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There is a Risk-Return Tradeoff After All

*Eric Ghysels, Pedro Santa-Clara,
Rossen Valkanov*

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There is a Risk-Return Tradeoff After All *

Eric Ghysels[†], Pedro Santa-Clara[‡], Rossen Valkanov[§]

Résumé / Abstract

Nous étudions le modèle ICAPM à l'aide d'un nouvel estimateur MIDAS, basé sur un mélange de données temporelles échantillonnées à différentes fréquences. Nous trouvons une relation positive et significative avec cet estimateur. Nous analysons également des modèles avec asymétries.

Mots clés : estimation avec mélange de fréquence de séries temporelles, relation risque-rendement.

This paper studies the ICAPM intertemporal relation between conditional mean and conditional variance of the aggregate stock market return. We introduce a new estimator that forecasts monthly variance with past daily squared returns — the Mixed Data Sampling (or MIDAS) approach. Using MIDAS, we find that there is a significantly positive relation between risk and return in the stock market. This finding is robust in subsamples, to asymmetric specifications of the variance process, and to controlling for variables associated with the business cycle. We compare the MIDAS results with other tests of the ICAPM based on alternative conditional variance specifications and explain the conflicting results in the literature. Finally, we offer new insights about the dynamics of conditional variance.

Keywords: *mixed data sampling, risk-return trade-off.*

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[†] Department of Economics, University of North Carolina and CIRANO, Gardner Hall CB 3305, Chapel Hill, NC 27599-3305, tel.: (919) 966-5325, e-mail: eghysels@unc.edu.

[‡] The Anderson School, UCLA, Los Angeles, CA 90095-1481, e-mail: pedro.santa-clara@anderson.ucla.edu, tel.: (310) 206-6077.

[§] The Anderson School, UCLA, Los Angeles, CA 90095-1481, e-mail: rossen.valkanov@anderson.ucla.edu, tel.: (310) 825-7246.

1 Introduction

Merton’s (1973) ICAPM suggests that the conditional expected excess return on the stock market should vary positively with the market’s conditional variance:

$$E_t[R_{t+1}] = \mu + \gamma \text{Var}_t[R_{t+1}], \quad (1)$$

where γ is the coefficient of relative risk aversion of the representative agent and, according to the model, μ should be equal to zero. This risk-return tradeoff is so fundamental in financial economics that it could well be described as the “first fundamental law of finance.”¹ Unfortunately, the tradeoff has been hard to find in the data. The relation between risk and return has often been found insignificant, and sometimes even negative.

Baillie and DeGennaro (1990), French, Schwert, and Stambaugh (1987), Chou (1992), and Campbell and Hentschel (1992) do find a positive albeit mostly insignificant relation between the conditional variance and the conditional expected return. In contrast, Campbell (1987) and Nelson (1991) find a significantly negative relation. Glosten, Jagannathan, and Runkle (1993), Harvey (2001), and Turner, Startz, and Nelson (1989) find both a positive and a negative relation depending on the method used.² The main difficulty in testing the ICAPM relation is that the conditional variance of the market is not observable and must be filtered from past returns.³ The conflicting findings of the above studies are mostly due to differences in the approach to modeling the conditional variance.

In this paper, we take a new look at the risk-return tradeoff by introducing a new estimator of the conditional variance. Our *Mixed Data Sampling*, or MIDAS, estimator forecasts the monthly variance as a weighted average of lagged daily squared returns. We use a flexible functional form to parameterize the weight given to each lagged daily squared return and show that a parsimonious weighting scheme with only three parameters to estimate works quite well. We estimate the coefficients of the conditional variance process jointly with μ

¹However, Abel (1988), Backus and Gregory (1993), and Gennotte and Marsh (1993) offer models where a *negative* relation between return and variance is consistent with equilibrium. Campbell (1993) discusses general conditions under which the risk-return relation holds as an approximation.

²See also Chan, Karolyi, and Stulz (1992), Lettau and Ludvigson (2002), Merton (1980), and Pindyck (1984). Goyal and Santa-Clara (2002) find a positive tradeoff between market return and *average stock variance*.

³We could think of using option implied volatilities as do Santa-Clara and Yan (2001) to make variance “observable.” Unfortunately, option prices are only available since the early 1980’s which is insufficient to reliably make inferences about the conditional mean of the stock market.

and γ from the expected return equation (1) using quasi-maximum likelihood.

Using monthly and daily market return data from the post-WWII period (1946-2000) and with MIDAS as a model of the conditional variance, we find a *positive and statistically significant* relation between risk and return. The estimate of γ is about four, which lines up well with economic intuition about a reasonable level of risk aversion. The MIDAS estimator explains more than eight percent of the variation of realized variance in the next month, which compares favorably with other models of conditional variance such as GARCH. The estimated weights on the lagged squared daily returns decay slowly, thus capturing the persistence in the conditional variance process. More impressive still is the fact that, in the ICAPM risk-return relation, the MIDAS estimator of conditional variance explains 2.4 percent of the variation of next month's stock market returns. This is quite substantial given previous results about forecasting the stock market return.⁴ Finally, the above results are qualitatively similar when we split the sample into two subsamples of approximately equal sizes, 1946-1972 and 1973-2000, or when we exclude the October 1987 crash from the full sample.

The success of the MIDAS estimator in forecasting the stock market variance and explaining the risk-return tradeoff resides in the use of high-frequency data to estimate the conditional variance and the flexible parameterization of the weights on the lagged squared daily returns. In particular, the weight function determines the persistence of the conditional variance process and the statistical precision with which variances are estimated. To better understand MIDAS and its success in testing the ICAPM risk-return tradeoff, we compare our approach to previously used models of conditional variance. French, Schwert, and Stambaugh (1987) propose a simple and intuitive rolling-windows estimator of the monthly variance. They forecast monthly variance by the sum of daily squared returns in the previous month. Their method is similar to ours in that it uses daily returns to forecast monthly variance. However, when French, Schwert, and Stambaugh use that method to test the ICAPM, they find a negative γ coefficient. We replicate their results but also find something rather interesting and new. When the length of the rolling window is increased from one month to a longer period of three or four months, we again obtain a positive and statistically significant estimate of γ . This nicely illustrates the point that the window length plays a crucial role in forecasting variances and detecting the tradeoff between risk and return. By

⁴For instance, the forecasting power of the dividend yield for the market return does not exceed 1.5 percent (see Campbell, Lo, and MacKinlay (1997) and references therein).

optimally choosing the weights on lagged squared returns, MIDAS implicitly selects the optimal window size to forecast the variance, and that in turn leads to a positive test of the ICAPM equation.

The ICAPM risk-return relation has also been tested using several variations of GARCH-in-mean models. However, the evidence from that literature is inconclusive and sometimes conflicting. Using simple GARCH models, we confirm the finding of French, Schwert, and Stambaugh (1987) and Glosten, Jagannathan, and Runkle (1993), among others, of a positive but insignificant γ coefficient in the risk-return tradeoff. Actually, except for the lack of significance, the estimated γ in these tests are similar to our MIDAS results. The absence of statistical significance comes undoubtedly from GARCH's use of monthly return data in estimating the conditional variance process. The use of daily data in the MIDAS estimator provides the power needed to find statistical significance in the risk-return tradeoff.

It has long been recognized that volatility tends to react more to negative returns than to positive returns. Nelson (1991) and Engle and Ng (1993) show that GARCH models that incorporate this asymmetry perform better in forecasting the market variance. However, Glosten, Jagannathan, and Runkle (1993) show that when such asymmetric GARCH models are used in testing the risk-return tradeoff, the γ coefficient is estimated to be negative (sometimes significantly so). This stands in sharp contrast with the positive and insignificant γ obtained with symmetric GARCH models and creates a puzzle to researchers. To investigate this issue, we extend the MIDAS approach to capture asymmetries in the dynamics of conditional variance by allowing lagged positive and negative daily squared returns to have different weights in the estimator. Contrary to the asymmetric GARCH results, we still find a large positive estimate of γ that is statistically significant. This discrepancy between the asymmetric MIDAS and asymmetric GARCH tests of the ICAPM turns out to be quite interesting.

We find that what matters for the tests of the risk-return tradeoff is not so much the asymmetry in the conditional variance process but rather the persistence of that process. In this respect, asymmetric GARCH and asymmetric MIDAS models prove to be very different. Consistent with the GARCH literature, negative shocks have a larger immediate impact on the MIDAS conditional variance estimator than do positive shocks. However, we find that the impact of negative returns on variance is only temporary and lasts no more than one month. Positive returns, on the other hand, have an extremely persistent impact on the variance process. In other words, while short-term fluctuations in the conditional variance are mostly

due to negative shocks, the persistence of the variance process is primarily due to positive shocks. This is an intriguing finding about the dynamics of variance. Although asymmetric GARCH models allow for a different response of the conditional variance to positive and negative shocks, they constrain the persistence of both types of shocks to be the same. Since the asymmetric GARCH models “load” heavily on negative shocks and these have little persistence, the estimated conditional variance process shows almost no persistence at all.⁵ In contrast, by allowing positive and negative shocks to have different persistence, the asymmetric MIDAS model still obtains high persistence for the overall conditional variance process. Since only persistent variables can capture variation in expected returns, the difference in persistence of asymmetric MIDAS and asymmetric GARCH conditional variance estimators explains their success and lack thereof in finding a risk-return tradeoff.

