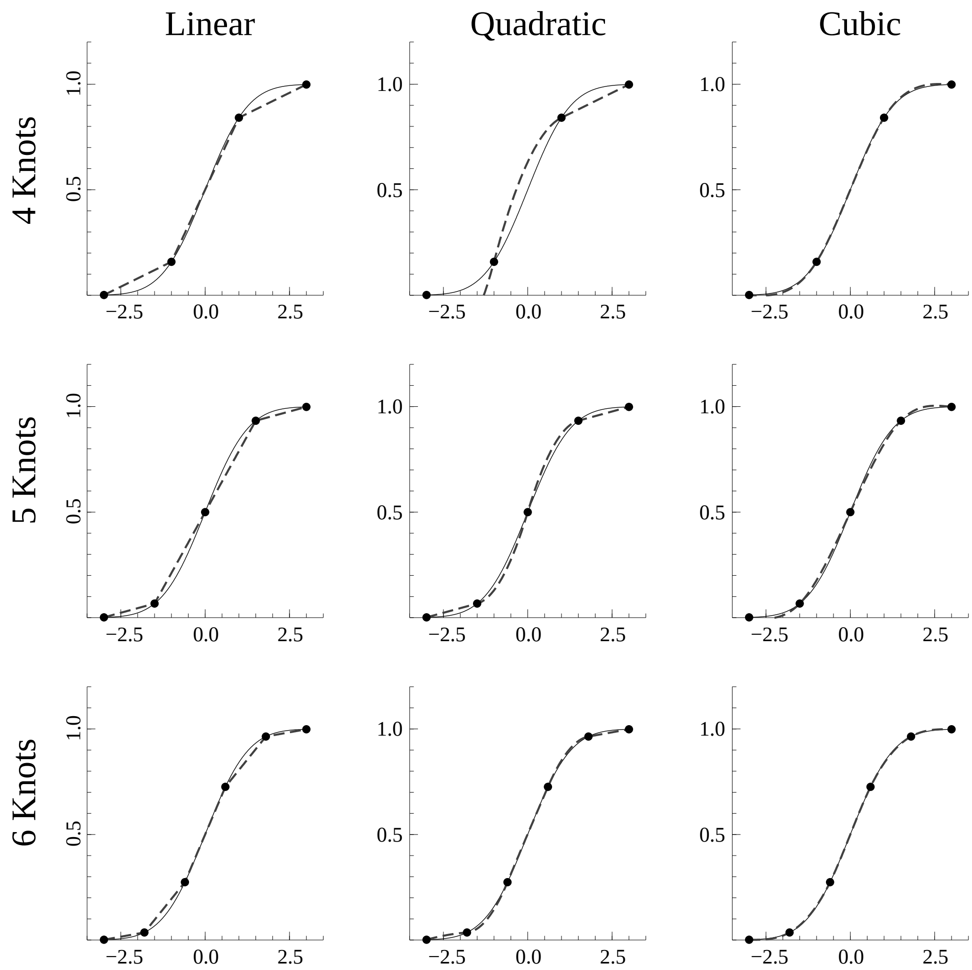

Figure 3: Fitted normal distributions using linear, quadratic and cubic spline interpolations for 4, 5, and 6 knots. The full line represents the true distribution. The dashed line represents fitted distributions using the number of knots (dark points).

