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Selective Penalization Of Polluters: An Inf-Convolution Approach*

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Résumé / Abstract

On modélise un oligopole hétérogène : les firmes ont des coûts différents et des paramètres de pollution différents. On montre que les taux de taxes optimales imposées sur les émissions ne sont pas les mêmes. On appelle cette propriété la pénalisation sélective. Il existe donc un conflit entre l'équité et l'efficacité. Le résultat principal de notre article est Le Théorème de la Distorsion Optimale. La structure des taxes optimales exige que les firmes aux coûts les plus élevés paient les taxes les plus élevées. Un autre résultat s'appelle le Théorème sur le motif pro-concentration.

In this paper, we consider an asymmetric polluting oligopoly: firms have different production costs, and their pollution characteristics may also be different. We will demonstrate that, in this case, optimal tax rates per unit of emission are not the same for all firms. We call this property ``selective penalization'', or ``favoritism in penalties.'' Thus, the ``efficiency'' objective may be served only at the expense of ``fairness'. One of our main results is the Optimal Distortion Theorem.. We show that even in the case w here the rates of emission per unit of output are identical for all firms, the efficient tax structure requires that high cost firms pay a higher tax rate on emissions. Our result implies that the efficient tax structure favors the efficient firms, but the magnitude of the favors is a decreasing function of the marginal cost of public fund. Another characterization of optimal tax structure is our Pro-concentration Motive Theorem. Optimal taxes penalize the inefficient firms more, and thus increases the concentration of the industry, as measured by the Herfindahl index. In fact, we show that the variance of the distribution of the firms' tax-inclusive marginal costs after the imposition of efficient taxes exceeds the variance that would be obtained if there were no taxes. We call this the Magnification Effect: the variance of marginal costs is magnified by a factor which depends on the marginal cost of public fund.

Key words: Pollution, environmental regulation, oligopoly

Mots-clés : Pollution, réglementations environnementales, oligopole

JEL classifications: Q20, D60, D63

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1. Introduction

When production generates pollution as a by-product, competitive …rms over-produce in the sense that marginal social cost exceeds price. Under perfect competition, a Pigouvian tax equal to marginal damage cost is called for. This rule applies whether …rms are identical or not. Thus, under perfect competition, all polluting …rms are treated fairly by a regulator that seeks to achieve e¢ciency: the tax rate per unit of emission is the same for all …rms, even under heterogeneity of production costs. When the market is not competitive[1], however, as we will show below, it is no longer true that e¢ciency can be achieved by a uniform tax rule.

In this paper, we consider an asymmetric polluting oligopoly: …rms have di¤erent production costs, and their pollution characteristics may also be di¤erent. We will demonstrate that, in this case, optimal tax rates per unit of emission are not the same for all …rms. We call this property "selective penalization", or "favoritism in penalties." Thus, the "e¢ciency" objective may be served only at the expense of "fairness." We do not propose to resolve the issue of fairness, or equity, here. Our objective is to analyze the direction of "favoritism in penalties": how the e¢ciency-inducing …rm-speci…c tax structure depends on the structure of (heterogenous) production costs, and heterogenous pollution-output ratio. We wish to determine which types of …rms (low cost, or high cost …rms) should be penalized more.

Asymmetry is important, because it is a prevalent real world feature, and because it introduces another source of distortion: in a Cournot equilibrium, marginal production costs are not equalized across …rms, resulting in production ine¢ciency at any given total output.

[1] Buchanan (1969) and Barnett (1980) have shown that the optimal tax per unit of emission under monopoly is less than the marginal damage cost (and it can be negative). Katsoulacos and Xepapadeas (1995), consider the case of a symmetric polluting oligopoly (i.e., they assume that …rms are identical) and show that if the number of …rms is endogenous and if there are …xed costs, the optimal Pigouvian tax could exceed the marginal damage cost, because free entry may result in an excessive number of …rms.

In this context, pollution taxes or pollution standards must seek to remedy both the environmental problem and the intra-industry production ine¢ciency problem. With asymmetric oligopoly, the regulator would want to be able to correct distortion on a …rm-speci…c basis. While de jure di¤erential treatments to …rms in the same industry (in the sense that di¤erent standards apply to di¤erent …rms), may be politically unacceptable in most economies, de facto di¤erential treatments (e.g., di¤erent degrees of enforcement and veri…cation) may be feasible. In what follows, whenever the terms "…rm-speci…c tax rates", or "…rm-speci…c pollution standards" are used, they should be interpreted in the de facto sense.

The objective of this paper two-fold. Our …rst aim is to characterize the structure of optimal …rm-speci…c emission tax rates, and to provide an intuitive explanation of our results on di¤erential treatments. Here, we go a step further than just establishing the conditions for unequal treatment of equals, which have been provided elsewhere (Salant and Scha¤er, 1996, 1999, Long and Soubeyran, 1997a,b, 2001). We fully characterize the direction of bias. Our second aim is to highlight the con‡ict between e¢ciency and equity. We show that, in the case of taxation, …rms that are equally polluting (i.e., their emissions per unit of output are identical) may be taxed di¤erently, when their production costs are di¤erent. In a sense, this is inequitable. In the case where regulation is by means of pollution standards, we again show a con‡ict between e¢ciency and equity: e¢ciency may require that di¤erent standards be imposed on ex-ante identical …rms.

One of our main results is the Optimal Distortion Theorem. We show that even in the case where the rates of emission per unit of output are identical for all …rms, the e¢cient tax structure requires that high cost …rms pay a higher tax rate on emissions. Our result implies that the e¢cient tax structure favors the e¢cient …rms, but the magnitude of the favors is a decreasing function of the marginal cost of public fund. More generally, we de…ne the Pigouvian distortion for …rm i as the di¤erence between the tax rate on its output and the adjusted[2] marginal pollution damage caused by an extra unit of its output, and show that the optimal Pigouvian distortion for …rm i is greater than that for …rm j, if and only if the marginal social cost of …rm i is greater than that of …rm j. (Marginal social cost is the sum of marginal production cost and adjusted marginal damage cost.) Another characterization of optimal tax structure is our Pro-concentration Motive Theorem. Optimal taxes penalize the ine¢cient …rms more, and thus increases the concentration of the industry, as measured by the Her…ndahl index. In fact, we show that the variance of the distribution of the …rms'tax-inclusive marginal costs after the imposition of e¢cient taxes exceeds the variance that would be obtained if there were no taxes. We call this the Magni…cation Effect: the variance of marginal costs is magni…ed by a factor which depends on the marginal cost of public fund. From a mathematical point of view, our result has a nice geometric interpretation: We show that optimal taxes can be obtained as a solution of minimizing the distance between a hyperplane and a reference point. Our approach is an application of the duality theory using conjugate function and inf-convolution.

Our main focus in on …rm-speci…c taxation on emissions. A brief section on …rm-speci…c standards is included to show the general applicability of our method[3].Our derivation of optimal taxes and optimal standards (in slightly di¤erent models) shows that there is a uni…ed framework for analyzing …rm-speci…c penalties.

In the models we present below, we use a two-stage game framework. In the …rst stage, the regulator sets …rm-speci…c emission taxes or standards. In the second stage, …rms compete in the …nal good market. To …x ideas, we focus on the case where …rms produce a

[2] An adjustment factor is applied to account for the marginal cost of public fund.

[3] We do not seek to compare taxes to standards, because there exists already a large literature on that subject. In fact, the model we develop to analyze optimal standards is quite di¤erent from the model we use to analyze optimal taxes, and therefore it would not make sense to discuss, using our results, the relative attractiveness of these two policy measures.

homogenous good, and compete à la Cournot. However, our analysis can easily be adapted to deal with other cases, such as Bertrand competition with di¤erentiated products, spatial competition (as in the Hotelling model), and even markets in which some …rms are Stackelberg leaders.

The games considered in this paper belong to the class of games called "cost manipulation games with costs of manipulating" (see Long and Soubeyran, 2001a). The regulator uses the chosen policy instrument to a¤ect, on a discriminatory basis, the marginal costs of individual …rms. This in turn a¤ects their equilibrium outputs and market shares. The costs of manipulating can take di¤erent forms. In the case of taxation, when the marginal cost of public funds exceeds unity[4], these costs include the loss of tax revenue when the regulator changes the tax structure In the case of standards, …rms are induced to acquire costly equipment to reduce the pollution generated by their production process. Such equipment alters the marginal production costs.