Campbell (1987) and Scruggs (1998) point out that the difficulty in measuring a positive risk-return relation might be due to misspecification of equation (1). Following Merton (1973), they argue that if changes in the investment opportunity set are captured by state variables in addition to the conditional variance itself, then those variables must be included in the equation of expected returns. In parallel, an extensive literature on the predictability of the stock market finds that variables that capture business cycle fluctuations are also good forecasters of market returns (see Campbell (1991), Campbell and Shiller (1988), Fama (1990), Fama and French (1988, 1989), Ferson and Harvey (1991), and Keim and Stambaugh (1986), among many others). We include business cycle variables together with both the symmetric and asymmetric MIDAS estimators of conditional variance in the ICAPM equation and find that the tradeoff between risk and return is virtually unchanged. Indeed, the explanatory power of the conditional variance for expected returns is orthogonal to the other predictive variables.

We conclude that the ICAPM is alive and well.

The rest of the paper is structured as follows. Section 2 explains the MIDAS model and details the main results. Section 3 offers a comparison of MIDAS with rolling-window and GARCH models of conditional variance. In Section 4, we discuss the asymmetric MIDAS model and use it to test the ICAPM. In Section 5, we include several often-used predictive variables as controls in the risk-return relation. Section 6 concludes the paper.

⁵The only exception is the two-component GARCH model of Engle and Lee (1999) who report findings similar to our asymmetric MIDAS model. They obtain persistent estimates of conditional variance while still capturing an asymmetric reaction of the conditional variance to positive and negative shocks.

2 MIDAS Tests of the Risk-Return Tradeoff

In this section, we introduce the *Mixed Data Sampling*, or MIDAS, estimator of conditional variance and use it to test the ICAPM between risk and return of the stock market.

2.1 Methodology

The MIDAS approach mixes daily and monthly data to estimate the conditional variance of stock market returns. The returns on the “left-hand side” of equation (1) are measured at monthly intervals since a higher frequency would be too noisy to use in a study of conditional means. On the other hand, we use daily returns in the variance estimator to exploit the advantages of high-frequency data in the estimation of second moments due to the well-known continuous-record argument of Merton (1980).⁶ Furthermore, we allow the estimator to load on a large number of past daily squared returns with optimally chosen weights.

The MIDAS estimator of the conditional variance of *monthly* returns, $\text{Var}_t[R_{t+1}]$, is based on prior *daily* squared return data:

$$V_t^{\text{MIDAS}} = 22 \sum_{d=1}^{\infty} w_d r_{t-d}^2 \quad (2)$$

where w_d is the weight given to the squared return of day $t - d$. We use the lower case r to denote *daily* returns, which should be distinguished from the upper case R used for *monthly* returns; the corresponding subscript $t - d$ stands for the date t minus d days. With weights that sum up to one, the factor 22 insures that the variance is expressed in monthly units (since there are typically 22 trading days in a month). We postulate a flexible form for the weight given to the squared return on day $t - d$:

$$w_d(\kappa_0, \kappa_1, \kappa_2) = \frac{\exp\{\kappa_0 + \kappa_1 d + \kappa_2 d^2\}}{\sum_{i=1}^{\infty} \exp\{\kappa_0 + \kappa_1 i + \kappa_2 i^2\}} \quad (3)$$

This scheme has several advantages. First, the specification (3) guarantees that the weights are positive which in turn ensures that the conditional variance in (2) is also positive. Second,

⁶Recently, various authors, including Andersen, Bollerslev, Diebold, and Ebens (2001), Andreou and Ghysels (2002), Barndorff-Nielsen and Shephard (2002), and Taylor and Xu (1997) suggest using higher-frequency intra-daily data to estimate variances. Alizadeh, Brandt, and Diebold (2002) propose an alternative measure of realized variance using the daily range of the stock index.

the weights add up to one. Third, the functional form in (3) can produce a wide variety of shapes for the weights for different values of the parameters. Fourth, the specification is parsimonious, with only three parameters to be estimated. Fifth, as long as the coefficient κ_2 is negative, the weights go to zero as the lag length increases. The speed with which the weights decay controls the effective number of observations used to estimate the conditional variance. Finally, we can increase the order of the polynomial in (3) or consider other functional forms. For instance, all the results shown below are robust to parameterizing the weights as a Beta function instead of the exponential form in (3).⁷ As a practical matter, the infinite sum in (2) and (3) needs to be truncated at a finite lag. In all the results that follow, we use 260 days (which corresponds to roughly one year of trading days) as the maximum lag length. Extensive experimentation shows that the results are not sensitive to increasing the maximum lag length beyond one year.

The weights of the MIDAS variance estimator capture implicitly the dynamics of the conditional variance since the number of past returns effectively used in the estimator determines the persistence of the conditional variance process. The weighting function also determines the amount of data effectively used to estimate the conditional variance thereby controlling the statistical precision of the variance estimator. When the function decays slowly, a large number of observations enter in the forecast of the variance and the measurement error is low. Conversely, a fast decay corresponds to using a small number of daily returns to forecast the variance with potentially large measurement error. To some extent there is a tension between capturing the dynamics of variance and minimizing measurement error. Since variance changes through time, and given the persistence of the process, we would like to use more recent observations to forecast the level of variance in the next month. However, to the extent that measuring variance precisely requires a large number of daily observations, the estimator may still place significant weight on more distant observations.

We estimate the parameters in the weight function (3) to maximize the likelihood of monthly returns. We use the variance estimator (2) with the weight function (3) in the ICAPM relation (1) and estimate the parameters κ_0 , κ_1 , and κ_2 jointly with μ and γ by maximizing

⁷See Ghysels, Santa-Clara, and Valkanov (2002) for a general discussion of the functional form of the weights.

the likelihood function, assuming that the conditional distribution of returns is normal:⁸

$$R_{t+1} \sim N(\mu + \gamma V_t^{\text{MIDAS}}, V_t^{\text{MIDAS}}) \quad (4)$$

Since the true conditional distribution of returns may depart from a normal, our estimator really is only quasi-maximum likelihood. The parameter estimators are nevertheless consistent and asymptotically normally distributed. Their variance covariance matrix is estimated using the Newey-West approach with twelve monthly lags to account for heteroskedasticity and serial correlation.

2.2 Empirical Analysis

We estimate the ICAPM with the MIDAS approach using excess returns on the stock market in the post-WWII period, from January of 1946 to December of 2000. We use the CRSP value-weighted portfolio as a proxy for the stock market and the yield of the three-month Treasury bill as the risk-free interest rate. The daily market returns are obtained from CRSP for the period 1963 to 2000, and from William G. Schwert’s website for the period 1946 to 1962. The daily risk-free rate is constructed by assuming that the Treasury bill rates stay constant within the month and suitably compounding them. Monthly returns are obtained by compounding the daily returns. In what follows, we refer to excess returns simply as returns.

Table 1 displays summary statistics for the monthly returns and the monthly realized variance of returns computed from within-month daily data (as explained in equation (5) below). We show the summary statistics for the full 1946-2000 sample and, for robustness, we also analyze three subsamples. We consider two subsamples of approximately equal length, 1946 to 1972 and 1973 to 2000, and a sample that excludes the crash of 1987, which is simply the full sample period excluding the months of October and November of 1987. The October 1987 crash was the largest stock market decline since the Great Depression, and it is likely to have had a unique impact on market volatility and the risk-return tradeoff.

The monthly market return has a mean of about 0.7 percent and a standard deviation of 4.12 percent (variance of 0.174×10^2). Market returns are negatively skewed and slightly

⁸Alternatively, we could use GMM for more flexibility in the relative weighting of the conditional moments in the objective function.

leptokurtic. The first order autoregressive coefficient of monthly market returns is negligible, at 0.029. The summary statistics of the entire sample of 660 observations are similar to those obtained in the sub-samples. The average market return during 1946-1972 is slightly lower than that observed during 1973-2000. The returns in the second sub-period are also slightly more skewed and leptokurtic. The monthly variance is higher and exhibits more skewness and kurtosis in the second sub-period, while it seems to be more serially correlated in the first sub-period. This is largely due to the October 1987 crash, as can be seen from the last sample. Excluding the two months around the 1987 crash increases average market returns significantly, lowers their variance, and decreases dramatically the skewness and kurtosis of both series. The results from these summary statistics are well-known in the empirical finance literature.

Table 2 contains the main result of the paper, the estimation of the risk-return tradeoff equation with the MIDAS estimator of conditional variance. The estimated ICAPM coefficient of the risk-return tradeoff γ is 4.007 in the full sample, with a highly significant t -statistic (corrected for heteroskedasticity and serial correlation with the Newey and West method) of 2.647. Most importantly, the *magnitude* of γ lines up well with the theory. According to the ICAPM, γ is the coefficient of relative risk aversion of the representative investor and a risk aversion coefficient of four matches a variety of empirical studies (see Hall (1988) and references therein). The significance of γ is robust in the subsamples, with estimated values of 8.397 and 1.428, both statistically significant. The lower value in the second subsample is largely due to the 1987 crash. Indeed, the estimate of γ in the no-crash sample is 4.254 with a t -statistic of 2.950. The estimated magnitude and significance of the γ coefficient in the ICAPM relation are remarkable in light of the ambiguity of previous results. The intercept μ is always significant, which, in the framework of the ICAPM, may capture compensation for covariance of the market return with other state variables (which we address in section 5) or compensation for jump risk (Pan (2002)).

