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**Constant Consumption and the  
Economic Depreciation of Natural  
Capital : The Non-Autonomous  
Case**

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# Constant Consumption and the Economic Depreciation of Natural Capital : The Non-Autonomous Case

*John M. Hartwick<sup>†</sup>, Ngo Van Long<sup>‡</sup>*

## Résumé / Abstract

Nous décrivons une formule pour comptabiliser la dépréciation économique des stocks de capital (y compris la qualité de l'environnement) dans les cas où les prix, la technologie et les taux d'intérêt ne sont pas stationnaires. Dans les deux premiers cas, le taux de consommation est constant si l'on adopte une stratégie d'investissement pour compenser la dépréciation économique. Dans le cas où le taux d'intérêt n'est pas stationnaire, on a besoin d'une stratégie modifiée pour s'assurer un taux de consommation constant.

*We investigate economic depreciation of natural capital for cases of non-stationary output prices, technology and interest rates. For the former two cases (exogenous movements in prices and technology), constant consumption emerges under a strategy of investing to cover off economic depreciation. The interest rate case requires a modified sinking fund strategy.*

**Mots Clés :** Dépréciation économique, capital naturel, environnement, consommation constante

**Keywords :** Economic Depreciation, Natural Capital, Environment, Constant Consumption

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# 1 Introduction

Efficient use of capital stocks implies a path of economic depreciation of stocks that can be calculated using suitable prices. If all production functions are time independent, if the discount rate is stationary, and if there are no exogenous price movements, then economic depreciation is simply the sum of  $n$  terms, each being a product of the current change in a stock and its current marginal value; see Hartwick (1977), and Dixit, Hammond and Hoel (1980), for example.

In this paper, we investigate the non-autonomous case in which current profit is shifted by time. This includes both the case of exogenous changes in the terms of trade for a small open economy (see Vincent, Panayotou and Hartwick (1995), Kemp and Long<sup>1</sup>(1995)), and the case of exogenous technical change (Kemp and Long (1982), Weitzman<sup>2</sup> (1995)). We derive new expressions for economic depreciation and show that if the rate of interest is time independent, then when a sum equal to economic depreciation is invested in a sinking fund, constant consumption paths emerge. Constant wealth is preserved in such cases. We then turn to the problem of exogenously moving interest rates and perfect foresight (see Asheim (1995), Kemp and Long (1995), and Lozada<sup>3</sup> (1995)). We show that constant consumption emerges in a “modified sinking fund” strategy.

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<sup>1</sup>Kemp and Long (1995) deal with the case of utility maximization where the rate of discount is dependent on current and past levels of utility, in the Epstein-Uzawa framework. They focus on the evaluation of national income, and are not concerned with constant consumption. In contrast, we deal with the case where the rate of discount is the market interest rate that is taken as exogenous and is time-dependent. We seek an investment rule that yields constant consumption.

<sup>2</sup>Weitzman (1995) provides an analysis of the case of a time-dependent technology, previously treated by Kemp and Long (1982), and untreated in Weitzman (1976). He aims at providing semi-operational formulas for adjusting NNP. His sustainability concept relates to hypothetical (possibly infeasible) constant consumption streams.

<sup>3</sup>Lozada (1995) focusses on the theory of the mine. His general expression for economic depreciation does not bring to the fore the crucial role of the gap between current and future interest rates. Asheim (1995) contrasts short term and long term interest rates, and like Weitzman, is concerned only with hypothetical constant consumption. Kemp and Long (1995) point out the importance of the rate of change in the short term interest rate in an ideal evaluation of NNP.

## 2 Economic Depreciation

The economy has  $n$  types of capital goods. Let  $s = (s_1, s_2, \dots, s_n)$  denote the quantities of the capital goods. Let  $q = (q_1, q_2, \dots, q_m)$  be the vector of control variables. These may represent the rates of re-forestation, pollution abatement, harvesting, investment, maintenance of equipment etc. The rate of change in the stock  $s_i$  is

$$\dot{s}_i = \Phi_i(s, q, t) \quad , \quad i = 1, \dots, n \quad (2.1)$$

or, in vector notation,

$$\dot{s} = \Phi(s, q, t) \quad (2.2)$$

The variable  $t$  appears explicitly in (2.2) to allow for exogenous technical changes. Equation (2.1) is general enough to capture, at one extreme, interactions between species, and, at the other extreme, simple neoclassical net investment of the form  $\dot{s}_i = q_i - \delta s_i$ .