We are able to provide a uni…ed treatment of …rm-speci…c pollution policies because we transform variables in such a way that all modes of intervention (in distinct models of emission generation) can be seen to have the same basic structure. We show that maximizing the …rst stage objective with respect to one of the environmental instruments (such as Pigouvian taxes, speci…c pollution standards, tradable pollution permits) is equivalent to choosing the Cournot equilibrium quantities. This is because the discriminatory use of policy instruments in the …rst stage amounts to the same thing as manipulating the marginal costs of production which in turn a¤ect second-stage equilibrium outputs.

2. Selective Penalization by Pigouvian Taxes

In this section present our basic model of an asymmetric polluting oligopoly, and derive the optimal …rm-speci…c Pigouvian taxes.

[4] See Ballard et al. (1985) for estimates of the marginal cost of public …nance.

Here, our major task is to show how the optimal firm-specific taxes are related to the structure of heterogenous costs and heterogenous emission-output ratios. Our main results are summarized in Proposition ODT (Optimal Distortion Theorem). According to this theorem, in a oligopoly consisting of firms with non-identical costs, the optimal Pigouvian tax for each firm must deviate from its adjusted marginal damage per unit of output, and such deviations vary among firms: the deviation should be the greatest for the most inefficient firm, and lowest for the most efficient firms. In general, inefficient firms are penalized more, relative to efficient firms.

2.1. The basic model

We consider a polluting oligopoly consisting of n non-identical firms producing a homogenous final good. Let $I = \{1, 2, ..., n\}$. The total output of the final good is $Q = \sum_{i \in I} q_i$. The inverse demand function for the final good is $P = P(Q)$ where $P'(Q) < 0$. In this model, emission is proportional to output: $e_i(q_i) = \varepsilon_i q_i$. In general, $\varepsilon_i \neq \varepsilon_j$. This is the first source of heterogeneity. Firm i has production cost $c_i(q_i)$. The subscript i in $c_i(.)$ indicates that in general firms are heterogenous also in production cost. Firms sell their good in the same market place, but they are located at different points. We represent this third source of heterogeneity (distance from the central market place) by assuming that firm i must incur a transport cost d_i per unit of output (in general $d_i \neq d_j$). We assume that firm i must pay a tax t_i per unit of its emission. (The regulator must set the tax rates optimally.) Th profit function is

$$\pi_i = P(Q)q_i - c_i(q_i) - t_i \varepsilon_i q_i - d_i q_i$$

We define

$$\tau_i \equiv t_i \varepsilon_i \tag{1}$$

so that τ_i is the tax per unit of output of firm i. It is convenient to define $\omega_i \equiv d_i + \tau_i$. The profit function becomes

$$\pi_i = P(Q)q_i - C_i(q_i; \omega_i)$$

where $C_i(q_i; \omega_i)$ is the (tax-inclusive) total cost function:

$$C_i(q_i; \omega_i) \equiv \omega_i q_i + c_i(q_i)$$

The (tax-inclusive) average variable cost function is:

$$\rho_i(q_i; \omega_i) = \omega_i + \frac{c_i(q_i)}{q_i} \qquad (2)$$

and the (tax-inclusive) marginal cost function is

$$\theta_i(q_i; \omega_i) = \omega_i + c_i'(q_i) \qquad (3)$$

The difference between θ_i and ρ_i, defined as r_i, measures the degree of convexity of the cost function. We have

$$r_i = \theta_i - \rho_i = c_i'(q_i) - \frac{c_i(q_i)}{q_i}$$

If $c_i(.)$ is linear, then r_i equals zero identically. If $c_i(.)$ is strictly convex, then r_i is positive for all $z_i > 0$.

We will show how the government can optimally manipulate the tax-inclusive costs of the firms so as to maximize social welfare. To do this, we set up the problem as a two-stage game. In the first stage, the government sets firm-specific taxes, and in the second stage, firms compete as Cournot rivals, taking tax rates as given. As usual, to solve for the optimal taxes, we must first analyse the equilibrium of the game in stage two.

2.2. Stage two: Cournot equilibrium given tax rates

The first order condition for an interior equilibrium for firm i is

$$\frac{\partial \pi_i}{\partial q_i} = P'(Q)q_i + P(Q) - \theta_i = 0; \qquad i \in I \qquad (4)$$

We assume that these conditions determine a unique[5] Cournot equilibrium $(\hat{Q}; \hat{q}_i, i \in I)$; where the hat over a symbol indicates that it is the

[5] For assumptions ensuring existence and uniqueness of equilibrium, see Long and Soubeyran (2000).

Cournot equilibrium value. It is convenient to express the equilibrium output of firm i as a function of the equilibrium output of the industry, and of the parameters of firm i's (tax-inclusive) cost function:

$$\hat q_i = \hat q_i(\hat Q; \omega_i) \tag{5}$$

Inserting (3) and (5) into (4), we obtain

$$P'(\hat Q)\hat q_i(\hat Q; \omega_i) + P(\hat Q) = \omega_i + c_i'\left[\hat q_i(\hat Q; \omega_i)\right] \tag{6}$$

Summing (6) over all i, we obtain the identity

$$P'(\hat Q)\hat Q + nP(\hat Q) = n\omega_I + \sum_{i\in I} c_i'\left[\hat q_i(\hat Q; \omega_i)\right] \tag{7}$$

where $\omega_I \equiv (1/n)\sum_{i\in I}\omega_i$. Equation (7) indicates that the equilibrium output can be determined from the knowledge of the ω_i's. Given the ω_i's, we assume that there exists a unique $\hat Q$ that satisfies (7). (See Long and Soubeyran (2000) for sufficient conditions for uniqueness). Thus we write

$$\hat Q = \hat Q(\omega) \tag{8}$$

where $\omega \equiv (\omega_1; \omega_2; \dots; \omega_n)$:

We now express the equilibrium profit of firm i as follows

$$\begin{aligned}\hat\pi_i &= \left(\hat P - \hat\tau_i\right)\hat q_i = \left[\left(\hat P - \hat\mu_i\right) + \left(\hat\mu_i - \hat\tau_i\right)\right]\hat q_i \\ &= [-\hat P']\hat q_i^2 + b_i(\hat q_i)\hat q_i \\ &= [-\hat P']\hat q_i^2 + \hat q_i c_i'(\hat q_i) - c_i(\hat q_i)\end{aligned} \tag{9}$$

where we have made use of the Cournot equilibrium condition

$$\hat P - \hat\mu_i = [-\hat P']\hat q_i \tag{10}$$

Expression (9) deserves some comments. Since the profit expression in (9) incorporates the Cournot equilibrium condition (10), it indicates that, in the first stage of the game, while the government can manipulate the $\hat{q}_i$ and $\hat{Q}$ via the choice of the policy parameters τ_i, it cannot violate the Cournot equilibrium condition. (Technically, this is very much like the incentive compatibility constraint in principal-agent problems: the principal cannot ignore economic agents' equilibrium conditions.) We now turn to a complete analysis of the first stage of the game.

2.3. The first stage: optimization by the government

The objective of the government is to maximize a weighted sum of profits, consumers' surplus, and tax revenue, minus the damage cost caused by pollution. The weight given to consumers' surplus is $\beta > 0$. The weight $\gamma \geq 1$ is a measure of the marginal cost of public fund. We will restrict attention to the empirically relevant range of $\gamma : 1 \leq \gamma \leq 2$ (see Ballard et al. (1985) for discussion, and for estimates of γ for the US economy). Thus, welfare is

$$W = \sum_{i \in I} \pi_i + \beta S + \gamma \sum_{i \in I} t_i e_i - D(E), \text{ where } 1 \leq \gamma \leq 2, \qquad (11)$$

where $E = \sum_{i \in I} e_i$, $D(E)$ is the damage cost, and S is the consumers' surplus

$$S = \int_0^Q P(\tilde{Q})d\tilde{Q} - P(Q)Q$$

In what follows, we assume that the damage cost function is linear, $D(E) = \sigma E > 0$. We define the adjusted marginal damage per unit of output of firm i as follows:

$$\delta_i \equiv \sigma \varepsilon_i / \gamma \qquad (12)$$

Here, the adjustment factor is $1/\gamma \leq 1$. The difference between τ_i, the optimal tax per unit of firm i's output, and δ_i, its adjusted marginal damage per unit of output, will be called the optimal distortion for firm i. We will show that (i) if firms are identical, the optimal distortions are equal for all firms, and (ii) if firms are heterogenous, the optimal distortions are no longer equal: higher cost firms will be penalized more, relative to lower cost firms.