Table 2 also reports the estimated parameters of the MIDAS weight function (3). We should point out that some of the coefficients are not individually significant. However, a likelihood ratio test of their joint significance, $\kappa_0 = \kappa_1 = \kappa_2 = 0$, has a p -value of 0.000. Since the restriction $\kappa_0 = \kappa_1 = \kappa_2 = 0$ corresponds to placing equal weights on all lagged squared daily returns, we conclude that the estimated weight function is statistically different from a simple equally-weighted scheme. We cannot interpret the magnitudes of the coefficients κ_0 , κ_1 , and κ_2 individually but only jointly in the weighting function (3). In Figure 1, we

plot the estimated weights, $w_d(\kappa_0, \kappa_1, \kappa_2)$, of the conditional variance on the lagged daily squared returns, for the full sample and the subsamples. For the full sample, we observe that the weights are a slowly declining function of the lag length. For example, only 26 percent of the weight is put on the first lagged month of daily data (22 days), 46 percent on the first two months, and it takes more than four months for the cumulative weight to reach 75 percent. The weight profiles for the subsamples are very similar. We thus conclude that it takes a substantial amount of daily return data to accurately forecast the variance of the stock market. This result stands in contrast with the common view that one month of daily returns is sufficient to reliably estimate the variance.

To assess the predictive power of the MIDAS variance estimator for the market return we run a regression of the realized return in month $t + 1$, R_{t+1} , on the forecasted variance for that month, V_t^{MIDAS} (which uses only daily returns up to time t). The coefficient of determination for the regression using the entire sample, R_R^2 , is 2.4 percent, which is a reasonably high value for a predictive regression of returns at monthly frequency. Similar coefficients of determination obtain in the first two sub-samples, but we notice sizeable forecasting improvement if the 1987 crash is eliminated from the full sample.

We also examine the ability of the MIDAS estimator to forecast realized variance. We estimate realized variance from within-month daily returns as:

$$\sigma_{t+1}^2 = \sum_{d=1}^{22} r_{t+1-d}^2 \quad (5)$$

Table 2 reports the coefficient of determination, $R_{\sigma^2}^2$, from regressing the realized variance, σ_{t+1}^2 , on the MIDAS forecasted variance, V_t^{MIDAS} . MIDAS explains over eight percent of the fluctuations of the realized variance in the entire sample. Given that σ_{t+1}^2 in (5) is only a noisy proxy for the true variance in the month, the $R_{\sigma^2}^2$ obtained is actually quite high.⁹ If we eliminate the 1987 crash from the sample, the $R_{\sigma^2}^2$ jumps to an impressive 0.251. Figure 2 displays the realized variance together with the MIDAS forecast for the entire sample. We see that the estimator does a remarkable job of forecasting next month's variance.

⁹The high standard deviation of the realized variance and the relatively low persistence of the process, shown in Table 1, indicate a high degree of measurement error.

3 Why MIDAS Works: Comparison with Other Tests

To understand why tests based on the MIDAS approach support the ICAPM so strongly when the extant literature offers conflicting results, we compare the MIDAS estimator with previously used estimators of conditional variance. We focus our attention on rolling windows and GARCH estimators of conditional variance. For conciseness, we focus on the entire sample, but the conclusion also hold in the subsamples.

3.1 Rolling Window Tests

As an example of the rolling window approach, French, Schwert, and Stambaugh (1987) use within-month daily squared returns to forecast the variance:

$$V_t^{\text{RW}} = 22 \sum_{d=1}^D \frac{1}{D} r_{t-d}^2 \quad (6)$$

where D is the number of days used in the estimation of variance.¹⁰ (Again, daily squared returns are multiplied by 22 to measure the variance in monthly units.) French, Schwert, and Stambaugh choose the window size to be one month, or $D = 22$. Besides its simplicity, this approach has a number of advantages. First, as with the MIDAS approach, the use of daily data increases the precision of the variance estimator. Second, the stock market variance is very persistent (see Officer (1973) and Schwert (1989)), so the realized variance on a given month ought to be a good forecast of next month's variance.

However, it is not clear that we should confine ourselves to using data from the last month only to estimate the conditional variance. We may want to use a larger window size D in equation (6), corresponding to more than one month worth of daily data. Interestingly, this choice has a large impact on the estimate of γ .

We estimate the parameters μ and γ of the risk-return tradeoff (1) with maximum likelihood using the rolling window estimator (6) for the conditional variance. Table 3 reports the

¹⁰French, Schwert, and Stambaugh (1987) include a correction for serial correlation in the returns that we ignore for now. We follow their example and do not adjust the measure of variance by the squared mean return as this is likely to have only a minor impact with daily data. Additionally, French, Schwert, and Stambaugh actually use the fitted value of an ARMA process for the one-month rolling window estimator to model the conditional variance. This does not change the results substantially due to the high persistence of the variance process. See Goyal (2000).

estimates of the risk-return tradeoff for different sizes D of the window used to estimate the conditional variance. The first line corresponds to using daily data from the previous month only and is similar to the one reported in French, Schwert, and Stambaugh (1987). The estimate of γ is *negative and insignificant*. The parameter we estimate, -0.342, is actually almost identical to the value estimated by French, Schwert, and Stambaugh (1987), of -0.349. (Their t -statistic is lower than ours due to their smaller sample size.) However, as we increase the window size to two through six months, the sign of γ becomes *positive and significant* and the R_R^2 increases substantially. In fact, the estimated coefficient is negative only when the variance estimator uses a single month of data.¹¹ Finally, as the window size increases beyond six months (not shown in the table), the magnitude of the estimated γ decreases as does the likelihood value. This suggests that there is an optimal window size to estimate the risk-return tradeoff.

These results are quite striking. They confirm our MIDAS finding, namely, that there is a positive and significant tradeoff between risk and return. Indeed, the rolling window approach can be thought of as a robust check of the MIDAS regressions since it is such a simple estimator of conditional variance with no parameters to estimate. Moreover, Table 3 helps us reconcile the MIDAS results with the findings of French, Schwert, and Stambaugh (1987) using the rolling-window approach. That paper missed out on the tradeoff by using too small a window size (one month) to estimate the variance. One month worth of daily data simply is not enough to reliably estimate the conditional variance and to measure its impact on expected returns.

The maximum likelihood across window sizes is obtained with a four-month window. This window size implies a constant weight of 0.0114 in the lagged daily squared returns of the previous four months. Of the different window lengths we analyze, these weights are closest to the optimal MIDAS weights shown in Figure 1, which puts roughly three quarters of the weight in those first four months of past daily squared returns.

The rolling window estimator is similar to MIDAS in its use of daily squared returns to forecast monthly variance. But it differs from MIDAS in that it constrains the weights to be

¹¹These findings are consistent with Brandt and Kang (2003), Harrison and Zhang (1999), and Whitelaw (1994) who report a lagged relation between the conditional variance and the conditional mean. Brandt and Kang (2003) use a latent VAR approach and find the relation between conditional mean and conditional variance to be negative contemporaneously but positive with a lag. Whitelaw (1994) argues that the lags in the relation between conditional mean and conditional variance are due to the different sensitivity of both conditional moments to the phase of the business cycle.

constant and inversely proportional to the window length. This constraint on the weights affects the performance of the rolling window estimator compared to MIDAS. For instance, the rolling window estimator does not perform as well as the MIDAS estimator in forecasting realized returns or realized variance. The coefficient of determination for realized returns is less than one percent compared to 2.4 percent for MIDAS, and for realized variance it is 6.2 percent which is lower than the 8.2 percent obtained with MIDAS. Similarly, the MIDAS estimate of γ is 4.0 which is substantially higher than the estimate of 2.4 obtained with the rolling-window approach.

3.2 GARCH Tests

By far the most popular approach to study the ICAPM risk-return relation has been with GARCH-in-mean models estimated with *monthly* return data (French, Schwert, and Stambaugh (1987), Campbell and Hentschel (1992), Glosten, Jagannathan, and Runkle (1993), among others). The simplest model in this family can be written as:¹²

$$V_t^{\text{GARCH}} = \omega + \alpha \epsilon_t^2 + \beta V_{t-1}^{\text{GARCH}} \quad (7)$$

where $\epsilon_t = R_t - \mu - \gamma V_{t-1}^{\text{GARCH}}$. The squared innovations ϵ_t^2 in the variance estimator play a role similar to the monthly squared return in the MIDAS or rolling window approaches and, numerically, they are very similar (since the squared average return is an order of magnitude smaller than the average of squared returns). For robustness, we also estimate an absolute GARCH model, ABS-GARCH:

$$(V_t^{\text{ABSGARCH}})^{1/2} = \omega + \alpha |\epsilon_t| + \beta (V_{t-1}^{\text{ABSGARCH}})^{1/2} \quad (8)$$

As a further robustness check, we estimated higher order GARCH(p,q) models (not shown for brevity), with $p = 1 \dots 3$ and $q = 1 \dots 3$, and the estimates of γ remain virtually unchanged and are still insignificant.