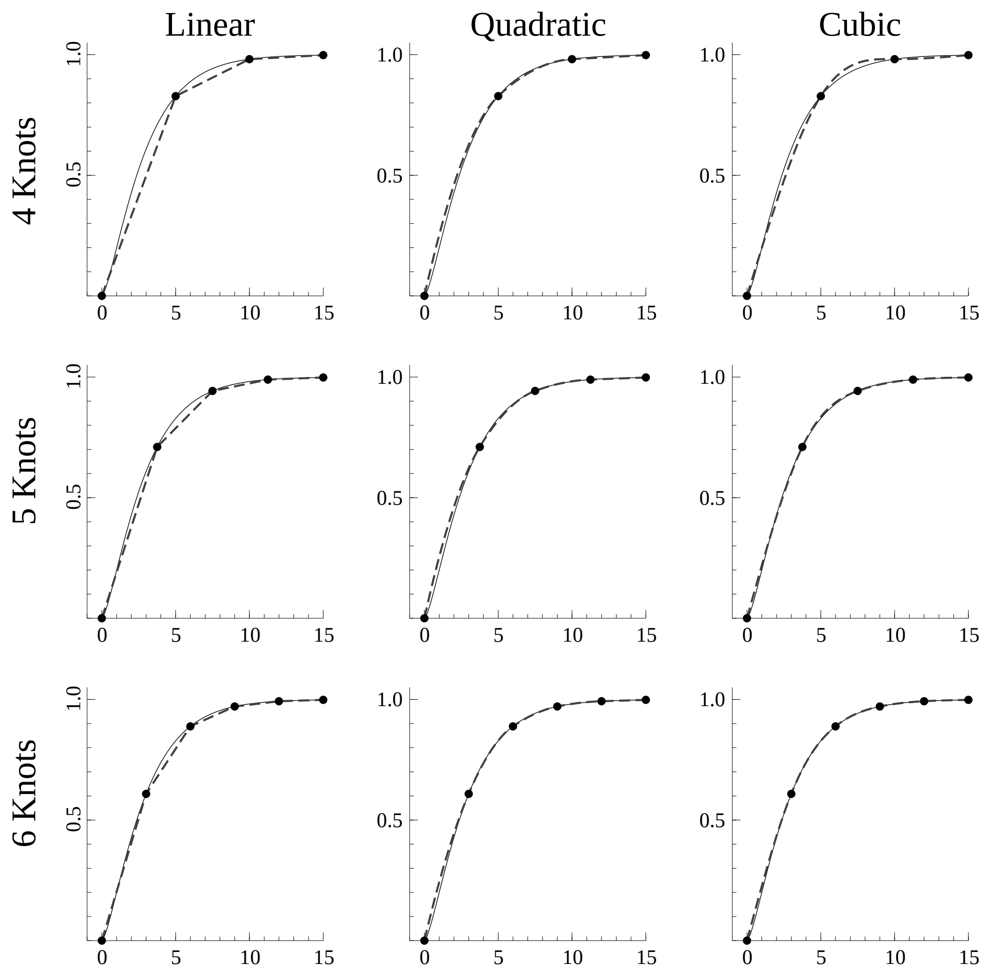

Figure 4: Fitted $\chi^2(3)$ distribution truncated to the [0,15] interval using linear, quadratic and cubic spline interpolations for 4, 5, and 6 knots. The full line represents the true distribution. The dashed line represents fitted distributions using the number of knots (dark points)