The social welfare at a Cournot equilibrium may be written as

$$\hat{W} = \sum_{i \in I} \hat{\pi}_i + \beta S(\hat{Q}) + \gamma \sum_{i \in I} (\tau_i - \delta_i)\hat{q}_i(\hat{Q}; \omega_i) \qquad (13)$$

where $\hat{Q} = \hat{Q}(\omega)$ and $\hat{\pi}_i$ is given by (9). Note that the first term on the right-hand side of (13) contains tax payments by firm (in the expression $\hat{\pi}_i$) and the third term contains social value of tax revenue $\gamma\tau_i\hat{q}_i$. These two tax terms do not cancel each other out when $\gamma \neq 1$.

From expression (13), we see that welfare can be maximized by an appropriate choice of the firm-specific tax rates τ_i. However, as we demonstrate below, it is analytically much more convenient to solve the welfare maximization problem by using the equilibrium outputs $\hat{q}_i$ as choice variables, and afterward infer the optimal taxes. The two methods yield the same solution. We now transform variables so that the τ_i's are no longer explicitly present in the objective function. We will below how to replace the τ_i's in (13) by equilibrium quantities. From the equilibrium condition (4),

$$\hat{P}'\hat{q}_i + \hat{P} = d_i + \tau_i + c'(\hat{q}_i)$$

we get

$$\tau_i - \delta_i = \hat{q}_i\hat{P}' + (\hat{P} - d_i - \delta_i) - c'(\hat{q}_i) \qquad (14)$$

Substituting (14) into (13), we get

$$\hat{W} = F(\hat{Q}) - \sum_{i \in I} f_i(\hat{q}_i; \hat{Q}) \qquad (15)$$

where

$$F(\hat{Q}) \equiv \beta S(\hat{Q}) + \gamma \hat{P}\hat{Q} \qquad (16)$$

$$f_i(\hat{q}_i; \hat{Q}) \equiv (d_i + \delta_i)\gamma\hat{q}_i + (\gamma - 1)[-\hat{P}']\hat{q}_i^2 + \phi_i \qquad (17)$$

$$\phi_i \equiv (\gamma - 1)\hat{q}_i c_i'(\hat{q}_i) + c_i(\hat{q}_i)$$

Note that for given $\hat{Q}$, f_i is strictly convex in $\hat{q}_i$ if $\gamma > 1$, $c_i'' \geq 0$ and $c_i''' \geq 0$: Expression (15) shows that welfare is directly dependent on the $\hat{q}_i$'s. The tax rates do not (explicitly) appear in this expression. Thus $\hat{W}$ can be maximized by the direct choice of the equilibium outputs. Afterwards, the taxes can be inferred from (14).

We have thus obtained a very useful lemma:

Lemma 1: In the welfare maximization problem, there is a one-to-one correspondence between determining firm-specific emission tax rates to maximize welfare, expression (13), and determining Cournot equilibrium outputs to maximize welfare, expression (15).

We note that taxing firms is a way of manipulating their marginal costs. Thus, our Pigouvian taxation problem lies within the framwork of the "cost manipulation approach" that we have explained in our previous work (Long and Soubeyran, 1997a, 1997b, 2001a).

2.4. A benchmark case: perfect competition

Before solving for optimal emission taxes in an asymmetric oligopoly, it is useful to consider a benchmark case, with perfect competition. We assume in this subsection that the marginal cost of public fund is unity, $\gamma = 1$, and the weight given to consumers' surplus is also unity, $\beta = 1$. In this case, we obtain the well-known formula for optimal tax $\tau_i = \delta_i$, i.e., $t_i = \sigma$ for all $i \in I$. (see Appendix A1 for details.)

Thus, under perfect competition, all firms are treated equally. (We will later show that this "equal treatment" result does not apply to the oligopoly case.) Even though this result is well known, it is useful for future reference to state it as a proposition:

Proposition 1: (Benchmark Pigouvian Tax: fair treatment)Under perfect competition, the optimal Pigouvian tax t_i (per unit of emission) is the same for all firms and equal to the marginal damage σ:

$$t_i = \sigma \text{ for all } i \in I. \tag{18}$$

Thus all firms are treated fairly.

Recall that by definition, $\tau_i = \varepsilon_i t_i$. Thus, proposition B1 implies that, under perfect competition, the optimal tax per unit of output is $\tau_i^B = \sigma\varepsilon_i$ where the superscript B indicates the optimal value for the benchmark case.

2.5. An oligopoly with constant marginal cost

Now we turn to the case of an oligopoly with constant marginal cost: $c_i(q_i) = \alpha_i q_i$. (The increasing marginal cost case is treated in section 4.) In this case, (17) becomes

$$f_i(q_i; Q) \equiv (d_i + \delta_i + \alpha_i)\gamma q_i + (\gamma - 1)[-P']q_i^2 \tag{19}$$

We define the marginal social cost of firm i's output as

$$s_i \equiv d_i + \delta_i + \alpha_i \tag{20}$$

Thus, marginal social cost consists of production cost, α_i, transport cost, d_i, and adjusted marginal damage, δ_i.

We consider two sub-cases: (a) $\gamma = 1$ and (b) $\gamma > 1$.

2.5.1. Sub-case (a): $\gamma = 1$

In this sub-case, the marginal cost of public fund is unity. It is easy to show that the optimal policy is to design taxes so that only the firm with the lowest marginal social cost will produce. (See Appendix A2).

2.5.2. Sub-case (b): $\gamma > 1$

In this sub-case, (17) becomes

$$f_i(q_i; Q) \equiv (d_i + \delta_i + \alpha_i)\gamma q_i + (\gamma - 1)[-P']q_i^2 \equiv a_i q_i + b(Q) q_i^2 \tag{21}$$

where $a_i \equiv \gamma s_i$ is the weighted marginal social cost of firm i's output, and

$$b(\hat{Q}) \equiv (\gamma - 1)[-P'(\hat{Q})].$$

For given $\hat{Q}$, the function $f_i(\hat{q}_i; \hat{Q})$ is quadratic and strictly convex in $\hat{q}_i$. A very effective way to characterize the optimal outputs is to use the following two-step procedure. (See Appendix A2).

Our result can be summarized as follows:

(i) The optimal industry output is given by $\hat{Q}$ which is the solution of the following first order condition (please see Appendix A2 for details):

$$F'(\hat{Q}) - a_I - \frac{1}{n} b'(\hat{Q})\hat{Q}^2 - \frac{2}{n} b(\hat{Q})\hat{Q} - \frac{b'(\hat{Q})}{4b^2(\hat{Q})} \sum_{i \in I} (a_i - a_I)^2 = 0$$

(ii) The optimal output for firm i is

$$\hat{q}_i^* = \frac{\lambda\left(\hat{Q}\right) - a_i}{2b(\hat{Q})} \text{ for all } i \in I \qquad (22)$$

where $\lambda\left(\hat{Q}\right)$ is given by:

$$\lambda\left(\hat{Q}\right) = a_I + \frac{2}{n} b(\hat{Q}) Q\hat{Q}$$

and this implies that the optimal firm-specific tax rate firm i is given by

$$\hat{t}_i = \hat{q}_i^* P'(\hat{Q}) + (P(\hat{Q}) - d_i) - \alpha_i \qquad (23)$$

where $\hat{q}_i^*$ is given by 22

(iii) Let

$$\Delta_i \equiv \hat{t}_i - \delta_i \qquad (24)$$

denote the gap between the optimal firm-specific output tax τ_i under oligopoly and the adjusted damage cost δ_i. Then

$$\Delta_i = \tilde{q}_i P'(\tilde{Q}) + P(\tilde{Q}) - s_i = \frac{(2-\gamma)s_i}{2(\gamma-1)} + P(\tilde{Q}) - \frac{\lambda\left(\tilde{Q}\right)}{2(\gamma-1)} \quad (25)$$

We thus can state:

Proposition 2: (Selective penalization) For any pair of firm $(i; j)$, we have

$$\Delta_i - \Delta_j = (\tilde{\tau}_i - \delta_i) - (\tilde{\tau}_j - \delta_j) = \frac{2-\gamma}{2(\gamma-1)}(s_i - s_j): \quad (26)$$

Proof: This follows from equation 25

To understand the formula (26), consider first the special case where $\varepsilon_i = \varepsilon_j$, so that the marginal damage per unit of output is the same for both firms $\delta_i = \delta_j$: Then, if $s_i < s_j$, the optimal selective tax rates are such that $\tilde{\tau}_i < \tilde{\tau}_j$, provided $1 < \gamma < 2$. That is, the more efficient firm pays a lower tax rate if $1 < \gamma < 2$. However, note that $(2-\gamma)/2(\gamma-1)$ is a decreasing function of γ; therefore the gap $\tilde{\tau}_i - \tilde{\tau}_j$ becomes narrower as γ increases (because with a greater γ, revenue considerations become more important, and therefore the government increases the tax rate on the bigger firms). When $\gamma = 2$, the tax rates are equal[6].