¹²For simplicity, we restrict our attention to GARCH(1,1) models, including only one autoregressive term and one moving average term.

Note that the GARCH model (7) can be rewritten as:

$$V_t^{\text{GARCH}} = \frac{\omega}{1 - \beta} + \alpha \sum_{i=0}^{\infty} \beta^i \epsilon_{t-i}^2 \quad (9)$$

The GARCH conditional variance model is thus approximately a weighted average of past *monthly* squared returns. Compared to MIDAS, the GARCH model uses monthly rather than daily squared returns. Moreover, the functional form of the weights implied by the dynamics of variance in GARCH models exhibits less flexibility than the MIDAS weighting function.

Table 4 shows the coefficient estimates of the GARCH models which are also estimated with quasi-maximum likelihood. Both models yield similar results, so we concentrate on the simple GARCH model. For that model, the estimate of γ is large, 6.968, but *insignificant*, with a *t*-statistic of only 0.901.¹³ This result is similar to French, Schwert, and Stambaugh (1987) who, in a sample similar to ours, obtain an estimate of γ of 7.809, which they also find to be statistically insignificant. Using a symmetric GARCH model, Glosten, Jagannathan, and Runkle (1993) estimate γ to be 5.926 and insignificant with. Although GARCH models find an estimate of γ with a magnitude similar to the MIDAS tests, they lack the power to find statistical significance for the coefficient.

The success of MIDAS relative to GARCH in finding a significant risk-return tradeoff resides in the extra power that MIDAS obtains from the use of daily data in the conditional variance estimator. Put differently, MIDAS has more power than GARCH because it estimates the same number of parameters with a lot more observations. Also, relative to GARCH, MIDAS has a more flexible functional form for the weights on past squared returns. These two differences explain the much higher *t*-statistics obtained by MIDAS. Also, the coefficients of determination from predicting returns, R_R^2 , and realized variances, $R_{\sigma^2}^2$, are 1.0 and 7.0 percent, respectively, for the GARCH models, which appear low when compared with the coefficients of 2.4 and 8.2 percent obtained with MIDAS.

It may seem unfair to compare MIDAS with GARCH tests since the former uses daily data while the latter uses only monthly data. Presumably, if we estimate a daily GARCH model and then use the estimated process for daily variance to compute the variance over the next

¹³As a further robustness check, we estimated higher order GARCH(*p,q*) models (not shown for brevity), with $p = 1 \dots 3$ and $q = 1 \dots 3$, and the estimates of γ remain virtually unchanged and are still insignificant.

month in a multi-step ahead forecast, we might obtain a result similar to MIDAS.¹⁴ However, our point is that the lack of significance of tests of the ICAPM in the literature is due to the use of GARCH models estimated with monthly data.

4 Asymmetries in the Conditional Variance

In this section, we present a simple and natural extension of the MIDAS specification that allows positive and negative returns to have not only an asymmetric impact on the conditional variance, but also to exhibit different persistence. We compare the asymmetric MIDAS model to previously used asymmetric GARCH models in tests of the ICAPM. Our results shed light on the puzzling results in the literature.

4.1 Asymmetric MIDAS Tests

It has long been recognized that volatility is persistent and increases more following negative shocks than positive shocks.¹⁵ Using asymmetric GARCH models, Nelson (1991) and Engle and Ng (1993) confirm that volatility reacts asymmetrically to positive and negative return shocks. Following that idea, Glosten, Jagannathan, and Runkle (1993) use an asymmetric GARCH-in-mean formulation to capture the differential impact of negative and positive lagged returns on the conditional variance and use it to test the relation between the conditional mean and the conditional variance of returns.¹⁶ They find that the sign of the tradeoff changes from insignificantly positive to significantly negative when asymmetries are included in GARCH models of the conditional variance. This result is quite puzzling and below we explain its provenance.

To examine whether the risk-return tradeoff is robust to the inclusion of asymmetric effects

¹⁴Although the results in French, Schwert, and Stambaugh (1987) are not very encouraging in that respect. Also, the multi-step ahead forecast of variances is not easy for GARCH models other than the simple GARCH. Even for the ABSGARCH model, this exercise is not trivial.

¹⁵This is the so-called “feedback effect,” based on the time-variability of the risk-premium induced by changes in variance. See French, Schwert, and Stambaugh (1987), Pindyck (1984) and Campbell and Hentschel (1992). Alternatively, Black (1976) and Christie (1982) justify the negative correlation between returns and innovations to the variance by the “leverage” effect. Bekaert and Wu (2000) conclude that the feedback effect dominates the leverage effect.

¹⁶See also Campbell and Hentschel (1992) for an examination of the risk-return tradeoff with asymmetric variance effects.

in the conditional variance, we introduce the asymmetric MIDAS estimator:

$$V_t^{\text{ASYMIDAS}} = 22 \left[\phi \sum_{d=1}^{\infty} w_d(\kappa_0^-, \kappa_1^-, \kappa_2^-) \mathbf{1}_{t-d}^- r_{t-d}^2 + (2 - \phi) \sum_{d=1}^{\infty} w_d(\kappa_0^+, \kappa_1^+, \kappa_2^+) \mathbf{1}_{t-d}^+ r_{t-d}^2 \right] \quad (10)$$

where $\mathbf{1}_{t-d}^+$ denotes the indicator function for $\{r_{t-d} \geq 0\}$, $\mathbf{1}_{t-d}^-$ denotes the indicator function for $\{r_{t-d} < 0\}$, and ϕ is in the interval $(0, 2)$. This formulation allows for a differential impact of positive and negative shocks on the conditional variance. The coefficient ϕ controls the total weight of negative shocks in the conditional variance. A coefficient ϕ between zero and two ensures that the total weights sum up to one since the indicator functions are mutually exclusive and each of the positive and negative weight functions adds up to one. A value of ϕ equal to one places equal weight on positive and negative shocks. The two sets of parameters $\{\kappa_0^-, \kappa_1^-, \kappa_2^-\}$ and $\{\kappa_0^+, \kappa_1^+, \kappa_2^+\}$ characterize the time profile of the weights from negative and positive shocks, respectively.

Table 5 reports the estimates of the risk-return tradeoff (1) with the conditional variance estimator in equation (10). The estimated coefficient γ is 3.314 and highly significant in the entire sample. In contrast to the findings of Glosten, Jagannathan, and Runkle (1993) with asymmetric GARCH models, in the MIDAS framework, allowing the conditional variance to respond asymmetrically to positive and negative shocks does *not* change the sign of the risk-return tradeoff. Hence, asymmetries in the conditional variance are consistent with a positive coefficient γ in the ICAPM relation.

In agreement with previous studies, we find that asymmetries play an important role in driving the conditional variance. The statistical significance of the asymmetries can easily be tested using a likelihood ratio. The restricted likelihood function under the null hypothesis of no asymmetries is presented in Table 2, whereas the unrestricted likelihood with asymmetries appears in Table 5. The null of no asymmetries, which is a joint test of $\kappa_0^+ = \kappa_0^-$, $\kappa_1^+ = \kappa_1^-$, $\kappa_2^+ = \kappa_2^-$, and $\phi = 1$, is easily rejected with a p-value of 0.001.

The κ coefficients are of interest only because they parameterize the weight functions $w_d(\kappa_0^-, \kappa_1^-, \kappa_2^-)$ and $w_d(\kappa_0^+, \kappa_1^+, \kappa_2^+)$. We plot these weight functions in Figure 3. Interestingly, the weight profiles of negative and positive shocks are markedly different. All the weight of negative shocks (dash-dot line) on the conditional variance is concentrated in the first 30 daily lags. In other words, negative shocks have a strong impact on the conditional variance, but that impact is transitory. It disappears after only one month. In contrast, positive

returns (dash-dash line) have a much smaller immediate impact, but their effect persists up to a year after the shock. This decay is much slower than the usual exponential rate of decay obtained in the case of GARCH and ARMA models.

We find that the estimated value of ϕ is less than one. Since ϕ measures the *total* impact of negative shocks on the conditional variance, our finding implies that *positive* shocks have overall a greater weight on the conditional variance than do *negative* shocks. This asymmetry is statistically significant. A *t*-test of the null hypothesis of $\phi = 1$ is rejected with a *p*-value of 0.028. The combined effect of positive and negative shocks, weighted by ϕ , is plotted as a thick solid line in Figure 3 (the symmetric weight is also plotted for reference as a thin solid line). In the short run, negative returns actually have a higher impact on the conditional variance since their estimated weight in the first month is so much larger than the weight on positive shocks in the same period. For longer lag lengths, the coefficient ϕ determines that positive shocks actually become more important.

We thus find that the asymmetry in the response of the conditional variance to positive and negative returns is more complex than previously documented. Negative shocks have a higher immediate impact but are ultimately dominated by positive shocks. Also, there is a clear asymmetry in the persistence of positive and negative shocks, with positive shocks being responsible for the persistence of the conditional variance process beyond one month.