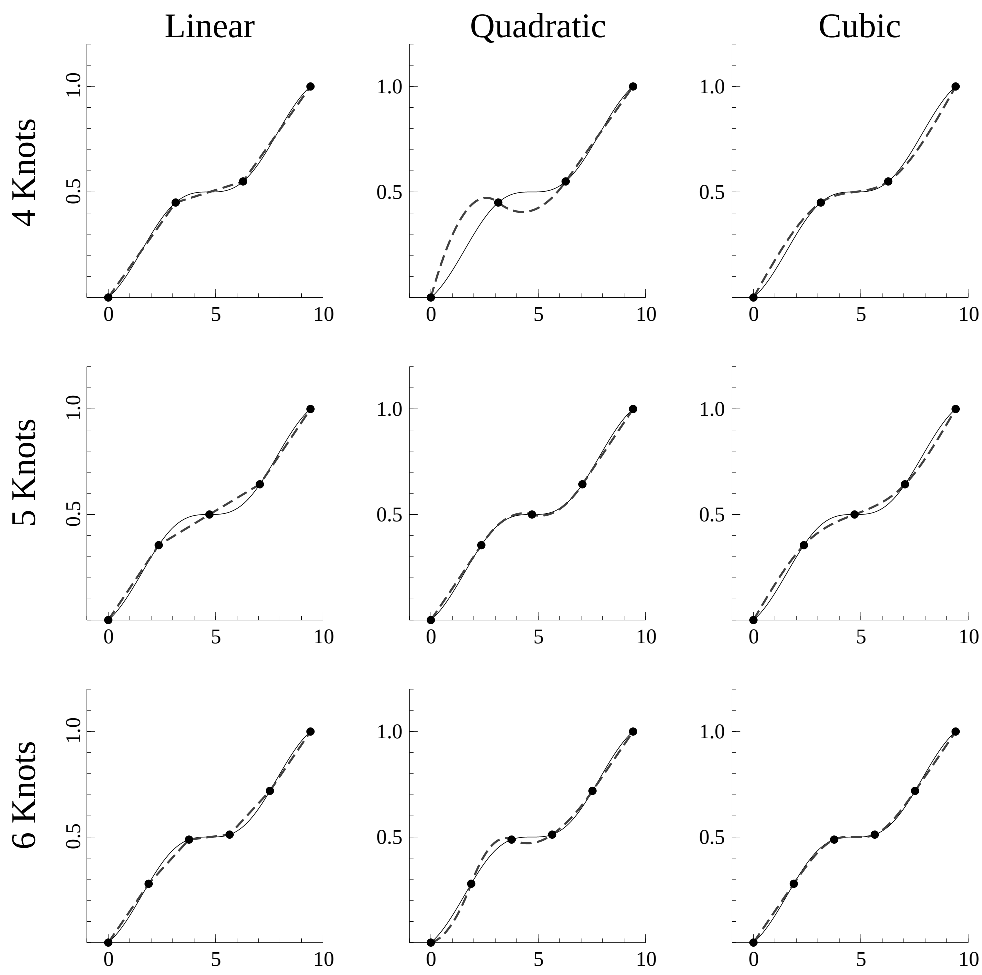

Figure 5: Fitted bimodal distribution using linear, quadratic and cubic spline interpolations for 4, 5, and 6 knots. The full line represents the true distribution. The dashed line represents fitted distributions using the number of knots (dark points). The bimodal density if given by $f(x) = \frac{sin(x)+1}{A}$ where $A = 2 + 3\pi$ and is truncated to the $[0,3\pi]$ interval.

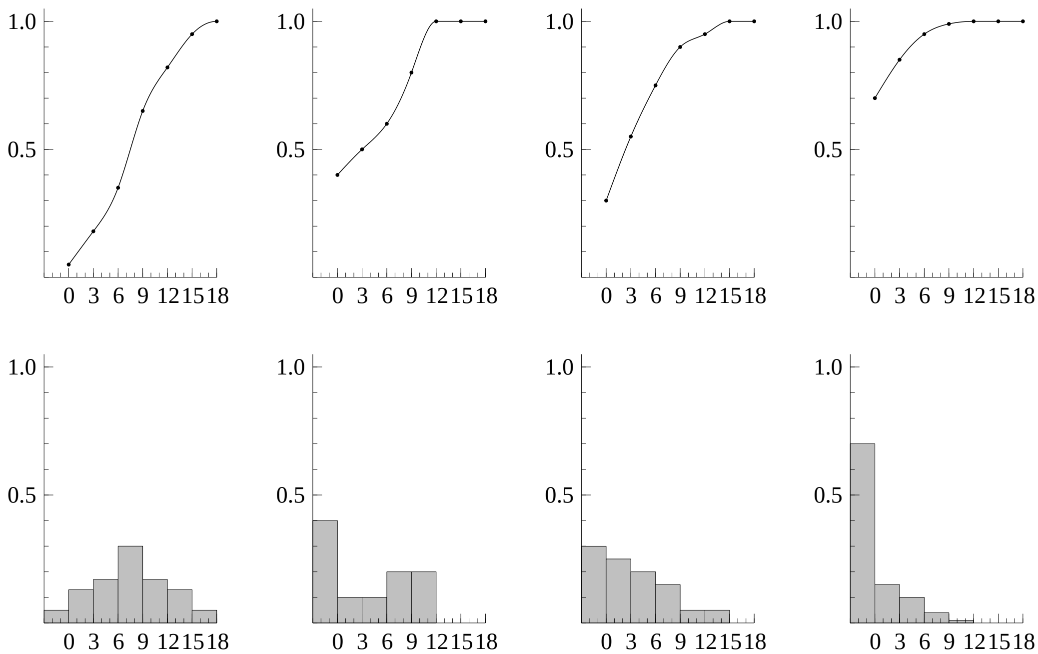

Figure 6: Spline approximation of subjective cumulative distributions when investing. Example of four of our 38 first players in the experiment.