The intuition behind proposition1 is as follows. Recall that in an oligopoly with firms having different production costs, it is in general not optimal to tax firms equally for their pollution. This is because the Pigouvian taxes now serve two purposes: correction for pollution externalities, and correction for market power and for production inefficiency (because oligopolists do not equalize marginal production costs among themselves) while taking into account the marginal cost of public funds ($\gamma > 1$.) We now seek to characterize the optimal departure from the benchmark Pigouvian taxes δ_i obtained in Proposition1.

[6] For $\gamma > 2$; the more efficient firm must pay a higher tax rate. Recall that in our specification of the welfare function, we exclude the case $\gamma > 2$:

From (67) in the appendix, at the optimal solution, the taxes are such that the more efficient firms (those firms with low s_i) always produce more than the less efficient ones. The quantity Δ_i measures the deviation of optimal firm-specific tax under oligopoly from the firm-specific marginal damage cost caused by a unit of output of firm i. Recall that in this section we assume that $2 > \gamma > 1$ (to be in line with the empirical estimation of the marginal cost of public fund by Ballard et al.,1985) and that $c_i(z) = \alpha_i z$. We will call Δ_i the optimal Pigouvian distortion for firm i.

We now characterize the deviation of Δ_i from the industry average Δ_I. From (26),

$$\Delta_i - \Delta_I = \frac{2-\gamma}{2(\gamma-1)}(s_i - s_I) \tag{27}$$

where s_i is the marginal social cost of firm i's output. This result shows that the efficient tax structure favors the efficient firms, but this favor falls as γ increases.

From this, we can compute the variance of the statistical distribution of the Pigouvian distortions:

$$Var\,\Delta = \left[\frac{2-\gamma}{2(\gamma-1)}\right]^2 Var\,[s] \tag{28}$$

Proposition 3: (Optimal distortion theorem)

Optimal Pigouvian distortions (the gaps between optimal tax and adjusted marginal damage) are not equalized in a heterogeneous oligopoly. In the empirically relevant range of the marginal cost of public finance γ, i.e., for $1 < \gamma < 2$, if the marginal social cost s_i of firm i is greater than the industry average, the Pigouvian distortion for firm i will be greater than average Pigouvian distortion. The optimal tax structure penalizes inefficient firms.

The variance of the distribution of the Pigouvian distortions is given by (28), and it is a decreasing function of γ.

Remark: In the rather extreme case where $\gamma > 2$ (which is unlikely from empirical data) if the marginal social cost s_i of firm i is greater than the industry average, the Pigouvian distortion for firm i will be smaller than average Pigouvian distortion. It remains true that the optimal solution implies that the more efficient firms have greater outputs, see (67).

The Optimal Distortion Theorem provides a link between the ex-ante heterogeneity of the oligopoly's cost structure and the ex-post dispersion of the firm-specific Pigouvian tax rates.

3. Further Interpretation

Our results on firm-specific pollution taxes can be given an interesting geometric interpretation (see the Projection Theorem below) and an industrial organization interpretation (see the Concentration Motive Theorem below).

3.1. A geometric interpretation: the Projection Theorem

We now provide a geometric interpretation of the optimal choice of outputs. Consider the first step in the two-step procedure explained in section 2.5.2. That step is equivalent to the program of choosing the $\hat{q}_i$ $(i \in I)$ to

$$\min \sum_{i \in I} f_i(\hat{q}_i; \hat{Q})$$

subject to $\sum_{i \in I} \hat{q}_i = \hat{Q}$ (given) and $\hat{q}_i$ positive. This step can be described by the following Projection Theorem.

Proposition 4 (Projection Theorem) The determination of the optimal composition of industry output is equivalent to choosing a vector $\hat{q} \equiv (\hat{q}_1; \ldots; \hat{q}_n)$ from an $n - 1$ dimensional simplex S so as to minimize the distance between the vector $\hat{q}$ and a reference vector $q^* \equiv (q_1^*; \ldots; q_n^*)$ where

$$q_i^* \equiv -\frac{a_i}{2b(\hat{Q})} \tag{29}$$

and where

$$S \equiv \{\hat{q} \geq 0 : \sum_{i\in I} \hat{q}_i = \hat{Q}\}$$

Proof:

$$f_i(\hat{q}_i;\hat{Q}) = a_i\hat{q}_i + b(\hat{Q})\hat{q}_i^2 = b(\hat{Q})\left[\left(\hat{q}_i + \frac{a_i}{2b}\right)^2 - \frac{a_i^2}{4b^2}\right]$$

$$= b\left[\left(\hat{q}_i - q_i^*\right)^2 - \left(q_i^*\right)^2\right]$$

Thus

$$\sum_{i\in I} f_i(\hat{q}_i;\hat{Q}) = b\|\hat{q} - q^*\|^2 - b\sum_{i\in I}\left(q_i^*\right)^2$$

where the second term on the right-hand side depends only on $\hat{Q}$, which is fixed, and the first term on the right-hand side is b times the square of the distance of the point $\hat{q}$ in the set S (which is an $n-$simplex) to the given point q^*. Given $\hat{Q}$, both b and q^* are fixed. It follows that the first step (62) of the program is equivalent to finding the minimal distance between $\hat{q}$ and the given point q^*:

The optimal $\tilde{q}$ which achieves the minimal distance $\|\hat{q} - q^*\|$ is the projection of q^* on the $n-$simplex S. Its components are given by

$$\tilde{q}_i = \tilde{q}_I - \frac{1}{2b(\hat{Q})}(a_i - a_I) \tag{30}$$

which is (66). Figure 1 illustrates the case $n = 2$. The projection $\tilde{q}$ satisfies

$$\tilde{q} = q^* + (\tilde{q}_I - q_I^*)u$$

where $u = (1;1;\ldots;1)$ where $q_I^* = -\frac{a_I}{2b(\hat{Q})}$. ■

3.2. The Concentration Motive

Our result shows that firm-specific Pigouvian taxes in a polluting oligopoly serve two functions: the usual function of correcting for externalities, and the function of correcting for production efficiency, while taking into account the marginal cost of public funds. For this second function, the optimal tax vector depends on two elements (i) the degree of unit-cost asymmetry in the oligopoly, and (ii) the cost of public fund. The first element is measured by the variance of the statistical distribution of unit costs before and after taxation (this variance is related to the Herfindahl index.) The second element is measured by γ and reflects the trade-off between profits and tax revenue.

Does the optimal Pigouvian tax structure increase or decrease the concentration of the industry? Before answering this question, it is necessary to examine the relationships among the variance of the distribution of the unit costs, the Herfindahl index of concentration, industry profit, and welfare. We now state a number of lemmas concerning these relationships. First, recall that the Herfindahl index of concentration is

$$H = \sum_{i \in I} \left(\frac{q_i}{Q} \right)^2$$

Given that there are n firms, this index attains its maximum value ($H = 1$) when one firm produces the whole industry's output and the remaning $n - 1$ firms produce zero output, and it attains its mimimum value ($H = 1/n$) when each of the n firms produces $q_i = Q/n$. Now all firms will produce the same amount of output if they have the same tax-inclusive marginal costs.

Lemma 2: For a given output level $\hat{Q}$; the Cournot equilibrium industry profit is an increasing function of the Herfindahl index of concentration.

Proof: Recall that at a Cournot equilibrium, firm i's profit is $\hat{\pi}_i = [-\hat{P}'] \hat{q}_i^2 + \hat{q}_i c_i'(\hat{q}_i) - c_i(\hat{q}_i)$ With $c_i(q_i) = \alpha_i q_i$, the industry profit is

$$\hat\pi = \sum_{i\in I}\hat\pi_i = \left[-\hat P'\hat Q^2\right]\hat H + \text{constant}$$

where

$$\hat H = \sum_{i\in I}\left(\frac{\hat q_i}{\hat Q}\right)^2 \tag{31}$$

Lemma 3: Given the output level $\hat Q$, the Herfindahl index of concentration is an increasing function of the variance $Var(\hat\mu)$ of the distribution of the tax-inclusive marginal costs in a Cournot equilibrium.

$$\hat H = \frac{1}{n}\left[1 + \frac{Var(\hat\mu)}{\left[(-\hat P')\bar{\hat q}\right]^2}\right] \geq \frac{1}{n} \tag{32}$$

Thus any policy that maximizes [respectively, minimizes] the variance of the distribution of tax-inclusive marginal costs will maximize [respectively, minimizes] the concentration of the industry, and, for a given $\hat Q$, maximizes the profit of the industry.