Our results are consistent with a recent literature on multi-factor variance models (Alizadeh, Brandt, and Diebold (2002), Chacko and Viceira (2003), Chernov, Gallant, Ghysels, and Tauchen (2003), Engle and Lee (1999) and Gallant, Hsu, and Tauchen (1999), among others) which finds reliable support for the existence of two factors driving the conditional variance. The first factor is found to have high persistence and low volatility, whereas the second factor is transitory and highly volatile. The evidence from estimating jump-diffusions with stochastic volatility points in a similar direction. For example, Chernov, Gallant, Ghysels, and Tauchen (2003) show that the first factor, the diffusive component, is highly persistent and has low variance, whereas the second factor, the jump component, is by definition not persistent.

Using the asymmetric MIDAS specification, we are able to identify the first factor with lagged positive returns and the second factor with lagged negative returns.¹⁷ Indeed, if we decompose the conditional variance estimated with equation (10) into its two components,

¹⁷Engle and Lee (1999) have a similar finding using a two-component asymmetric GARCH model.

$\phi \sum_{d=1}^{\infty} w_d(\kappa_0^-, \kappa_1^-, \kappa_2^-) \mathbf{1}_{t-d}^- r_{t-d}^2$ and $(2 - \phi) \sum_{d=1}^{\infty} w_d(\kappa_0^+, \kappa_1^+, \kappa_2^+) \mathbf{1}_{t-d}^+ r_{t-d}^2$, we verify that their time series properties match the results in the literature. More precisely, the positive shock component is very persistent, with an AR(1) coefficient of 0.981, whereas the negative shock component is temporary, with an AR(1) coefficient of only 0.129. Also, the standard deviation of the negative component is twice the standard deviation of the positive component. (These results are robust in the subsamples.) The results from the asymmetric MIDAS model are thus consistent with the literature on two-factor models of variance.

4.2 Asymmetric GARCH Tests

For comparison, we estimate three different asymmetric GARCH-in-mean models: asymmetric GARCH, exponential GARCH, and quadratic GARCH. For conciseness, we use the acronyms ASYGARCH, EGARCH, and QGARCH to refer to these models. The ASYGARCH and EGARCH formulations are widely used to model asymmetries in the conditional variance. We use the specifications of Glosten, Jagannathan, and Runkle (1993) (without seasonal dummies) for these two models. The QGARCH model was introduced by Engle (1990) and is used in the risk-return tradeoff literature by Campbell and Hentschel (1992).

The ASYGARCH model is specified as:

$$V_t^{\text{ASYGARCH}} = \omega + \alpha \epsilon_t^2 + \lambda \epsilon_t^2 \mathbf{1}_t^+ + \beta V_{t-1}^{\text{ASYGARCH}} \quad (11)$$

where $\epsilon_t = R_t - \mu - \gamma V_{t-1}^{\text{ASYGARCH}}$ and $\mathbf{1}_t^+$ is an index function that equals to one when ϵ_t is positive and zero otherwise. The coefficient λ captures the asymmetry in the reaction of the conditional variance to positive and negative returns. A negative λ indicates that negative returns have a stronger impact on the conditional variance. When $\lambda = 0$, the ASYGARCH reduces to a simple GARCH.

The EGARCH process is similar in nature, but imposes an exponential form on the dynamics of the conditional variance as a more convenient way of imposing positiveness. It is specified as:

$$\ln(V_t^{\text{EGARCH}}) = \omega + \alpha u_t + \lambda u_t \mathbf{1}_t^+ + \beta \ln(V_{t-1}^{\text{EGARCH}}) \quad (12)$$

where $u_t = (R_t - \mu - \gamma V_{t-1}^{EGARCH}) / (V_{t-1}^{EGARCH})^{1/2}$ are the normalized innovations and $\mathbf{1}_t^+$ is equal to one when u_t is positive. Again, when λ is negative, the variance reacts more to negative returns.

The QGARCH model incorporates asymmetries as:¹⁸

$$V_t^{\text{QGARCH}} = \omega + \alpha(\epsilon_t + \lambda)^2 + \beta V_{t-1}^{\text{QGARCH}} \quad (13)$$

where $\epsilon_t = R_t - \mu - \gamma V_{t-1}^{\text{QGARCH}}$. For $\lambda = 0$, the QGARCH model collapses into the simple GARCH specification.

The estimated coefficients of the three asymmetric GARCH models are shown in Table 6. We confirm the finding in Glosten, Jagannathan, and Runkle (1993) that asymmetries in the ASYGARCH and EGARCH produce a *negative*, albeit statistically insignificant, estimate of the risk-return tradeoff parameter γ . In fact, our estimates of the model are similar to the ones reported in Glosten, Jagannathan, and Runkle (1993). The QGARCH model also produces a negative and statistically insignificant estimate of γ , which is comparable (although slightly lower in absolute terms) to the negative and statistically insignificant estimates obtained in Campbell and Hentschel (1992).¹⁹ In all three models, the estimates of λ are negative and statistically different from zero, indicating that the asymmetries are important and that, in asymmetric GARCH models, negative shocks tend to have a higher impact on the conditional variance than positive shocks. Finally, if we compare the $R_{\sigma^2}^2$ from Tables 4 and 6, we notice that the asymmetric GARCH models produce forecasts of the realized variance that are slightly better than those from the symmetric GARCH models. The improvement in the forecasting power of returns is negligible to non-existent.

The persistence of the conditional variance in the above asymmetric GARCH models is driven by the β parameter. It is important to note that the asymmetric GARCH specifications do not allow for differences in the persistence of positive and negative shocks. In other words, both positive and negative shocks decay at the same rate, determined by β . Furthermore, the estimated conditional variance in such asymmetric GARCH processes loads heavily on negative shocks, which we know from the MIDAS results (Figure 3) have a strong immediate

¹⁸The formulation of Campbell and Hentschel (1992) has a negative sign in front of the λ term. We write the QGARCH model differently to maintain the interpretation of a negative λ corresponding to a higher impact of negative shocks on the conditional variance.

¹⁹In addition to this result, Campbell and Hentschel (1992) estimate the risk-return tradeoff imposing a constraint from a dividend-discount model. In that case, they obtain a positive and significant estimate of γ .

impact on volatility. However, we have also seen that the impact of negative shocks on variance is transitory. Hence, it is not surprising that the estimates of the persistence parameter β in the asymmetric GARCH models shown in Table 6 (similar to Glosten, Jagannathan, and Runkle (1993)) are much lower than in the symmetric GARCH models.²⁰ This implicit restriction leads Glosten, Jagannathan, and Runkle to conclude that “the conditional volatility of the monthly excess return is not highly persistent.” In contrast, the asymmetric MIDAS model allows the persistence of positive and negative shocks to be different, resulting in overall higher persistence of the variance process.

To demonstrate the implications of the asymmetric GARCH restriction on the persistence of positive and negative shocks, we can compute the AR(1) coefficient of the filtered variance processes. The AR(1) coefficients of the ASYGARCH, EGARCH, and QGARCH estimated conditional variance processes are only -0.091, 0.004, and 0.100, respectively. These coefficients are surprisingly low given what we know about the persistence of variance (Officer (1973) and Schwert (1989)). The constraint that asymmetric GARCH models place on the equality of persistence of positive and negative shocks imposes a heavy toll on the overall persistence of the forecasted variance process. In contrast, the AR(1) coefficient of the symmetric GARCH and the symmetric MIDAS conditional variance estimators are 0.91 and 0.88, respectively. It is worth noting that the lack of persistence is not due to the asymmetry in the variance process. The AR(1) coefficient of the asymmetric MIDAS variance process is still high at 0.85, showing that the conditional variance process can have both asymmetries and high persistence.

Given the lack of persistence of the asymmetric GARCH models, it is not surprising to find that their estimated conditional variance processes are incapable of explaining expected returns in the ICAPM relation.²¹ The persistence of symmetric GARCH and both symmetric and asymmetric MIDAS estimators allows them to capture the relation between risk and return in the ICAPM. This explains the puzzling findings of Glosten, Jagannathan, and Runkle (1993) that the risk-return tradeoff turns negative when we take into account asymmetries in the conditional variance. Their results are not driven by the asymmetries. Instead, they depend on the lack of persistence in the conditional variance induced by the

²⁰This constraint can be relaxed in the GARCH framework. Using a two-component GARCH model, Engle and Lee (1999) show that only the persistent component of variance has explanatory power for stock market returns.

²¹Indeed, Poterba and Summers (1986) show that persistence in the variance process is crucial for it to have any economically meaningful impact on stock prices.

restriction in the GARCH processes. To adequately capture the dynamics of variance, we need both asymmetry in the reaction to negative and positive shocks and a different degree of persistence of those shocks. When we model the conditional variance in this way, the ICAPM continues to hold.

5 The Risk-Return Tradeoff with Additional Predictive Variables

In this section, we extend the ICAPM relation between risk and return to include other predictive variables. Specifically, we modify the ICAPM equation (1) as:

$$E_t[R_{t+1}] = \mu + \gamma \text{Var}_t[R_{t+1}] + \theta^\top Z_t \quad (14)$$

where Z_t is a vector of variables known to predict the return on the market and θ is a conforming vector of coefficients. The variables in Z_t are known at the beginning of the return period, but they might be observed at various frequencies (monthly, weekly, daily).