The value of net output that can be consumed, or, in another interpretation, exchanged in the world market, is denoted by

$$x(t) = \Pi(s, q, t) \quad (2.3)$$

This specification captures possible exogenous terms of trade changes.

Let  $r(t)$  be the exogenous positive discount rate which is time dependent (one may think of the world rate of interest that small countries take as given). Let the discount factor be

$$D(t) = \exp \int_0^t -r(\theta) d\theta \quad (2.4)$$

We assume that  $r(t)$  is differentiable, and that all the functions in (2.1) and (2.3) have continuous second order partial derivatives.

Consider the problem of choosing the time path of  $q$  to maximize the integral of discounted value of net output:

$$\int_0^{\infty} \Pi(s, q, t) D(t) dt \quad (2.5)$$

subject to initial conditions  $s(0) = s_0$  and to the transition equation (2.2).

We assume that a solution exists and that the integral in (2.5) converges. This implies, in particular, that any favourable time trend of technical progress or terms of trade improvement is sufficiently weak relative to the discount rate.

Define the present value Hamiltonian

$$H = D(t)\Pi(s, q, t) + \gamma\Phi(s, q, t) \quad (2.6)$$

where  $\gamma$  is the vector of  $n$  costate variables. The corresponding current value costates and Hamiltonian are

$$\Psi = \gamma D(t)^{-1} \quad (2.7)$$

$$H^c = HD(t)^{-1} \quad (2.8)$$

Let  $J(s, t)$  denote the value of the integral of discounted value of net output, starting from time  $t$ , with stock levels  $s$ :

$$J(s, t) = \int_t^\infty D(\theta)\Pi(s^*, q^*, \theta)d\theta \quad (2.9)$$

where  $(s^*(\theta), q^*(\theta))$  are the optimal values of  $(s, q)$  at time  $\theta$ . The asterisk will be dropped in what follows, for notational simplicity. Let  $V(s, t)$  be the current value of  $J(s, t)$ :

$$V(s, t) = J(s, t)D(t)^{-1} \quad (2.10)$$

We define *economic depreciation* to be the negative of the rate of change of  $V$ . The following proposition decomposes economic *appreciation* ( $\dot{V}$ ) into three terms: (a) the value of stock accumulation,  $\Psi(t)\dot{s}(t)$ , (b) the improvement in future income flows brought about by exogenous technological and terms of trade changes, and (c) the gain in the capitalized value of the future income brought about by interest rate changes.

**Proposition 1:** Economic appreciation is given by

$$\dot{V} = \Psi(t)\dot{s}(t) + M(t) + G(t) \quad (2.11)$$

where

$$M(t) \equiv \int_t^\infty [\Pi_\theta + \Psi\Phi_\theta] D(\theta)D(t)^{-1}d\theta \quad (2.12)$$

$$G(t) \equiv \int_t^{\infty} [r(t) - r(\theta)] \Pi(s, q, \theta) D(\theta) D(t)^{-1} d\theta \quad (2.13)$$

**Proof:** See the Appendix.

Let us try to interpret (2.11). In the simple case where the functions  $\Pi$  and  $\Phi$  do not contain the independent time term  $t$  as an argument of the functions, then the partial derivatives  $\Pi_\theta, \Phi_\theta$  are identically zero. If  $r$  is time independent, then  $r(t) - r(\theta) = 0$ , and hence  $G(t) = 0$ . Under these conditions, economic *appreciation* is simply  $\Psi(t)\dot{s}(t)$ . In the general case,  $M(t)$  measures the present value of the incremental change in income caused merely by the passage of time. To fix ideas, suppose that  $\Phi$  does not contain  $t$ , and  $\Pi(q, s, t)$  is simply  $p(t)f(q, s)$ , where  $p(t)$  is the exogenous price path of an exportable good and  $f(q, s)$  is the net output of that good. If  $p(t)$  is an increasing path, then, other things being equal, the country's outlook at time  $t + \Delta t$  is better than its outlook at  $t$ . This increase in value is an economic appreciation. The meaning of  $G(t)$  is also quite clear. For any flow of revenue, the higher is the rate of interest, the lower is its present value. Therefore, if the time path of interest rate is increasing, the mere passage of time reduces the value of the flow of revenue. In other words, if  $r(\theta) > r(t)$  for all  $\theta > t$ , then  $G(t)$  is negative, representing a reduction of value.