Proof:

From (4),

$$\hat q_i = \frac{\hat P - \hat\mu_i}{(-\hat P')} \tag{33}$$

we obtain

$$\sum_{i\in I}\hat q_i^2 = \sum_{i\in I}\frac{1}{(-\hat P')^2}\left[(\hat P - \bar{\hat\mu}) - (\hat\mu_i - \bar{\hat\mu})\right]^2$$

$$= \frac{1}{(-\hat P')^2}\left[n(\hat P - \bar{\hat\mu})^2 + \sum_{i\in I}(\hat\mu_i - \bar{\hat\mu})^2\right]$$

$$= \frac{1}{(1-\hat{b}^0)^2}\left[n\left((1-\hat{b}^0)\bar{\theta}\right)^2 + nVar(\hat{\theta})\right] \qquad (34)$$

The result (32) follows from (34) and (31). ■

We now can state an important result: with $\gamma > 1$, the optimal tax structure increases the concentration of the industry. (See proposition CM below for a precise statement.) This means inefficient firms are penalized more, relative to efficient firms. Recall that for the case $\gamma = 1$, in section 2.5.1, at the social optimum, only the firm with the lowest social cost (s) will produce, giving rise to the maximum level of industry concentration (i.e., the Herfindahl index will take on its highest possible value, 1). Here, with $2 > \gamma > 1$, tax revenue is an important consideration, and hence there is a tradeoff between productive efficiency and tax revenue. It is still the case that the optimal tax structure increases the Herfindahl index, by increasing the variance of the distribution of tax-inclusive marginal costs. We call this the Magnification effect:The magnification factor, denoted by $\Omega(\gamma)$, is defined by

$$\Omega(\gamma) \equiv \left[\frac{\gamma}{2(\gamma-1)}\right]^2 > 1 \text{ for } 2 > \gamma > 1 \qquad (35)$$

This magification factor is a decreasing function of γ. As γ approaches the value 2, the magification factor falls to 1.

Proposition 5 (A pro-concentration motive theorem)

Assume that all firms have the same emission coefficients: $\epsilon_i = \epsilon$ for all i. Given $2 > \gamma > 1$, the optimal firm-specific Pigouvian tax structure increases the variance of the statistical distribution of tax-inclusive marginal costs within the oligopoly relative to the variance of the statistical distribution of pre-tax marginal costs[7]. The relationship between the two variances is given by:

$$Var(\tilde{\theta}) = \Omega(\gamma)Var(\theta^0) \qquad (36)$$

[7] In the empirically unlileky case where $\gamma > 2$; replace "increases" by "decreases".

where $\theta_i^0 = d_i + \alpha_i$ is the equilibrium marginal cost of firm i in a Cournot equilibrium where all the taxes are zero, and where

Proof: First, note that if all the taxes are zero, then

$$\theta_i^0 - \theta_l^0 = (d_i - d_l) - (d_l + \alpha_l) \tag{37}$$

Recall that $\hat{q}_i$ denote the equilibrium Cournot output of firm i given an arbitrary vector of firm-specific taxes, and $\tilde{q}_i$ is the equilibrium Cournot output of firm i when the taxes are optimized. From (66), and (33), which is true also when the tilda replaces the hat,

$$\tilde{q}_i - \tilde{q}_l = -\frac{(\tilde{\theta}_i - \tilde{\theta}_l)}{-\tilde{P}'^0} = -\frac{(a_i - a_l)}{2(\gamma - 1)\left(-\tilde{P}'^0\right)}$$

Hence

$$\tilde{\theta}_i - \tilde{\theta}_l = \frac{\gamma}{2(\gamma - 1)}\left[(d_i + \alpha_i) - (d_l + \alpha_l) + (\delta_i - \delta_l)\right] \tag{38}$$

Therefore

$$\tilde{\theta}_i - \tilde{\theta}_l = \frac{\gamma}{2(\gamma - 1)}\left[(\theta_i^0 - \theta_l^0) + (\delta_i - \delta_l)\right]$$

$$Var(\tilde{\theta}) = \left(\frac{\gamma}{2(\gamma - 1)}\right)^2\left[Var(\theta^0) + Var\,\delta + 2cov(\theta^0; \delta)\right] \tag{39}$$

If $\epsilon_i = \epsilon$ for all i, then, in view of (1) and (12), (39) reduces to (36). Note that $\Psi(\gamma) > 1$ if $1 < \gamma < 2$.

Remark: The intuition behind the pro-concentration motive theorem is as follows. If $\delta_i = \delta$ for all i, the marginal cost of public fund is within the empirically likely range ($1 < \gamma < 2$), then, for any given industry output level, the optimal firm-specific tax structure increases the variance of marginal costs (from $Var(\theta^0)$ to $\Psi(\gamma)Var(\theta^0)$) by taxing more efficient firms at a lower rate, see (26), because this helps the lower cost firms to expand output relative to the higher cost firms, and as a result improves productive efficiency. However, if γ is great, the tax revenue becomes a very important consideration, and it becomes optimal to tax more efficient firms at a higher rate, so as to generate more revenue. Take for example the case of a duopoly, where firm 2 has higher production cost. For a given level of industry output $\hat{Q}$, we must maintain $t_1 + t_2 =$ constant, say $2\bar{t}$. From an initial assignment $(t_1; t_2) = (\bar{t}; \bar{t})$, consider deviation of t_2 from $\bar{t}$, say $t_2 = \bar{t} + \epsilon$, and hence $t_1 = \bar{t} - \epsilon$. An increase in ϵ yields marginal gain in production efficiency, because the same level of industry output $\hat{Q}$ is produced, but the lower cost firm increases its output and the higher cost firm reduces its output. However, an increase in ϵ by $\cdot\,\epsilon$ implies reduced tax revenue, by approximately $(\hat{z}_1 - \hat{z}_2) \cdot \epsilon$; (plus the effect of induced changes in composition of industry output) and this implies increased distortion cost, approximately $(\gamma - 1) \cdot \epsilon \cdot (\hat{z}_1 - \hat{z}_2)$. For a given $\gamma > 1$, the optimal deviation ϵ^* is at the point where the marginal gain in productive efficiency is equated to the marginal increase in distortion cost. Clearly, a higher γ shifts the marginal distortion cost upwards, implying a smaller ϵ^*.

4. Selective Penalization under Non-linear Costs

We now examine the the case where $c_i(z_i)$ is strictly convex. To simplify the exposition, we assume that the marginal cost of public fund is unity: $\gamma = 1$. In this case, the functions $f_i(\hat{z}_i; \hat{Q})$ become

$$f_i(\hat{z}_i; \hat{Q}) = (d_i + \delta_i)\,\hat{z}_i + c_i(\hat{z}_i) = g_i(\hat{z}_i)$$

The first stage of the game can be solved in two steps: In step (i), we solve

$$\max_{\hat{z}_i} \widehat{W} = F\left(\hat{Q}\right) - \sum_{i \in I} f_i(\hat{z}_i; \hat{Q})$$

where

$$F\left(\hat{Q}\right) \equiv \beta S(\hat{Q}) + \hat{Q}P(\hat{Q}) \tag{40}$$

subject to

$$\sum_{i\in I} \hat{q}_i = \hat{Q}$$

where $\hat{Q}$ is given, and $q_i \geq 0$. In step (ii), we determine the optimal $\hat{Q}$.

To solve step (i), we form the Lagrangian

$$\hat{L} = F\left(\hat{Q}\right) - \sum_{i\in I} f_i(\hat{q}_i;\hat{Q}) + \lambda\left[\sum_{i\in I} \hat{q}_i - \hat{Q}\right]$$

or

$$\hat{L} = F\left(\hat{Q}\right) - \lambda\hat{Q} + \sum_{i\in I}\left[\lambda\hat{q}_i - f_i(\hat{q}_i;\hat{Q})\right]$$

We obtain the first order conditions

$$\frac{\partial f_i}{\partial \hat{q}_i} = d_i + \delta_i + c_i'(\hat{q}_i) = \lambda$$

hence

$$\tilde{q}_i = c_i^{x}(\lambda - d_i - \delta_i)$$

where $c_i^{x}(.)$ is the inverse function of $c_i'(.)$. Then $\tilde{q}_i(\lambda) = c_i^{x}(\lambda - d_i - \delta_i)$. The equation $\sum_{i\in I} \tilde{q}_i(\lambda) = \hat{Q}$ determines a unique $\lambda(\hat{Q})$.