Campbell (1991), Campbell and Shiller (1988), Chen, Roll, and Ross (1986), Fama (1990), Fama and French (1988, 1989), Ferson and Harvey (1991), and Keim and Stambaugh (1986), among many others, find evidence that the stock market can be predicted by variables related to the business cycle. At the same time, Schwert (1989, 1990) shows that the variance of the market is highly counter-cyclical. Therefore, our findings about the risk-return tradeoff could simply be due to the market variance proxying for business cycle fluctuations. To test this “proxy” hypothesis, we examine the relation between the expected return on the stock market and the conditional variance using macro variables as controls for business cycle fluctuations.

Alternatively, the specification (14) can be understood as a version of the ICAPM with additional state variables. When the investment opportunity set changes through time, Merton shows that:

$$E_t[R_{t+1}] = \mu + \gamma \text{Var}_t[R_{t+1}] + \pi^\top \text{Cov}_t[R_{t+1}, S_{t+1}], \quad (15)$$

where the term $\text{Cov}_t[R_{t+1}, S_{t+1}]$ denotes a vector of covariances of the market return with

innovations to the state variables, S , conditional on information known at date t . If the relevant information to compute these conditional covariances consists of the variables in the vector Z_t , we can interpret the term $\theta^\top Z_t$ in (14) as an estimate of the conditional covariance, $\pi \text{Cov}_t[R_{t+1}, S_{t+1}]$ in (15).²²

The predictive variables that we study are the dividend-price ratio, the relative Treasury bill rate, and the default spread (all available at monthly frequency). The dividend-price ratio is calculated as the difference between the log of the last twelve month dividends and the log of the current level of the CRSP value-weighted index. The three-month Treasury bill rate is obtained from Ibbotson Associates. The relative Treasury bill stochastically detrends the raw series by taking the difference between the interest rate and its twelve-month moving average. The default spread is calculated as the difference between the yield on BAA- and AAA-rated corporate bonds, obtained from the FRED database. We standardize these three macro variables (subtracting the mean and dividing by the standard deviation) to ensure comparability of the μ coefficients in equations (1) and (14).

Once the effect of the control variables in the conditional expected return is removed, γ shows the magnitude of the risk-return tradeoff, while the MIDAS weight coefficients still determine the lag structure of conditional covariance. Table 7 presents the results from estimating equation (14) with both the simple MIDAS weights (3) and the asymmetric MIDAS weights (10). The results strongly suggest that business cycle fluctuations do not account for our findings. Indeed, the coefficients of the risk-return relation with controls are remarkably similar to those estimated without controls (shown in Tables 2 and 5). The estimates of μ and γ are almost identical in the two tables across all four sample periods. This indicates that the explanatory power of the forecasted variance for returns is largely orthogonal to the additional macro predictive variables. Moreover, the estimates of κ_0 , κ_1 , and κ_2 are also very similar, implying that the weights the conditional variance places on past squared returns are not changed.

The three macro variables enter significantly in the ICAPM conditional mean either in the sample or in the subsamples. A likelihood ratio test of their joint significance in the entire sample has a p -value of 0.002. The coefficient of determination of the regression of realized returns on the conditional variance *and* the macro variables, R_R^2 , is 4.7 percent in the full

²²Scruggs (1998) emphasizes this version of the ICAPM, which predicts only a *partial* relation between the conditional mean and the conditional variance after controlling for the other covariance terms. Scruggs uses the covariance between stock market returns and returns on long bonds as a control and finds a significantly positive risk-return tradeoff.

sample. This is significantly higher than the corresponding coefficient without the macro variables, which is only 2.4 percent. The coefficient $R_{\sigma^2}^2$ is unchanged by the inclusion of the macro predictive variables.

We conclude that the risk-return tradeoff is largely unaffected by including extra predictive variables in the ICAPM equation and the forecasting power of the conditional variance is not merely proxying for the business cycle.

6 Conclusion

This paper takes a new look at Merton's ICAPM, focusing on the tradeoff between conditional variance and conditional mean of the stock market return. In support of the ICAPM, we find a *positive* and *significant* relation between risk and return. This relation is robust in subsamples, does not change when the conditional variance is allowed to react asymmetrically to positive and negative returns, and is not affected by the inclusion of other predictive variables.

Our results are more conclusive than those from previous studies due to the added power obtained from the new MIDAS estimator of conditional variance. This estimator is a weighted average of past daily squared returns, where the average is taken over an extended window of time, and the weights are parameterized with a flexible functional form. We find that the MIDAS estimator is a better forecaster of the stock market variance than rolling window or GARCH estimators, which is the reason why our tests of the ICAPM can robustly find the ICAPM's risk-return tradeoff.

We obtain new results about the asymmetric reaction of volatility to positive and negative return shocks. We find that, compared to negative shocks, positive shocks: (i) have a bigger impact overall on the conditional mean of returns; (ii) are slower to be incorporated into the conditional variance; and (iii) are much more persistent and indeed account for the persistent nature of the conditional variance process. Quite surprisingly, negative shocks have a large initial, but very temporary effect on the variance of returns. This feature of conditional variance has not been detected in previous studies.

The MIDAS estimator offers a powerful and flexible way of estimating economic models by taking advantage of data sampled at various frequencies. While the advantages of the

MIDAS approach have been demonstrated in the estimation of the ICAPM and conditional volatility, the method itself is quite general in nature and can be used to tackle several other important questions.

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Table 1: Summary Statistics of Returns and Realized Variance

This table shows summary statistics of monthly excess returns R_t of the stock market, and realized monthly variance computed from within-month daily data, σ_t^2 . The proxy for the stock market is the CRSP value-weighted portfolio and the risk-free rate is the yield on the three-month Treasury bill. The table shows the mean, variance, skewness, kurtosis, and first-order serial correlation for each of the two variables. The statistics are shown for the full sample, two subsamples of approximately equal length, and a sample without the months of October and November 1987.

Panel A: Monthly Returns (R_t)						
Sample	Mean ($\times 10^2$)	Variance ($\times 10^2$)	Skewness	Kurtosis	AR(1)	T
1946:01-2000:12	0.692	0.174	-0.562	5.121	0.039	660
1946:01-1972:12	0.839	0.136	-0.395	3.142	0.055	324
1973:01-2000:12	0.660	0.208	-0.599	5.657	0.021	336
1946:01-2000:12 (No 1987 Crash)	0.741	0.165	-0.341	4.024	0.018	658
Panel B: Monthly Realized Variances (σ_t^2)						
Sample	Mean ($\times 10^2$)	Variance ($\times 10^4$)	Skewness	Kurtosis	AR(1)	T
1946:01-2000:12	0.144	0.063	14.559	292.628	0.285	660
1946:01-1972:12	0.104	0.013	4.098	25.793	0.392	324
1973:01-2000:12	0.181	0.110	12.144	185.491	0.234	336
1946:01-2000:12 (No 1987 Crash)	0.136	0.021	3.339	17.938	0.564	658

Table 2: MIDAS Tests of the Risk-Return Tradeoff

This table shows estimates of the risk-return tradeoff (1) with the MIDAS estimator of conditional variance in equations (2) and (3). The coefficients and corresponding t -statistics (in brackets) are shown for the full sample and the three subsamples. R_R^2 and $R_{\sigma^2}^2$ quantify the explanatory power of the MIDAS variance estimator in predictive regressions for realized returns and variances, respectively. LLF is the log-likelihood value. The t -statistics are computed using Newey-West robust standard errors with a kernel of 12 monthly lags.

Sample	μ ($\times 10^3$)	γ	k_0 ($\times 10^5$)	k_1 ($\times 10^2$)	k_2 ($\times 10^9$)	R_R^2	$R_{\sigma^2}^2$	LLF
1946.01-2000.12	4.800 [2.419]	4.007 [2.647]	-4.363 [-0.004]	-1.353 [-1.903]	-3.984 [-0.092]	0.024	0.082	1221.837
1946.01-1972.12	1.565 [0.766]	8.397 [3.598]	-2.819 [-0.002]	-1.445 [-2.397]	-2.273 [-0.110]	0.029	0.101	624.008
1973.01-2000.12	9.050 [2.100]	1.428 [1.981]	-1.001 [-0.001]	-0.922 [-0.454]	-2.183 [-0.098]	0.022	0.056	580.097
1946.01-2000.12 (No 1987 Crash)	4.809 [2.515]	4.254 [2.950]	-3.960 [-0.002]	-1.402 [-1.959]	-3.293 [-0.011]	0.041	0.251	1239.100

Table 3: Rolling Window Tests of the Risk-Return Tradeoff

This table shows estimates of the risk-return tradeoff (1) with the rolling windows estimators of conditional variance (6). The coefficients and corresponding t -statistics (in brackets) are shown for the full sample and the three subsamples. R_R^2 and $R_{\sigma^2}^2$ quantify the explanatory power of the MIDAS variance estimator in predictive regressions for realized returns and variances, respectively. LLF is the log-likelihood value. The t -statistics are computed using Newey-West robust standard errors, with a kernel equal to the horizon (and the overlap) in the regression.