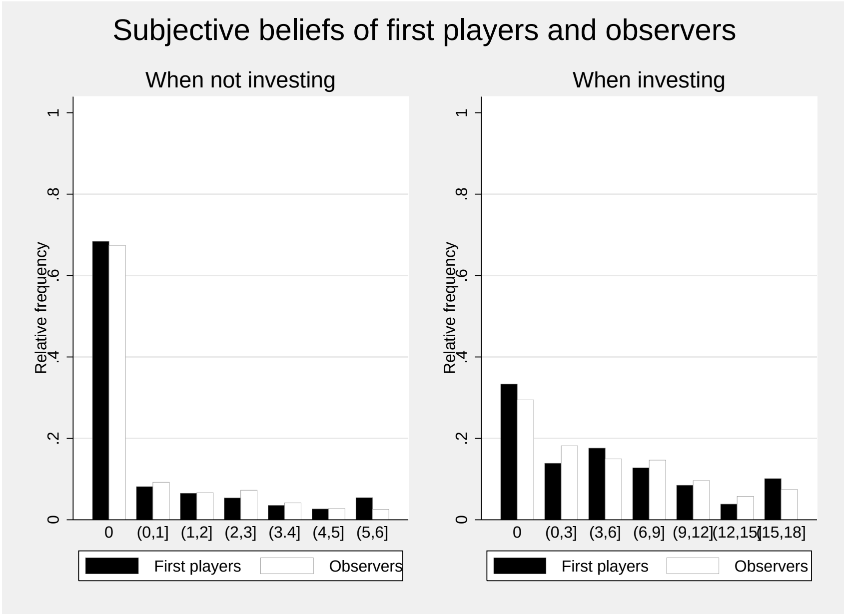

Figure 7: Subjective beliefs about the amount returned of all first players (dark bars, $N = 38$) and of observers (light bars, $N = 46$) in the event first players would not invest (left panel) and in the event they would invest (right panel).

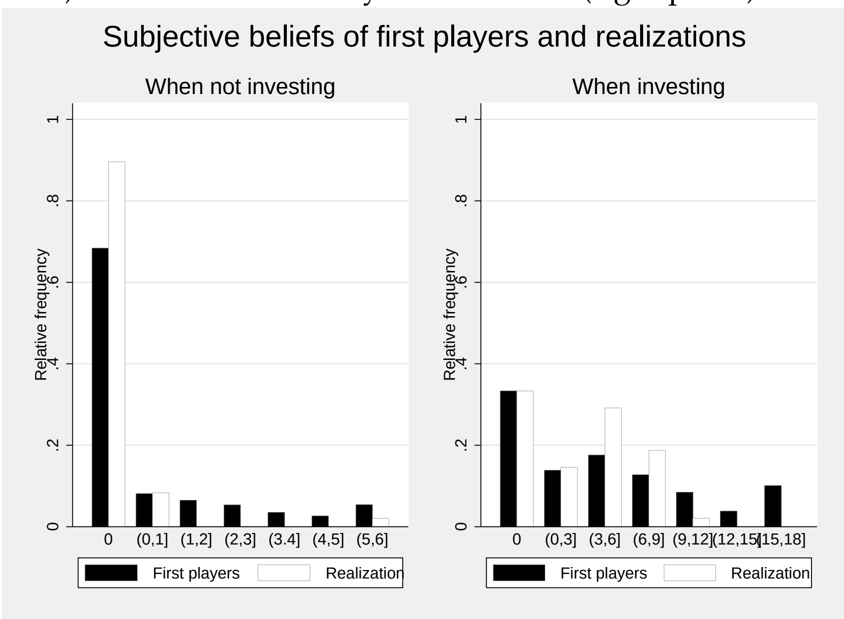

Figure 8: Subjective beliefs of all first players about the amount returned (dark bars, $N = 38$) and observed responses of all second players (light bars, $N = 38$) in the event first players would not invest (left panel) and in the event they would invest (right panel).