The second step: We now determine the optimal $\hat{Q}$. We follow the duality method used in Rockafellar (1970). Following Rockafellar, we define the conjugate function

$$f_i^*(\lambda;\hat{Q}) = \sup_{\hat{q}_i}\left[\lambda\hat{q}_i - f_i(\hat{q}_i;\hat{Q})\right]$$

then

$$f_i^*(\lambda;\hat{Q}) = \lambda\tilde{q}_i(\lambda) - f_i(\tilde{q}_i(\lambda);\hat{Q})$$

It follows that the optimal value of the Lagrangian (optimized with respect to the $\hat{q}_i$) is

$$\tilde{L}(\hat{Q}) = \left[F(\hat{Q}) - \lambda(\hat{Q})\hat{Q}\right] + \sum_{i \in I} f_i^*(\lambda(\hat{Q}); \hat{Q})$$

Differentiating $\tilde{L}(\hat{Q})$ with respect to $\hat{Q}$ and equating it to zero yields

$$F'(\hat{Q}) - \hat{Q}\frac{d\lambda}{d\hat{Q}} - \lambda(\hat{Q}) + \sum \frac{\partial f_i^*}{\partial \lambda}\frac{d\lambda}{d\hat{Q}} + \sum \frac{\partial f_i^*}{\partial \hat{Q}} = 0 \qquad (41)$$

Since

$$\frac{\partial f_i^*}{\partial \lambda} = \tilde{q}_i(\lambda) + \left[\lambda - \frac{\partial f_i}{\partial \hat{q}_i}\right]\frac{\partial \tilde{q}_i}{\partial \lambda} = \tilde{q}_i(\lambda)$$

(41) reduces to

$$F'(\hat{Q}) - \hat{Q}\frac{d\lambda}{d\hat{Q}} - \lambda(\hat{Q}) + \sum_{i \in I} \tilde{q}_i(\lambda(\hat{Q}))\frac{d\lambda}{d\hat{Q}} = 0$$

or

$$F'(\hat{Q}) - \lambda(\hat{Q}) = 0 \qquad (42)$$

This equation determines the optimal $\tilde{Q}$.

Now, from (40)

$$F'(\hat{Q}) = \hat{P} - (1 - \beta)\left[-\hat{P}'\right]\hat{Q} \qquad (43)$$

From (42) and (43),

$$\hat{P} - \lambda = (1 - \beta)\left[-\hat{P}'\right]\hat{Q} \qquad (44)$$

.

The difference between the optimal per unit tax and the marginal damage is given by

$$\tau_i - \delta_i = \tilde{P} + \tilde{P}'\tilde{q}_i - [(d_i + \delta_i) + c_i'(\tilde{q}_i)]$$

But, recall that

$$(d_i + \delta_i) + c_i'(q_i) = \lambda \qquad (45)$$

Therefore

$$\tau_i - \delta_i = \left[P - \lambda\right] + P' q_i \qquad (46)$$

From (44) and (46),

$$\Delta_i = \tau_i - \delta_i = \left[-P'Q\right]\left\{(1-\beta) - \frac{q_i}{Q}\right\} \qquad (47)$$

Thus

$$\Delta_i - \Delta_j = (\tau_i - \varepsilon_i \sigma) - (\tau_j - \varepsilon_j \sigma) = \left[-P'\right](q_j - q_i)$$

From (47) we can state the following proposition

Proposition 6: (Optimal Pigouvian Distortion under Strictly Convex Costs and $\gamma = 1$)

Under strictly convex cost, the optimal tax structure favors lower cost firms.

Remark: Under linear cost, if $\gamma = 1$, the optimal tax structure will eliminate all inefficient firms (only the lowest cost firm will survive). Under strictly convex cost, such extreme penalization does not emerge; rather, all firms will produce, but the more efficient firms are more favorably treated.

Thus the specific Pigouvian tax t_i on pollution by firm i is

$$t_i \equiv \frac{\tau_i}{\epsilon_i} = \delta + \frac{1}{\varepsilon_i} S'(Q)\left\{(1-\beta) - \frac{q_i}{Q}\right\} \qquad (48)$$

We conclude that (i) t_i is greater, the greater is the marginal damage cost, (ii) t_i is negatively related to the weight attached to consumers' surplus, and (iii) in equilibrium, among all firms that have the same emission coefficient ε_i, smaller firms are taxed at a higher rate. This is because smaller firms are less efficient, and optimal policy seeks to reduce their outputs.

It is easy to generalize the result to the case where firm i has u_i identical plants. Thus, let $z_i = q_i/u_i$ and denote cost at the plant level by $c_i(z_i)$. Then optimal policy also favors firms with more plants. To see this, consider two firms, say firm i and firm j with $d_i = d_j$, $\delta_i = \delta_j$, and the same cost function at the plant level, i.e., $c_i(.) = c_j(.)$. Then, equation (45), appropriately modified, gives $\hat{z}_i = \hat{z}_j$. It follows that $u_i > u_j$ then $\hat{q}_i > \hat{q}_j$, and therefore, from (48), firm i will pay less tax per unit of output than firm j. Intuitively, this is because, at the firm level, firm i has a lower marginal cost curve. It is in this sense a more efficient firm, and accordingly it is better treated. (This happens only under oligopoly; under perfect competition, both firms would be taxed at the same rate.)

Corollary: Firms with more plants will be more favorably treated.

5. Pollution Standards and Abatement Costs

We now turn to a model in which firms can reduce emission at any given output level, by incurring abatement costs (which is a function of both the output level and the emission level). We will focus on the use of firm-specific pollution standards.

We assume that for a given pollution standard $\bar{e}_i$ (the maximum level of emission that firm i is allowed), the cost of output q_i is

$$A_i(\bar{e}_i; q_i) = a_i(\bar{e}_i)v(q_i) \tag{49}$$

with $a_i(\bar{e}_i) > 0$ for all $\bar{e}_i \geq 0$, $a_i'(\bar{e}_i) < 0$, $a_i''(\bar{e}_i) \geq 0$, $v'(q_i) > 0$, $v''(q_i) \geq 0$, and $v(0) = 0$. Thus A_i is convex in both arguments, and $A_i(\bar{e}_i; 0) = 0$. Then firm i's profit is

$$\pi_i = q_i P(Q) - c_i q_i - A_i(q_i; e_i) \tag{50}$$

We assume that the regulating agency specifies an amount $\bar{e}_i$ (i.e., maximum pollution per period) that firm i must not exceed. We take it that the fines for violation are sufficiently high to ensure perfect compliance. It follows that if the firm wants to produce quantity q_i

then it must spend the amount $A_i(\bar{e}_i; q_i)$. We call $\bar{e}_i$ the "firm-specific emission standard" set by the regulatory agency.

We wish to determine the optimal configuration of firm-specific standards that maximizes social welfare, given the constraints that firms are oligopolists.

We now show that welfare can be increased by setting non-identical firm-specific standards.

Given e_i, firm i's marginal cost of production is $c_i + a_i(e_i)v'(q_i)$ Then, if $\hat{Q}$ is the Cournot equilibrium industry output, firm i's equilibrium output satisfies

$$\hat{q}_i P'(\hat{Q}) + P(\hat{Q}) = c_i + a_i(e_i)v'(\hat{q}_i) \equiv \mu_i \tag{51}$$

where μ_i is firm i's marginal cost at a Cournot equilibrium. We will exploit the following equilibrium relationship between e_i and $\hat{q}_i$; for a given $\hat{Q}$:

$$a_i(e_i) = \frac{P'(\hat{Q})\hat{q}_i + P(\hat{Q}) - c_i}{v'(\hat{q}_i)} \tag{52}$$

That is,

$$e_i(\hat{q}_i; \hat{Q}) = a_i^{-1}\left[\frac{\hat{q}_i \hat{P}' + \hat{P} - c_i}{v'(\hat{q}_i)}\right] \tag{53}$$

Thus in equibrium, firm i's abatement cost is

$$a_i(e_i)v(\hat{q}_i) = \frac{[P'(\hat{Q})\hat{q}_i + P(\hat{Q}) - c_i]\hat{q}_i}{\epsilon(\hat{q}_i)} \tag{54}$$

where $\epsilon(\hat{q}_i)$ is defined as the elasticity of $v(q_i)$: $\epsilon(q_i) = q_i v'(q_i)/v(q_i)$.