Let us consider some examples.

Example 1: Suppose  $s_i$  represents the stock of fish of type  $i$ , and  $q_i$  the corresponding rate of harvest. Let

$$\Pi(s, q, t) = \sum p_i(t)q_i(t) - c_i(q_i(t)) \quad (2.14)$$

where  $c_i(q_i)$  is the cost of harvesting and  $p_i(t)$  is the exogenous price. Assume that the interest rate is time invariant. Then the economic appreciation is

$$\dot{V} = \sum [p_i - c'_i(q_i)] \dot{s}_i + \int_t^{\infty} [\sum \dot{p}_i(\theta)q_i(\theta)] \exp[-r(\theta - t)] d\theta \quad (2.15)$$

Example 2: Suppose that the equation system  $\dot{s} = \Phi(s, q, t)$  can be inverted to yield

$$q = q(s, \dot{s}, t) \quad (2.16)$$

Substituting (2.16) into  $\Pi$ , we get the simplified Hamiltonian  $H^c = \Pi(s, \dot{s}, t) + \Psi\dot{s}$ . One may interpret  $\Pi$  as the output of the consumption good, and  $\dot{s}$  as net outputs of the capital goods, so that  $H^c$  is a measure of national income. Economic appreciation is then

$$\dot{V} = (-\Pi_{\dot{s}})\dot{s} + \int_t^{\infty} \Pi_{\theta} D(\theta) D(t)^{-1} d\theta + \int_t^{\infty} [r(t) - r(\theta)] \Pi(\theta) D(\theta) D(t)^{-1} d\theta \quad (2.17)$$

where  $-\Pi_{\dot{s}}$  are the prices of the investment goods in terms of the consumption good.

So far, we have not assumed that the function  $V(s, t)$  has partial derivatives. The existence of  $\dot{V}$  does not imply the existence of  $V_s$  and  $V_t$ . Let us now assume that these partial derivatives exist. Then

$$\dot{V} = V_s \dot{s} + V_t \quad (2.18)$$

On the other hand,

$$\dot{V} = r(t)V - \Pi \quad (2.19)$$

Therefore

$$r(t)V = \Pi + V_s \dot{s} + V_t \quad (2.20)$$

Comparing (2.18) with (2.19), it follows that

$$V_t = M(t) + G(t) \quad (2.21)$$

If  $r$  is *constant*, we may regard  $rV$  as the sustainable consumption level, in the sense that the future flow  $\Pi$  has the same capitalized value as that of a constant flow of consumption equal to  $rV$ . Under this interpretation, from (2.20) sustainable consumption is equal to the sum of two terms: (a) the standard NNP term,  $\Pi + V_s \dot{s}$ , and (b) the capitalized value of technical progress and terms of trade improvement,  $V_t$ . The term  $V_t$  may be thought of as an adjustment to NNP.<sup>4</sup>

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<sup>4</sup>How important is this adjustment? We can show that at  $t = 0$ ,  $V_t = (\frac{\lambda}{r-g})NNP(0)$ , where  $g$  and  $\lambda$  are, respectively, the long run average of future growth rates of NNP and of future growth rates of the Solow residual. The proof proceeds as follows. Define

### 3 Offsetting Depreciation with a Sinking Fund

Consider the following investment strategy: to invest an amount, denoted by  $\dot{K}$ , from the revenue  $\Pi$  at each instant, equal to the economic depreciation ( $-\dot{V}$ ) in a sinking fund. That is

$$\dot{K} = -\dot{V} \quad (3.1)$$

Suppose the residual income plus current interest is consumed. Then consumption is

$$c(t) = \Pi(s, q, t) + \dot{V}(t) + rK(t) \quad (3.2)$$

This leads to the following results:

**Proposition 2:** (Constant consumption). If the interest rate is constant, and the strategy of investing an amount equal to the economic depreciation ( $-\dot{V}$ ) is followed, then along an efficient stock use path, consumption is constant.