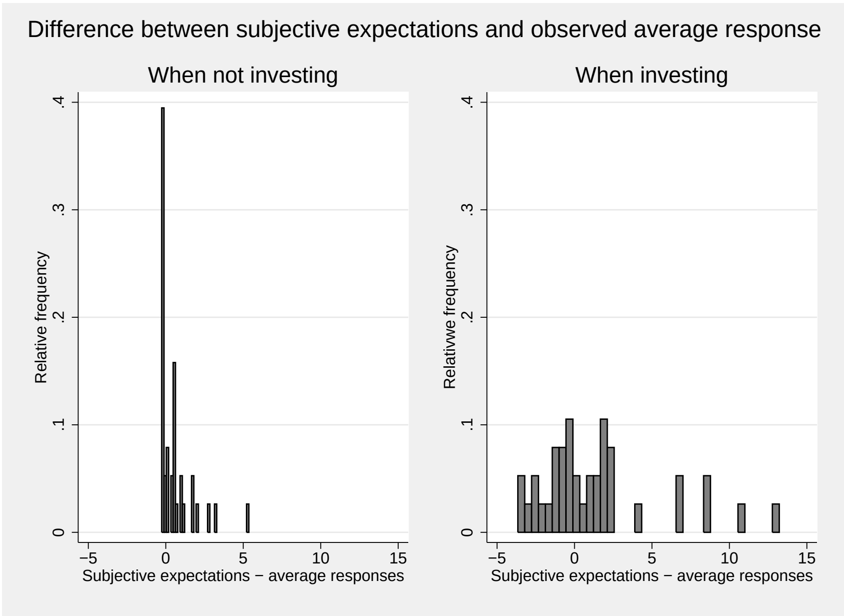

Figure 9: Distribution of the difference between subjective expectations of all first players and observed average response of all second players in the event of investing (top graph, $N = 38$) and in the event of not investing (bottom graph, $N = 38$).

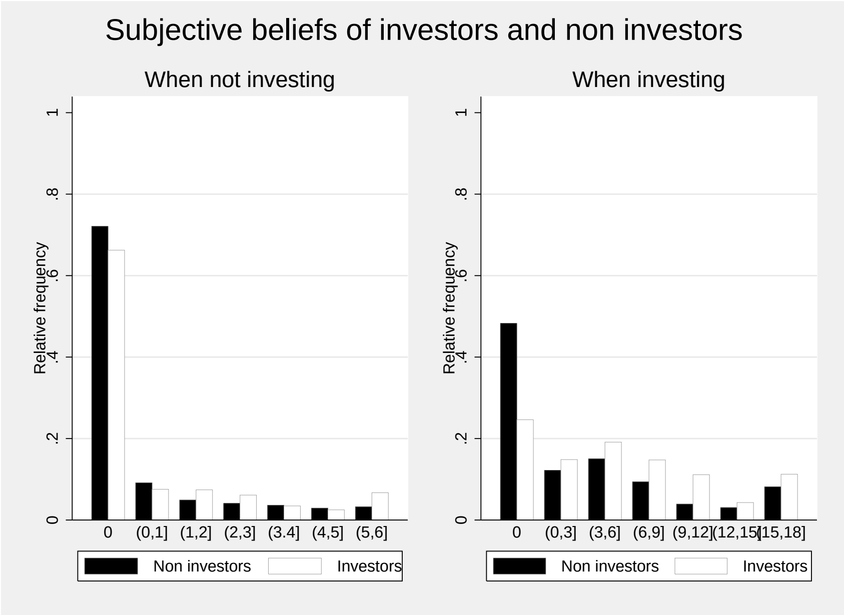

Figure 10: Subjective beliefs about the amount returned separately for investors (light bars, $N = 24$) and non investors (dark bars, $N = 14$). The left panel presents the distributions in the event first players would not invest, while the right hand side panel presents the distributions in the event they would invest.

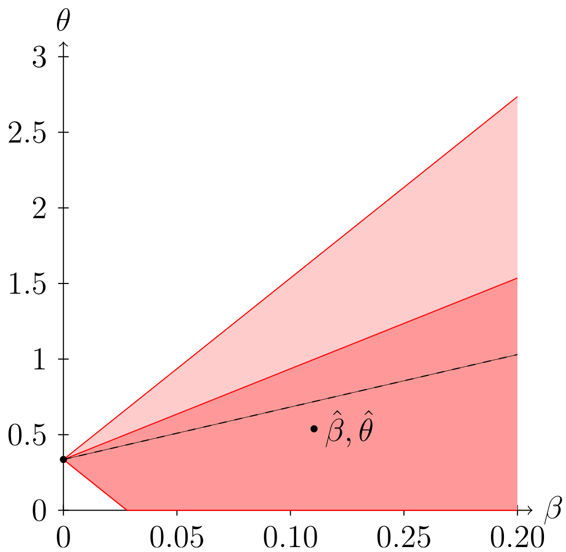

Figure 11: Estimated identified region of (β, θ) without information on subjective expectations (both shaded areas) and with the assumption that $\mathbf{E}(r_i^{invest}) \geq \mathbf{E}(r_i^{keep})$ (dark shaded area only). Region derived using an endowment of $w = 6$ and a proportion of investments of 63%. The dashed line represents the identified region under the assumption that proposers have rational expectations. The point $(\hat{\beta}, \hat{\theta})$ denotes the parameter estimates obtained using the subjective expectations data.