Equilibrium profit of firm i is, from (50) and (54),

$$\begin{aligned} \pi_i &= \hat{q}_i\{\hat{P} - c_i\} + \{[-\hat{P}']\hat{q}_i - (\hat{P} - c_i)\}[\hat{q}_i/\epsilon(\hat{q}_i)] \\ &= (\hat{P} - c_i)\left[1 - \frac{1}{\epsilon(\hat{q}_i)}\right]\hat{q}_i + [-\hat{P}']\frac{\hat{q}_i^2}{\epsilon(\hat{q}_i)} \end{aligned} \tag{55}$$

Industry profit in equilibrium is

$$\hat{\Pi} = \sum_i \hat{\pi}_i = \hat{Q}^2[-\hat{P}']\hat{H} + \sum_i (\hat{P} - c_i)\left[1 - \frac{1}{\eta(\hat{q}_i)}\right]\hat{q}_i$$

where $\hat{H}$ is a "modified Herfindahl index" of concentration:

$$\hat{H} = \sum_{i \in I} \frac{\hat{q}_i^2}{\eta(\hat{q}_i)\hat{Q}^2}$$

We can express social welfare as

$$\hat{W} = \beta S(\hat{Q}) - \sum_{i \in N} f_i(\hat{q}_i; \hat{Q}) \tag{56}$$

where

$$f(\hat{q}_i; \hat{Q}) \equiv -[P(\hat{Q}) - c_i]\left[1 - \frac{1}{\eta(\hat{q}_i)}\right]\hat{q}_i$$

$$+\hat{P}'\frac{\hat{q}_i^2}{\eta(\hat{q}_i)} + \sigma e_i(\hat{q}_i; \hat{Q}) \tag{57}$$

For any given $\hat{Q}$, the regulator can choose the $\hat{q}_i$'s to maximize social welfare subject to $\sum_{i \in I} \hat{q}_i = \hat{Q}$. An interesting property of the social welfare function (56) is that, under certain reasonable assumptions, it is convex in the $\hat{q}_i$'s, for a given $\hat{Q}$. For example, we obtain this convexity property if $v(q) = q$, $a(e) = B - e$ where $B > 0$, and $P(Q) = 1 - Q$. We can now the following proposition.

Proposition 7: When pollution abatement cost is of the form given by (49), optimal standards satisfy the following properties:

(i) If the social welfare function is concave in the $\hat{q}_i$'s, the optimal firm-specific pollution standards are

$$\tilde{e}_i = a_i^{-1}\left[\frac{\tilde{q}_i \tilde{P}'(\hat{Q}) + \tilde{P}(\hat{Q}) - c_i}{v'[\tilde{q}_i]}\right]$$

(ii) (Unequal treatment of equals): If the social welfare function (56) is convex in the θ_i's, the optimum choice of the θ_i's is achieved by giving non-identical treatments to identical firms.

Remark: For lack of space, we do not present results on the direction of bias here. Some examples are provided in Long and Soubeyran (2001b). Similar results apply to the case of tradeable permits, see Long and Soubeyran (2000).

6. Concluding remarks

We have characterized the optimal structure of penalties for polluting firms in an oligopoly with heterogenous costs. We have shown that there is a bias in favor of efficient firms. In achieving efficiency, a structure of systematic biases emerges.

Our paper goes beyond the existing result of unequal treatment to ex ante identical firms. Several important insights emerge. It is shown that optimal firm-specific regulations are partly driven by the motive to increase the industry concentration, because increased concentration can enhance productive efficiency. However, tax revenue can be an important consideration, and any increase in the marginal cost of public funds would lead to an increased tax rate on the more efficient firms. The degree of industry concentration is increased by the structure of efficient taxes.

Our analysis can be extended to study the role of strategic trade policy in the presence of a polluting international oligopoly. There are a number of insightful papers that deal with this topics (Conrad (1993), Barrett (1994), Kennedy (1994), Ulph and Ulph (1996), Ulph (1996a,b), Neary (1999)). However, the optimal structure of taxes was not explored in these papers, because the models did not allow for asymmetry within the domestic industry, and firm-specific taxes or standards were ruled out.

APPENDIX A1

The benchmark case: perfect competition.

Perfect competition means that each firm thinks that its output has no effect on the price, i.e., the term $P'(Q)$ does not appear (i.e., is assigned the value zero) in the firm's first order condition. Therefore (14) reduces to

$$\tau_i = \hat{P} - d_i - c'(\hat{q}_i) \tag{58}$$

With $\gamma = \beta = 1$ social welfare (15) becomes

$$\hat{W} = S(\hat{Q}) + \hat{P}\hat{Q} - \sum_{i \in I} [(d_i + \delta_i)\hat{q}_i + c_i(\hat{q}_i)] \tag{59}$$

Writing $\hat{Q} = \sum_{i \in I} \hat{q}_i$ and maximizing (59) with respect to the $\hat{q}_i$'s, we obtain

$$\hat{P} - d_i - \delta_i - c_i'(\hat{q}_i) = 0 \tag{60}$$

which says that marginal social cost of firm i's output must be equated to price, a standard result. From (60) and (58), we get $\tau_i = \delta_i$, and hence, using (1) and (12),

$$t_i = \frac{\sigma}{\gamma} \equiv \delta \text{ for all } i \in I \tag{61}$$

(where $\gamma = 1$) that is, the tax per unit of pollutant discharged by firm i is equal to the marginal damage cost.

APPENDIX A2

Oligopoly with constant marginal cost.

Subcase (a): $\gamma = 1$:

Without loss of generality, assume $s_1 < s_2 < s_3 < \dots < s_n$. Then, if $\partial \hat{W}/\partial \hat{q}_1 = F'(\hat{Q}) - s_1 = 0$ it must be true that, for all $j > 1$, $\partial \hat{W}/\partial \hat{q}_j = F'(\hat{Q}) - s_j = s_1 - s_j < 0$, implying that $\hat{q}_j = 0$. It follows that at the social optimum, only firm 1 produces.

An intuitive explanation of this result is as follows. Suppose that at an equilibrium both firms 1 and 2 produce positive outputs, and they satisfy the Cournot equilibrium conditions

$$P(\hat{Q}) + P'(\hat{Q})\hat{q}_1 = d_1 + \alpha_1 + \tau_1$$

and

$$P(\hat{Q}) + P'(\hat{Q})\hat{q}_2 = d_2 + \alpha_2 + \tau_2$$

Then, social welfare can be increased by raising $\tau_2/[-P'(\hat{Q})]$ by $\Delta > 0$ and reducing $\tau_1/[-P'(\hat{Q})]$ by Δ, so that firm 1's output will increase by Δ and firm 2's output will fall by Δ, leaving industry output and price unchanged. Social welfare increases because the total cost of producing the given output $\hat{Q}$ is now lower. Tax revenue will change, but industry profit, defined as sales revenue, minus production cost, minus tax payment) will change by the same amount, therefore, given that $\gamma = 1$, the tax revenue change does not matter.

Subcase (b) $\gamma > 1$

The two-step procedure:

In step 1, we fix an arbitrary level of industry output, $\hat{Q}$; and maximize welfare by choosing the $\hat{q}_i$'s subject to the constraint that $\sum_{i \in I} \hat{q}_i = \hat{Q}$. This gives the optimal value of $\hat{q}_i$, conditional on the given $\hat{Q}$. In the second step, we determine the optimal industry output.

Step 1:

Given $\hat{Q}$, we write the Lagrangian as

$$L = F(\hat{Q}) - \sum_{i \in I} f_i(\hat{q}_i; \hat{Q}) + \lambda \left[\sum_{i \in I} \hat{q}_i - \hat{Q} \right] \tag{62}$$

From this we obtain the conditions

$$-a_i - 2b(\hat{Q})\tilde{q}_i + \lambda = 0; \text{ for all } i \in I \tag{63}$$

where $\tilde{q}_i$ denotes the optimal value of $\hat{q}_i$, conditional on the given $\hat{Q}$. From (63),

$$\tilde{q}_i = \frac{\lambda - a_i}{2b(\hat{Q})} \text{ for all } i \in I \tag{64}$$

Summing (64) over all i, we obtain an expression showing that λ is uniquely determined by $\widehat{Q}$:

$$\lambda = \widehat{\lambda}(\widehat{Q}) = a_I + \frac{2}{n} b(\widehat{Q})\widehat{Q} \tag{65}$$

where $a_I \equiv (1/n)\sum_{i \in I} a_i$. Substituting (65) into (64), and letting $\widehat{q}_I \equiv \widehat{Q}/n$, we get

$$\widehat{q}_i(\widehat{Q}) = \widehat{q}_I - \frac{1}{2b(\widehat{Q})}[a_i - a_I] \quad \text{for all } i \in I \tag{66}$$

This equation gives us:

Lemma 2: The optimal deviation of the output of firm i from average industry output is a linear function of the deviation of its marginal social cost from the industry average.