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$y = \Pi(s, \dot{s}, t) + \Psi\dot{s}$ , and let

$$\lambda \equiv \frac{\int_0^{\infty} e^{-rt} (\partial y / \partial t) dt}{\int_0^{\infty} e^{-rt} y(t) dt}$$

$$g \equiv \frac{\int_0^{\infty} e^{-rt} (dy / dt) dt}{\int_0^{\infty} e^{-rt} y(t) dt}$$

Then

$$\frac{\lambda y(0)}{r - g} = \frac{y(0) \int_0^{\infty} e^{-rt} (\partial y / \partial t) dt}{\int_0^{\infty} r e^{-rt} y(t) dt - \int_0^{\infty} e^{-rt} \frac{dy}{dt} dt}$$

The denominator is equivalent to  $\int_0^{\infty} -\frac{d}{dt} [e^{-rt} y] dt$  which is equal to  $y(0)$ , if we assume that  $\lim_{t \rightarrow \infty} e^{-rt} y(t) = 0$ . Therefore  $\frac{\lambda y(0)}{r - g} = M(0) = V_t$  at  $t = 0$ . This result is due to Weitzman (1995). Our proof is simpler because it takes advantage of the relationships (2.20) and (2.21).

**Proposition 3:** (Constant wealth). If the interest rate is constant, then along an efficient stock use path, the investment strategy (3.1) implies that  $\dot{V} + \dot{K} = 0$ .

What happens if the interest rate is time dependent? The answer is provided in Proposition 4, of which Propositions 2 and 3 may be regarded as corollaries:

**Proposition 4:** (Constant consumption under time-dependent interest rate). Suppose the interest rate is time-dependent. Then along an efficient stock use path, consumption is constant if and only if the following investment rule is used:

$$\dot{K} = -\dot{V} - \int_t^\infty \dot{r}(\theta) [K(\theta) + V(\theta)] D(\theta) D(t)^{-1} d\theta \quad (3.3)$$

**Proof:** Let

$$\dot{K} = -\dot{V} + W \quad (3.4)$$

where  $W(t)$  is to be determined. By definition,

$$c = \Pi + rK - \dot{K} \quad (3.5)$$

From (3.4) and (3.5)

$$c = \Pi + rK + \dot{V} - W \quad (3.6)$$

Differentiate (3.6) with respect to time:

$$\dot{c} = \Pi_s \dot{s} + \Pi_q \dot{q} + \Pi_t + \ddot{V} + \dot{r}K + r\dot{K} - \dot{W} \quad (3.7)$$

From (2.11),

$$\ddot{V} = \dot{\Psi} \dot{s} + \Psi [\Phi_s \dot{s} + \Phi_q \dot{q} + \Phi_t] + \dot{G} + \dot{M} \quad (3.8)$$

where

$$\dot{M} = rM - \Pi_t - \Psi \Phi_t \quad (3.9)$$

Recall that along an efficient path,

$$\dot{\Psi} = (r - \Phi_s) \Psi - \Pi_s \quad (3.10)$$

$$\Pi_q + \Psi\Phi_q = 0 \quad (3.11)$$

Making use of (3.8), (3.9), (3.10) and (3.11), we can express (3.7) as:

$$\dot{c} = r\Psi\dot{s} + \dot{G} + rM + r\dot{K} + \dot{r}K - \dot{W} \quad (3.12)$$

Now using (3.4) and (2.11),

$$r\dot{K} = -r\dot{V} + rW = -r\Psi\dot{s} - rG - rM + rW \quad (3.13)$$

From (3.12) and (3.13),

$$\dot{c} = \dot{G} - rG + \dot{r}K + rW - \dot{W} \quad (3.14)$$

But from (2.13)

$$\dot{G} = \dot{r}V + rG \quad (3.15)$$

Therefore  $\dot{c} = 0$  if and only if

$$\dot{W} - rW = \dot{r}(K + V) \quad (3.16)$$

Integrating (3.16) to obtain

$$W(t) = - \int_t^\infty \dot{r}(\theta) [K(\theta) + V(\theta)] D(\theta) D(t)^{-1} d\theta \quad (3.17)$$

Substituting (3.17) into (3.4), we obtain the investment rule of Proposition 4. Notice that when this rule is followed, wealth is not constant:

$$\dot{K} + \dot{V} = - \int_t^\infty \dot{r}(\theta) [K + V] D(\theta) D(t)^{-1} d\theta \quad (3.18)$$

This completes the proof of Proposition 4.