Horizon (Months)	μ ($\times 10^3$)	γ	R_R^2	$R_{\sigma^2}^2$	LLF
1	10.719 [5.693]	-0.342 [-0.537]	0.001	0.072	1090.701
2	8.520 [4.215]	1.233 [1.504]	0.003	0.079	1113.584
3	7.291 [3.431]	2.033 [2.173]	0.007	0.076	1118.392
4	6.689 [3.027]	2.410 [2.360]	0.009	0.062	1119.985
5	6.628 [2.888]	2.441 [2.208]	0.007	0.057	1118.402
6	6.781 [2.861]	2.384 [2.027]	0.006	0.058	1116.664

Table 4: GARCH Tests of the Risk-Return Tradeoff

This table shows estimates of the risk-return tradeoff (1) with the GARCH estimators of conditional variance (7) and (8). The t -statistics (in brackets) are shown below the coefficient estimates. R_R^2 and $R_{\sigma^2}^2$ quantify the explanatory power of the MIDAS variance estimator in predictive regressions for realized returns and variances, respectively. LLF is the log-likelihood value. The t -statistics are computed using Newey-West robust standard errors with a kernel of 12 monthly lags.

Model	μ ($\times 10^3$)	γ	ω ($\times 10^3$)	α	β	R_R^2	$R_{\sigma^2}^2$	LLF
GARCH(1,1)-M	-0.740 [-0.370]	6.968 [0.901]	0.125 [0.244]	0.069 [1.398]	0.860 [18.323]	0.010	0.070	1152.545
ABS-GARCH(1,1)-M	1.727 [0.424]	6.013 [0.873]	2.751 [0.947]	0.099 [1.764]	0.858 [17.323]	0.010	0.071	1156.142

Table 5: Asymmetric MIDAS Tests of the Risk-Return Tradeoff

This table shows estimates of the risk-return tradeoff (1) with the Asymmetric MIDAS estimator of conditional variance (10). The coefficients and corresponding t -statistics (in brackets) are shown for the full sample and the three subsamples. R_R^2 and $R_{\sigma^2}^2$ quantify the explanatory power of the MIDAS variance estimator in predictive regressions for realized returns and variances, respectively. LLF is the log-likelihood value. The t -statistics are computed using Newey-West robust standard errors with a kernel of 12 monthly lags.

Sample	μ ($\times 10^3$)	γ	k_0^- ($\times 10^3$)	k_1^- ($\times 10^2$)	k_2^- ($\times 10^3$)	k_0^+ ($\times 10^3$)	k_1^+ ($\times 10^2$)	k_2^+ ($\times 10^4$)	ϕ	R_R^2	$R_{\sigma^2}^2$	LLF
1946:01-2000:12	5.766 [2.057]	3.314 [2.695]	0.523 [0.211]	9.573 [0.507]	-7.640 [-0.929]	0.033 [0.354]	0.073 [0.133]	-0.210 [-0.539]	0.606 [3.381]	0.025	0.085	1231.308
1946:01-1972:12	4.297 [0.245]	5.857 [3.173]	0.122 [0.021]	-1.921 [-0.699]	-0.029 [-0.080]	0.061 [0.109]	23.813 [2.203]	-6.654 [-0.836]	0.523 [2.993]	0.032	0.137	677.559
1973:01-2000:12	10.403 [2.858]	1.665 [2.109]	1.194 [0.024]	26.356 [1.099]	-13.177 [-1.218]	1.317 [0.027]	0.879 [0.953]	-0.360 [-0.535]	0.728 [2.372]	0.021	0.048	587.580
1946:01-2000:12 (No 1987 Crash)	5.550 [1.989]	3.735 [2.868]	0.513 [0.233]	-7.541 [-1.775]	-0.234 [-0.342]	0.021 [0.003]	-0.214 [-0.613]	-0.091 [-0.498]	0.716 [5.010]	0.043	0.363	1244.200

Table 6: Asymmetric GARCH Tests of the Risk-Return Tradeoff

The table shows estimates of the risk-return tradeoff in equation (1) with the asymmetric GARCH estimators of conditional variance in (11), (12), and (13). The coefficients and corresponding t -statistics (in brackets) are shown for the full sample and the three subsamples. R_R^2 and $R_{\sigma^2}^2$ quantify the explanatory power of the MIDAS variance estimator in predictive regressions for realized returns and variances, respectively. LLF is the log-likelihood value. The t -statistics are computed using Newey-West robust standard errors with a kernel of 12 monthly lags.

Model	μ ($\times 10^3$)	γ	ω ($\times 10^2$)	α	β	λ ($\times 10^2$)	R_R^2	$R_{\sigma^2}^2$	LLF
EGARCH(1,1)-M	14.978 [6.277]	-2.521 [-1.285]	-640.708 [-1.790]	-0.325 [-2.977]	0.497 [5.938]	-3.339 [-2.206]	0.011	0.071	1159.102
ASYGARCH(1,1)-M	1.117 [0.913]	-3.248 [-1.811]	0.056 [0.202]	0.018 [1.980]	0.609 [7.842]	-28.723 [-2.131]	0.010	0.077	1164.023
QGARCH(1,1)-M	13.970 [2.378]	-1.994 [-0.171]	0.060 [0.356]	0.086 [3.565]	0.145 [3.269]	-9.320 [-7.188]	0.010	0.072	1161.173

Table 7: MIDAS Tests of the Risk-Return Tradeoff Controlling for Other Predictive Variables

The table shows estimates of the risk-return tradeoff in equation (14) with the MIDAS estimator of conditional variance (2) and other predictive variables: the default spread (δ_1), the stochastically detrended risk-free interest rate (δ_2), and the market's dividend yield (δ_3). To facilitate comparison of the MIDAS coefficients with previous tables, the three control variables are normalized to have mean zero and unit variance. The coefficients and corresponding t -statistics (in brackets) are shown for the full sample and the three subsamples. R_R^2 and $R_{\sigma^2}^2$ quantify the explanatory power of the MIDAS variance estimator in predictive regressions for realized returns and variances, respectively. LLF is the log-likelihood value. The t -statistics are computed using Newey-West robust standard errors with a kernel of 12 monthly lags.

Panel A: No Asymmetries

Sample	μ ($\times 10^3$)	γ	k_0 ($\times 10^5$)	k_1 ($\times 10^2$)	k_2 ($\times 10^9$)	δ_1 ($\times 10^3$)	δ_2 ($\times 10^3$)	δ_3 ($\times 10^3$)	R_R^2	$R_{\sigma^2}^2$	LLF
1946.01-2000.12	5.510 [0.071]	3.516 [2.600]	-3.223 [-0.000]	-1.436 [-0.145]	-0.234 [-0.093]	6.825 [1.030]	-7.851 [-1.792]	3.821 [1.862]	0.047	0.083	1229.368
1946.01-1972.12	-0.672 [-0.031]	10.490 [15.858]	0.867 [0.001]	-1.524 [-0.536]	-0.834 [-0.070]	0.431 [0.205]	-13.840 [-3.856]	10.427 [3.799]	0.050	0.104	688.598
1973.01-2000.12	3.529 [0.064]	2.527 [2.606]	-0.850 [-0.001]	-1.470 [-0.292]	-1.092 [-0.504]	11.268 [1.761]	-5.049 [-0.926]	3.265 [0.746]	0.035	0.054	588.205
1946.01-2000.12 (No 1987 Crash)	2.840 [0.150]	5.805 [6.433]	-5.314 [-0.006]	-1.530 [-0.413]	-0.921 [-0.430]	0.692 [0.222]	-3.779 [-1.314]	2.087 [1.206]	0.053	0.274	1246.900

Table continued on next page ...

Panel B: With Asymmetries

Sample	μ ($\times 10^3$)	γ	k_0^- ($\times 10^5$)	k_1^- ($\times 10^2$)	k_2^- ($\times 10^9$)	k_0^+ ($\times 10^5$)	k_1^+ ($\times 10^2$)	k_2^+ ($\times 10^5$)	ϕ	δ_1 ($\times 10^3$)	δ_2 ($\times 10^3$)	δ_3 ($\times 10^3$)	R_R^2	$R_{G^2}^2$	LLF
1946.01-2000.12	6.690 [0.161]	3.995 [2.691]	-0.242 [-0.011]	-6.387 [-0.793]	-38.437 [-0.204]	0.695 [0.380]	0.654 [0.296]	-3.790 [-0.200]	0.742 [4.319]	7.143 [0.976]	-7.871 [-1.557]	4.000 [2.368]	0.049	0.103	1238.23
1946.01-1972.12	-0.355 [-0.011]	10.219 [12.818]	0.483 [0.881]	-1.713 [-1.371]	-11.805 [-0.083]	0.605 [0.093]	3.951 [4.673]	-15.528 [-3.483]	0.535 [2.362]	0.648 [0.330]	-14.125 [-3.926]	10.512 [3.630]	0.053	0.141	691.967
1973.01-2000.12	1.696 [0.023]	2.770 [3.745]	-0.958 [-1.009]	-3.821 [-0.194]	-185.918 [-7.440]	0.231 [0.101]	40.201 [7.268]	-226.586 [-8.597]	0.872 [2.777]	10.375 [1.689]	-3.475 [-0.633]	4.501 [1.098]	0.037	0.065	601.700
1946.01-2000.12 (No 1987 Crash)	3.540 [0.014]	5.373 [3.169]	0.243 [0.430]	-6.945 [-0.028]	-1.232 [-0.893]	-1.333 [-2.021]	-0.269 [-0.285]	-0.827 [-0.028]	0.744 [4.395]	1.050 [0.113]	-3.878 [-0.989]	2.321 [0.589]	0.057	0.341	1253.80

Table continued from previous page.