Remark: To illustrate, consider a pair of firms $(1, 2)$ with marginal social costs $s_1 < s_2$. Then (66) gives

$$\widehat{q}_1(\widehat{Q}) - \widehat{q}_2(\widehat{Q}) = \frac{\gamma}{2(\gamma - 1)[-\widehat{P}']}[s_2 - s_1] \tag{67}$$

That is, the solution of the optimization problem has the property that the firm with higher marginal social cost produces less than the firm with lower marginal social cost. Note that, for a given $\widehat{Q}$, a greater γ implies a smaller gap between $\widehat{q}_1$ and $\widehat{q}_2$, but this gap is always positive and greater than $[s_2 - s_1]/2[-\widehat{P}']$. This may be explained as follows: a greater γ implies that a greater weight is given to tax revenue. Thus, for any given $\widehat{Q}$, a marginal increase in γ would increase the government's desire to increase tax revenue at the cost of reduced productive efficiency (here, productive efficiency includes not only private cost considerations, but also environmental cost). The government would therefore raise τ_1 by some small amount $\epsilon > 0$ and at the same time reduce τ_2 by ϵ, thus leaving total output $\widehat{Q}$ constant. The increase in tax revenue is approximately $(\widehat{q}_1 - \widehat{q}_2)\epsilon$ and this must be balanced against the marginal loss in productive efficiency associated with the increase in the output of the high-cost firm and the reduced output of the low-cost firm.

Step 2:

Using the results in step 1, we are now ready to find the optimal industry output. We make use of the fact that the optimal value of the Lagrangian, given $\hat{Q}$, is equal to the maximized W, given $\hat{Q}$. Thus

$$\tilde{L}(\hat{Q}) = F(\hat{Q}) - \tilde{\lambda}(\hat{Q})\hat{Q} + \sum_{i\in I} f_i^*\left[\tilde{\lambda}(\hat{Q}); \hat{Q}\right]$$

where

$$f_i^*(\lambda;\hat{Q}) \equiv \sup_{q_i} \lambda \hat{q}_i - f_i(\hat{q}_i;\hat{Q})$$

($f_i^*(\lambda;\hat{Q})$ is called the conjugate function of $f_i(\hat{q}_i;\hat{Q})$, see Rockafellar, 1970, section 12.) In the present case,

$$f_i^*(\lambda;\hat{Q}) = [a_I + 2b\hat{q}_I]\,\tilde{q}_i - \left\{a_i\tilde{q}_i + b\tilde{q}_i^2\right\}$$

$$= -(a_i - a_I)\tilde{q}_i - b[\tilde{q}_i - \hat{q}_I]^2 + b\hat{q}_I^2$$

Thus, using (66)

$$f_i^*(\lambda;\hat{Q}) = -(a_i - a_I)\hat{q}_I + b\hat{q}_I^2 + \frac{1}{4b(\hat{Q})}(a_i - a_I)^2$$

and, using (65), the maximized welfare, for given $\hat{Q}$, is

$$\tilde{W}(\hat{Q}) = F(\hat{Q}) - \left[a_I + 2b(\hat{Q})\frac{\hat{Q}}{n}\right]\hat{Q}$$

$$+ nb(\hat{Q})\left(\frac{\hat{Q}}{n}\right)^2 + \frac{1}{4b(\hat{Q})}\sum_{i\in I}(a_i - a_I)^2$$

Hence

$$\widehat{W}(\widehat{Q}) = F(\widehat{Q}) - a_I\widehat{Q} - \frac{1}{n}b(\widehat{Q})\widehat{Q}^2 + \frac{1}{4b(\widehat{Q})}\sum_{i\in I}(a_i - a_I)^2 \qquad (68)$$

Maximizing (68) with respect to $\widehat{Q}$, we get the necessary condition

$$F'(\widehat{Q}) - a_I - \frac{1}{n}b'(\widehat{Q})\widehat{Q}^2 - \frac{2}{n}b(\widehat{Q})\widehat{Q} - \frac{b'(\widehat{Q})}{4b^2(\widehat{Q})}\sum_{i\in I}(a_i - a_I)^2 = 0$$

This equation determines the optimal value of $\widehat{Q}$, which we denote by $\widetilde{Q}$. The optimal output for firm i is

$$\tilde{q}_i^* = \frac{\lambda\left(\widetilde{Q}\right) - a_i}{2b(\widetilde{Q})} \text{ for all } i \in I \qquad (69)$$

From this, we derive the optimal firm-specific tax $\tilde{\tau}_i$, using (14):

$$\tilde{\tau}_i = \tilde{q}_i^* P'(\widetilde{Q}) + (P(\widetilde{Q}) - d_i) - c'(\tilde{q}_i^*): \qquad (70)$$

where $\tilde{q}_i^*$ is given by 69

APPENDIX A3
The Duality Approach

The following is the outline of the duality approach contained in Rockafellar (1970). Consider the problem

$$\max_{x_i} J = F(X) - \sum_{i\in I} f_i(x_i; X)$$

subject to

$$\sum_{i\in I} x_i = X, \qquad x_i \geq 0; i \in I$$

where X is given, $f_i(x_i; X)$ convex with respect to x_i and differentiable with respect to $(x_i; X)$, and proper ($f_i(x_i; X)$ is never $-\infty$, and is not identically $+\infty$).

To solve this problem, define the extended functions $g_i(x_i;X) = f_i(x_i;X)$ if $x_i \geq 0$ and $g_i(x_i;X) = +\infty$ if $x_i < 0$. Then we have the program

$$\max_{x_i} J = F(X) - \sum_{i \in I} g_i(x_i;X)$$

subject to

$$\sum_{i \in I} x_i = X, \qquad i \in I$$

For a given X; the Lagrangian of this problem is

$$L(x;\lambda;X) = [F(X) - \lambda X] + \sum_{i \in I} [\lambda x_i - g_i(x_i;X)]$$

where $x = (x_1;\ldots;x_n)$.

The saddlepoint duality theorem (see Rockafellar, 1970, pp 284-5) states that $\tilde{x} = (\tilde{x}_1;\ldots;\tilde{x}_n)$ is an optimal solution of the program if and only if (i) given $\tilde{\lambda}$, $\tilde{x}$ maximixes the function $L(x;\tilde{\lambda};X)$, and (ii) $\tilde{\lambda}$ minimizes $L(\tilde{x}(\lambda;X);\lambda;X)$ with respect to λ, where $\tilde{x}(\lambda;X)$ achieves the minimum of $L(x;\lambda;X)$ for each given λ.

The determination of $\tilde{x}_i(\lambda;X)$ is given by the first order condition of the program

$$\sup_{x_i} [\lambda x_i - g_i(x_i;X)] = g_i^*(\lambda;X)$$

$g_i^*(\lambda;X)$ is called the conjugate function of $g_i(x_i;X)$. We have

$$\frac{\partial g_i^*}{\partial \lambda} = \tilde{x}_i(\lambda;X) + \left[\lambda - \frac{\partial g_i}{\partial x_i}\right] \frac{\partial \tilde{x}_i}{\partial \lambda} = \tilde{x}_i(\lambda;X)$$

We also have

$$\tilde{L} = L[\tilde{x}(\lambda;X);\lambda;X] = [F(X) - \lambda X] + \sum_{i \in I} g_i^*(\lambda;X)$$

and $\tilde{\theta}(X) = \tilde{\theta}$ achieves the minimum of $\tilde{E}$ with respect to θ. The first order condition for that is

$$\sum_{i \in I} \frac{\partial g_i^*}{\partial \theta}(\tilde{\theta}; X) = X$$

that is,

$$\sum_{i \in I} \tilde{x}_i(\theta; X) = X$$

This equation gives $\tilde{\theta}(X)$. The optimal $\tilde{x}_i$ follows: $\tilde{x}_i(X) = \tilde{x}_i\left[\tilde{\theta}(X); X\right]$; $i \in I$:

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