C Tables

	Linear	Quadratic	Cubic
	Standard normal distribution		
4 knots	0.000	0.175	0.000
5 knots	0.000	0.000	0.000
6 knots	0.000	-0.002	0.000
	χ^2_3 distribution		
4 knots	0.076	-0.018	0.008
5 knots	0.037	-0.017	-0.006
6 knots	0.020	-0.012	-0.008
	Bimodal distribution		
4 knots	0.000	-0.019	0.000
5 knots	0.000	0.000	0.000
6 knots	0.000	0.003	0.000

Table 1: Bias of approximated expectations relative to the theoretical variances of the underlying distributions. The chi-square distribution is truncated to the [0,15] interval, while the bimodal density if given by $f(x) = \frac{sin(x)+1}{A}$ where $A = 2 + 3\pi$ and is truncated to the [0,3π] interval.

Interpolation	Linear	Quadratic	Cubic	Cubic
Monotonicity imposed	-	-	No	Yes
β	0.121**	0.116**	0.116**	0.116**
	(0.066)	(0.064)	(0.065)	(0.065)
θ	0.578**	0.566**	0.569**	0.569**
	(0.241)	(0.239)	(0.241)	(0.241)

Table 2: Maximum likelihood estimates of the model parameters with asymptotic standard errors in parenthesis. Significance at the 5% level is denoted by '**', whereby the estimate of β is evaluated against the one-sided alternative that $\beta > 0$ and the estimate of θ is evaluated against the two-sided alternative that $\theta \neq 0$.

References

ASHRAF, N., I. BOHNET, AND N. PIANKOV (2006): "Decomposing Trust and Trustworthiness," *Experimental Economics*, 9, 193–208.

BARR, A. (2003): "Trust and Expected Trustworthiness: Experimental Evidence from Zimbabwean Villages," *The Economic Journal*, 113, 614–630.

BELLEMARE, C., S. KRÖGER, AND A. VAN SOEST (2005): "Actions and Beliefs: Estimating Distribution-Based Preferences using a Large Scale Experiment with Probability Questions on Expectations," *CIRPÉE Discussion Paper no. 05-23, Laval University*.

BERG, J., J. DICKHAUT, AND K. MCCABE (1995): "Trust, Reciprocity, and Social History," *Games and Economic Behavior*, 10, 122–142.

BOOR, C., AND B. SWARTZ (1977): "Piecewise Monotone Interpolation," *Journal of Approximation Theory*, 21(4), 411–416.

BRANDTS, J., AND G. CHARNESS (2000): "Hot Vs. Cold: Sequential Responses and Preference Stability in Experimental Games," *Experimental Economics*, 2, 227–238.

CHARNESS, G., AND M. RABIN (2002): "Understanding Social Preferences with Simple Tests," *Quarterly Journal of Economics*, 117, 817–869.

COX, J. C. (2004): "How to Identify Trust and Reciprocity," *Games and Economic Behavior*, 46, 260–281.

DAWES, R. M., J. MCTAVISH, AND H. SHAKLEE (1977): "Behavior, Communication, and Assumptions About Other People's Behavior in A Commons Dilemma Situation," *Journal of Personality and Social Psychology*, 35(1), 1–11.

DELAVANDE, A. (2005): "Pill, Patch or Shot ? Subjective Expectations and Birth Control Choice," *CEPR Discussion Paper 4856*.

DOMINITZ, J., AND C. F. MANSKI (1997): "Using Expectations Data to Study Subjective Income Expectations," *Journal of the American Statistical Association*, 92, 855–867.

DUFWENBERG, M., AND U. GNEEZY (2000): "Measuring Beliefs in an Experimental Lost Wallet Game," *Games and Economic Behavior*, 30, 163–182.

ECKEL, C., AND R. K. WILSON (2004): "Is Trust a Risky Decision ?," *Journal of Economic Behavior and Organization*, 55, 447–465.