Proposition 4 is intuitively reasonable. In particular, it says that if the rate of interest tends to rise in the future, then to ensure constant consumption it is not necessary to invest an amount equal to economic depreciation ( $-\dot{V}$ ). For example, if an infinitely-lived consumer has  $x$  dollars in a bank account that pays the market rate of interest  $r(t)$ , and  $r(t)$  is rising continuously, then his additional investment,  $\dot{K}$ , should be negative if he wants to achieve the maxi-min consumption stream. Along his constant consumption path, his bank balance will be falling.

## 4 Conclusion

It has been known for some time that to achieve a constant consumption path under time-independent technology and terms of trade, the value of net investment must be zero; see Hartwick (1977), Dixit, Hammond and Hoel (1980). Exogenous time trend can in principle be taken into account (Kemp and Long (1982)) in the evaluation of income, but it was generally thought that no simple rule was available that ensures constant consumption. This paper has provided such a rule under fairly general conditions. It is a generalization of the former rule and affirms the same basic principle: even with time-dependent technology and terms of trade, for constant consumption, the accumulation of one stock (the sinking fund) must exactly compensate for the aggregate decumulation, in value terms, of all other stocks- provided that the rate of interest is time-invariant. We have provided a simple formula for evaluating such decumulation.

The case of a time-dependent rate of interest turns out to be quite different, but an easily interpretable rule is shown to apply.<sup>5</sup> Depending on whether the rate of interest is falling or rising, investment in a sinking fund must over-compensate or under-compensate for the aggregate decumulation of other stocks to ensure constant consumption.

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<sup>5</sup>Using the non-modified rule would result in non-constant consumption. This answers the query of Svensson (1986).

## Appendix

### Proof of Proposition 1:

Following a standard procedure (see Léonard and Long (1992, p.152) for instance), we rewrite  $J(s, t)$  as follows:

$$J(s, t) = \int_t^\infty [D(\theta)\Pi(s^*, q^*, \theta) - \gamma\Phi(s^*, q^*, \theta) - \gamma\dot{s}^*] d\theta \quad (\text{A.1})$$

that is,

$$J(s, t) = \int_t^\infty [H(\theta) - \gamma\dot{s}^*(\theta)] d\theta \quad (\text{A.2})$$

Differentiate (A.2) with respect to  $t$ :

$$\dot{J}(t) = \gamma\dot{s}^*(t) - H(t) = \gamma\dot{s}^*(t) + \int_t^\infty \dot{H}(\theta)d\theta - \lim_{T \rightarrow \infty} H(T) \quad (\text{A.3})$$

It is known that  $\lim_{T \rightarrow \infty} H(T) = 0$  (see Michel (1982)). Furthermore, the following equality holds almost everywhere along the optimal path:

$$\dot{H}(t) = \frac{\partial H}{\partial t} \quad (\text{A.4})$$

(see Hestenes (1966, Theorem 11.1) or Léonard and Long (1992, p. 211) for example). Therefore (dropping the asterisk for simplicity),

$$\dot{J} = \gamma\dot{s} + \int_t^\infty H_\theta d\theta \quad (\text{A.5})$$

Differentiate (2.10):

$$\dot{V} = \dot{J}D(t)^{-1} - \dot{D}(t)D(t)^{-2}J = \dot{J}D(t)^{-1} + r(t)V(t) \quad (\text{A.6})$$

Hence

$$\dot{V} = \Psi\dot{s} + D(t)^{-1} \int_t^\infty H_\theta d\theta + r(t)V(t) \quad (\text{A.7})$$

Now

$$H_\theta = \dot{D}(\theta)\Pi + D(\theta)\Pi_\theta + \gamma\Phi_\theta \quad (\text{A.8})$$

Substitute (A.8) into (A.7) to obtain

$$\dot{V} = \Psi \dot{s} - \int_t^\infty r(\theta) \Pi(\theta) D(\theta) D(t)^{-1} d\theta + \int_t^\infty [\Pi_\theta + \Psi \Phi_\theta] D(\theta) D(t)^{-1} d\theta + rV(t) \quad (\text{A.9})$$

Finally, recall that

$$V(t) = \int_t^\infty \Pi(\theta) D(\theta) D(t)^{-1} d\theta \quad (\text{A.10})$$

Multiply (A.10) by  $r(t)$  and substitute into (A.9) to obtain  $\dot{V} = \Psi \dot{s} + G(t) + M(t)$ .

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