Figure 1: MIDAS Weights

This figure plots the weights that the MIDAS estimator (2) and (3) places on lagged squared returns. The weights are calculated by substituting the estimated values of κ_0 , κ_1 , and κ_2 into the weight function (3). The exact estimates of κ_0 , κ_1 , and κ_2 are shown in Table 2. The figure displays the weights for the entire sample and for the three subsamples.

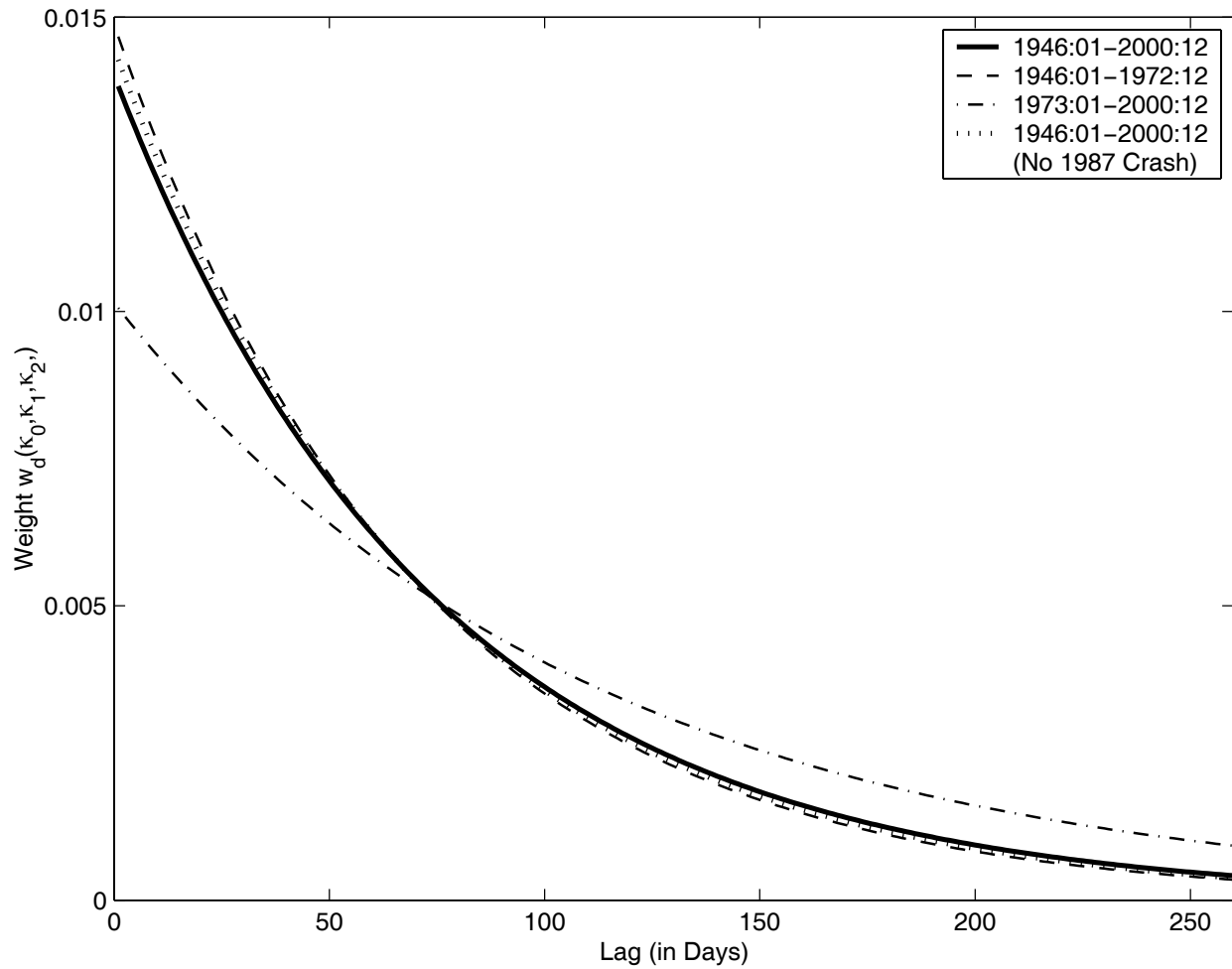


Figure 2: MIDAS Forecasted and Realized Variance

This figure plots the forecasted variance with the MIDAS estimator (2) and (3) and compares it with the realized variance (5). The parameter values are shown in Table 2 (1946-2000 sample). The realized variance in October of 1987 has been truncated. The actual value is 0.05.

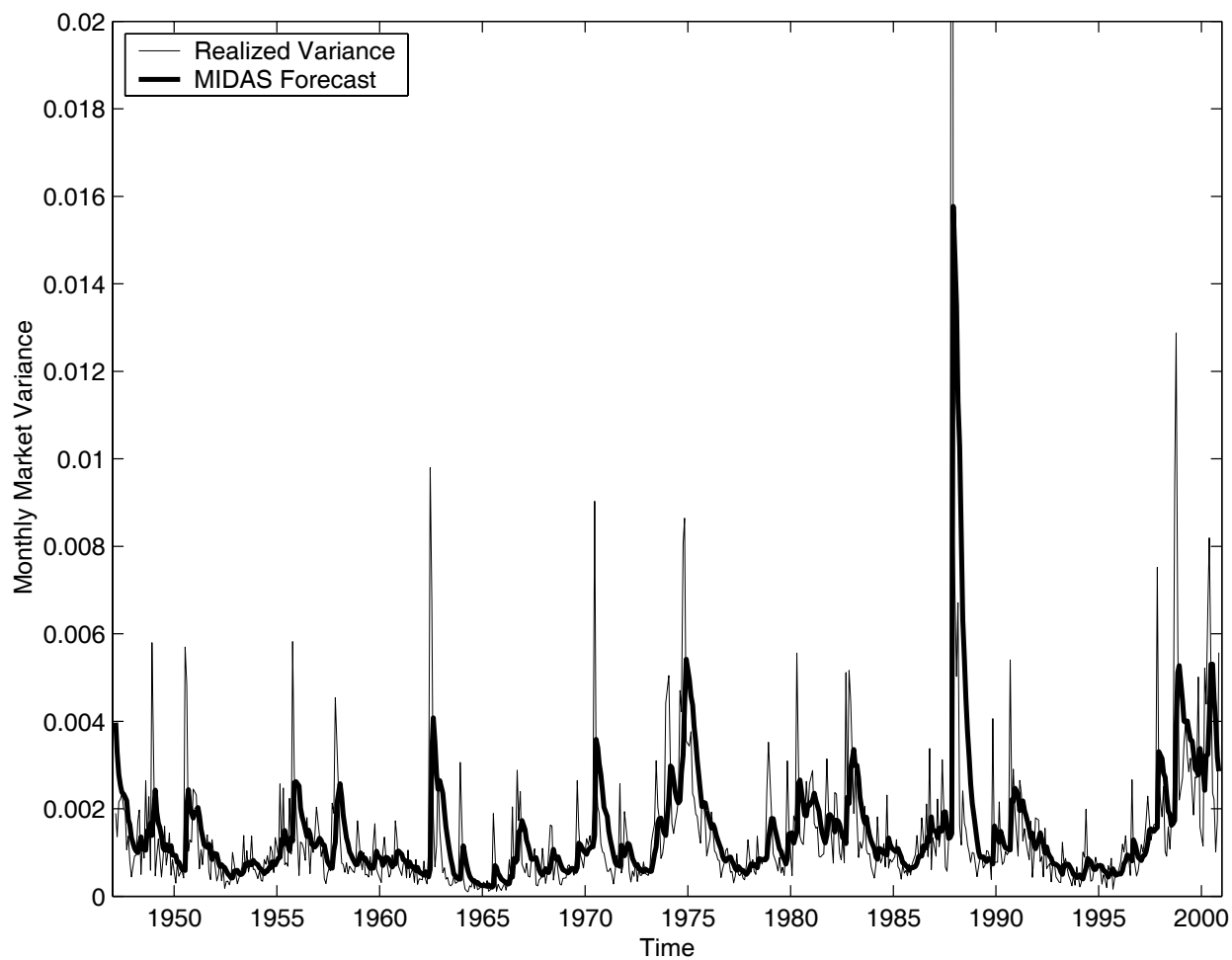


Figure 3: Asymmetric MIDAS Weights

This figure plots the weights, estimated from the entire sample, that the Asymmetric MIDAS estimator (10) and (3) places on lagged squared returns conditional on the sign of the returns. The weights on the negative shocks ($r < 0$) are calculated by substituting the estimated values of κ_0^- , κ_1^- , and κ_2^- into (3). Similarly, the weights on the positive shocks ($r \geq 0$) are calculated by substituting the estimated values of κ_0^+ , κ_1^+ , and κ_2^+ into (3). The total asymmetric weights, plotted using equation (10), take into account the overall impact of asymmetries on the conditional variance through the parameter ϕ . The exact estimates of all parameters are shown in Table 5. The symmetric weights from Figure 1 are also plotted for comparison.