ENGELBERG, J., C. MANSKI, AND J. WILLIAMS (2007): "Comparing the Point Predictions and Subjective Probability Distributions of Professional Forecasters," *forthcoming, Journal of Business and Economic Statistics*.

ENGELMANN, D., AND M. STROBEL (2004): "Inequality Aversion, Efficiency and Maximin Preferences in Simple Distribution Experiments," *American Economic Review*, 94(4), 857–869.

FISCHBACHER, U. (2007): "z-Tree: Zurich Toolbox for Ready-made Economic Experiments," *Experimental Economics*, 10(2), 171–178.

FRIEDMAN, D., AND D. W. MASSARO (1998): "Understanding Variability in Binary and Continuous choice," *Psychonomic Bulletin and Review*, 5, 370–389.

HOFFRAGE, U., S. LINDSEY, R. HERTWIG, AND G. GIGERENZER (2000): "Communicating Statistical Information," *Science*, 290 (5500), 2261–2262.

HOLT, C. A., AND S. K. LAURY (2002): "Risk Aversion and Incentive Effects," *American Economic Review*, 92(5), 1644 – 1655.

HOUSER, D., D. SCHUNK, AND J. WINTER (2006): "Trust or Risk? Experimental Evidence that Trust Games Measure Trust," *Munich Department of Economics WP 38*.

HYMAN, J. M. (1983): "Accurate Monotonicity Preserving Cubic Interpolation," *SIAM Journal on Scientific Computing*, 4(4), 645 – 654.

JUDD, K. L. (1998): *Numerical Methods in Economics*. The MIT Press, Cambridge, MA.

KARLAN, D. (2005): "Using Experimental Economics to Measure Social Capital and Predict Financial Decisions," *American Economic Review*, 95(5), 1688–1699.

LAPORTA, R., F. L. DE SILANES, A. SHLEIFER, AND R. W. VISHNY (1997): "Trust in Large Organizations," *American Economic Review*, 87(2), 333–338.

MAGNAC, T., AND D. THESMAR (2002): "Identifying Dynamic Discrete Decision Processes," *Econometrica*, 70(2), 801–816.

MANSKI, C. F. (2002): "Identification of Decision Rules in Experiments of Simple Games of Proposal and Response," *European Economic Review*, 46, 880–891.

——— (2004): "Measuring Expectations," *Econometrica*, 72(5), 1329–1376.

——— (2007): "Random Utility Models with Bounded Ambiguity," *forthcoming, International Economic Review*.

MCKENZIE, D., J. GIBSON, AND S. STILLMAN (2007): "A Land of Milk and Honey with Streets Paved with Gold: Do Emigrants have Over-optimistic Expectations about Incomes Abroad?," *CReAM Discussion Paper No 09/07*.

MCLEISH, K. N., AND R. OXOBY (2004): "Specific Decision and Strategy Vector Methods in Ultimatum Bargaining: Evidence on the Strength of Other-Regarding Behavior," *Economics Letters*, 84(5), 399–405.

NYARKO, Y., AND A. SCHOTTER (2002): "An Experimental Study of Belief Learning Using Elicited Beliefs," *Econometrica*, 70(3), 971–1005.

OFFERMAN, T., J. SONNEMANS, AND A. SCHRAMM (1996): "Value orientations, expectations and voluntary contributions in public goods," *The Economic Journal*, 106, 817–845.

PAGAN, A., AND A. ULLAH (1999): *Nonparametric Econometrics*. Cambridge University Press, Cambridge, U.K.

RUST, J. (1994): *Structural Estimation of Markov Decision Processes*vol. 4 of *Handbook of Econometrics*, chap. 51, pp. 3081–4143. Elsevier Science, North-Holland Publishers.

SONNEMANS, J., AND T. OFFERMAN (2001): "Is the Quadratic Scoring Rule Really Incentive Compatible?," Working paper CREED, University of Amsterdam.

TRAIN, K. E. (2003): *Discrete Choice Methods with Simulation*. Cambridge University Press.

WINKLER, R. L. (1967): "The Assessment of Prior Distributions in Bayesian Analysis," *Journal of the American Statistical Association*, 62(319), 776–800.

ZAK, P. J., AND S. KNACK (2001): "Trust and Growth," *Economic Journal*, 111(4), 295–321.